



JAVAD SATTARVAND\*, CHRISTIAN NIEMANN-DELIUS\*\*

**A NEW METAHEURISTIC ALGORITHM FOR LONG-TERM OPEN-PIT PRODUCTION PLANNING****NOWY META-HEURYSTYCZNY ALGORYTM WSPOMAGAJĄCY DŁUGOTERMINOWE  
PLANOWANIE PRODUKCJI W KOPALNI ODKRYWKOWEJ**

Paper describes a new metaheuristic algorithm which has been developed based on the Ant Colony Optimisation (ACO) and its efficiency have been discussed. To apply the ACO process on mine planning problem, a series of variables are considered for each block as the pheromone trails that represent the desirability of the block for being the deepest point of the mine in that column for the given mining period. During implementation several mine schedules are constructed in each iteration. Then the pheromone values of all blocks are reduced to a certain percentage and additionally the pheromone value of those blocks that are used in defining the constructed schedules are increased according to the quality of the generated solutions. By repeated iterations, the pheromone values of those blocks that define the shape of the optimum solution are increased whereas those of the others have been significantly evaporated.

**Keywords:** open pit optimization, production planning, ant colony optimization, metaheuristics

W artykule zaprezentowano nowy meta-heurystyczny algorytm oparty na zasadach optymalizacji mrowiska i zbadano jego skuteczność w zastosowaniach do planowania wydobywania w kopalniach. Uwzględniono szereg zmiennych w każdym bloku schematu i przeanalizowano „ślady feromonów” które przedstawiają „dążność” poszczególnych bloków w danej kolumnie do stania się najgłębszym punktem kopalni w trakcie określonego okresu prowadzenia prac wydobywczych. W ramach kolejnych iteracji generuje się kilka harmonogramów prowadzenia wydobywania. Następnie wartości poziomu feromonów przypisane do kolejnych bloków redukowane są do wielkości wyrażonych w procentach a wartości poziomu feromonów przypisane do bloków wykorzystywanych do wygenerowania danego harmonogramu zostają powiększone, zgodnie z wymogami odnośnie jakości uzyskanych rozwiązań. Droga kolejnych iteracji, wartości poziomu feromonów przypisane do bloków generujących rozwiązania optymalne zostają powiększane podczas gdy wartości przypisane do bloków pozostałych zostają odpowiednio pomniejszone.

**Słowa kluczowe:** optymalizacja kopalni odkrywkowych, planowanie produkcji, optymalizacja metodą kolonii mrówek, metaheurystyka

\* INSTITUTE OF SURFACE MINING AND DRILLING, RWTH AACHEN UNIVERSITY, 4-20 LOCHNERSTR, 52064 AACHEN, GERMANY. E-mail: [sattarvand@bbk3.rwth-aachen.de](mailto:sattarvand@bbk3.rwth-aachen.de)  
MINING ENGINEERING FACULTY, SAHAND UNIVERSITY OF TECHNOLOGY, SAHAND NEW TOWN, TABRIZ, IRAN.  
E-mail: [Sattarvand@sut.ac.ir](mailto:Sattarvand@sut.ac.ir)

\*\* INSTITUTE OF SURFACE MINING AND DRILLING, RWTH AACHEN UNIVERSITY, 4-20 LOCHNERSTR, 52064 AACHEN, GERMANY. E-mail: [niemann-delius@bbk3.rwth-aachen.de](mailto:niemann-delius@bbk3.rwth-aachen.de)

## 1. Introduction

Hard rock open pit mining is a mineral exploitation method by which the deposit is accessed by digging a large opening, called pit in the ground surface, to uncover the ore to air. The initial mining phase starts with a small pit, and then develops to a larger pit which encloses it. The process proceeds until a final shape of the mine called “ultimate pit limit” (UPL) is reached. These sequences of pits are known as mining sequences or push backs. Mining operations in each push back starts from the upper part and proceed towards its bottom. The objective of long-term pit optimisation is to find the sequence that will maximise the economic rewards. The results of these calculations are used as a guide for short-term production planning which may be for a quarter, month or week.

The last 30 years have seen a widely-publicised revolution in the application of numerical methods in the mining industry in order to produce better mine plans on more complicated and often lower grade deposits, and with staffing levels that would have been unthinkable prior to the early 1980s. Recent researches in the field of open-pit optimisation have been focused on developing new algorithms which are firstly less complex in terms of comprehensibility and programming, secondly require shorter computing times in order to be applicable to the large deposits and finally allow the incorporation of real mining complexities such as variable slopes, working slopes, time value of money, quality and quantity of planned material and related uncertainties.

