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AGNIESZKA SUROWIAK*, MARIAN BROŻEK*

METHODOLOGY OF CALCULATION THE TERMINAL SETTLING VELOCITY DISTRIBUTION OF SPHERICAL PARTICLES FOR HIGH VALUES OF THE REYNOLD'S NUMBER

METODOLOGIA WYLICZANIA ROZKŁADU GRANICZNEJ PRĘDKOŚCI OPADANIA ZIAREN SFERYCZNYCH DLA WYSOKICH WARTOŚCI LICZB REYNOLDSA

The particle settling velocity is the feature of separation in such processes as flowing classification and jigging. It characterizes material forwarded to the separation process and belongs to the so-called complex features because it is the function of particle density and size. i.e. the function of two simple features. The affiliation to a given subset is determined by the values of two properties and the distribution of such feature in a sample is the function of distributions of particle density and size. The knowledge about distribution of particle settling velocity in jigging process is as much important factor as knowledge about particle size distribution in screening or particle density distribution in dense media beneficiation.

The paper will present a method of determining the distribution of settling velocity in the sample of spherical particles for the turbulent particle motion in which the settling velocity is expressed by the Newton formula. Because it depends on density and size of particle which are random variable of certain distributions, the settling velocity is a random variable. Applying theorems of probability, concerning distributions function of random variables, the authors present general formula of probability density function of settling velocity for the turbulent motion and particularly calculate probability density function for Weibull's forms of frequency functions of particle size and density. Distribution of settling velocity will calculate numerically and perform in graphical form.

The paper presents the simulation of calculation of settling velocity distribution on the basis of real distributions of density and projective diameter of particles assuming that particles are spherical.

Keywords: jigging process, settling velocity, turbulent motion, random variable, function of random variable

Prędkość opadania ziarna jest cechą rozdziału w takich procesach przeróbki surowców jak klasyfikacja czy wzbogacanie w osadzarce. Cecha ta opisuje materiał kierowany do procesu rozdziału i należy do tzw. cech złożonych, ze względu na to, że jest funkcją dwóch cech prostych, którymi są: wielkość ziarna i gęstość ziarna. Przynależność do określonego podzbioru ziaren jest określona przez wartość dwóch cech, a rozkład tych cech w próbce jest funkcją rozkładów gęstości i wielkości ziarna. Znajomość rozkładu

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, DEPARTMENT OF MINERAL PROCESSING AND ENVIRONMENT PROTECTION, AL. MICKIEWICZA 30, 30-065 KRAKÓW, POLAND. E-mail: asur@agh.edu.pl, brozek@agh.edu.pl

prędkości opadania ziaren w osadzarce jest istotnym parametrem jak znajomość rozkładu wielkości ziarna w procesie przesiewania czy znajomość rozkładu gęstości w procesie wzbogacania w cieczach ciężkich.

W artykule przedstawiono metodykę wyliczania rozkładu prędkości opadania ziaren sferycznych w warunkach ruchu turbulentnego wyrażonego przy pomocy równania Newtona.

Zarówno gęstość jak i wielkość ziarna są zmiennymi losowymi o określonych rozkładach. W związku z tym prędkość opadania ziarna jako funkcja cech prostych tj. gęstości i wielkości ziarna będzie również zmienną losową o rozkładzie, który jest funkcją rozkładów argumentów prostych. Wykorzystując twierdzenia rachunku prawdopodobieństwa odnoszące się do rozkładów funkcji zmiennych losowych przedstawiono ogólny wzór na funkcję gęstości rozkładu prędkości opadania w warunkach ruchu turbulentnego. Empiryczne rozkłady wielkości i gęstości ziaren aproksymowano rozkładem Weibulla. Rozkład prędkości opadania wyliczono numerycznie i przedstawiono w postaci graficznej.

W artykule przedstawiono symulację wyliczania rozkładu prędkości opadania w oparciu o rzeczywiste rozkłady gęstości i średnicy projekcyjnej ziaren zakładając, że ziarna mają kształt sferyczny.

