# STRESS AND STRAIN MEASUREMENTS IN STATIC TENSILE TESTS 

Stanislaw Adamczak, Jerzy Bochnia, Czeslaw Kundera<br>Kielce University of Technology, Chair of Mechanical Technology and Metrology, Al. 1000-lecia P. P. 7, 25-314 Kielce, Poland, ( $\boxtimes$ adamczak@tu.kielce.pl, +48413424477, jbochnia@tu.kielce.pl, kundera@tu.kielce.pl )


#### Abstract

The paper deals with the accuracy of measurements of strains (elongation and necking) and stresses (tensile strength) in static room-temperature tensile strength tests. We present methods for calculating measurement errors and uncertainties, and discuss the determination of the limiting errors of the quantities measured for circular and rectangular specimens, which is illustrated with examples.


Keywords: static tensile strength test, measurement error, measurement uncertainty.

## 1. Introduction

The static tensile test performed at ambient temperature is one of the fundamental standardized methods used for determining the mechanical properties of metals and metal alloys, which enables the comparison of metallic materials, the classification of their applications and the evaluation of the efficiency of the material production process. The test involves fixing specimens in grips and stretching them uniaxially, generally until they break, by increasing stress at a predetermined rate, with the shape of specimens being defined by the appropriate standard. The tensile force or tensile stresses are registered in the function of specimen elongation. The mechanical properties of the material tested are determined from the stress-strain curve and the geometrical parameters of the specimens measured before and after fracture by using the guidelines and equations provided in the standard document.

The EN ISO 6892-1:2009 standard specifies the method for tensile testing of metallic materials and defines the mechanical properties that can be determined at room temperature $[1,2]$. The uncertainty of measurement results for parameters such as tensile strength $R_{m}$ or yield strength $R_{e}$ can be estimated using:

- one specimen or a series of specimens sampled from the same piece of material, e.g. a rod,
- a series of specimens sampled for one type of material but different batches and different semi-finished products.
In the second case, we can expect higher values of measurement uncertainty, as described in [3]. For specimens sampled from one rod, the repeatability of the yield strength $\mathrm{R}_{\mathrm{e}}$ was $1 \%$. For specimens made of the same type of material but sampled from two hundred different rods, the repeatability was $4 \%$, which was mainly due to material variety. Reference [3] describes an experiment conducted for five different materials, i.e. two ferritic steels, one austenitic steel and two nickel base alloys. The uncertainties of measurement performed under the same conditions for the same number of specimens ranged from $2.3 \%$ to $4.6 \%$. Reference [4] describes the general procedures for the evaluation of uncertainty of
measurement results obtained during a tensile strength test, the typical sources of uncertainty and their probable influence on the final results for cold-rolled steel.

This paper will include examples of calculations used for the evaluation of errors and uncertainties of measurements of strains and tensile strength obtained in static tensile strength tests for one specimen or a small series of specimens. These examples can supplement the current methods of analysis of errors in the measurement of stresses and strains. The standard used for the evaluation of measurement uncertainty in materials metrology does not solve all the problems that might arise in this area.

## 2. Stress and strain measurement errors

Numerical errors are assumed to fall into two categories:

- an absolute error, defined as a measurement result minus the true value of the measured quantity [5],
- a relative error determined by the ratio of the absolute error to the true value of the measured quantity.
An absolute error is expressed in the units of the measured quantity. However, one must not confuse 'an absolute error', which may have a positive or a negative sign, with 'an absolute value of the error', being the error modulus.

