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VERSITA

Arch. Min. Sci., Vol. 58 (2013), No 3, p. 941–952

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2013-0066

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BINARY LINEAR PROGRAMMING IN THE MANAGEMENT OF MINE RECEIVABLES

BINARNE PROGRAMOWANIE LINIOWE W ZARZĄDZANIU NALEŻNOŚCIAMI KOPALNI

This paper presents a method of binary linear programming for the selection of customers to whom a rebate will be offered. In return for the rebate, the customer undertakes payment of its debt to the mine by the deadline specified. In this way, the company is expected to achieve the required rate of collection of receivables. This, of course, will be at the expense of reduced revenue, which can be made up for by increased sales. Customer selection was done in order to keep the overall cost to the mine of the offered rebates as low as possible:

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \to \min$$

where:

K_{cR} — total cost of rebates granted by the mine;
k_j — cost of granting the rebate to a j^{th} customer;
x_j — decision variables;
$j = 1, \ldots, n$ — particular customers.

The calculations were performed with the *Solver* tool (Excel programme). The cost of rebates was calculated from the formula:

$$k_j = \Delta P_j - K_k^{(j)}$$

where:

ΔP_j — difference in revenues from customer j;
$K_k^{(j)}$ — cost of the so-called trade credit with regard to customer j.

The cost of the trade credit was calculated from the formula:

$$K_k = \sum_{s=1}^{T} \frac{r}{100} \cdot \frac{t_s}{360} \cdot N_s$$

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where

r — interest rate on the bank loan, %

t_s — collection time for the receivable in days (e.g. $t_1 = 30$, $t_2 = 45, \ldots, t_{12} = 360$);

N_s — value of the receivable at collection date t_s.

This paper presents the general model of linear binary programming for managing receivables by granting rebates. The model, in its general form, aims at:

– minimising the objective function:

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \to \min$$

– with the restrictions:

$$\sum_{j=1}^{n} (N_{t\,ji} + x_j \cdot N_{n\,ji}) \geq q \cdot N_i \qquad i = 1, \ldots, m$$

– and:

$$x_j \in (0, 1)$$

where:

$N_{t\,ji}$ — value of the timely payments of a customer j in an i^{th} month of the period analysed;

$N_{n\,ji}$ — value of the overdue receivables of a customer j in an i^{th} month of the period analysed;

q — the assumed minimum percentage of timely payments collected;

N_i — summarised value of all receivables in the month i;

m — the number of months in the period analysed.

The general model was used for application to the example of the operating Mine X. Furthermore, the study has been extended through the presentation of a binary model of linear programming, in which the objective function should minimise the anticipated value of the cost of rebates:

$$E(K_{cR}) = \sum_{j=1}^{n} p_j \cdot x_j \cdot k_j$$

where:

p_j — denotes the probability of use of the rebate by customer j.

The paper presents two mathematical models. One is a determinist model which can be used under certainty conditions, whereas the other considers the risk of the rebates not being used by the customers. The paper also describes some random experiments with the Monte Carlo method.

Keywords: linear programming method, risk and uncertainty, rebate, discount, receivables management

W artykule zastosowano metodę binarnego programowania liniowego w celu wyboru odbiorców, którym zostanie zaproponowany rabat. W zamian za proponowany rabat, dany odbiorca zobowiązuje się spłacać w założonym terminie należność kopalni. W ten sposób przedsiębiorstwo ma osiągnąć odpowiedni poziom ściągalności należności terminowych. Stanie się to oczywiście kosztem zmniejszenia przychodów, które można zrekompensować poprzez zwiększenie sprzedaży. Wybór odbiorców dokonany został w taki sposób aby sumaryczny koszt zaproponowanego rabatu był dla kopalni jak najmniejszy, czyli:

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \to \min$$

gdzie:

K_{cR} — całkowity koszt udzielonych rabatów przez kopalnię

k_j — koszt udzielenia rabatu j-temu odbiorcy,

x_j — zmienne decyzyjne,

$j = 1, \ldots, n$ — poszczególni odbiorcy.