Almost all computerised hard rock open-pit mine planning methods are based on block models. A block model divides the whole ore body and surrounding waste rocks into 3D blocks adjacent to each other. The model may have millions of blocks depending on the size of deposit and the size of blocks. The average ore grade of each block is estimated using geostatistical approaches or conditional simulation methods. The long-term open-pit mine production scheduling problem can be defined as specifying the sequence in which the blocks should be removed from the mine as a certain material type, in order to maximise the total discounted profit from the mine subject to a variety of physical and economic constraints.

## 2. Problem statement

### 2.1. Mathematical formulation

Integer Linear Programming (IP) with binary variables can be effectively used to model this problem. The model has binary variables and its objective function could be expressed as the maximisation of the net present return by mining and processing of the blocks. This is subject to a variety of constraints. First of all the total tonnage of extracted material should be between a pre-determined upper and lower limit. Secondly the quantity of each material type should also be between the defined boundaries. Furthermore the average grade of each production element should be between pre-determined limits. Moreover the sequencing constraints are necessary to ensure that a block could only be removed if all overlaying blocks have been removed in the previous or current periods. Finally some reserve constraints are applied to mathematically guarantee that a block is mined only once.

Several approaches have been proposed in literature to solve this model. (Dagdelen & Johnson, 1986) and (Caccetta, et al., 1998) used lagrangian parameterisation in order to relax mining

and milling constraints into objective function. Consequently the problem could be handled by repetition of any Ultimate Pit Limit (UPL) algorithm such as (Lerchs & Grossmann, 1965) graph theory based algorithm. In this process lagrange multipliers were utilised to omit the mining and milling constraints and solved the model using subgradient optimisation method. Later (Caccetta & Hill, 2003) proposed a branch and bound technique to solve the formulated scheduling problem. (Dowd and Onur, 1992) and (Onur & Dowd, 1993) formulated the problem as a dynamic programming model. (Ramazan et al., 2005) described the application of fundamental tree algorithm to reconstruct the mining blocks and decrease the number of variables in scheduling problems without reducing the resolution of the model or optimality of the results. They defined the fundamental tree as any combination of the blocks such that they can be profitably mined respecting slope constraints. Recently (Ramazan & Dimitrakopoulos, 2004) have added a new aspect related to the uncertainties involved in estimation of geological block models. (Osanloo et al., 2008) published a comprehensive review of various open pit production scheduling approaches. Lately (Sayadi et al., 2011) have used a new 3D open pit optimization algorithm based on the artificial neural networks. Consideration of the optimum cut-off grade in production scheduling is one of the other issues which might add more complexity to the problem. (Azimi & Osanloo, 2011) have studied a combination of nonlinear programming and genetic algorithm methods for finding of the optimum cut-off grade strategy.

## 2.2. Metaheuristics

A metaheuristic is a set of algorithmic concepts that can be used to improve heuristic methods applicable to difficult problems. These concepts are usually inspired by biology and nature. The use of metaheuristics has significantly increased the ability of finding very high quality solutions for hard combinatorial problems (that are often easy to state but very difficult to solve) in a reasonable time. This is particularly true for large and poorly understood problems. The family of the metaheuristics includes, but not limited to, genetic algorithm, simulated annealing, tabu search, ant colony optimisation, and particle swarm optimisation. Two research studies addressed so far in literature regarding the application of metaheuristic algorithms in long-term open pit mine production planning.

Denby and Schofield (1994) described the process of the application of Genetic Algorithm (GA) in optimisation of an open-pit mine production planning shown in Figure 1a. The main advantage of their method was in its ability to solve ultimate pit limit and long-term planning problems simultaneously. By choosing proper values for genetic parameters, the method was capable of producing good results for a small block model in an acceptable time. Later Denby and Schofield (1995) continued to consider risk assessment in their scheduling process. They also extended the algorithm from 2D to 3D (1996) and used it for a flexible scheduling operation (1998).

Kumral and Dowd (2002, 2005) investigated solving of the open pit mine production scheduling problem by use of Simulated Annealing (SA) metaheuristic as shown in figure 1b. The main advantage of this routine is that it utilises a multi-objective function comprised of three minimisation components, on the other hand, the separate determination of UPL and production schedule would be counted as a disadvantage for this method.

