SUPPLY LEAD TIME UNCERTAINTY IN A SUSTAINABLE ORDER QUANTITY INVENTORY MODEL

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ABSTRACT
Transport plays a key role in inventory management since it affects logistic costs as well as environmental performance of the supply chain. Expected value and variability of supply lead time depend on the transportation means adopted, and influence the optimal values of order quantity, reorder level, and safety stock to be adopted. Fast transportation means allow reducing expected value of the lead time; they are characterized by the highest costs of externalities (i.e. air pollutant emission, noise, congestion, accidents). On the contrary, slow transportation means require high inventory level due to large order quantity; in this case costs of externalities tend to decrease. The Sustainable Order Quantity (SOQ) [1] allows identifying optimal order quantity, reorder level, safety stock as well as transportation means which minimize the sum of the logistic and environmental costs in case of stochastic variability of product demand. In this paper, the authors propose a new SOQ analytical model considering stochastic variability of supply lead time (LT). A solution procedure is suggested for solving the proposed model. The approach is applied to a real industrial case study in order to evaluate the benefits of applying it if compared with the traditional one.

KEYWORDS
inventory management, SOQ, loss factor of transport, stochastic variability of supply lead time.

Introduction

The Economic Order Quantity (EOQ) model [2] is one of the most investigated inventory management models aiming at identifying the lot size of a given inventory item as a trade-off between holding and order costs. A wide scientific literature is available which has been devoted to include other cost figures in the optimization function or to remove restrictive hypotheses.

A joint lot sizing and inspection policy problem under an EOQ model is investigated in [3], where the replacing of a random quantity of defective items is considered. A continuous inventory model in case of imperfect quality items is developed in [4]; in the model, the percentage of these items has a known probability density function, and it is assumed that they can be used in another production/inventory situation, generating less revenue than good ones. An extension of this model is proposed in [5], where the probability of misclassification in inspection activities is considered.

An EOQ model for exponentially decaying inventory considering a constant product demand is developed in [6]; an extension of this model is proposed in [7]. A mathematical model allowing evaluating the EOQ value in case of deteriorating items and permissible delay in payments is proposed in [8].

The inflation effects on the EOQ model is investigated in [9], where it is demonstrated that when prices vary with inflation, optimal order quantity is higher than the one obtained from [2]. The role and the impact of inflation uncertainty on inventory decisions are investigated in [10]. Inflation and deterio-

rating items effects are simultaneously considered in models proposed in [11] and [12].

The EOQ model is modified in order to explicitly consider transportation costs in [13]. Freight rate functions available in the scientific literature are adopted in [14] in order to include transportation costs into inventory replenishment decisions. An optimal solution procedures for solving the EOQ models is provided in case of transport costs are explicitly considered and shaped as all-unit-discount costs in [15].

Optimum lot-sizing algorithms are proposed in [16] in case of quantity and freight discounts are applied; both all-units and incremental discounts are considered; the model is extended to the case of a demand dependent upon price and of a demand varying stochastically in [17] and in [18], respectively.

Many contributions are available in which traditional EOQ model is extended to the case of stochastic variability of product demand. In case of uncertain product demand the safety stock sizing problem is integrated into the EOQ model. A model allowing obtaining optimal service level and safety stock level as a function of the ratio Q/σ_x, being σ_x the standard deviation of the forecasted lead time demand, is defined in [19]. The EOQ model is adopted to evaluate the optimal safety stock levels of components assembled to obtain the finished product in [20]. In case of a variable product demand a deterministic inventory model is defined in order to evaluate the effects of partial backordering on the EOQ solution [21]; in the model, during stock out periods, a fraction of the demand is backordered and the remaining fraction generates shortage costs. A solution procedure to compute EOQ in case of backordering is provided in [22]; two different optimization problems, providing the optimal value of the maximum inventory level and the optimal value of the backorder level are defined. Potential benefits of Vendor-Managed Inventory (VMI) implementation in EOQ model are investigated in [23].