Do obliczeń wykorzystano narzędzie *Solver* (program Exel). Koszty rabatów wyliczono ze wzoru:

$$k_j = \Delta P_j - K_k^{(j)}$$

gdzie
ΔP_j — różnica w przychodach od odbiorcy j,
$K_k^{(j)}$ — koszt tak zwanego kredytu kupieckiego ponoszonego względem odbiorcy j.

Koszt kredytu kupieckiego wyznaczono ze wzoru:

$$K_k = \sum_{s=1}^{T} \frac{r}{100} \cdot \frac{t_s}{360} \cdot N_s$$

gdzie
r — stopa procentowa zaciągniętego kredytu bankowego, %,
t_s — okres ściągania należności, dni (np. $t_1 = 30$, $t_2 = 45,\ldots, t_{12} = 360$),
N_s — wartość należności o terminie ściągalności t_s.

W artykule zaprezentowano ogólny model binarnego programowania liniowego do zarządzania należnościami, z wykorzystaniem rabatu. Model ten w ogólnej postaci ma:
– zminimalizować funkcję celu:

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \rightarrow \min$$

– przy ograniczeniach:

$$\sum_{j=1}^{n} (N_{t\,ji} + x_j \cdot N_{n\,ji}) \geq q \cdot N_i \qquad i = 1,\ldots, m$$

– oraz:

$$x_j \in (0,1)$$

gdzie:
$N_{t\,ji}$ — wartość należności terminowych odbiorcy j w i-tym miesiącu analizowanego okresu,
$N_{n\,ji}$ — wysokość należności nieterminowych odbiorcy j w i-tym miesiącu okresu,
q — założony, minimalny odsetek ściągalności należności terminowych,
N_i — sumaryczna wartość wszystkich należności w miesiącu i,
m — liczba miesięcy w analizowanym okresie.

Ogólny model posłużył do jego aplikacji na przykładzie danych funkcjonującej kopalni „X”. Ponadto rozważania te rozszerzono przedstawiając binarny model programowania liniowego, w którym funkcja celu powinna minimalizować oczekiwaną wartość kosztów rabatów, czyli:

$$E(K_{cR}) = \sum_{j=1}^{n} p_j \cdot x_j \cdot k_j$$

gdzie p_j oznacza prawdopodobieństwo skorzystania z rabatu przez odbiorcę j.

W artykule zostały sformułowane dwa modele matematyczne. Pierwszy z nich to model deterministyczny, który może być stosowany w warunkach pewności, zaś drugi uwzględnia ryzyko nieskorzystania z rabatów przez odbiorców. W artykule przeprowadzono również pewne losowe eksperymenty za pomocą metody Monte Carlo.

Słowa kluczowe: metoda programowania liniowego, ryzyko i niepewność, rabat, zarządzanie należnościami

1. Introduction

Linear programming has been applied to a number of optimisation problems (Czopek, 2001; Osiadacz, 2000; Jaśkowski, 1998). It can be also applied in the management of receivables which result from trade credits granted in the course of business, including various forms of rebates (Cooke, 2001; Jasiukiewicz at.al., 2001; Klonowska, 2005; Lewandowska, 2000).

Conventional receivables-management methods are based on the so-called incremental method, in which the increase in revenue after the rebate compensates for the cost of granting the rebate.

However, linear programming enables methods of receivables management to be optimised, for example, as follows, through:

- maximising revenue from sales, and thus profit;
- optimising the periods of rebates granted;
- minimising the cost of trade credit granted;
- minimising the risk of overdue or lost receivables;
- restructuring timely payments and overdue receivables.