By using systematic limiting errors with an unknown sign, called apparatus errors, we can determine the interval containing the unknown value of the measured quantity. These errors result from the imperfection of the apparatus used in a measurement. The measure of imperfection is the class of accuracy defined for a given instrument. The numerical indicator of the instrument accuracy class $\delta_{\mathrm{kl}}$ [6] defines the limiting value of the absolute error $\pm \Delta_{\mathrm{g}}$ expressed in percentage of the apparent value $\mathrm{W}_{\mathrm{u}}$, which can be the nominal value (length scale) $\mathrm{W}_{\mathrm{n}}$ or the maximum value (gauge range) $\mathrm{W}_{\text {max }}$. If the apparent value is equal to the measured value, then the error determined by the indicator of the class is the measurement error.

$$
\begin{equation*}
\delta_{p}=\delta_{k l}=\frac{ \pm \Delta_{g}}{W_{n}} \times 100 \% \tag{1}
\end{equation*}
$$

In the case of indirect measurements, for example, measurements of stresses in a tensile strength test, when the quantity measured is a function of many variables $y=f\left(x_{i}\right)$ and $\mathrm{i}=1,2, \ldots ., \mathrm{n}$, the absolute and relative limiting errors are determined by means of the following relationships:

$$
\begin{equation*}
\Delta y_{g}=\sum_{i=1}^{n}\left|\frac{\partial f}{\partial x_{i}} \Delta x_{g i}\right| \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta y_{g}=\sum_{i=1}^{n}\left|\frac{\partial f}{\partial x_{i}} \frac{x_{i}}{y} \delta x_{g i}\right|, \tag{3}
\end{equation*}
$$

respectively.
The maximum error of the tensile strength $R_{m}$ of one specimen with a circular crosssectional area $S_{0}$ is calculated in the way described below. For example, if the maximum force
acting on the specimen $\mathrm{F}_{\mathrm{m}}$ is 10 kN and the specimen diameter before tension $\mathrm{d}_{0}$ is 4 mm , then the tensile strength is: $\mathrm{R}_{\mathrm{m}}=\mathrm{F}_{\mathrm{m}} / \mathrm{S}_{0}=4 \mathrm{~F}_{\mathrm{m}} / \pi \mathrm{d}_{0}{ }^{2} \rightarrow \mathrm{R}_{\mathrm{m}}=796.2 \mathrm{MPa}$.

The absolute maximum error $\Delta y_{g}=\Delta R_{m}$ determined from (2) is:

$$
\begin{equation*}
\Delta R_{m}= \pm\left[\left|\frac{\partial R_{m}}{\partial F} \Delta F\right|+\left|\frac{\partial R_{m}}{\partial d_{0}} \Delta d_{0}\right|\right]= \pm\left[\frac{4}{\Pi d_{0}^{2}} \Delta F+\frac{8 F}{\Pi d_{0}^{3}} \Delta d_{0}\right], \tag{4}
\end{equation*}
$$

where:
$\Delta \mathrm{F}$ is the limiting error of the measured value of the force resulting from the class of accuracy of the measurement apparatus, i.e. the universal testing machine, and that of the cross-head used in the test, thus $\Delta \mathrm{F}=50 \mathrm{~N}=0.05 \mathrm{kN}$, and
$\Delta \mathrm{d}_{0}$ is the limiting error of the measured value of the specimen diameter corresponding to the smallest graduation on the scale of the measuring instrument, which, in this case, is a micrometer; thus $\Delta \mathrm{d}_{0}=0.01 \mathrm{~mm}$.

After substituting the measured values of the force and the specimen diameter and their limiting errors to (4), we obtain the value of the absolute limiting error of the tensile strength: $\Delta \mathrm{R}_{\mathrm{m}}= \pm 7.96 \mathrm{MPa}$.