Słowa kluczowe: proces osadzania, prędkość opadania, ruch turbulentny, zmienna losowa, funkcja zmiennej losowej

1. Introduction

Terminal velocity is understood as a value of settling velocity of particles in steady state, i.e. when the geometrical sum of all forces acting upon a particle is zero. Many papers were devoted to the problem and methods of calculating the free settling velocity of particles (Finkey, 1924; Laszczenko, 1940; Olewskij, 1953; Heiss & Coull, 1952; Akkerman, 1966; Christiansen & Barker, 1965; Bedran et al., 1976; Concha & Almendra, 1979; Heider & Levenspiel, 1989; Briens, 1991; Saha et al., 1992; Madhav & Chhabra, 1994, 1995; Nguyen et al., 1994, 1997; Olajossy, 1995; Tsakalakis & Stamboltzis, 2001; Merinov, 2001; Sztaba, 2004). According to the previous considerations of the authors the free settling velocity of spherical particles is given by the following equation (Brożek & Surowiak, 2010):

$$v = 5,33\sqrt{x}\sqrt{d} \tag{1}$$

where:

$x = \dfrac{\rho - \rho_0}{\rho_0}$ — denotes reduced relative density,

ρ — particle density,

ρ_o — liquid density,

d — particle size.

In this paper the projective diameter of the particle was accepted as particle size d because of the fact that during calculation of distribution of particle settling velocity in working cell of the jig this factor is being applied. Here, the methodology of calculating distribution of settling velocity was applied assuming that the distribution of spherical particle size is the same as distribution of projective diameter for irregular particles.

Settling velocity is the separation feature in flowing classification process and jigging. The influence of particle settling velocity on separation efficiency in jig is given by the following empirical equation (Samylin et al., 1976):

$$I = I_o + \mathrm{atg}\alpha \tag{2}$$

where:

$I = \frac{E_p}{v_r}$ — imperfection,

E_p — probable error,

v_r — partition velocity or specific velocity of separation,

I_o — constant characteristic for certain jig,

a — scale parameter occurring from applied units system,

tgα — slope of tangent to distribution curve of settling velocity in point $v = v_r$.

The lower is imperfection value the higher is separation efficiency. Because of this the knowledge about distribution of particle settling velocity in feed is required for precise evaluation of separation efficiency.

For unequivocal materials considering its granulation and density the feature of separation during flow classification and jigging is particle settling velocity. The state for which physically and geometrically unequivocal material in jig bed is separated ideally by settling velocity is one of the lowest potential energy. It occurs from energy Mayer's theory (Mayer, 1964). In case of separation into density fractions and each density fraction additionally into size fraction according to granulation pyramid from the highest till the lowest value the porosity of the layer would be the biggest. That means that the height of the layer would be bigger than before separation (Kuprin et al., 1983). According to this fact, the potential energy after such separation would be higher than before. This is against the rule of smallest action according to which processes courses in direction of the minimal potential energy of the system. The lowering of the porosity occurs in the situation when empty spaces between bigger particles would be filled by smaller ones. It will occur when in ideally separated material smaller particles of higher density would be transferred to higher sub-layer of bigger particles of smaller density or vice versa. The perfect stratification according to settling velocity will occur which would cause the phenomenon of dissipation of particles of certain density to other layers.

In mineral processing the phenomenon of dissipation of particles to not proper layers can be described by means of partition function and its graphical form, so-called partition curve.

The differential form of partition function or particle dissipation function during separation into concentrate and tailings on the level z can be described by the general formulae:

$$t\,(v|z) = \frac{dT(v|z)}{dv} \tag{3}$$

where: $T(v|z)$ — conditional probability that particle of settling velocity v would be directed to tailings under condition that separation point is on the level z.

The product $t(v|z)dz$ presents the probability that particle of settling velocity v will be found in the layer $(z, z + dz)$ of jig bed or it is a part of total number of particles of velocity v, directed to the layer $(z, z + dz)$.

The knowledge about the function $t(v|z)$ is practically significant because it characterizes separating device and also creates possibilities of forecasting separation results by known feed characteristics.

If function $f(v)$ represents frequency of distribution of feed particles settling velocity then the product $t(v|z)dz \cdot f(v)dz = g(v,z)dvdz$ presents the yield of particles of fraction $(v, v + dv)$ (measured in ratio to total feed) being present in the layer $(z, z + dz)$. The following expression:

$$g(z)dz = dz \int_{v_{\min}}^{v_{\max}} t(v|z) f(v) dv \tag{4}$$

presents the yield of the separation product contained in the layer $(z, z + dz)$. By separation into two products the yield of heavy product (tailings in case of coal) is equal to:

$$\gamma_c(z_r) = \int_0^{z_r} g(z)dz = \int_0^{z_r} dz \int_{v_{\min}}^{v_{\max}} t(v|z) f(v) dv \tag{5}$$

where: z_r — location of separation point.