The linear programming problem involves finding an optimal solution to the linear objective function $f(x)$ with n arguments $x_1, x_2, ..., x_n$, given certain restrictions including the variables x_j ($j = 1, 2, ..., n$). This can be presented in general form, as follows:

- to optimise the objective function:

$$f(x_1, x_2, ..., x_n) = \sum_{j=1}^{n} c_j \cdot x_j \rightarrow optimum\,(\max, \min) \tag{1}$$

with the restrictions:

$$\sum_{j=1}^{n} a_{ij} \cdot x_j \leq b_i \tag{2}$$

or

$$\sum_{j=1}^{n} a_{ij} \cdot x_j \geq b_i, \quad \text{for} \quad i = 1, 2, ..., m \tag{3}$$

$$x_j \geq 0, \quad \text{for} \quad j = 1, 2, ..., n \tag{4}$$

$$m < n \tag{5}$$

where:

$f(x_1, x_2, ..., x_n)$ — objective function;
c_j — coefficients of objective function;
a_{ij}, b_j — direction coefficients, real numbers;
m — number of restrictions.

In the group of linear programming problems under consideration, it is naturally necessary to limit the range of acceptable solutions (Gass, 1980; Jasiukiewicz et. al., 2001; Radzikowski, 1979), since two further conditions limiting the solution usually have to be satisfied. In the case of a mine, such restrictions might be, for example, limited resources or limited mining output.

In receivables management, restrictions (2) and (3), depending on the rebate scheme chosen, may involve (Lewandowska, 2000; Pluta & Michalski, 2005; Portalska & Kornatowicz, 2003; Sierpińska, 2005; Szyszko, 2000) the following:

- the acceptable value of receivables for the entire mine or an individual customer;
- the acceptable value of the overall cost of the trade credit granted, or granted individually to each of the customers;
- the expected revenue of the mine after applying the rebate;
- minimum prices needed to guarantee required revenue;
- minimum rate of collection of timely payments, etc.

In the case of management of a mining company, generally speaking, the decision-maker, even in the case of receivables management, is often faced with a 'yes or no' choice.

If this is the case, binary problems are involved, usually decision-related problems, with the natural interpretation $x_j = 0$, when a decision is made not to execute a project or $x_j = 1$ when decided otherwise. This paper addresses this very issue, which depends on the selection of customers to whom the mine will and will not grant the rebate.

The paper formulates two mathematical models. One can be applied under deterministic conditions, whereas the other considers the risk of the rebates not used by the customers. Some completely random experiments were also conducted (using the Monte Carlo method). Therefore, the following analysis of the problem can be understood as how to make decisions under certainty, risk or uncertainty conditions. It is worth mentioning that strategies for the above-mentioned situations have also been investigated (Cyrul, 2004) in a paper presenting different methods.

2. Formulation of the general binary model of the linear programming

Let us suppose that the mine accurately monitors its customers and is able to specify (at least approximately) the following values:

k_j — cost of rebate granted to customer j, $j = 1, 2, ..., n$.
$N_{t\,ji}$ — value of the timely payments of customer j in the i^{th} month of the period analysed;
$N_{n\,ji}$ — value of the overdue receivables of a customer j in the i^{th} month of the period analysed;
n — number of customers.

Let N_{ti} denote the overall value of all timely payments from all customers in the month i,

$$N_{ti} = \sum_{j=1}^{n} N_{t\,ji}, \qquad i = 1, 2, ..., m \tag{6}$$

and N_{ni} denote the overall value of all overdue receivables from all customers in the month i,

$$N_{ni} = \sum_{j=1}^{n} N_{n\,ji} \tag{7}$$

and N_i denote the overall value of all receivables in the month i:

$$N_i = N_{ti} + N_{ni} \tag{8}$$

It is further assumed that a decision on granting a rebate to a given customer applies to the entire period analysed. The customer, in return for the rebate, undertakes to pay all receivables in timely fashion every month. The objective of the mine is to grant rebates to some of the customers, so that the overall percentage of timely payments from all customers will increase to the assumed minimum level in every month. Such a minimum percentage will be denoted by q. In addition, the overall cost of rebates should be as low as possible.