The absolute maximum error of $\mathrm{R}_{\mathrm{m}}$ determined from (3) is:

$$
\begin{equation*}
\delta y_{g}=\delta R_{m}= \pm\left[\left|\frac{\partial R_{m}}{\partial F} \frac{F_{m}}{R_{m}} \delta F_{m}\right|+\left|\frac{\partial R_{m}}{\partial d_{0}} \frac{d_{0}}{R_{m}} \delta d_{o}\right|\right]= \pm\left[\frac{4}{\Pi d_{0}^{2}} \frac{F_{m}}{R_{m}} \delta F_{m}+\frac{8 F_{m}}{\Pi d_{0}^{3}} \frac{d_{0}}{R_{m}} \delta d_{0}\right] \tag{5}
\end{equation*}
$$

and, after transformation, we have:

$$
\begin{equation*}
\delta R_{m}= \pm\left[\delta F_{m}+2 \delta d_{0}\right] . \tag{6}
\end{equation*}
$$

The limiting error of the measured force depends on the class of accuracy of the cross-head $\delta F_{m}=0.5 \%$, whereas the limiting error of the measured diameter of the specimen is $\delta d_{0}=0.25 \%$, according to (3). Thus, the relative maximal error of the tensile strength in the example considered here is: $\delta R_{m}= \pm[0.5+2 \cdot 0.25] \%=1 \%$.

In the case of rectangular specimens, the cross-sectional area before fracture $S_{0}$ is calculated from results being the arithmetic mean of measurements of the appropriate dimensions:

$$
\begin{equation*}
S_{0}=\bar{a}_{0} \bar{b}_{0}, \tag{7}
\end{equation*}
$$

where: $\bar{a}_{0}$ - the mean thickness of a flat specimen, $\bar{b}_{0}$ - the mean width of a flat specimen.
Thus, the absolute maximum error of stress $\mathrm{R}=\mathrm{F} / \mathrm{S}_{0}$ for rectangular specimens is:

$$
\begin{equation*}
\Delta R= \pm\left[\left|\frac{\partial R}{\partial F} \Delta F\right|+\left|\frac{\partial R}{\partial a_{0}} \Delta a_{0}\right|+\left|\frac{\partial R}{\partial b_{0}} \Delta b_{0}\right|\right]= \pm\left[\frac{1}{\bar{a}_{0} \bar{b}_{0}} \Delta F+\frac{F}{\bar{a}_{0}^{2} \bar{b}_{0}} \Delta a_{0}+\frac{F}{\bar{a}_{0} \bar{b}_{0}^{2}} \Delta b_{0}\right], \tag{8}
\end{equation*}
$$

where: $\Delta F$ - the limiting error of the measured value of the force, $\Delta a_{0}, \Delta b_{0}$ - the limiting errors of the specimen cross-sectional dimensions.

In the case of tubular specimens, the cross-sectional area before fracture $S_{0}$ is calculated using the results being the arithmetic mean of measurements of the appropriate dimensions:

$$
\begin{equation*}
S_{0}=\pi \bar{a}\left(\bar{D}_{0}-\bar{a}\right) \tag{9}
\end{equation*}
$$

where: $\bar{a}$ - the mean thickness of a tubular specimen, $\bar{D}_{0}$ - the mean outer diameter of a tubular specimen.

Thus, the absolute maximum error of stress $R=F / S_{0}$ for tubular specimens is:

$$
\begin{equation*}
\Delta R= \pm\left[\frac{1}{\pi \bar{a}\left(\bar{D}_{0}-\bar{a}\right)} \Delta F+\frac{F\left(\bar{D}_{0}-2 \bar{a}\right)}{\pi \bar{a}^{2}\left(\bar{D}_{0}-\bar{a}\right)^{2}} \Delta a+\frac{F}{\pi \bar{a}\left(\bar{D}_{0}-\bar{a}\right)^{2}} \Delta D_{0}\right], \tag{10}
\end{equation*}
$$

where: $\Delta F$ - the limiting error of the measured value of the force, $\Delta a, \Delta D_{0}$ - the limiting errors of the dimensions of the specimen cross-section.

When the data is read off the universal testing machine monitor, the value of the maximum error depends on the amount of force applied and the class of accuracy defined for the crossheads. The value of the maximum error for stress read directly off a printed plot (which refers to a graphical method of determination of apparent stresses) is: $\Delta \mathrm{R}=7 \mathrm{MPa}$ - measurement with a ruler where $\Delta \mathrm{L}=1 \mathrm{~mm}$ or $\Delta \mathrm{R}=3.5 \mathrm{MPa}$ - measurement with a caliper where $\Delta \mathrm{L}=0.05 \mathrm{~mm}$.