In the model proposed in [24], lead time variability is investigated; lead time is assumed as a decision variable, and its optimal value is obtained by means of minimizing crashing costs, defined as extra costs to be charged in order to reduce lead time. Optimal lead time value, as well as order quantity and safety stock values are obtained in case of crashing lead time costs and price discounts of backorders in [25]. An algorithm allowing solving the single vendor single buyer problem in case of stochastic variability of lead time demand and a lead time varying linearly is proposed in [26]. Stochastic variability of lead time is assumed in [27] in order to solve the EOQ problem in case of a deterministic demand rate. Optimal order quantity and reorder level are obtained in case of random lead times by assuming the possibility of obtain expediting orders, that is orders with a shorter-than-average lead time at an extra cost [28].

Methods to reduce supply lead time variability are explored in [29]; in case of a lot size-dependent supply lead time, order splitting is identified as the optimal solution. In case of a gamma distributed lead time, the effect of reducing lead time variability on safety stock level is investigated in [30]. In case of a deterministic product demand, in [31] it is shown how the reduction of lead time variability is more effective than the reduction of its expected value.

Nowadays, increasing attention is being paid to sustainable manufacturing. The importance of inventory planning on the environment is widely discussed, and a simple extension of the traditional EOQ model is proposed in [32] in order to include sustainable aspects into the solution provided by [2]. The impacts of carbontrade, carbonprice, and carboncap mechanisms on optimal order quantity value, carbon emissions and total costs are investigated in [33].

Environmental performances of inventory systems are mainly due to transport. Environmental and social damages caused by transport are widely recognized. Related costs are defined transport "externalities" since they are not fully accounted or compensated by the transport user. Estimates of externalities costs due to transport are available in scientific literature [34, 35] and in EU official guidelines [36, 37], where the pollutant-to-pay principle is highlighted, and internalization strategies (reflecting the external costs in the price of transport) are suggested. In classical inventory models available in scientific literature costs of 'externalities', like freight transport emission, noise, accidents, and waste disposal, are neglected or limited to greenhouse gas emissions [32, 33, 38].

The Sustainable Order Quantity model [39] considers the logistic and the environmental costs of transport. Optimal lot-size and transportation means allowing to minimize logistic and environmental costs of transport are identified in case of a deterministic product demand; costs of disposal of an assembly line spare parts are also considered to identify the optimal order quantity of spare parts policy (replace vs. repair) [40].

The SOQ model in case of stochastic variability of the product demand is defined in [1]. The solution of the analytical model provides the optimal order quantity, the safety stock level, and the transportation means minimizing logistic and environmental

costs. The model is applied to a spare parts inventory case study from automotive industry in [41].

In this paper, the authors propose a new SOQ analytical model considering stochastic variability of lead time of suppliers. In the objective function of the model all the externalities generated from freight transport are accounted. The effects of the lead time uncertainty on whole logistic costs are investigated. The model is applied to a real industrial case study in order to evaluate the benefits of applying it if compared with traditional approaches.

The SOQ model in case of stochastic lead time variability

In this Section the SOQ model, in case of stochastic variability of supply lead time, is defined. Notations and assumptions adopted are listed below.

Notation

f	loss factor (decision variable) [–]
G	expected annual demand [unit/year]
H	production hours per year [h/year]
D	product demand [unit/h]
I	inventory level [unit]
Q	order quantity (decision variable) [unit]
r	reorder level [unit]
LT	supply lead time [h]
$E(LT)$	expected value of the supply lead time [h]
σ_{LT}	std. dev. of the supply lead time [h]
$pdf(LT)$	probability density function of LT [h^{-1}]
CT	consumption time [h]
SS	safety stock level (decision variable) [unit]
SL	service level [–]
N_S	number of shortages in one ordering cycle [unit]
L	transportation distance [km]
v	speed of transport [km/h]
m	mass of one product [t/unit]
c_H	unitary holding cost [€/unit·year]
c_O	fixed ordering cost [€/order]
c_T	unitary transport cost [€/t]
c_S	unitary shortage cost [€/unit·order]
ε	unitary external cost [€/t·km]

Assumptions

The inventory replenishment model is defined under the following assumptions:

a. the product demand (D) is deterministic and stationary, $D = G/H$;

b. orders do not cross;

c. backordering is not allowed;

d. the expected value of the supply lead time $(E(LT))$ is evaluated as the sum of the expected transport time $(E(T_T))$ and the expected value of the time required for the material handling, order management and quality control $(E(T_L))$; $E(T_L)$ is evaluated as a fraction $(k < 1)$ of $E(T_T)$; consequently, the speed of transport is obtained from the free flow or cruise speed (v_{act}) of different transportation means as $v = v_{act}/(1 + k)$, and the expected value of the supply lead time as $E(LT) = L/v$;

e. supply lead time (LT) is a continuous random variable normally distributed.