To create a mathematical model for the above problem, the following decisive variables will be introduced:

$$x_j = \begin{cases} 1, & \text{customer } j \text{ granted the discount} \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Let us analyse how timely payments change after rebates are applied. While granting a rebate to the customer j (i.e., when $x_j = 1$), the timely payments of that customer will increase by $N_{n\,ji}$ in each month i, and thus yield $N_{t\,ji} + N_{n\,ji}$, which equals $N_{t\,ji} + x_j \cdot N_{n\,ji}$. If no rebate is granted to the customer j, i.e. when $x_j = 0$, the timely payments of that customer will not change and will, therefore, remain at the level $N_{t\,ji}$, which, again, equals $N_{t\,ji} + x_j \cdot N_{n\,ji}$. Therefore, the condition under which the timely payments in each month i achieve the assumed minimum percentage of the overall receivables of that month will be fulfilled by the restriction:

$$\sum_{j=1}^{n} \left(N_{t\,ji} + x_j \cdot N_{n\,ji} \right) \geq q \cdot N_i \tag{10}$$

Assuming that the cost of the rebate granted to customer j is denoted by k_j, then the actual cost of the rebate granted to customer j is $x_j \cdot k_j$, which means zero cost when the rebate is not granted (because then $x_j = 0$); if the rebate is granted, it is simply k_j (because then $x_j = 1$). Hence, the overall cost of the rebates given is:

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \tag{11}$$

To sum up, the mathematical model of binary linear programming of the problem is as follows:

– minimise

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \tag{12}$$

– with the restrictions

$$\sum_{j=1}^{n} \left(N_{t\,ji} + x_j \cdot N_{n\,ji} \right) \geq q \cdot N_i \tag{13}$$

and:

$$x_j \in \{0,1\} \qquad j = 1, 2, ..., n \tag{14}$$

It is known that granting the rebate to customer j means, in practice, reducing the sales price for that customer. If no additional terms were added to the contract save the requirement to reduce the term of payment for customer j, then the revenue of the mine would be reduced. To prevent that, extra provisions should be added to the contract.

Therefore, let the following denote:

A_{oj} — the revenue of the mine received so far from customer j;
A_j — the expected revenue from customer j, given the rebate.

Then the additional provision will take the following form:

- for $x_j = 1$

$$A_j \cdot x_j \geq A_{oj} \tag{15}$$

- for $x_j = 0$

$$A_j = A_{oj} \tag{16}$$

Note, however, that conditions (15) and (16) do not constitute part of the model (12)-(14), but are included only in a modified contract with a given customer.

3. Application of the binary model of the linear programming

Let us present the above model in a hypothetical example. Table 1 shows the data regarding the payment of receivables by ten customers within a period of 3 months in the past year. The mine anticipates the same values in the first quarter of the year analysed.

TABLE 1

Payment of mine receivables by 10 selected customers, PLN

Customer	Month of the 1st quarter year A	Receivables paid on time	Overdue receivables, months						
			Overall overdue receivables	1	2	3	6	9	12
1	2	3	4	5	6	7	8	9	10
customer 1	1	80,744	112,597	92,597	20,000	0	0	0	0
	2	36, 718	133,032	53,008	80,024	0	0	0	0
	3	0	156,749	23,445	66,280	67,024	0	0	0
customer 2	1	17,859	39,031	19,420	19,611	0	0	0	0
	2	19,821	35,745	7,613	18,475	9,657	0	0	0
	3	21,858	42,478	17,719	9,714	15,045	0	0	0
customer 3	1	6,598	0	0	0	0	0	0	0
	2	50,656	34,945	33,303	1,642	0	0	0	0
	3	10,470	63,179	48,180	13,357	1,642	0	0	0
customer 4	1	29,733	30,405	30,405	0	0	0	0	0
	2	26,245	23,174	23,174	0	0	0	0	0
	3	17,780	27,797	21,345	6,452	0	0	0	0
customer 5	1	0	0	0	0	0	0	0	0
	2	33,063	0	0	0	0	0	0	0
	3	26,785	6,278	6,278	0	0	0	0	0