The percentage elongation after fracture is:

$$
\begin{equation*}
A=\frac{L_{u}-L_{0}}{L_{0}} \cdot 100 \%, \tag{11}
\end{equation*}
$$

where: $L_{0}$ - the original gauge length of a specimen, $L_{u}$ - the gauge length of a specimen after fracture.

The maximum error of the percentage elongation after fracture is:

$$
\begin{equation*}
\Delta A= \pm\left|\frac{\partial A}{\partial L_{0}} \Delta L_{0}+\frac{\partial A}{\partial L_{1}} \Delta L_{1}\right|= \pm\left[\left|\frac{-L_{u}}{L_{0}^{2}} \Delta L_{0}\right|+\left|\frac{1}{L_{0}} \Delta L_{u}\right|\right] \times 100 \% . \tag{12}
\end{equation*}
$$

For example, the maximum error of the percentage elongation A calculated from (12) was determined for the original gauge length $L_{0}=45 \mathrm{~mm}$ and the gauge length after fracture $L_{u}=55 \mathrm{~mm}$ of one rectangular specimen. For $\Delta L_{0}=\Delta L_{u}=0.1 \mathrm{~mm}$, the limiting error $\Delta A$ is $\pm 0.28 \%$.

The measurement error for each dimension of the specimen cross-section should not exceed $\pm 0.5 \%$. The cross-sectional area $S_{u}$ of a flat specimen after fracture at the point of greatest necking is calculated from the equation:

$$
\begin{equation*}
S_{u}=0.25\left(a_{u}+a_{u}{ }^{\prime}\right)\left(b_{u}+b_{u}{ }^{\prime}\right), \tag{13}
\end{equation*}
$$

where: $a_{u}$ and $a_{u}{ }^{\prime}$ - the maximum and minimum thickness of the specimen at fracture point, respectively, $b_{u}$ and $b_{u}{ }^{\prime}$ - the maximum and minimum width of the specimen at fracture point, respectively (Fig. 1).


Fig. 1. Dimensions of the rectangular specimen at fracture point; $a_{0}$ - thickness of the specimen before fracture; $b_{0}$ - width of the specimen before fracture; $a_{u}$ and $a_{u}{ }^{\prime}$ - the maximum and minimum thickness of the specimen at fracture point, respectively; $b_{u}$ and $b_{u}{ }^{\prime}$ - the maximum and minimum width of the specimen at fracture point, respectively.

The accuracy of measurement of the quantities $a_{0}, \bar{a}_{0}, a_{u}$ and $a_{u}{ }^{\prime}$ and $b_{0}, \bar{b}_{0}, b_{u}$ and $b_{u}{ }^{\prime}$ is:
-0.1 mm if $a_{0}$ and $b_{0}>20 \mathrm{~mm}$,

- 0.05 mm if $10 \leq a_{0}$ and $b_{0} \leq 20 \mathrm{~mm}$,
- 0.02 mm if $4 \leq a_{0}$ and $b_{0}<10 \mathrm{~mm}$,
- 0.01 mm if $2 \leq a_{0}$ and $b_{0}<4 \mathrm{~mm}$,
-0.005 mm for $a_{0}, \bar{a}_{0}, a_{u}$ and $a_{u}{ }^{\prime}$ if $a_{0}<2 \mathrm{~mm}$.
The maximum error of the percentage necking $Z$ of a specimen with a rectangular crosssection is determined as follows:

$$
\begin{equation*}
Z=\frac{S_{u}-S_{0}}{S_{0}} \times 100 \% ; \quad S_{u}=a_{u s} \times b_{u s} ; \quad a_{u s}=\frac{a_{u}+a_{u}{ }^{\prime}}{2} ; \quad b_{u s}=\frac{b_{u}+b_{u}{ }^{\prime}}{2} . \tag{14}
\end{equation*}
$$

