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Arch. Min. Sci., Vol. 60 (2015), No 4, p. 921–929

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.1515/amsc-2015-0060

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# POINT MOVEMENT TRACE VS. THE RANGE OF MINING EXPLOITATION EFFECTS IN THE ROCK MASS

# ŚLAD PRZEMIESZCZENIA PUNKTU A ZASIĘG WPŁYWÓW EKSPLOATACJI GÓRNICZEJ W GÓROTWORZE

The geometric-integral theories of the rock mass point movements due to mining exploitation assume the relationship between the progress of subsidence and horizontal movement.

By analysing the movement trace of a point located on the surface, and the influence of the mining exploitation in the rock mass, an equation describing the relationship between the main components of the deformation conditions was formulated. The result is consistent with the in situ observations and indicates the change of the rock mass component volume due to mining exploitation. The analyses and in situ observations demonstrate clearly that the continuity equation adopted in many solutions in the form: $\sum_{i=1}^{i=3} \varepsilon_{ii} = 0$ is fundamentally incorrect.

**Keywords:** point movement trace, horizontal movement, exploitation influence range

Teorie geometryczno-całkowe ruchów punktów górotworu spowodowanych eksploatacją górniczą zakładają zależność pomiędzy przebiegiem osiadania i przebiegiem przemieszczenia poziomego.

Analizując przebieg śladu przemieszczenia punktu położonego na powierzchni oraz przebieg zasięgu wpływów eksploatacji górniczej w górotworze otrzymano wzór opisujący zależność pomiędzy głównymi składowymi stanu odkształcenia. Wynik ten jest zgodny z obserwacjami in situ i wskazuje na zmianę objętości elementu górotworu spowodowanej eksploatacją górniczą. Z przeprowadzonych rozważań oraz z obserwacji in situ wynika jednoznacznie, że przyjmowane w wielu rozwiązaniach równanie ciągłości w postaci: $\sum_{i=1}^{i=3} \varepsilon_{ii} = 0$ jest z zasady niewłaściwe.

***Słowa kluczowe:*** *ślad przemieszczenia punktu, przemieszczenia poziome, zasięg wpływu eksploatacji*

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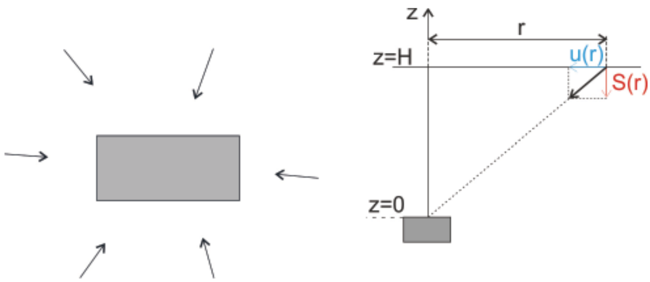
# 1. Point movement trace in vertical plane

The first theory describing the method for calculating the values of horizontal movement due to mining exploitation is the centre of gravity method developed by Keinhorst (1925). It was formulated by in situ observations in German mines where, for relatively small exploitation sites, the projections of the horizontal movement vectors on the horizontal plane were oriented towards the centre of the areas selected (Fig. 1a).




a) Horizontal projection b) Vertical projection

Fig. 1. Horizontal point movement towards the centre of gravity according to Keinhorst theory for elementary exploitation

Bearing the above in mind, Keinhorst assumed that the trace of movement of a point located on the land surface aligns with a straight line connecting that point with the centre of gravity of the "elementary" exploitation (Fig. 1b). The movement trace is a curve along which the measuring point moves in the space due to mining exploitation. For the assumption made by Keinhorst it yields:

$$u(r) = -\frac{r}{H} \cdot s(r) \tag{1.1}$$

where:

| | | |
|---|---|---|
| $r$ | — | horizontal distance of the calculation point from the exploitation element; |
| $H$ | — | deposition depth of the selected seam component; |
| $s(r)$ | — | subsidence; |
| $u(r)$ | — | horizontal movement. |

The symbols of the Glossary of the 4th Mining Damage Committee of the International Society for Mine Surveying were adopted in this publication (Pielok, 1992).