TABLE 1

1	2	3	4	5	6	7	8	9	10
customer 6	1	48,603	16,279	16,279	0	0	0	0	0
	2	50,702	16,279	0	16,279	0	0	0	0
	3	88,168	0	0	0	0	0	0	0
customer 7	1	43,100	5,926	5,926	0	0	0	0	0
	2	37,581	7,725	7,725	0	0	0	0	0
	3	43,882	0	0	0	0	0	0	0
customer 8	1	99,750	64,287	15,210	13,455	16,928	18,694	0	0
	2	64,647	61,288	29,750	12,274	4,381	14,883	0	0
	3	76,108	81,106	29,818	19,750	12,274	19,264	0	0
customer 9	1	23,204	40,222	0	0	0	0	40,222	0
	2	49,330	40,303	13,081	0	0	0	27,222	0
	3	25,444	76,578	36,275	13,081	0	0	0	27,222
customer 10	1	35,020	0	0	0	0	0	0	0
	2	19,392	29,054	29,054	0	0	0	0	0
	3	26,725	29,211	15,965	13,246	0	0	0	0
TOTAL		1,109,986	1,173,668						
Percentage, %		49	51						

As shown above, timely payments constitute only 49% of all receivables. Assuming that the goal of the company's management is to plan a strategy which would result in a receivable collection rate of at least 80% each month, that goal can be achieved by giving rebates in return for timely payment of the receivables.

Considering, for example, customer 9 and the third month, assuming that the annual interest rate of a bank loan is $r = 9.5\%$ and the rate of the rebate is $u = 9\%$, then, the profit achieved through paying off the trade credit would be:

$$K_k = \frac{0.095}{12} \cdot 36275 + \frac{0.095}{6} \cdot 13081 + 0.095 \cdot 27222 = 3080.38 \text{ PLN}$$

The loss in the third month due to the rebate would be:

$$\Delta P = 0.09 \cdot (25444 + 76578) = 9692.09 \text{ PLN}$$

Therefore, in the third month, the cost of the rebate granted to customer 9 would be:

$$\Delta P - K_k = 6611.71 \text{ PLN}$$

The value k_9 would be achieved after adding up the cost of the rebate in all 3 months. Table 2 below presents the cost calculated.

TABLE 2

Cost of rebate to individual customers in three analysed months in PLN

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
41,222.32	14,213.49	14,004.77	13,266.755	5,901.639	19,416.16	12,331.19	35,627.12	14,967.96	11,980.05

The current status of overdue receivables and timely payments is shown in tables 3 and 4.

TABLE 3

Value of overdue receivables paid by individual customers, PLN

Customer	1	2	3	4	5	6	7	8	9	10	Overall overdue receiveables
Month 1	112,597	39,031	0	30,405	0	16,279	5,926	64,287	40,222	0	308,747
Month 2	133,032	35,745	34,945	23,174	0	16,279	7,725	61,288	40,303	29,054	**381,545**
Month 3	156,749	42,478	63,179	27,797	6,278	0	0	81,106	76,578	29,211	**483,376**

TABLE 4

Value of timely payments paid by individual customers, PLN

Customer	1	2	3	4	5	6	7	8	9	10	Overall timely payments
Month 1	80,744	17,859	6,598	29,733	0	48,603	43,100	99,750	23,204	35,020	**384,611**
Month 2	36,718	19,821	50,656	26,245	33,063	50,702	37,581	64,647	49,330	19,392	**388,155**
Month 3	0	21,858	10,470	17,780	26,785	88,168	43,882	76,108	25,444	26,725	**337,220**

As shown above, the overall performing and overdue receivables are respectively PLN 693,358, 769,700 and 820,596 in months 1, 2 and 3, and 80% (the assumed minimum percentage q) of that sum is respectively PLN 554,686.40 in month 1, PLN 615,760 in month 2 and PLN 656,476.80 in month 3.