$S_{0}$ calculated from (7). The necking is:

$$
\begin{equation*}
Z=\frac{a_{u s} b_{u s}-\bar{a}_{0} \bar{b}_{0}}{\bar{a}_{0} \bar{b}_{0}} \times 100 \%, \tag{15}
\end{equation*}
$$

whereas the maximum error is:

$$
\begin{align*}
& \Delta Z= \pm\left[\frac{\partial Z}{\partial a_{o}} \Delta a_{o}+\frac{\partial Z}{\partial b_{o}} \Delta b_{o}+\frac{\partial Z}{\partial a_{u s}} \Delta a_{u s}+\frac{\partial Z}{\partial b_{u s}} \Delta b_{u s}\right] \times 100 \%=  \tag{16}\\
& = \pm\left[\left|\frac{-\bar{b}_{0} a_{u s} b_{u s}}{\bar{a}_{0}^{2} \bar{b}_{0}^{2}}\right| \Delta a_{o}+\left|\frac{-\bar{a}_{0} a_{u s} b_{u s}}{\bar{a}_{0}^{2} \bar{b}_{0}^{2}}\right| \Delta b_{o}+\left|\frac{b_{u s}}{\bar{a}_{0}^{2} \bar{b}_{0}^{2}}\right| \Delta a_{u s}+\left|\frac{a_{u s}}{\bar{a}_{0}^{2} \bar{b}_{o}^{2}}\right| \Delta b_{u s}\right] \times 100 \%,
\end{align*}
$$

where: $\Delta a_{u}=\Delta a_{u}{ }^{\prime}=\Delta b_{u}=\Delta b_{u}{ }^{\prime}=0.01 \mathrm{~mm} ; \Delta a_{0}=\Delta b_{0}=0.01 \mathrm{~mm}$.

$$
\begin{align*}
\Delta a_{u s} & = \pm\left[\frac{\partial a_{u s}}{\partial a_{u}} \Delta a_{u}+\frac{\partial a_{u s}}{\partial a_{u}^{\prime}} \Delta a_{u}{ }^{\prime}\right]= \pm\left[\frac{1}{2} \Delta a_{u}+\frac{1}{2} \Delta a_{u}{ }^{\prime}\right]= \pm 0.01, \\
\Delta b_{u s} & = \pm\left[\frac{\partial b_{u s}}{\partial b_{u}} \Delta b_{u}+\frac{\partial b_{u s}}{\partial b_{u}^{\prime}} \Delta b_{u}^{\prime}\right]= \pm\left[\frac{1}{2} \Delta b_{u}+\frac{1}{2} \Delta b_{u}^{\prime}\right]= \pm 0.01 . \tag{17}
\end{align*}
$$

For example, the absolute maximum error $\Delta Z$ of the percentage necking $Z$ calculated from (16) was determined on the basis of the measurement of parameters $a_{0}$ and $b_{0}$ of the specimen cross-sectional area before fracture and the parameters $a_{u}$ and $b_{u}$ of the specimen cross-sectional area after fracture. For $\bar{a}_{0}=6 \mathrm{~mm}, \bar{b}_{0}=10 \mathrm{~mm}, \bar{a}_{u}=4 \mathrm{~mm}, \bar{b}_{u}=7 \mathrm{~mm}$, and $\Delta a_{0}=\Delta b_{0}=\Delta \bar{a}_{u}=\Delta \bar{b}_{u}=0.01 \mathrm{~mm}$, the error $\Delta Z$ was $0.31 \%$.

## 3. Uncertainty of stress and strain measurement

Measurement uncertainty $u$ is a parameter that can be used to determine the limits of an interval containing, with an assumed probability, the unknown true value of the measured quantity. Measurement uncertainty is affected by many components of uncertainty, which can be determined by evaluating the estimators of standard deviations for the dispersion of results obtained in a series of measurements. In the case of a single measurement, the components of uncertainty are specified by means of standard deviations based on the forecast probability distributions.