Theoretical formulation

In the inventory model defined in [39], annual holding cost (Φ_H), transport cost (Φ_T), and environmental cost of transport (Φ_E) are considered in order to define a logistic cost function (Φ_L):

$$\Phi_L = \Phi_H + \Phi_T + \Phi_E \quad [€/\text{year}]. \quad (1)$$

Starting from (1), in [39] a logistics, no-dimensional cost factor is defined as the ratio between the total annual costs and the annual inventory cost occurring in a limited situation where the order quantity (Q) is equal to the annual requirement (G):

$$F_L = 2 \cdot \Phi_L/(G \cdot c_H). \quad (2)$$

In case of deterministic and stationary values of the product demand and the supply lead time, the general logistic optimization problem is defined as:

$$\min F_L(f, Q), \quad (3)$$

with f the loss factor, defined in [42] as "*the ratio of the work required to overcome the frictional resistance during the transport and the transport performance*". Parameter f depends only on the means of transport adopted, as it measures the loss in energy occurring during materials shipping:

$$f = \frac{E}{W \cdot L}, \quad (4)$$

with E – energy required for the transport [kJ]; W – weight of the load transported [N].

As in [39], in this paper a taxonomy of f factor for discontinuous transport systems is adopted in order to univocally identify a given means of transport throughout its specific energy consumption; taxonomy allows shaping transport cost $(c_T(f, L)$ [€/t]), speed of transport $(v(f)$ [km/h]), and externality costs $(\varepsilon(f)$ [€/t·km]) as a function of the loss factor. Loss factor values are strictly related to the energy conversion technology of transportation means. Conversion technologies significantly evolved due to technical improvements (e.g. new fuel injection technologies), as well as to policy regulations (e.g. fuel composition, emission limits). Accordingly, the up-to-date f taxonomy as per [1] is adopted where more

details on transportation means performances data sets can be found.

By solving problem (3), the loss factor value (f_{OPT}) minimizing the logistic cost function (1), the optimal (Sustainable) value of the Order Quantity (SOQ), and of the reorder level ($r(f_{OPT})$) are obtained.

The SOQ model has been modified in order to take into account stochastic variability of supply lead time. The logistic cost function has been modified as follow:

$$\Phi_L = \Phi_H + \Phi_O + \Phi_T + \Phi_S + \Phi_{Ex} \quad [\text{€/year}] \quad (5)$$

where Φ_O is the yearly ordering cost, Φ_S is the yearly shortage cost generated by stock-out events, and Φ_{EX} is the yearly cost of all the externalities generated by transport.

In case of stochastic variability of supply lead time, the optimal values of the loss factor (f_{OPT}), the order quantity (SOQ), and the corresponding values of the reorder level ($r(f_{OPT})$), and of the safety stock ($SS(f_{OPT})$) jointly minimizing the logistic and external costs can be obtained by solving problem (3).

In the following, cost figures in (5) are discussed.

Cost functions

Holding cost

In order to compute holding cost, the expected inventory level ($E(I)$) in one ordering cycle has to be computed. It is affected by both the order quantity size (Q) and the safety stock level (SS), as well as by the supply lead time variability.

The SS value consistent with an assigned service level, SL, can be evaluated as:

$$SS = D \cdot [LT^* - E(LT)], \quad (6)$$

where

$$SL = prob\left(D_{TOT} \leq LT^* \cdot D\right)$$
$$= \int_{-\infty}^{LT^*} pdf(LT)dLT = F(LT^*) \quad (7)$$

with LT^* the maximum value of the supply lead time not causing a stock out event at a given service level (SL) (see Fig. 1); D_{TOT} the lead time demand.