The relation mentioned above allows forecasting separation results and creates possibility of process monitoring (by change of separation point) dependably on requested products quality.

2. Methodology

2.1. Particle settling velocity as a random variable

The feed, forwarded to the separation process, is characterized by the distribution of physical and geometrical properties of particle and, respectively, of settling velocity. Terminal settling velocity of a spherical particle is a function of its density and size. Assuming that reduced density x and projection diameter d_p are random variables, particle settling velocity constitutes also a random variable which is the function of the mentioned random variables (Brożek & Surowiak, 2005). The form of the distribution function of settling velocity depends upon the probability density function of random variables contained in formula (1). The methodology is based on transforming the formulae of particle settling velocity into a form which is the product of two random variables (Brożek & Surowiak, 2004, 2005a,b; Surowiak, 2014).

Let the random variable of reduced density be marked by X while the random variable of projection diameter by D_p. Introducing new random variables:

$$Y_1 = \sqrt{X} \tag{6a}$$

$$Y_2 = 5,33 Y_1 \tag{6b}$$

$$U_1 = \sqrt{D_p} \tag{6c}$$

the random variable of settling velocity can be presented, according to formula (1), as a product of random variables Y_2 and U_1:

$$V = Y_2 \cdot U_1 \tag{7}$$

In order to calculate the distribution of random variable V, the theorems of probability calculus concerning the functions of random variables are used (Gerstenkorn & Śródka, 1972). The frequency function of random variable being the product of two independent random variables $V = Y_2 \cdot U_1$ is expressed by the following formula (Gerstenkorn & Śródka, 1972):

$$h(v) = \int f_2(y_2) g_1\left(\frac{v}{y_2}\right)\frac{1}{y_2} dy_2 \tag{8}$$

where $h(v)$, $f_2(y_2)$, and $g_1(u_1)$ – the frequency functions of random variables V, Y_2 and U_1 respectively.

The procedure of transforming two random variables into a product is carried out by stages.

2.2. Distribution of random variable Y_2

Let R, X and D_p denote random variables of particle density, reduced density and particle diameter, respectively. Reduced density x is connected with particle density by the dependence:

$$x = \frac{\rho - \rho_o}{\rho_o} \tag{9}$$

from which

$$\rho = \rho_o x + \rho_o = \rho(x) \tag{10}$$

Let random variable R have the distribution determined by the frequency function $f_\rho(\rho)$. Then random variable X has the following frequency function (Gerstenkorn & Śródka, 1972):

$$f(x) = f_\rho[\rho(x)]\left|\frac{d\rho(x)}{dx}\right| \tag{11a}$$

$$f(x) = \rho_o f_\rho(\rho = \rho_o x + \rho_o) \tag{11b}$$

After introducing a new random variable:

$$Y_1 = X^{1/2} \tag{12}$$

from which $x = y_1^2 = x(y_1)$, the distribution of random variable Y_1, analogically as above, is as follows:

$$f_1(y_1) = f[x(y_1)]\left|\frac{dx(y_1)}{dy_1}\right| \tag{13a}$$

$$f_1(y_1) = 2y_1 f(x = y_1^2) \tag{13b}$$

In this situation random variable Y_2 = 5,33 Y_1 $\left(y_1 = \frac{y_2}{5,33} = y_1(y_2)\right)$ has the frequency function:

$$f_2(y_2) = f_1[y_1(y_2)]\left|\frac{dy_1(y_2)}{dy_2}\right| \tag{14a}$$

$$f_2(y_2) = f_1\left(y_1 = \frac{y_2}{5,33}\right)\cdot\frac{1}{5,33} \tag{14b}$$

The frequency function of particle density, according to the dispersive particle model (Brożek, 1995), is expressed by a two-parameter function which belongs to the gamma distribution set. As it will be shown further on, this is Weibull's distribution function whose cumulative and frequency functions of density are as follows:

$$F_\rho(\rho) = 100\left\{1 - \exp\left[-\left(\frac{\rho}{\rho_c}\right)^{k_n}\right]\right\} \tag{15}$$

$$f_\rho(\rho) = \frac{100k_n}{\rho_c^{k_n}}\rho^{k_n-1}\exp\left[-\left(\frac{\rho}{\rho_c}\right)^{k_n}\right] \tag{16}$$

where

ρ_c — characteristic density ($F_\rho(\rho = \rho_c) = (63{,}21\%)$,
k_n — non-homogeneity coefficient.