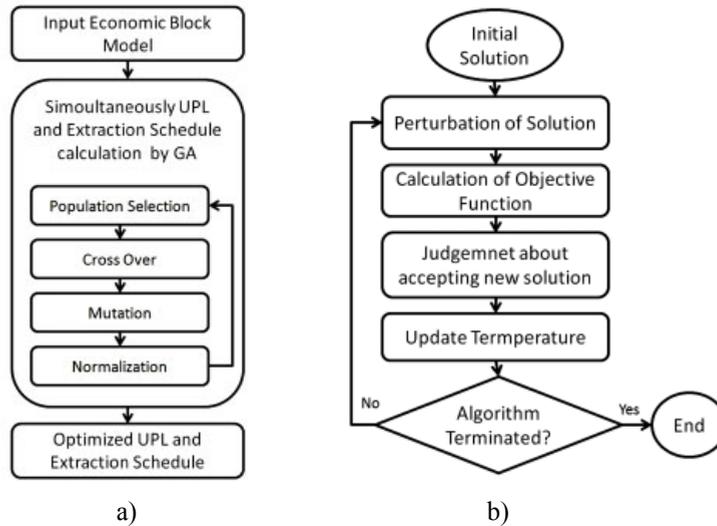


Fig. 1. The process of open pit scheduling by genetic algorithm (a) and simulated annealing (b)

### 2.3. Ant colony optimisation

Ant Colony Optimisation (ACO) is one of the most successful metaheuristic algorithms developed by Dorigo and Stützle (2004). It is inspired by the foraging behaviour of ant colonies. In nature, ants walk randomly, and upon finding food return to their colony while laying down chemical trails called pheromone. The pheromone trail transmits a message to other members of the colony. The other ants are likely follow the trail instead of randomly traveling. If they eventually find food then reinforce the trail by depositing more pheromone. Over the time the pheromone trail starts to evaporate and reduce its attraction. Magnitude of the evaporation in longer paths is higher than that of shorter routes. Thus the intensity of laid pheromone on shortest path, by comparison, gradually increases up to the level that balances with the evaporation rate. This makes the shortest path to be marched and almost all of the ants to follow this route.

A study was conducted in the Institute of Surface Mining and Drilling Technology, RWTH Aachen University, aiming of the application of ACO for optimisation of long-term open pit mine production planning, (Sattarvand, 2009). The process has the ability to optimise UPL and long-term planning problems simultaneously according to the multi-objective target and complex constraints by utilising a population of mine schedule solutions. Figure 2 shows the proposed process of long-term open-pit production planning.

### 3. Steps of the new algorithm

The algorithm consists of saving  $P$  variables for each block of the model,  $\tau_{ip}$ , which represent the pheromone value related to the mining of  $i^{\text{th}}$  block in  $p^{\text{th}}$  period. The magnitude of saved pheromones represents the desirability of a block to be the deepest point of the mine in that period. The initial values of these variables are assigned based on a sub-optimal mine schedule generated

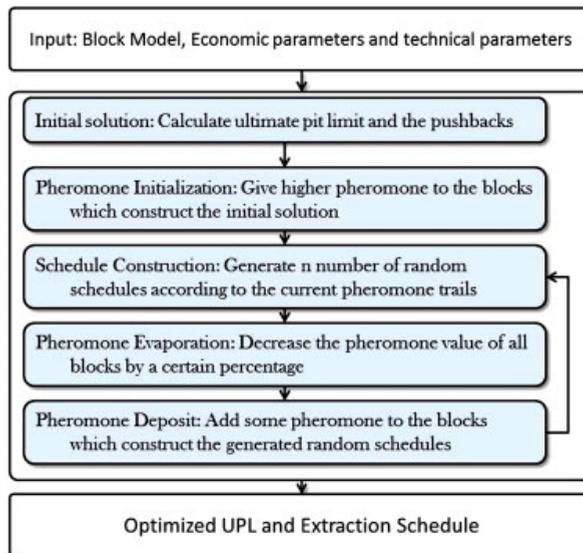


Fig. 2. The process of long-term open-pit production planning using ACO

by the algorithms proposed by Lerchs and Grossmann (1965) and Wang and Sevim (1995). Then the random mining schedules are constructed according to the initial pheromones. These schedules deposit an extra pheromone proportional to their economic quality. This action along with pheromone evaporation leads the algorithm towards the optimum boundary of mining push backs.