After substituting the data in the model (12)-(14) and solving it with Excel's *Solver* tool, the following solution was obtained:

$$\left.\begin{array}{r} x_1 = x_3 = x_4 = x_9 = 1, \\ x_2 = x_5 = x_6 = x_7 = x_8 = x_{10} = 0 \end{array}\right\}$$

and the total cost of the rebate granted was K_{cR} = PLN 83,461.80.

Therefore, the best choice would be to grant rebates to customers 1, 3, 4 and 9. Then the entire cost of the rebate would be PLN 83,461.80. For example, achieving the required level of timely payments would be possible if the company granted rebates to customers 1, 3, 8 and 10. Then, however, the overall cost of rebates would be PLN 102,834.26.

4. Binary linear programming method in granting rebates to mine customers, considering risk

In practice, the requirement that those customers given rebates pay their debt on time might not be met. Therefore, an additional random element has been introduced.

Let p_j be the approximate known (as a result of monitoring) probability that customer j will use the rebate.

The objective is to identify those customers to whom rebates will be granted, so that the expected value of timely payments is at least $q \cdot 100\%$ of all receivables (note that q is the minimum assumed percentage of collected timely payments) and the cost of the rebates given is as low as possible.

$\Delta N_{t\,ji}$ denotes a random variable which indicates an increase in the collection of timely payments from customer j in month i, when that customer was granted the rebate. Therefore:

$$\Delta N_{t\,ji} = N_{n\,ji} \text{ with the probability } p_j$$

$$\Delta N_{t\,ji} = 0 \text{ with the probability } 1 - p_j$$

(the above equations result from the fact that in cases where the rebate is used, the customer is obliged to pay overdue receivables on time).

Therefore, the expected value of the increase in timely payments from customer j in month i equals:

$$E(\Delta N_{t\,ji}) = N_{n\,ji} \cdot p_j \tag{17}$$

Finally, the expected value of the increase in timely payments from customer j in month i equals:

$$E(N_{t\,ji} + \Delta N_{t\,ji}) = N_{t\,ji} + E(\Delta N_{t\,ji}) = N_{t\,ji} + p_j \cdot N_{n\,ji} \tag{18}$$

owing to the linearity of the expected values (note that $N_{t\,ji}$ is not a random variable but a known value). On the other hand, when customer j is not granted the rebate, the performing receivables will not change and will remain $N_{t\,ji}$. In general, both situations can be described in a common formula:

$$E(N_{t\,ji} + \Delta N_{t\,ji}) = N_{t\,ji} + x_j \cdot p_j \cdot N_{n\,ji} \tag{19}$$

using again the interpretation of the decision variables x_j. Note that the overall cost is also a random variable with the expected value $E(K_{cR}) = \sum_{j=1}^{n} p_j \cdot x_j \cdot k_j$. Therefore, the following binary programming problem will be involved:

– minimise

$$E(K_{cR}) = \sum_{j=1}^{n} p_j \cdot x_j \cdot k_j \tag{20}$$

– with the restrictions

$$\sum_{j=1}^{n} \left(N_{t\,ji} + x_j \cdot p_j \cdot N_{n\,ji} \right) \geq q \cdot N_i \qquad i = 1, ..., m \tag{21}$$

$$x_j \in \{0,1\} \qquad j = 1, ..., n \tag{22}$$

The above model will be described again with the data from table 1. Moreover, all probabilities p_j were determined as 0.8. After plugging the data into the model (15)-(17) and solving it with Excel's *Solver* tool, the following solution was obtained:

$$\begin{cases} x_1 = x_3 = x_8 = x_9 = x_{10} = 1 \\ x_2 = x_4 = x_5 = x_6 = x_7 = 0 \end{cases}$$

and the expected cost of the rebate granted was K_{cR} = PLN 94,241.78.

5. Summary

Note that the cost of granting rebates k_j, $j = 1, ..., n$ can be understood in the broadest sense as the cost of debt collection resulting from other methods of receivables management (e.g. court fees, hiring a debt collection company, or the cost of credit insurance). Therefore, model (12)-(14) would also be applicable in such analysis (this refers also to model (20)-(22)).