With z the standardized variable of the stochastic variable LT, it is obtained:

$$SS = D \cdot z^* \cdot \sigma_{LT}, \quad (8)$$

$$SL = F(z*), \quad (9)$$

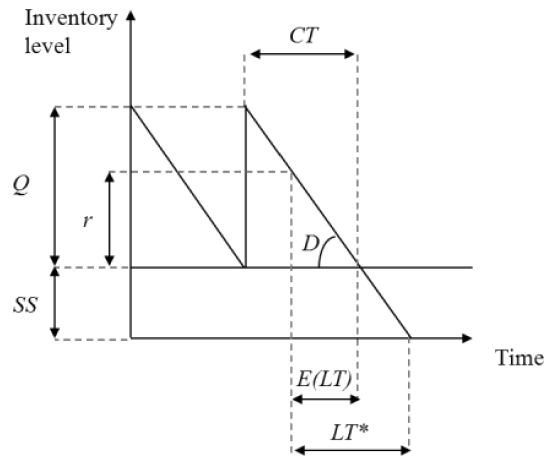with

$$z^* = \frac{LT^* - E(LT)}{\sigma_{LT}}. \quad (10)$$

Fig. 1. Inventory level over time in case of $LT = E(LT)$.

Considering the stochastic variability of LT, three different situations may occur:
1. $LT \leq E(LT)$ (see Fig. 1A),
2. $E(LT) < LT \leq LT^*$ (see Fig. 1B),
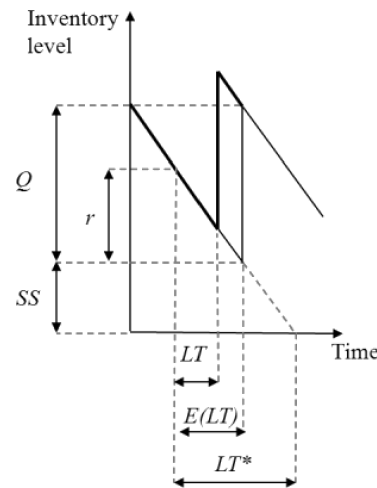3. $LT > LT^*$ (see Fig. 1C).

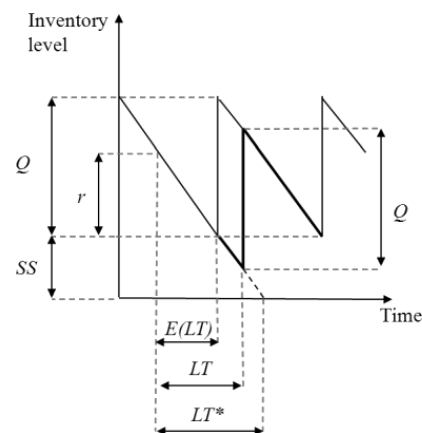Fig. 1A. Inventory level over time in case of $LT \leq E(LT)$.

Fig. 1B. Inventory level over time in case of $E(LT) < LT \leq LT^*$.

Table 1
Expected inventory level and ordering cycle length in the three cases considered.

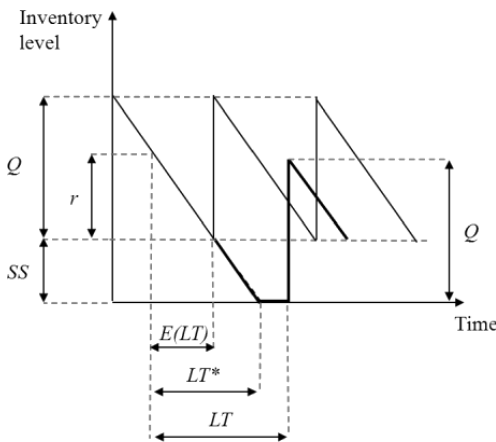|	Expected inventory level E(I)	Consumption time CT	Case probability
Case A $LT \leq E(LT)$	$E(I)_A = \frac{1}{2}Q + D \cdot [E(LT) - LT] + SS$	$\frac{Q}{D}$	$\int_{-\infty}^{0} pdf(z)dz$
Case B $E(LT) < LT \leq LT^*$	$E(I)_B = \frac{1}{2}Q - D \cdot [LT - E(LT)] + SS$	$\frac{Q}{D}$	$\int_{0}^{z^*} pdf(z)dz$
Case C $LT > LT^*$	$E(I)_C = \frac{1}{2}Q + \frac{1}{2}SS - \frac{1}{2}Q \cdot \frac{[LT^* - E(LT)]}{CT_C}$	$\frac{Q}{D} + [LT - LT^*]$	$\int_{z^*}^{+\infty} pdf(z)dz = 1 - SL$

Fig. 1C. Inventory level over time in case of $LT > LT^*$.