There are two types of uncertainty components [6-10]:

- components obtained by a type A evaluation, which is performed statistically,
- components obtained by a type B evaluation, which is conducted with other methods.

Measurement results contain type A and type B uncertainties. Their values can be comparable or one dominating considerably over the other. If the predominant uncertainty is:

- the type A standard uncertainty, then it is necessary to determine the type A combined uncertainty,
- the type B standard uncertainty, then it is necessary to determine the type B combined uncertainty.

When the two uncertainties have similar values, the combined uncertainty will be of the AB type. Type A and type B uncertainties are obtained while determining mechanical properties of materials.

In the case of direct measurements, the type A standard uncertainty is determined on the basis of results of a series of measurements:

$$
\begin{equation*}
u_{A}=\bar{S}_{\bar{x}}=\sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}, \tag{18}
\end{equation*}
$$

where:
$x_{i}$ - value of the i-th measurement, $\bar{x}$ - mean value, $n$ - number of measurements.
The expanded uncertainty $u_{c}$ for determining the limits of the confidence interval is:

$$
\begin{equation*}
u_{c}=k_{\alpha} \cdot u_{A}=k_{A}(\alpha) \cdot u_{A} \tag{19}
\end{equation*}
$$

with coverage factor $\mathrm{k}_{\mathrm{A}}(\alpha)$ assuming the values of:

- a standardized variable Z if the distribution of the random variable X is the normal distribution with a known standard deviation $\sigma$; when the sample size is large ( $n>30$ ),
- a standardized variable of the Student's $t$ distribution if the distribution of the random variable X is the normal distribution with unknown parameters; when the sample size is small ( $\mathrm{n}<30$ ), the value is obtained from the Student's $t$ distribution tables for the predetermined confidence level $\alpha$ and for the number of degrees of freedom $m$ equal to $\mathrm{n}-1$.
When the mechanical properties of a material are determined through a direct measurement, we can use the value of the force at a certain characteristic point of the plot, the so-called local peak or the magnitude of displacement (specimen elongation). When a test is conducted with an extensometer, the error of a single measurement of the value of strain displayed by the extensometer corresponds to the limiting error of this instrument. Generally, an extensometer is used to measure small displacements. If a test is performed without an extensometer, the result of the strain measurement contains a systematic error resulting from the elastic deformation of the components of the universal testing machine. This systematic error should be taken into account in the form of a correction. When strains are large, it is omitted as a negligible error.

In the case of indirect measurements, for instance, stress measurements, the type A uncertainty is evaluated using the results of a series of measurements performed separately for each quantity. It is necessary to determine the mean values $\bar{x}_{i}$ and the standard uncertainty $u_{A}$. The standard uncertainty for the mean $\bar{y}$ is calculated from the formula:

$$
\begin{equation*}
u_{A \bar{y}}=\sqrt{\sum_{i=1}^{n}\left(\frac{\partial y}{\partial \bar{x}_{i}}\right)^{2} \cdot u_{A i}^{2}} . \tag{20}
\end{equation*}
$$

For the mean value $\bar{y}$ of the quantity $Y$ measured in an indirect way and its standard uncertainty $u_{A \bar{y}}$, the expanded uncertainty is:

$$
\begin{equation*}
u_{A c}=k_{A}(\alpha) \cdot u_{A \bar{y}} . \tag{2}
\end{equation*}
$$

When the sample size used for determining the quantity $X_{i}$ is small ( $n<30$ ), the distribution of the mean $\overline{\mathrm{X}}$ is better approximated by a Student's $t$ distribution with effective degrees of freedom $\mathrm{m}_{\mathrm{e}}$.