This method is named the centre of gravity method after the assumption made by Keinhorst. The equations developed based on that assumption demonstrate that the maximum horizontal movement values for the so called infinite half-plane can be presented as follows:

$$u_{\max} = c_m(\gamma_m) \cdot a \cdot M \cdot \cot \gamma_m \tag{1.2}$$

where:

| | | |
|---|---|---|
| $c_m(\gamma_m)$ | — | constant depending on the calculation method; |

$a$ — subsidence factor;
$M$ — seam thickness;
$\gamma_m$ — angle of main influences depending on the calculation method.

However, it should be concluded that the values of the maximum horizontal movement calculated with the centre of gravity method are several times smaller for various theories than those observed in situ (Table 1).

According to Fläschenträger (1956), the ratio of the maximum horizontal movement value to the maximum subsidence value, for full troughs, falls within the range from 0.3 to 0.5.

TABLE 1

The relationship between the maximum horizontal movement values and the maximum subsidence for the infinite half-plane for various German calculation methods (Schleier, 1956)

| $\gamma_m$ [°] | Theory | | | | | |
|---|---|---|---|---|---|---|
| | Bals | Fläschenträger | Fläschenträger, Perz | Beyer | Sann | Keinhorst |
| 55 | 0.136 | 0.126 | 0.144 | 0.158 | 0.085 | 0.158 |
| 45 | 0.175 | 0.160 | 0.190 | 0.255 | 0.120 | — |
| 35 | 0.200 | 0.186 | 0.230 | 0.322 | 0.172 | — |

The values provided in the table correspond to the product of $c_m(\gamma_m) \cdot \cot\gamma_m$ (see the equation 1.2).

Adopting the assumption (1.1) for the subsidence described by Knothe's theory (1984), we receive the following relationship for the infinite half-plane:

$$u_{\max} = -\frac{a \cdot M}{2\pi} \cdot \cot\beta \tag{1.3}$$

where:

$\beta$ — angle of main influences;
thus the result is also much smaller than the observed values.

Sroka, Schober (1982) assumed that the point movement trace is curvilinear (so called curvilinear centre of gravity point model) (Fig. 2). By analogy to the radius of main influence range course in the rock mass, they assumed that the course could be described with the formula:

$$r(z) = r \cdot \left(\frac{z}{H}\right)^m \tag{1.4}$$

where:

$H$ — exploitation depth,
$z$ — vertical distance from the working roof,
$m$ — parameter describing the shape of the movement trace propagation.

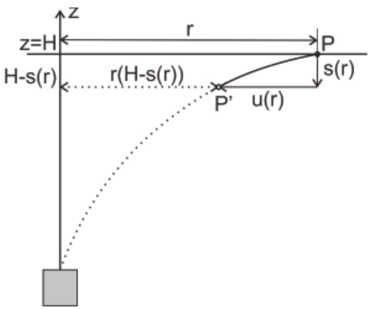



Fig. 2. Point P movement trace towards the exploitation element

It follows that:

$$r[H-s(r)] = r\cdot\left(\frac{H-s(r)}{H}\right)^m = r\cdot\left(1-\frac{s(r)}{H}\right)^m \tag{1.5}$$

For

$$u(r) = r\cdot[H-s(r)] - r \tag{1.6}$$

it yields:

$$u(r) = -m\cdot\frac{r}{H}\cdot s(r) \tag{1.7}$$

This result is qualitatively consistent with the 2D model tests performed by Krzysztoń (Fig. 3).

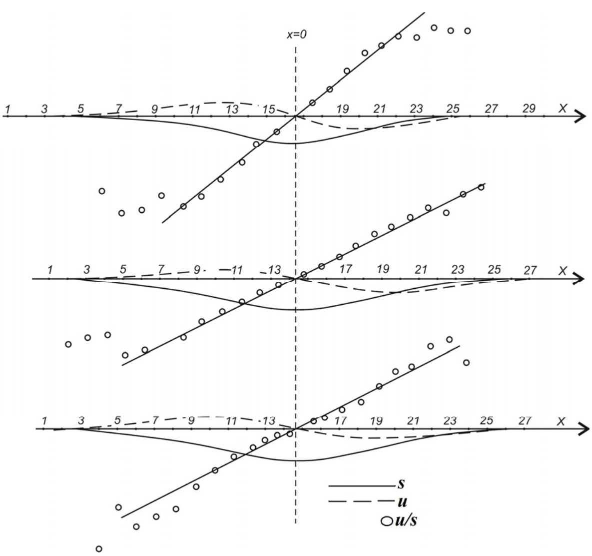



Fig. 3. Distributions of subsidence, horizontal movement and their relationship for elementary discharge (Krzysztoń, 1965)