### 3.1. Pheromone Initialisation

Experiments showed that the calculation time increased dramatically using the uniform initial pheromone pattern. Therefore a sub-optimal solution for the problem of long-term open-pit scheduling is firstly determined by means of Lerchs and Grossmann's algorithm (1965) and the Wang and Sevim's nested pits design algorithm (1995). Then, initial pheromone trails are assigned to the blocks according to this sub-optimal solution. Normally the shape of a desired pushback does not change drastically from a sub-optimal solution to the optimal one. Thus assigning of higher pheromones to a few numbers of blocks around the sub-optimal pit depth could be enough to lead the algorithm towards the optimal solution. During the process of pheromone initialisation, the pheromone values of the ore blocks close to the pit shape in initial solution (the highlighted blocks in Figure 3) are set to relatively higher values.

### 3.2. Construction of schedules

In order to construct a mine scheduling solution, a series of feasible pit shapes related to the different mining push backs should be created. Each one of these pits consists of a series of block columns. The shape of each pit could be defined by determination of the pit depth on the block columns. The pheromone value is the major element in determination of the pit depth on

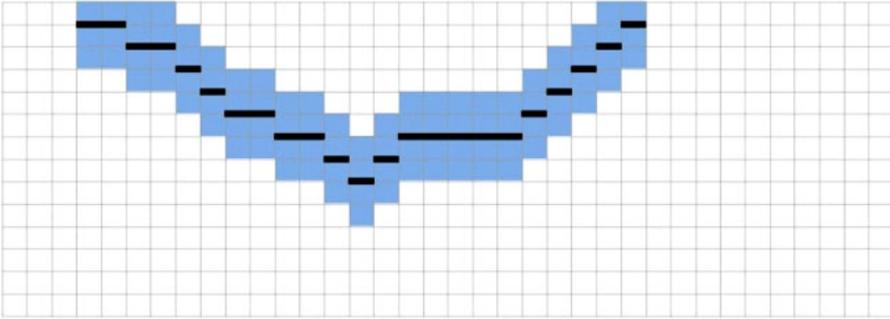


Fig. 3. Pheromone initialisation of the blocks

a column. Sometimes using heuristic information such as economic value of the blocks could also help the efficiency of the method. The probability with which ant  $k$ , chooses the block  $i$  is:

$$p_i^k = \frac{[\tau_i]^\alpha [\eta_i]^\beta}{\sum [\tau_i]^\alpha [\eta_i]^\beta}, \quad i \in N_i^k$$

Where  $\tau_i$  is the pheromone value of block  $i$ ,  $\eta_{ij}$  is the heuristic information,  $\alpha$  and  $\beta$  are two parameters which determine the relative influence of the pheromone trail and the heuristic information, and  $N_i^k$  is the feasible neighbourhood of ant  $k$ .

A numerical example of depth determination process has been explained in Table 1. The upper and lower boundary of the permitted pit depth should also be available for the column in determination of the pit depth. The maximum allowed depth defines the deepest possible mining depth on that column; while, the minimum depth is determined according to the shape of the mine in earlier push back, Figure 4.

It should be noted that the process of depth finding is done only for the columns containing at least one ore block. The depth of the pit in totally waste columns will be defined based on the neighbouring selected depths. Another important point is that the initial pheromones are assigned only to the ore blocks. Therefore, the selected depth will always coincide on an ore block. Similarly, there will be no pheromone update (evaporation or deposition) for waste blocks.

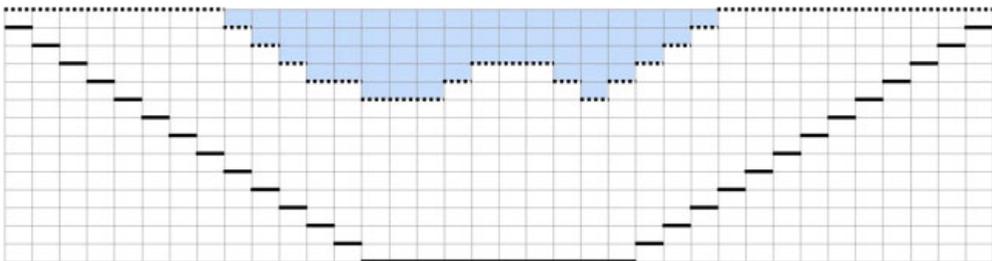


Fig. 4. Maximum and minimum depth definition in the process of depth determination

TABLE 1

An example of depth determination process

Block Column	Pheromone*	Heuristic Information*	Selection possibility**	Cumulative Possibility
	0	0	0.0000	0.0000
	280	8	0.0285	0.0285
 Min Depth	0	0	0.0000	0.0285
 W	0	0	0.0000	0.0285
 O	330	6	0.0297	0.0583
 O	540	7	0.0930	0.1514
 W	0	0	0.0000	0.1514
 O	670	6	0.1227	0.27424
 Selected Depth	890	8	0.2889	0.5631***
 O	750	9	0.2308	0.7939
 W	0	0	0.0000	0.7939
 Max Depth	870	5	0.1725	0.9664
	350	6	0.0335	1.0000
	0	0	0.0000	1.0000