$$
\begin{equation*}
m_{e}=\frac{u_{A \bar{y}}^{4}}{\sum \frac{1}{n_{i}} \cdot\left(\frac{\partial y}{\partial \bar{x}_{i}}\right)^{4} \cdot u_{A i}^{4}}<\sum_{i=1}^{N} n_{i} . \tag{22}
\end{equation*}
$$

Thus, the expanded uncertainty is:

$$
\begin{equation*}
u_{A c}=t_{q m e} \cdot u_{A \bar{y}}, \tag{23}
\end{equation*}
$$

where:
$t_{q m e}$ - the standardized variable from the Student's $t$ distribution tables.
The measurement uncertainty of the tensile strength $R_{m}$ was evaluated by performing a tensile test for five circular specimens with a nominal diameter d of 5 mm . The evaluation required measuring the specimens before the tension test and calculating the mean values of the diameters $\mathrm{d}_{0 \text { str }}$.

The results of the measurements of the cylindrical samples before stretching are presented in Table 1 where: $d_{0 i}$ - the values of the measured diameters, $\overline{\mathrm{d}}_{0}$ - average value of the diameter, $\Delta \mathrm{d}_{0 \mathrm{i}}$ - difference between the average value and the i -th measurement, $\mathrm{u}_{\mathrm{Ad} 0}$ uncertainty of the measurement of the sample diameter at the base (18).

Standard uncertainty of type B introduced by the measuring tool is: $u_{\text {Bd0 }}=\Delta \mathrm{d}_{0} / \sqrt{3}=$ $0.01 / \sqrt{3}=0.006 \mathrm{~mm}$. Comparing the value $\mathrm{u}_{\mathrm{Bd} 0}$ to the obtained uncertainties $\mathrm{u}_{\mathrm{Ad} 0}$ (Table 1) one can state that the uncertainty of type A and B is not dominant. In such case the combined uncertainty is calculated.

$$
\begin{equation*}
u_{(d 0)}=\sqrt{u_{A d O}^{2}+u_{B d O}^{2}} . \tag{24}
\end{equation*}
$$

The results of the calculated uncertainties are presented in Table 1.
Table 1. Values of the specimen diameter and the maximum force and results of uncertainties.

| Specimen <br> number | $\mathrm{d}_{0 \mathrm{i}}$ <br> $[\mathrm{mm}]$ | $\overline{\mathrm{d}}_{0}$ <br> $[\mathrm{~mm}]$ | $\sum \Delta \mathrm{d}_{0 \mathrm{i}}{ }^{2}$ <br> $\left[\mathrm{~mm}^{2}\right]$ | $\mathrm{u}_{\text {Ad0 }}$ <br> $[\mathrm{mm}]$ | $\mathrm{u}_{\mathrm{Bd} 0}$ <br> $[\mathrm{~mm}]$ | $\mathrm{u}_{(\mathrm{d} 0)}$ <br> $[\mathrm{mm}]$ | $\mathrm{F}_{\mathrm{i}}$ <br> $[\mathrm{N}]$ | $\left(\mathrm{F}_{\mathrm{i}}-\overline{\mathbf{F}}\right)^{2}$ <br> $\left[\mathrm{~N}^{2}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $5.02 ; 5.02 ; 5.04 ; 5.04 ; 5.06 ; 5.06$ | 5.04 | 0.0016 | 0.0074 | 0.006 | 0.0095 | 13460 | 4624 |
| 2 | $5.04 ; 5.05 ; 5.05 ; 5.05 ; 5.06 ; 5.06$ | 5.05 | 0.0003 | 0.0032 | 0.006 | 0.0068 | 13540 | 144 |
| 3 | $5.06 ; 5.05 ; 5.05 ; 5.05 ; 5.04 ; 5.03$ | 5.05 | 0.0006 | 0.0045 | 0.006 | 0.0075 | 13510 | 324 |
| 4 | $5.04 ; 5.05 ; 5.06 ; 5.06 ; 5.07 ; 5.06$ | 5.06 | 0.0006 | 0.0045 | 0.006 | 0.0075 | 13660 | 17424 |
| 5 | $5.08 ; 5.07 ; 5.05 ; 5.07 ; 5.04 ; 5.04$ | 5.06 | 0.0015 | 0.0071 | 0.006 | 0.0092 | 13470 | 3364 |
| $\Sigma$ |  | 25.26 |  |  |  |  | 67640 | 25880 |
| $\overline{\mathrm{X}}$ |  | 5.05 |  |  |  |  | 13528 |  |