The above model (12)-(14) was also applied to analysis of the receivables of a real mine ('X') and its 110 selected customers. The simulated level of the company's timely payments in Year 1 of the analysis was around 41%. The required share of timely payments was fixed, as in the example, at the level of at least 80% in each month of the year analysed. To solve the binary problem (12)-(14), the *Solver* tool was used again. However, due to the great number of variables (110 customers), Excel did not find an optimum solution within a reasonable time. Instead of optimising the original binary problem, it was decided to solve its linear relaxation, i.e. the problem of linear programming resulting from (12)-(14), by replacing the restrictions (14) with

$$0 \le x_j \le 1$$

The solution of the relaxation is the value of PLN 2,227,114 which is the lower limit for K_{cR}, i.e. $K_{cR} \ge$ PLN 2,227,114. Such a solution, however, is unacceptable, since the variables assume fractional values and have no practical interpretation. The solution was then rounded to the closest binary one.

One drawback of such rounding is that it may not fulfil all the required restrictions. Also, in this case, after rounding, the percentage of timely payments was maintained above the level of 80% in all months except one, in which it was slightly below 79%. Since that level only slightly differed from the assumed one, such rounding was considered acceptable. The cost of such a solution was PLN 2,354,076, so the relative error does not exceed 5.7%. The value of relative error was derived from the formula:

$$\frac{|Z^* - Z_{opt}|}{Z_{opt}} = \frac{Z^*}{Z_{opt}} - 1 \le \frac{2354076}{2227114} - 1 = 0.057$$

where:

Z^* — the accepted solution,
Z_{opt} — unknown value of the optimum solution.

The inequality in the formula results from the fact that $Z_{opt} \ge 2{,}227{,}114$ PLN. It is merely an estimation of the relative error of approximation. In practice, a much smaller error should be expected. The solution was, therefore, considered satisfactory.

To compensate for the resultant cost of the rebate by increasing sales, the mine would have to sell 48,956.84 Mg more product at an average price of PLN 48.08/Mg. In the mine in question, the average monthly sales in the year analysed amounted to 130,416.89 Mg, so the planned increase in sales would be 37.5% of the monthly average.

Some random experiments were also conducted, using the Monte Carlo method, in which rebates were granted to all customers, and the probability of using the rebate was set at 0.7 for each of the customers. The expected value of timely payments slightly exceeded 80% in each month. The expected value of the total cost of the rebates granted was PLN 3,020,984.61. As can be seen, careful selection (simplex method) of customers to whom rebates should be granted works, in this respect, better than random choice (Monte Carlo method). However, model (12)-(14) will produce good results only when all customers are carefully monitored and when most of the predictions prove to be correct. Calculations with the Monte Carlo method are less sensitive to unexpected changes in some values.

The model described by formulas (20)-(22) is a sort of combination of the optimum choice method with a certain degree of randomness in allowing the customers not to use the rebate. In contrast to the Monte Carlo method, however, a group of customers to whom we are inclined to grant the rebate is first identified. Then, random methods are applied to the selected group, whereas in the Monte Carlo method, random methods were applied to all customers. Model (20)-(22) is a kind of compromise (limited randomness) between optimum choice and random choice of the recipients of the rebates.

Model (20)-(22) was also applied to the analysed Mine X. A procedure was adopted similar to the case of model (12)-(14). The probability of using the rebate was set at 0.7. A solution meeting all conditions and with an expected value of the total cost of PLN 2,855,210 was obtained. Thus it can be stated that the attempt to safeguard against certain random occurrences resulted in increasing the overall expected cost of the operation. Note, however, that complete randomness in granting rebates resulted in an expected overall cost of PLN 3,020,984.61 (see the summary of model (12)-(14)).

The paper was written in 2013 within the framework of statutory research registered with the AGH University of Science and Technology in Krakow, under the number 11.11.100.481.

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Received: 23 November 2012