According to formulae (11a,b), the frequency and the cumulative distribution functions of reduced density are expressed by the following formulas:

$$f(x) = \frac{100k_n}{(x_c+1)^{k_n}}(x+1)^{k_n-1}\exp\left[-\left(\frac{x+1}{x_c+1}\right)^{k_n}\right] \tag{17}$$

$$F(x) = 100\left\{1 - \exp\left[-\left(\frac{x+1}{x_c+1}\right)^{k_n}\right]\right\} \tag{18}$$

where: $x_c = \dfrac{\rho_c - \rho_o}{\rho_o}$.

The distribution of random variable $Y_1 = \sqrt{X}$, according to formulas (13a,b), is as follows:

$$f_1(y_1) = \frac{200k_n}{(y_{1c}^2+1)^{k_n}}y_1\left(y_1^2+1\right)^{k_n-1}\exp\left[-\left(\frac{y_1^2+1}{y_{1c}^2+1}\right)^{k_n}\right] \tag{19}$$

$$F_1(y_1) = 100\left\{1 - \exp\left[-\left(\frac{y_1^2+1}{y_{1c}^2+1}\right)^{k_n}\right]\right\} \tag{20}$$

where $y_{1c} = \sqrt{x_c}$.

The distribution of random variable $Y_2 = 5{,}33\,Y_1$, according to formulas (14a,b), is characterized by the following functions:

$$f_2(y_2) = \frac{200k_n}{(y_{2c}^2+28{,}41)^{k_n}}y_2(y_2^2+28{,}41)^{k_n-1}\exp\left[-\left(\frac{y_2^2+28{,}41}{y_{2c}^2+28{,}41}\right)^{k_n}\right] \tag{21}$$

$$F_2(y_2) = 100\left\{1 - \exp\left[-\left(\frac{y_2^2 + 28,41}{y_{2c}^2 + 28,41}\right)^{k_n}\right]\right\} \tag{22}$$

where $y_{2c} = 5,33 y_{1c}$.

2.3. Distribution of random variable U_1

Let the distribution of random variable D_p be given by the frequency function $g(d_p)$. The new random variable $U_1 = \sqrt{D_p}$ $(d_p = u_1^2 = d_p(u_1))$ has the following distribution:

$$g_1(u_1) = g\left[d_p(u_1)\right]\left|\frac{dd_p(u_1)}{du_1}\right| \tag{23a}$$

$$g_1(u_1) = 2u_1 g(d_p = u_1^2) \tag{23b}$$

The frequency function of particle size of fine coal can be well approximated by Weibull's distribution (R-R-B). In other words, the random variable D_p has Weibull's distribution whose frequency function and cumulative distribution function are equal to:

$$g(d_p) = \frac{100k_p}{d_{op}} d_p^{k_p - 1} \exp\left[-\left(\frac{d_p}{d_{op}}\right)^{k_p}\right] \tag{24}$$

$$G(d_p) = 100\left[1 - \exp\left(-\frac{d_p}{d_{op}}\right)^{k_p}\right] \tag{25}$$

where:
d_{op} — characteristic diameter,
k_p — non-homogeneity coefficient.

Random variable $U_1 = \sqrt{D_p}$, according to formulae (23a,b), will have such a distribution whose frequency and cumulative distribution function are expressed by the formulas:

$$g_1(u_1) = \frac{200k_p}{u_{o1}^{2k_p}} u_1^{2k_p - 1} \exp\left[-\left(\frac{u_1^2}{u_{o1}^2}\right)^{k_p}\right] \tag{26}$$

$$G_1(u_1) = 100\left[1 - \exp\left(\frac{u_1^2}{u_{o1}^2}\right)^{k_p}\right] \tag{27}$$

where: $u_{o1} = \sqrt{d_{op}}$.

3. Experimental

The experiment was conducted on hard coal sample. The float and sink analysis of coal sample was performed in solutions of zinc chloride of densities, respectively: 1,3; 1,4; 1,5; 1,6; 1,7; 1,8; 2,0 [Mg/m^3]. Each density fraction was screened on sieved of meshes: 2,0; 3,15; 5,0; 6,3; 8,0; 10,0; 12,5; 16,0; 20,0 mm. In purpose of calculating the particles projective diameters their photos were done by means of digital camera in the most stable position. Using the image analysis of computer program SigmaScanPro the particles area and projective diameter was calculated.