\* without unit

\*\* based on  $p_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}$  formula ( $\alpha = 1$  and  $\beta = 1$ )

\*\*\* selected depth according to the random number (0.6328)

### 3.3. Normalisation

Normally the consequence of independent depth determination in each column is not always feasible due to the required slope angles; therefore, a normalisation stage based on the selected depths is necessary in order to generate a feasible pit shape. The normalisation step is implemented after determination of depths to ensure that the constructed pit shape covers all of the determined depths as well as the outline of earlier push backs. The feasible pit shape shown in Figure 5b is constructed based on the set of determined depths and the shape of earlier period displayed in Figure 5a.

Finally individual normalised pits which have been created for different mining periods are combined together to produce a mine schedule, Figure 6.

### 3.4. Pheromone update

Constructed mine schedules are manipulated by the ACO module in two steps as a series of decreases and increases in pheromone values of the blocks. The first step, called Pheromone

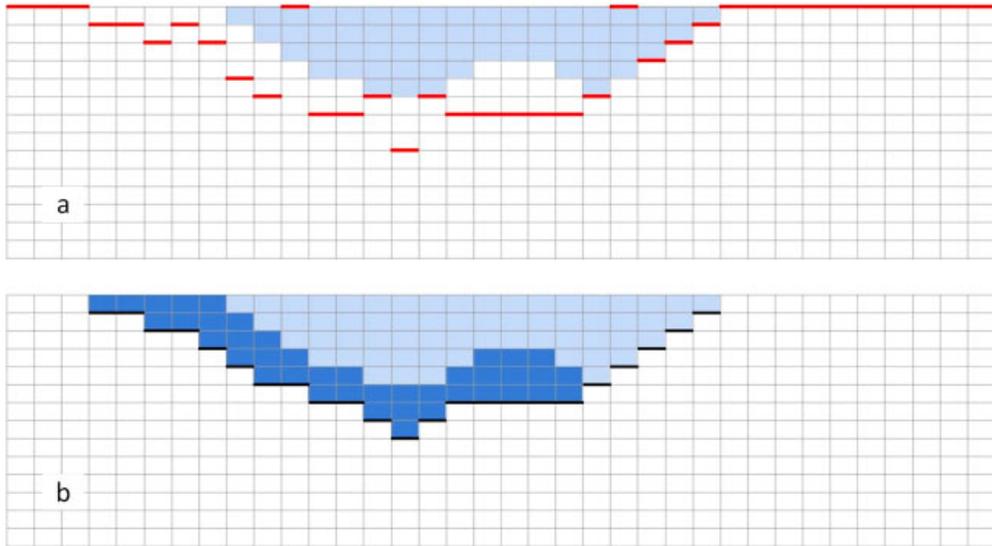


Fig. 5. Generation of feasible pit according to the selected depths (Normalisation)

Evaporation, consists of a uniform reduction in the value of all pheromones in order to help the ACO model disregard the bad solutions. In this stage, the pheromone value of all blocks corresponding to all production schedules should be decreased by a certain percentage. The next step, Pheromone Deposition, consists of adding additional pheromone to the blocks which have contributed in the construction of the schedules. It should be noted that the deposition action is applied only to the ore blocks. In cases where the pit depth falls on a waste block, the additional pheromone is assigned to an imaginary block on the ground surface. This action creates an imaginary block to compete against the other ore blocks which have not really contributed in construction of the schedules.

#### 4. Case study

To evaluate the applicability of the proposed algorithm for long-term planning of open-pit mines, a computer program has been developed in the Visual Studio 2005 programming environment for the implementation of calculations. In order to test the program, a hypothetical block model of an iron ore deposit containing 1000 blocks was created and the grades of Fe and SiO<sub>2</sub> were randomly assigned to all ore blocks. The grades of Fe and SiO<sub>2</sub> varied from 45 to 65 and from 5 to 15 percent respectively. The calculated UPL by the Lerchs and Grossmann's graph algorithm contained 455 ore and 161 waste blocks which led to 681 monetary units of undiscounted economic value. Then mining push backs were generated by the alternative to parameterisation algorithm of Wang and Sevim (1995). Through this, nine uniform push backs with the size of 70 blocks were constructed. Considering an annual interest rate of 10 percent and the mine life of 20 years, the discounted economic value of the constructed initial schedule was calculated as 323 units. As a simple scheduling condition, the values of mining and processing rates are considered