The data used for the calculation of the mean tensile strength $\overline{\mathrm{R}}_{\mathrm{m}}$ and the uncertainty of its measurement is provided in Table 1.
The mean tensile strength calculated from the data in Table 1 is: $\overline{\mathrm{R}}_{\mathrm{m}}=4 \overline{\mathrm{~F}} / \pi \overline{\mathrm{d}}_{0 \text { srr }}^{2}=675 \mathrm{MPa}$.
The uncertainty calculated from (20) is:

$$
\begin{equation*}
u_{\bar{R} m}=\sqrt{\left(\frac{\partial \bar{R}_{m}}{\partial \bar{d}_{0}}\right)^{2} u_{\left(\bar{d}_{0}\right)}^{2}+\left(\frac{\partial \bar{R}_{m}}{\partial \bar{F}}\right)^{2} u_{(\bar{F})}^{2}}=\sqrt{\left(\frac{8 \bar{F}}{\pi \bar{d}_{0}^{3}}\right)^{2} u_{\left(\bar{d}_{0}\right)}^{2}+\left(\frac{4}{\pi \bar{d}_{0}^{2}}\right)^{2} u_{(\bar{F})}^{2}}, \tag{25}
\end{equation*}
$$

where:
$\bar{R}_{m}, \bar{d}_{0}$ and $\bar{F}$ - the mean values of the tensile strength, the specimen diameter and the maximum force, respectively (Table 1), $u_{\left(\bar{d}_{0}\right)}$ - the combined uncertainty of measurement of the specimen diameter $\left(u_{\left(\bar{d}_{0}\right)}=0.0095\right.$ - Table 1), $u_{(\bar{F})}$ - the combined uncertainty of measurement of the maximum force $\left(u_{A \bar{F}}=34[N], u_{B \bar{F}}=39[N] ; u_{(\bar{F})}=\sqrt{u_{A \bar{F}}^{2}+u_{B \bar{F}}^{2}}\right.$, $\left.u_{(\bar{F})}=51.7 \mathrm{MPa}\right)$.

The standard uncertainty calculated from (25) is $u_{\bar{R} m}=11.6 \mathrm{MPa}$.
Thus, the measurement result of the tensile strength for the analyzed series of circular specimens can be written as $R_{m}=675 \mathrm{MPa} \pm 11.6 \mathrm{MPa}$.

## 4. Conclusion

The paper has discussed problems related to the accuracy of measurement of strains (e.g. elongation and necking) and stresses (e.g. tensile strength) during static tensile testing of metallic materials performed at room temperature. It has also described the methods used for the calculation of errors and the determination of measurement uncertainty. The analysis has focused on the determination of the limiting errors of quantities obtained in tensile tests for rectangular specimens because the number of semi-finished products with rectangular crosssections is constantly increasing.

Evaluation of errors is essential to the determination of measurement uncertainty. The calculation examples provided above suggest that it is necessary to apply state-of-the-art highaccuracy measuring equipment to ensure as low a value of measurement uncertainty as possible. The accuracy of specimen production is also of great importance as it affects the test quality, the measurement results and, accordingly, the measurement uncertainties. In the example described above, the measurement uncertainties obtained during tensile strength testing were evaluated using specimens made with such accuracy that the uncertainty of measurement of the working diameters was smaller than the limiting error of the measuring instrument, i.e. the micrometer. This guarantees correct measurement results because tensile strength is an apparent stress whose value is the ratio of the maximum force to the crosssectional area of the working part before the specimen is subjected to load.

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