4. Results and discussion

4.1. Distribution of particle size and density

It was assumed that particles are spheres whose diameters are equal to a projection diameter. In other words, determining this distribution is a simulation of a possible distribution of particles settling velocity in the analyzed feed sample if the particles were spherical. The cumulative distribution and frequency functions of projection diameter are presented in Fig. 1a and 1b, respectively. The parameters of this distribution, according to formula (25), are: $k_p = 1{,}52$; $d_{op} = 15{,}27$ [mm]. Analogically for sieve diameter distribution, the parameters are as following: $k_s = 1{,}52$; $d_{os} = 10{,}83$ [mm] and the curvilinear correlation coefficient is bigger than 0,95.

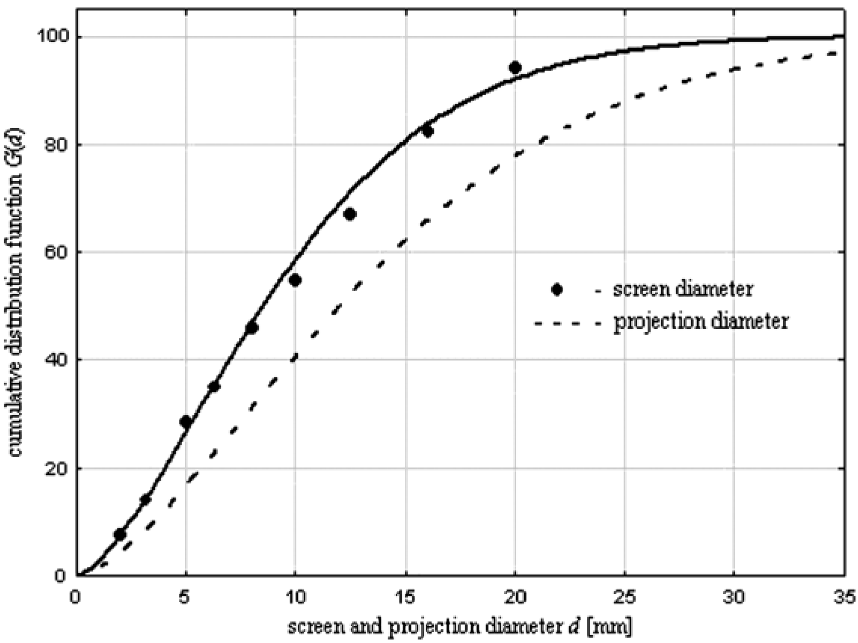

Fig. 1a. Cumulative distribution function of screen and projection diameters of particles in the feed $d_{os} = 10{,}83$ [mm], $d_{op} = 15{,}27$ [mm], $k_p = 1{,}52$

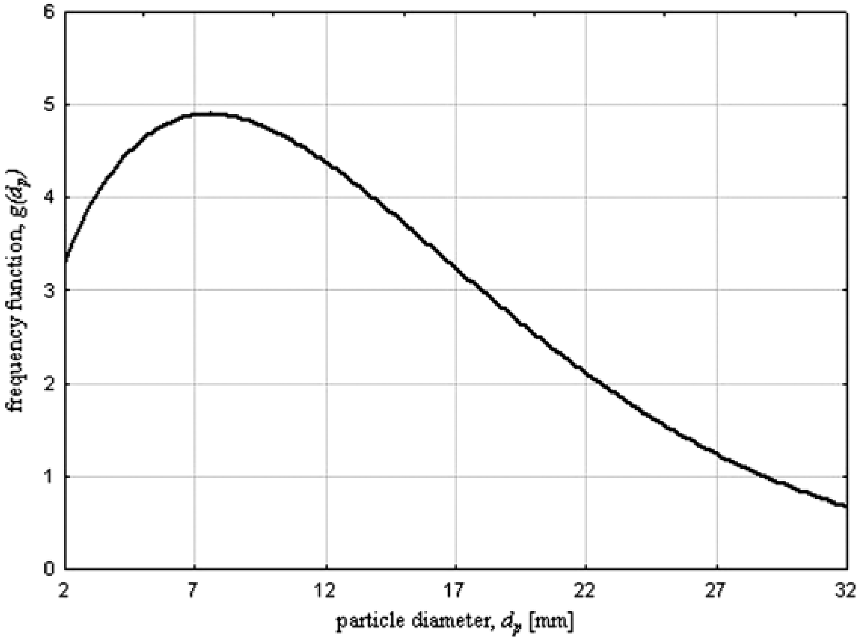

Fig. 1b. Frequency function of projection diameter of particles in the feed

For these parameters the frequency function of variable U_1, according to formula (26), is as follows:

$$g_1(u_1) = 175161{,}5u_1^{2{,}04} \exp\left[-\left(\frac{u_1^2}{0{,}015}\right)^{1{,}52}\right] \tag{28}$$

Figure 2 shows the frequency function of random variable U_1.

Figures 3a and 3b shows the cumulative distribution and frequency function of particle density in the feed, respectively. The parameters of this distribution, according to formula (15), are as follows: $k_n = 3{,}54$; $\rho_c = 2{,}17$ [Mg/m^3].

Therefore, the frequency function of variable Y_2, according to formula (21), is as follows:

$$f_2(y_2) = 2{,}43 \cdot 10^{-4}\, y_2 (y_2^2 + 28{,}41)^{2{,}68} \exp\left[-\left(\frac{y_2^2 + 28{,}41}{57{,}68}\right)^{3{,}54}\right] \tag{29}$$

while: $x_c = 0{,}985$; $y_{1c} = 0{,}992$; $y_{2c} = 5{,}29$.

Frequency function $f_2(y_2)$ is presented on Fig. 4.

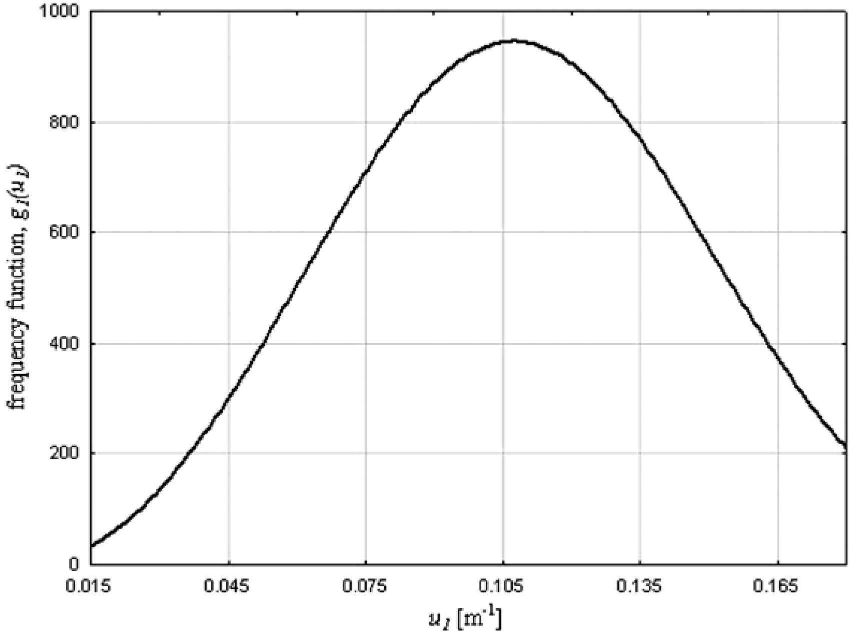

Fig. 2. Frequency function of variable U_1

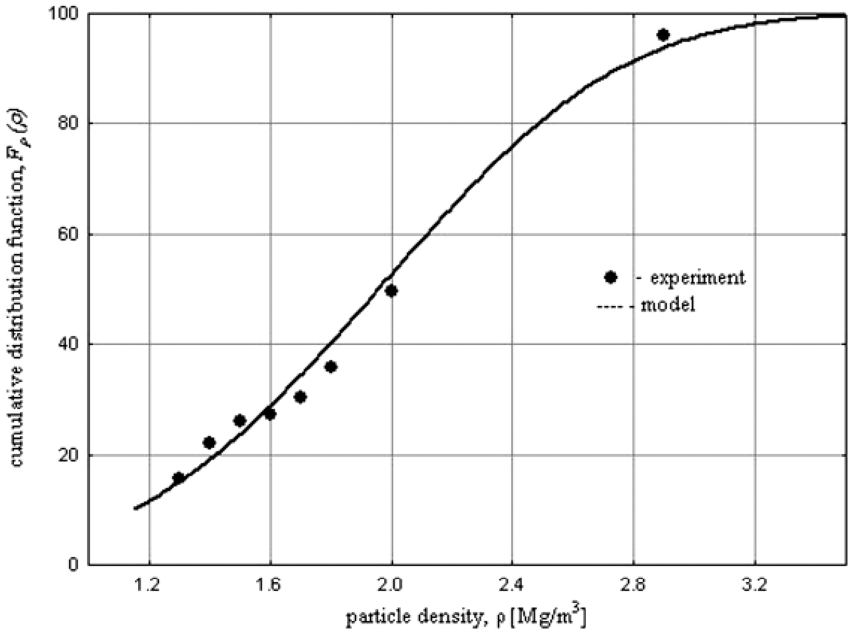

Fig. 3a. Cumulative distribution function of distribution of particle density in the feed $\rho_c = 2{,}17$ [Mg/m^3]; $k_n = 3{,}54$

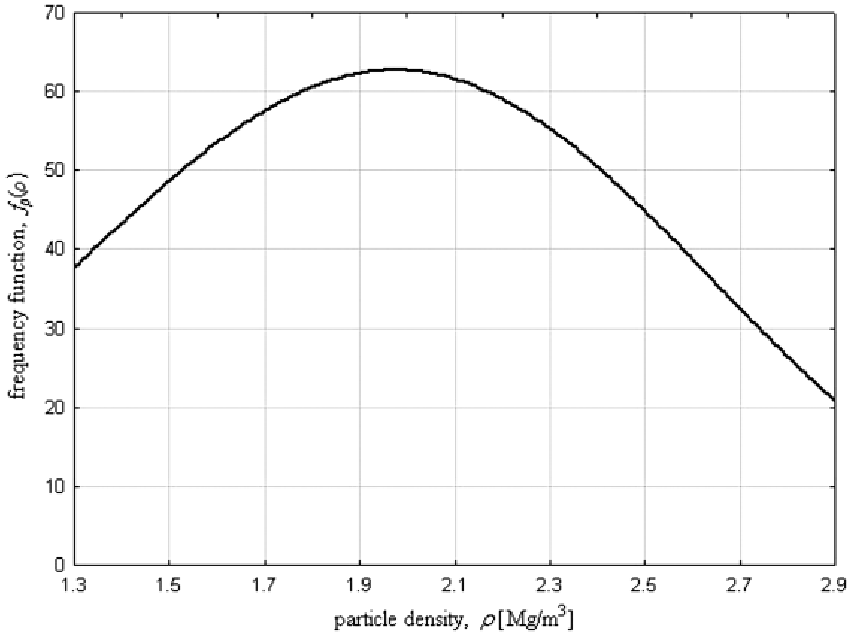

Fig. 3b. Frequency function of particle density

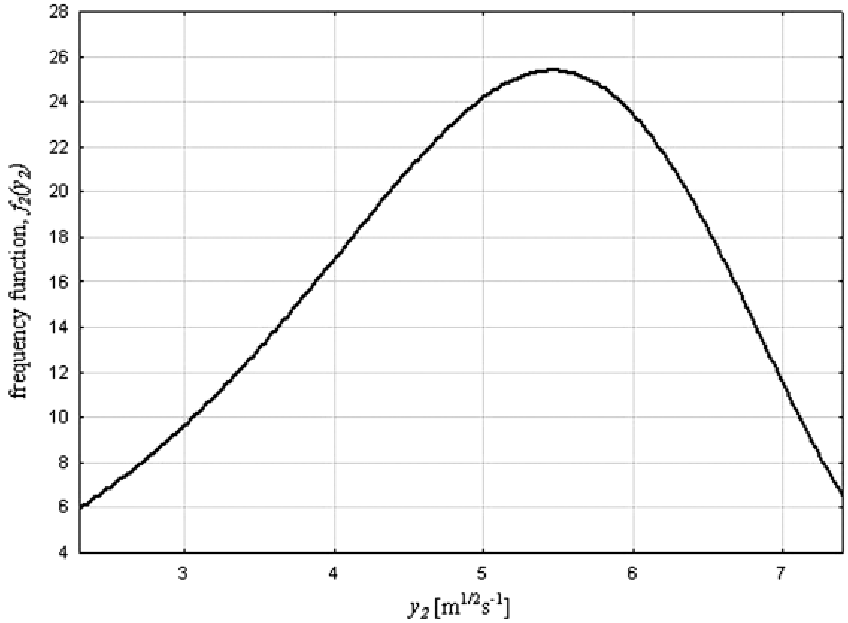

Fig. 4. Frequency function of random variable Y_2

4.2. Distribution of settling velocity

Consequently, according to formula (8) the frequency function of settling velocity in the feed of spherical particles is expressed by the following formula:

$$h(v) = 42,56 \int_{1.2}^{7,7} (y_2^2 + 28,41)^{2,68} \exp\left[-\left(\frac{y_2^2 + 28,41}{57,68} \right)^{3,54} \right] \left(\frac{v}{y_2} \right)^{2,04} \times \\ \times \exp\left[-\left(\frac{v^2}{0,015 y_2^2} \right)^{1,52} \right] dy_2 \tag{30}$$

Distribution (30) was determined numerically by means of algorithm created in C++ computer language.

Figure 5 presents the frequency function of settling velocity of spherical particles in the feed.

On the basis of figure 5a it is visible that the distribution of settling velocity is asymmetrical function with minima particles settling velocity equals to 0,12 m/s and the maximal spherical particles settling velocity is even equal to 1,4 m/s.

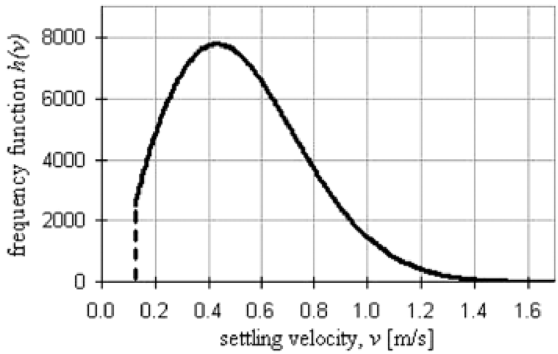

Fig. 5a. Frequency function of settling velocity of spherical particles in the feed

5. Conclusions

Assuming that settling velocity is a random variable being the function of random variables of physical and geometrical properties of a particle and using the theorems of probability calculus concerning the functions of random variables, it is possible to derive general formulas of the frequency function of free settling velocity of a particle for the conditions of turbulent motion, existing in the upgrading process in the jig.

The frequency function of terminal free settling velocity clearly depends upon the parameters of the frequency functions of density and projection diameters. When the distributions of density and particle diameter are known, it is possible, according to the algorithm presented in this paper, to determine the distribution of terminal particle settling velocity.

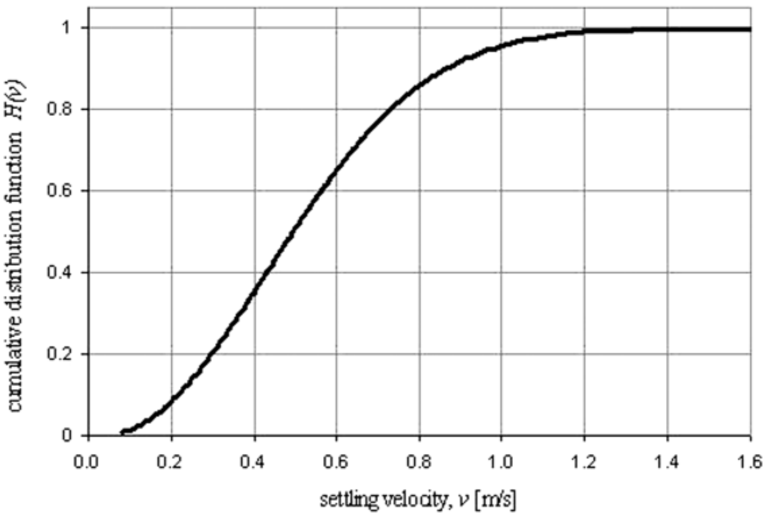

Fig. 5b. Cumulative distribution function of settling velocity of spherical particles in the feed

Terminal free settling velocity of particle is a complex feature of separation in the process of upgrading in the jig. The distribution of this feature in a sample is the function of distributions of simple features, i.e. physical and geometrical properties of a particle, which are particle density and diameter.

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