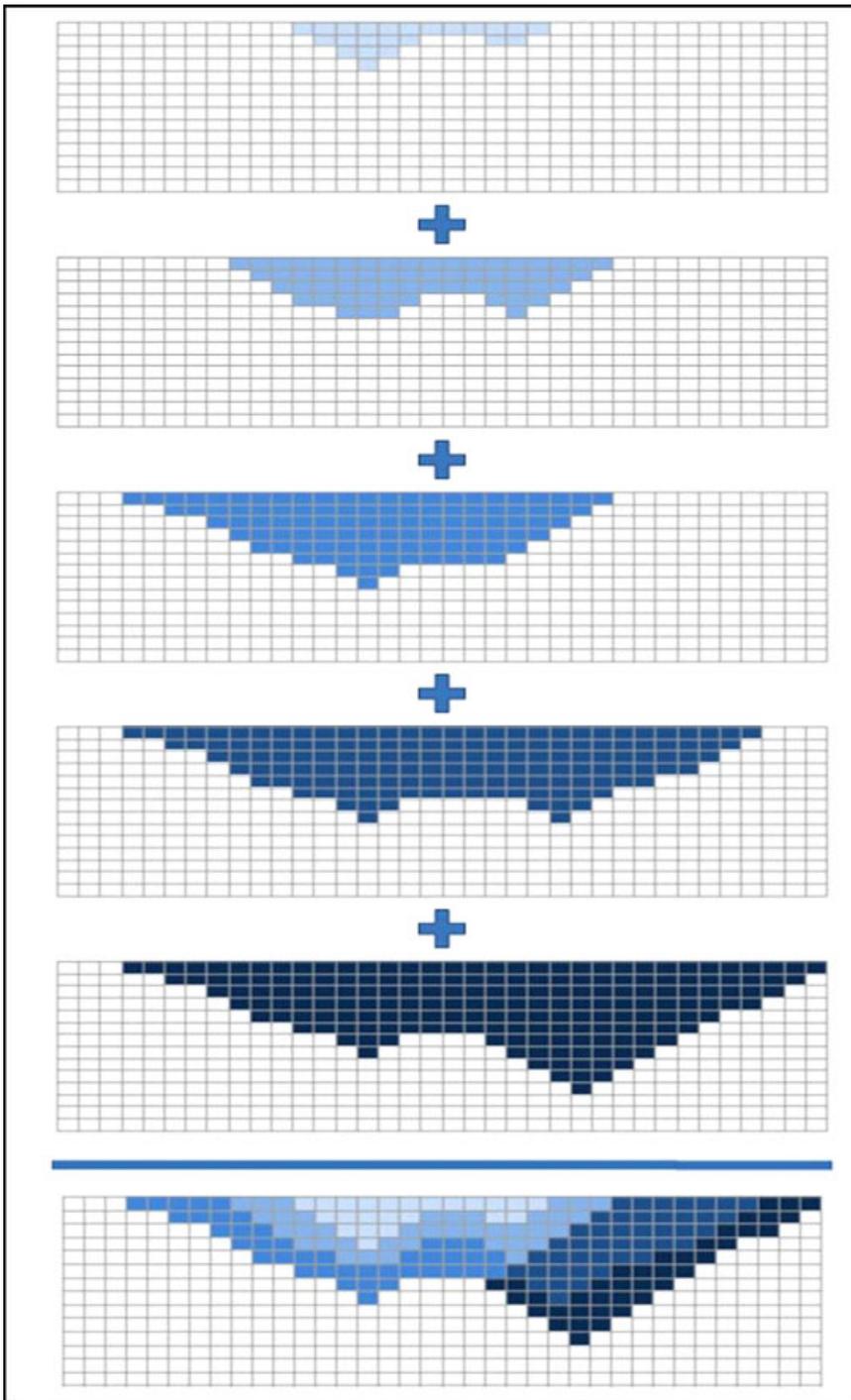


Fig. 6. Combination of generated pits to produce a mine schedule

from 59 to 64 and from 47 to 53 blocks per period respectively. The average allowed grades of Fe and SiO<sub>2</sub> supposed to be between 54 and 56 and between 9 and 11 percent respectively. Anything exceeding these limits has been considered to have 1 currency unit of penalty cost for each of the extra or fewer blocks. Consequently the value of the constructed initial scheduling solution received 79 currency units of penalty costs and its economic value dropped to 244 units.