The expected inventory level during one ordering cycle can be evaluated as:

$$E(I) = \int_{-\infty}^{E(LT)} E(I)_A \cdot pdf(LT)dLT$$
$$+ \int_{E(LT)}^{LT^*} E(I)_B \cdot pdf(LT)dLT \qquad (11)$$
$$+ \int_{LT^*}^{+\infty} E(I)_C \cdot pdf(LT)dLT.$$

In the reference case of $LT = E(LT)$ (see Fig. 1, and the thin lines in Fig. 1A), $E(I) = 1/2 \cdot Q + SS$, and $CT = Q/D$. In cases B and C lead time variability affects average inventory level in the next ordering cycle; in case C, the length of the ordering cycle is greater than Q/D. The expected inventory level, the consumption time, and the corresponding occurrence probability values in the three cases considered are in Table 1.

In case of high SL values (≥ 0.90):

- the term $[LT-LT^*]$ assumes small values;
- the occurrence probability of case C is very small;
- CT in case C can be approximated to Q/D;
- $E(I)_C = 1/2 \cdot Q$;

- the expected inventory level $E(I)$ in one ordering cycle is obtained as:

$$E(I) = \frac{1}{2}Q + D \cdot \sigma_{LT} \cdot [pdf(z^*) + z^* \cdot F(z^*)] \quad (12)$$

and the corresponding holding cost is:

$$\Phi_H = c_H \cdot \left\{ \begin{array}{l} \frac{1}{2}Q \\ +D \cdot \sigma_{LT} \cdot [pdf(z^*) + z^* \cdot F(z^*)] \end{array} \right\}. \quad (13)$$

Ordering cost

Since G/Q is the average number of ordering cycles in one year, the ordering cost is evaluated as:

$$\Phi_O = c_O \cdot \frac{G}{Q}. \qquad (14)$$

Transport cost

As in [1], the unit transport cost is expressed by means of quadratic functions shaping dependency of costs on loss factor for different route lengths:

$$c_T(f, L) = a \cdot f^2 + b \cdot f + c \ \ [\text{€/t}]. \qquad (15)$$

The annual cost of transport can be expressed as:

$$\Phi_T = c_T(f, L) \cdot G \cdot m. \qquad (16)$$

Shortage cost

Stock outs events occur in case C ($LT > LT^*$) discussed in the previous section. The corresponding number of shortage units can be evaluated as:

$$N_S = D \cdot \int_{LT^*}^{+\infty} (LT - LT^*) \cdot pdf(LT) \cdot dLT \qquad (17)$$

and the corresponding annual shortage cost can be evaluated as:

$$\Phi_S = c_S \cdot \frac{G}{Q} \cdot D \cdot \sigma_{LT} \cdot L(z^*) \qquad (18)$$

with $L(z^*)$ the standardized normal loss function defined as:

$$L(z^*) = \int_{z^*}^{+\infty} (z - z^*) \cdot pdf(z)dz \qquad (19)$$
$$= pdf(z^*) - z^* \cdot [1 - F(z^*)].$$

External cost

Environmental performances of inventory systems are mainly due to transport. EU Commission defines the "social cost" of transport as the sum of the private (or internal) and the external costs of transport. Internal costs consist of collection, handling, transshipment, and distribution of goods [36]. They are directly charged to the user. External costs represent the monetary value of the social damages caused by transport activities (e.g. noise, congestion, accidents, air pollution, global warming) and they are not fully accounted or compensated by the transport user.

Emissions from the transport sector increased in EU by 26% [43] in the period 1990–2005. In the attempt of reducing environmental costs of transport, thus achieving emission reduction goals stated by international agreements, severe emissions standards for new vehicles [44] and for fuels [45] have been adopted in EU, and new taxes on road freight transport have been introduced in some EU Countries. However, charges and taxes do not fully reflect the social costs of transport [37], and charging external costs of transport to the general taxpayer does not drive decision-makers of freight transport towards sustainable choices [46].

A "greening transport" policy should require the adoption of a pollutant-to-pay principle by an internalization strategy, i.e. adding the external costs in the price of transport, as suggested in [36].

Estimates of external costs of freight transport are in scientific literature [34, 35, 47–51] and in EU official guidelines [36, 37]. In this paper external cost data of water, rail, and road freight transport in [50] and cost data of air freight transport in [51] have been adopted. Discounting indexes available in [52] have been adopted to refer costs to 2013 values.