Based on the tests for the model, the following relationship can be formulated:

$$u(x) = -\alpha \cdot x \cdot s(x) \tag{1.8}$$

where:

$u(x)$ — horizontal movement of point.
$\alpha$ — some constant, depending on the medium properties,
$x$ — distance from the elementary exploitation point,
$s(x)$ — subsidence of that point.

For the assumed curvilinear movement trace (equation 1.7), for the subsidence distribution according to Knothe's theory, assuming the so called infinite half-plane, the following formula is obtained:

$$u_{\max} = -\frac{m}{H} \cdot a \cdot M \cdot \cot\beta \tag{1.9}$$

In a classical form of Knothe's theory, the horizontal movement vector is proportional to the inclination vector $T$:

$$u = -B \cdot T \tag{1.10}$$

where: $B$ — horizontal movement factor.

For the exploitation element, the inclination is described by the equation:

$$T(r) = \frac{\partial u_z(r)}{\partial r} = -\frac{\partial s(r)}{\partial r} \tag{1.11}$$

Assuming that:

$$s(r) = s_0 \cdot \exp\left(-\pi \frac{r^2}{R^2}\right) \tag{1.12}$$

where:

$u_z$ — horizontal movement,
$R$ — angle of main influences ($R = H \cdot \cot\beta$),
$s_0$ — maximum subsidence above the worked out seam component $\left(s_0 = \dfrac{a \cdot V}{R^2}\right)$,
$V$ — component volume,

it yields:

$$T(r) = 2\pi \cdot \frac{r}{R^2} \cdot s(r) \tag{1.13}$$

and, finally:

$$u(r) = -B \cdot T(r) = -B \cdot 2\pi \cdot \frac{r}{R^2} \cdot s(r) \tag{1.14}$$

According to the research of Budryk (1953), the value of the horizontal movement factor $B$ is:

$$B = \frac{R}{\sqrt{2\pi}} \tag{1.15}$$

which, taking into account that value, leads to the final result:

$$u(r) = -\sqrt{2\pi} \cdot \frac{r}{R} \cdot s(r) \tag{1.16}$$

Comparing the equation 1.16 to 1.7, it yields:

$$m = \sqrt{2\pi} \cdot \tan\beta \tag{1.17}$$

This result is also achieved comparing equation 1.9 to 1.18 for the maximum value of horizontal movement for the so called infinite half-plane:

$$u_{\max} = -\frac{a \cdot M}{\sqrt{2\pi}} = -0.40 \cdot a \cdot M \tag{1.18}$$

## 2. Range of mining exploitation effects in the rock mass

The range of mining exploitation effects in the rock mass is described with the equation:

$$R(z) = R \cdot \left(\frac{z}{H}\right)^n = H^{1-n} \cdot z^n \cdot \cot\beta \tag{2.1}$$

where: $n$ — influence area factor in the rock mass.

Figure 4 schematically shows the course of the radius of main influence range $R(z)$ in the rock mass.

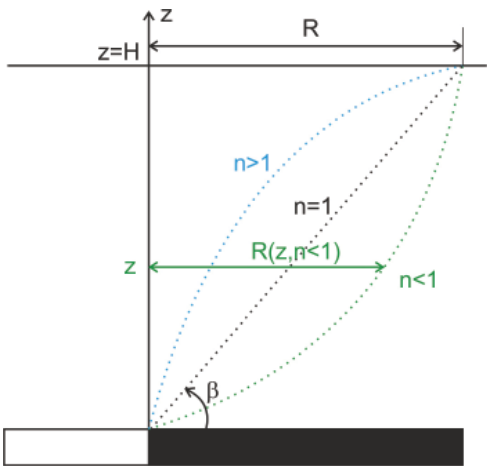



Fig. 4. The shape of mining influence range zones depending on the number $n$

The values of the factor $n$ resulting from the theoretical analyses, model examination and in situ observations are presented in table 2.