#### 4.1. Comparison of ACO variants

The efficiencies of different ACO variants are tested on the hypothetical block model in order to find the best method and the most favourable parameter values. Table 2 shows the quality of the solutions provided by each variant of ACO as well as the time spent for computation on a PC.

Ant system (AS) is the simplest ACO system in which all ants have the ability to deposit pheromone proportional to the quality of their constructed schedule. The results showed that AS has the ability of improving the quality of the initial solution, however it does not reach to the higher solutions by reasonable computing resources.

The Elitist Ant System (EAS) was the second tested variant in which a strong emphasis is considered to the best-so-far solution and it is allowed to deposit as much pheromone as that of several normal ants. The elitist ant strategy has eliminated the scattering behaviour of the AS, but still it suffers from the early stagnation in local optimums.

TABLE 2

Comparison of different ACO variants

	<b>Fitness value of the best solution</b>	<b>Calculation time</b>
Ant System	265.13	50
Elitist Ant System	274.10	200
Ranked based Ant System	279.06	90
Max-Min Ant System	284.53	200
Wide Max-Min Ant System	294.55	270
Ant Colony System	279.00	60

Another variant was the rank based ant system ( $AS_{rank}$ ) in which only a series of ranked ants and the best-so-far ant are permitted to deposit pheromones. Study revealed that the ranking strategy increases the exploration potential and prevents the algorithm from stagnating even after a hundred iterations and reach to high quality solution. It suffers from the large amount of required memory.

Max-Min Ant System (MMAS) was one of the most effective variants. In MMAS only the iteration-best ant or the best-so-far ant is allowed to deposit pheromones. The pheromone trail values are also limited to a certain interval and are initialised to the upper pheromone trail limit. The pheromone evaporation rate is very small in this variant and pheromone trails are reinitialised each time the system approaches stagnation or when no improved tour has been generated for a certain number of consecutive iterations. The main power of MMAS comes from its explorative nature which lets the program use higher perturbation distances (wide MMAS) which may lead to better solutions. However this will take more calculation time and higher scattering iterations before primary improvements are noticed.

The other successful variant was the Ant Colony System (ACS). In  $q$  percentages of the cases ACS ants choose the node with the highest pheromone and heuristic information (selection probability) and for the rest of the cases ( $1-q$  percentage), it uses the same routine as AS for the selection. In ACS only the best-so-far ant is allowed to add pheromones after each iteration and the evaporation process only applies to the arcs of the best-so-far tour. The ants use also a local pheromone update rule in which they apply immediately after having crossed an arc during the tour construction of ACS. Results of using ACS have shown that the calculation time of each iteration has been drastically reduced because of the reduction in the number of ants. Another factor that helps the speed of the ACS algorithm is the fact that pheromone evaporation and deposition happen only on the arcs of the best so far solution. Consequently, when compared to the other variants of ACO, ACS could reach much better solutions in a given time of calculation which is very beneficial when dealing with the large block models.

## 5. Conclusion

The analysis revealed that the ACO is able to improve the value of the initial mining schedule generated by the Lerchs and Grossmann algorithm and parameterisation by up to 34 percent in some cases in a reasonable computational time. Despite the fact that this is mainly contributed to the consideration of the penalties to the deviations of the capacities and the production qualities from their permitted limits, the magnitude of the pure improvements was also considerable. It was also proved that the MMAS variant is the most explorative variant, while ACS is the fastest method. These two variants also count as the only variants which could be applied to a large block model in respect to the amount of memory needed.

The proposed ACO algorithm is able to consider any kind of objective functions in the optimisation process. Variable slope angles can be modelled with ease in the generated schedules. It is also possible to consider working slope angles by supposing different values for the slopes of the inner periods and the most outer phase. The calculation time of the algorithm is highly dependent on the number of mine schedules being constructed in each iteration which is usually considered equal to the number of block columns in the model. In other words the calculation time is not very sensitive to the size of the model and for a double sized block model in each direction (eight times more blocks) is expected to be only four times longer. The memory usage in MMAS and ACS where only the best schedule needs to be saved during calculations is very low. For a block model with one million blocks, the capacity of 4MB will be sufficient for the MMAS and ACS variants, for example.