A regression analysis has been carried out in order to shape dependency of unit external costs on loss factor values. A quadratic dependency was found as an appropriate function to shape dependency of external costs function on loss factor for different types of externalities:

$$\varepsilon_j = \alpha_j \cdot f^2 + \beta_j \cdot f \quad [\text{€}/\text{t} \cdot \text{km}] \qquad (20)$$

with j = accidents, air pollution, noise, congestion, global warming, LCA, and other externalities.

More details on the regression analysis and parameters values obtained are in the next Section.

The annual external cost of transport can be computed as:

$$\Phi_{Ex} = \left[f^2 \cdot \sum_j \alpha_j + f \cdot \sum_j \beta_j \right] \cdot G \cdot m \cdot L. \qquad (21)$$

The logistic cost function

By substituting costs functions in (5), the logistic cost function can be rewritten as:

$$\Phi_L = c_H \cdot \left\{ \frac{1}{2}Q + D \cdot \sigma_{LT} \cdot [pdf(z^*) + z^* \cdot F(z^*)] \right\}$$
$$+ c_O \cdot \frac{G}{Q} + [a \cdot f^2 + b \cdot f + c] \cdot G \cdot m$$
$$+ c_S \cdot \frac{G}{Q} \cdot D \cdot \sigma_{LT} \cdot L(z^*) \qquad (22)$$
$$+ \left[f^2 \cdot \sum_j \alpha_j + f \cdot \sum_j \beta_j \right] \cdot G \cdot m \cdot L.$$

Let parameter p the ratio between the reorder level and the inventory level at the beginning of each ordering cycle. Parameter p also represents the ratio between the expected value of the supply lead time (see Fig. 1) and the duration of the ordering cycle:

$$p = \frac{r}{Q} = \frac{E(LT)}{CT} \qquad (23)$$

with $0 < p \leq 1$.

For a given transportation means, the corresponding value of the reorder level can be computed as:

$$r = D \cdot E(LT). \qquad (24)$$

The corresponding minimum value of the order quantity can be obtained by assuming $p = 1$ (or $Q = r$).

Starting from (23), under the assumption made, $E(LT) = L/v(f)$ and the order quantity can be obtained as:

$$Q = \frac{G \cdot L}{p \cdot H \cdot v(f)}. \qquad (25)$$

By adopting (25) and $\sigma_{LT} = cv_{LT} \cdot E(LT)$ in the logistic cost function (22), problem (3) can be reformulated as:

$$\min F_L(f, p), \qquad (26)$$

where the decision variable Q is replaced with p.

For an assigned set of (SL, cv_{LT}, L) values, the following procedure has been adopted to solve problem (26):

- *Step 1*: compute z^* from (9);
- *Step 2*: compute f_{OPT} and p_{OPT} by solving (26);
- *Step 3*: compute $E(LT) = L/v(f_{OPT})$;
- *Step 4*: compute $r(f_{OPT})$ from (24);
- *Step 5*: compute $SOQ = r(f_{OPT}) \cdot p_{OPT}$;
- *Step 6*: compute $\sigma_{LT} = cv_{LT} \cdot E(LT)$;
- *Step 7*: compute $SS(f_{OPT})$ by means of (8).

Step 2 has to be carried out by means of a numerical method.

An automotive supply chain case study

The model has been applied to a case study from the automotive industry [54, 55]. The case refers to a multi-site manufacturing system producing breaking equipment. The supply chain consists of three production sites (see Fig. 2): sites 1 and 2 are responsible of producing semi-finished products for sites 2 (P1C, P2C) and 3 (P1B, P2B) respectively; in site 3, three finite products

(P1, P2, P3) are assembled for a single customer. Sites 2 and 3 produce additional products (PX, PY) required by external customers of the aftermarket starting from semi-finished product (PXf, PYf) externally supplied. In [54, 55] a lot sizing and scheduling problem is solved for the manufacturing system by means of a simulation model and a hybrid model, respectively.

In this work, the logistic problem (26) has been solved for PX product in site 1. PXf product is supplied independently from P1f and P2f products. It is characterized by an annual demand (G) of 9224 [unit/year]. The number of working hours (H) in site 1 is 3520 [h/year], thus resulting in a product demand (D) of 2.62 [unit/h].