TABLE 2

Value of the parameter $n$ according to various hypotheses (Dżegniuk et al., 2003).

| Author | Year | Value |
|---|---|---|
| Budryk | 1953 | $n = \sqrt{2\pi} \cdot \tan\beta$ |
| Mohr | 1958 | $n = 0.65$ |
| Krzysztoń | 1965 | $n = 1.0$ |
| Drzęźla | 1972 | $n = 0.525$ |
| Sroka, Bartosik-Sroka | 1974 | $n = 0.50$ |
| Drzęźla | 1975 | $n = 0.665$ |
| Gromysz | 1977 | $n = 0.61$ |
| Drzęźla | 1979 | $0.47 \leq n \leq 0.49$ |
| Kowalski | 1984 | $0.48 \leq n \leq 0.66$ |
| Zych | 1985 | $n = 0.55$ |
| Drzęźla | 1989 | $0.45 \leq n \leq 0.70$ |
| Preusse | 1990 | $n = 0.54$ |

Note that only the value of the factor $n$ given by Budryk differs significantly from other values and corresponds to the value of the factor $m$ describing the course of point movement trace in horizontal plane. That incorrect conclusion resulted from the predominant assumption of the time that in the rock mass deformation process the continuity equation is satisfied in the form:

$$\sum_{i=1}^{i=3} \varepsilon_{ii} = 0, \text{ i.e.: } \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0 \tag{2.2}$$

However, this relationship was not confirmed in the in situ observations. The evidence for that was presented, among others by Dżegniuk (1970) and Sroka (1973).

Analysing the values of the horizontal and vertical deformations measured during the exploitation of shaft pillars, Dżeginiuk (1970) found that the values of the vertical deformations were from 5 to 10 times lower than the sum of horizontal deformations.

Analysing the values of 3 main components of deformation tensor, Sroka (1973), based on the in situ measurements, found that the sum of horizontal deformations was about 7.1 to 8.5 times higher than the vertical deformation value. According to Sroka, for the area horizon, this relationship can be described with the equation:

$$\left| \frac{\varepsilon_{xx} + \varepsilon_{yy}}{\varepsilon_{zz}} \right| = \frac{1 - v}{v} \tag{2.3}$$

where: $v$ — Poisson factor.

The application of Poisson factor within the soil medium is in fact very limited due to very high variability. According to the recommendations of Förster (1996), at the initial deformation phase $v = 0.1 \div 0.2$ can be assumed, whereas, for large deformations and multiple loading $v = 0.3 \div 0.4$.

Also, the observations performed by Deutsche Steinkohle AG in tensometric centres (3 horizontal and 1 vertical tensometer) prove the relationships given by Dżegniuk and Sroka. The above works demonstrate clearly that the ratio of the sum of horizontal deformations to the vertical deformation falls within the range 5 to 12.

Analysing the deformation of the exploitation component, we will receive the following expression for radial $\varepsilon_r(r)$ and tangential deformations $\varepsilon_t(r)$ of any point on the land surface:

$$\varepsilon_r(r) = -\frac{m}{H} \cdot s(r) \cdot \left[1 - 2\pi \cdot \frac{r^2}{R^2}\right]; \qquad \varepsilon_t(r) = \frac{u(r)}{r} = -\frac{m}{H} \cdot s(r) \tag{2.4}$$

The sum of both major horizontal deformations at any point on the land surface is:

$$\varepsilon_r(r) + \varepsilon_t(r) = -\frac{2m}{H} \cdot s(r) \cdot \left[1 - \pi \frac{r^2}{R^2}\right] \tag{2.5}$$

The value of the vertical deformation can be determined from the general formula:

$$\varepsilon_{zz}(r,z) = \frac{\partial u_z(r)}{\partial z} = -\frac{\partial s(r,z)}{\partial z} \tag{2.6}$$

Assuming:

$$a(z) = a = \text{const} \tag{2.7}$$

it yields:

$$\varepsilon_{zz}(r,z) = -\frac{\partial s(r,z)}{\partial R(z)} \cdot \frac{\partial R(z)}{\partial z} \tag{2.8}$$

and, finally:

$$\varepsilon_{zz}(r,z) = \frac{2n}{z} \cdot s(r,z) \cdot \left[1 - \pi \frac{r^2}{R^2(z)}\right] \tag{2.9}$$