On the other hand there are drawbacks related to the process. At the outset, the process is not mathematically proven to always reach the best schedule. Moreover the efficiency of the ACO algorithm is highly dependent on the parameters like number of ants, evaporation rates and deposited pheromones in each iteration. The found combinations of these parameters for this case study are not essentially the best combination for all deposits and block models. Hence a trial and error process might be necessary at the beginning to set the relevant combination of parameters for each individual case. Finally it is necessary to consider a relatively small perturbation distance during construction of the initial schedule in order to control the size of generated push backs. This lets the constructed solutions to be very close to the initial solution. Therefore the optimum solution will not always be reachable by all variants of the ACO if it would be far from the initial solution.

## REFERENCES

- Azimi Y., Osanloo M., 2011. *Determination of open pit mining cut-off grade strategy using combination of nonlinear programming and genetic algorithm*. Arch. Min. Sci., Vol. 56, No 2, p. 189-212.
- Caccetta L., Hill S.P., 2003. *An application of branch and cut to open pit mine scheduling*. Journal of Global Optimisation, 27, p. 349-365.
- Caccetta L., Kelsey P., Giannini L., 1998. *Open pit mine production scheduling*, in Proceedings 3<sup>rd</sup> Regional APCOM Symposium, p. 65-72 (The Australasian Institute of Mining and Metallurgy: Kalgoorlie).
- Dagdelen K., Johnson T.B., 1986. *Optimum open pit mine production scheduling by Lagrangian parameterization*, in Proceedings Application of Computer in Mineral Industry, p. 127-142, (Littleton, Colorado)
- Denby B., Schofield D., 1994. *Open pit design and scheduling by use of genetic algorithms*. Trans. Inst. Min. Metall. Sec. A: Min. Industry, 103, A21-A26.
- Denby B., Schofield D., 1995. *Inclusion of risk assessment in open pit design and scheduling*. Trans. Inst. Min. Metall. Sec. A: Min. Industry, 104, A67-A71.
- Denby B., Schofield D., 1996. *Genetic algorithms for open pit scheduling-extension into 3-dimensions*, in Proceedings Mine Planning and Equipment Selection Conference, Sao Paulo, Brazil, p. 177-185.
- Denby B., Schofield D., Surme T., 1998. *Genetic algorithms for flexible scheduling of open pit operations*, in Proceedings 27<sup>th</sup> Application of Computer in Mineral Industry, p. 473-483, (London).
- Dorigo M., Stützle T., 2004. *Ant Colony Optimisation*, p. 67-113, (A Bradford Book).
- Dowd P.A., Onur A.H., 1992. *Optimising open pit design and sequencing*, in Proceedings 23<sup>rd</sup> Application of Computer in Mineral Industry, p. 411-422, (Tucson, Arizona).
- Kumral, M and Dowd, P A, 2002. *Short-term mine production scheduling for industrial minerals using multi-objective simulated annealing*, in Proceedings 30<sup>th</sup> Application of Computer in Mineral Industry, p. 731-742, (Fairbanks, Alaska).
- Kumral M., Dowd P.A., 2005. *A simulated annealing approach to mine production scheduling*. Journal of the Operational Research Society, 56, p. 922-930.
- Lerchs H., Grossmann I.F., 1965. *Optimum design of open pit mines*. Canadian Institute of Mining Transactions, 68, p. 17-24.
- Onur A.H., Dowd P.A., 1993. *Open pit optimisation-part 2: production scheduling and inclusion of roadways*. Trans. Inst. Min. Metall. Sec. A: Min. Industry, 102, p. A105-A113.
- Osanloo M., Gholamnejad J., Karimi B., 2008. *Long-term open pit mine production planning: a review of models and algorithms*. International Journal of Mining, Reclamation and Environment, Vol. 221, p. 3-35.
- Ramazan S., Dagdelen K., Johnson T.B., 2005. *Fundamental tree algorithm in optimising production scheduling for open pit mine design*. Trans. Inst. Min. Metall. Sec. A: Mining Technol., 114, p. A45-A114.
- Ramazan S., Dimitrakopoulos R., 2004. *Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation*, in Proceedings Orebody Modelling and Strategic Mine Planning Spectrum Series, Vol. 14, p. 359-365.
- Sattarvand J., 2009. *Long-term open-pit planning by ant colony optimisation*. PhD thesis, RWTH Aachen University – Germany.
- Sayadi A., Fathianpour N., Mousavi A.A., 2011. *Open pit optimization in 3D using a new artificial neural network*. Arch. Min. Sci., Vol. 56, No 3, p. 389-403.
- Wang Q., Sevim H., 1995. *Alternative to parameterisation in finding a series of maximum-metal pits for production planning*. Mining Engineering, p. 178-182.