Determining the ratio of the sum of horizontal deformations to the vertical deformation for the area horizon ($z = H$), it yields:

$$\left|\frac{\varepsilon_{xx} + \varepsilon_{yy}}{\varepsilon_{zz}}\right| = \left|\frac{\varepsilon_r + \varepsilon_t}{\varepsilon_{zz}}\right| = \frac{m}{n} \tag{2.10}$$

is the equation in the form:

$$\varepsilon_{xx} + \varepsilon_{yy} + \frac{m}{n}\varepsilon_{zz} = 0 \tag{2.11}$$

where the ratio $m/n$, as demonstrated by the in situ tests, differs significantly from unity.

Adopting $\tan\beta = 2.0$ and $n = 0.5$ it yields: $m/n \cong 10$, which is the value close to those observed in situ.

It proves that the change of the rock mass volume is:

$$\frac{\Delta V(x,y,z)}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \left(1 - \frac{m}{n}\right) \cdot \varepsilon_{zz} \tag{2.12}$$

To conclude, it has to be noted that in view of the in situ observations, during mining exploitation, the volume within the deformed rock mass changes (inter alia Tajduś, 2013). It is, therefore, a fundamental mistake for many theoretical works to assume that the equation of form continuity is met in the rock mass deformation process.

The project was financed from the means of National Science Center granted on the grounds of decision No. DEC – 2011/01/D/ST8/07280

## References

Budryk W., 1953. *Wyznaczanie wielkości poziomych odkształceń terenu*. Archiwum Górnictwa i Hutnictwa, t. 1, z. 1, PWN, Warszawa.

Dżegniuk B., 1970. *Próba doświadczalnego ustalenia związku między odkształceniami poziomymi i pionowymi w górotworze*. Zeszyty Naukowe AGH, Geodezja, z. 17, Kraków.

Dżegniuk B., Sroka A., Niedojadło Z., 2003. *Podstawy wymiarowania i eksploatacji szybowych filarów ochronnych*. Wydawnictwo IGSMiE PAN Szkoła Eksploatacji Podziemnej 2003, Kraków.

Fläschenträger H., 1956. *Die Bodenbewegungsvorgänge im linksniederrheinischen Gebiet*. Der Deutsche Steinkohlenbergbau, Band 2. Vermessungs- und Risswesen, Bergschäden, Verlag Glückauf GmbH, Essen.

Förster W., 1996. *Mechanische Eigenschaften der Lockergesteine*. Teubner Studienbücher, Bauwesen, Verlag B.G.Teubner Stuttgart, Leipzig.

Keinhorst H., 1925. *Die Berechnung der Bodensenkungen im Emschergebiet*. 25 Jahre der Emschergenossenschaft 1900-1925, Essen.

Knothe S., 1984. *Prognozowanie wpływów eksploatacji górniczej*. Wydawnictwo Śląsk, Katowice.

Krzysztoń D., 1965. *Parametr zasięgu niecek osiadania w ośrodku sypkim*. Archiwum Górnictwa, t. 10, z. 1, Warszawa.

Pielok J., 1992. *Über die Tätigkeit der ISM-Kommission 4*. Das Marscheidewesen. Jahrgang 99 (1992), Heft 1, Verlag Glückauf GmbH, Essen.

Schleier O., 1956. *Vorausberechnung von Bodenbewegungen*. Der Deutsche Steinkohlenbergbau, Band 2,. Vermessungs- und Risswesen, Bergschäden, Verlag Glückauf GmbH, Essen.

Sroka A., 1973. *Związki pomiędzy składowymi stanu odkształcenia na powierzchni*. Rudy i Metale Nieżelazne, R: 18, nr 12, Katowice.

Sroka A., Schober F., 1982. *Die Berechnung der maximalen Bodenbewegungen über kavernenartigen Hohlräumen unter Berücksichtigung der Hohlraumgeometrie*. Kali und Steinsalz, August 1982.

Tajduś K., 2013. *Mining-induced surface horizontal displacement: the case of BW Prosper Haniel Mine*. Archives of Mining Sciences, Vol. 58. nr 4.

*Received: 19 March 2015*