The unit holding cost of product PXf in site 1 (c_H) is 18.98 [€$_{2013}$/unit·year], and the extra cost generated in case of stock out (c_S) is 12.32 [€$_{2013}$/unit·order] ($c_S/c_H = 0.65$). The mass of the product (m) PXf is 0.5 [kg], and the cost to place an order (c_O) is 100 [€/order]. The corresponding EOQ value is 312 [unit].
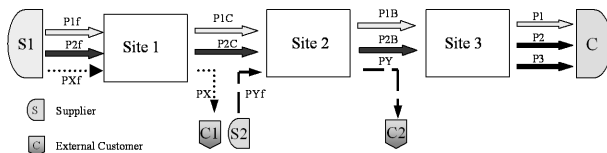

Fig. 2. The material flows in the supply chain

In solving (26), optimal loss factor values ($f_{OPT.}$) have been searched for in the set of loss factor values characterizing the available means of transport. The up-to-date f taxonomy of discontinuous transport systems as in [1] has been adopted. Values are in Table 5.

Cost data

Unit external cost data of water, rail and road freight transport in [50] and cost data of air freight transport in [51] have been adopted; updated values ([€$_{2013}$/t·km]) are in Table 2. Unit external costs of road transport are

provided for both LDV (Light Duty Vehicles) category, with a GWR (Gross Weight Rate) less than 3.5 [t], and HDV (Heavy Duty Vehicles) category, with a GWR greater than 3.5 [t]. In the LCA category only the unit external cost of energy production and distribution (well-to-tank) is considered. In the "other external cost" category the sum of the unit costs of nature and landscape effects, of biodiversity losses, of soil and water pollution, and of the urban effects (energy dependency cost) is provided.

The term "high scenario" in Table 2 refers to the hypothesis adopted to evaluate the external cost of the unit mass of CO_2 emitted. In general, two methodological approaches for the evaluation of climate change impacts could be adopted: assessment of damage costs and assessment of avoidance costs. In the former, economic impacts of the physical changes in the environment caused by greenhouse gases (GHG) emissions are evaluated. In the latter, the least costs option to achieve a reduction target of greenhouse gas emission is evaluated by means of a cost-effectiveness analysis. Assessment of avoidance costs is often preferred to the assessment of physical damages, because of the difficulties in assessing certain physical effects of GW [50]. In the assessment of avoidance cost, usually two different targets are considered: EU GHG reduction target for 2020 (corresponding to a reduction of 20% of GHG emissions compared to 1990 levels, "low scenario") and a longer term target for keeping concentration of CO_{2eq} in the atmosphere below 450 [ppm] (thus keeping global temperature rise below 2 [°C] relative to pre-industrial levels [53], "high scenario"). A higher cost for the unit mass of CO_{2eq} emitted is obtained when the "high scenario" target is considered.

Unitary cost data of GW, LCA, and other external cost in Table 2 have been obtained adopting the assessment of avoidance cost approach, in case of a "high scenario".

Parameter values adopted to evaluate unitary transport cost as in [39] are in Table 3.

Table 2
Unitary external cost [€$_{2013}$/t·km] of different transportation means [50, 51].

External cost category	Waterborne	Rail electric	Rail diesel	LDV	HDV	airplane
Accidents	0.00	0.22	0.22	62.27	11.30	0.00
Air Pollution	5.98	1.00	1.88	19.83	7.42	20.69
Noise	0.00	1.11	1.11	6.98	1.99	11.80
Congestion	0.00	0.00	0.00	107.01	28.50	0.00
GW high scenario	3.99	0.00	4.32	49.31	10.86	325.93
LCA high scenario	1.44	4.43	5.65	15.84	3.32	10.23
Other external cost	1.00	0.55	0.55	7.09	2.77	5.04

Table 3
Parameters values for unitary transport cost evaluation per transportation distance.

L [km]	a	b	c
	[€/kg]		
200	391.37	−402.35	108.69
500	329.84	−259.17	107.63
1000	227.28	−20.54	105.87

Table 4

Observed free flow speeds in UK in 2011 [UK Dft 2012].

	Light Good Vehicles	Heavy Good Vehicles				
	<3.5 [t]	3.5–18 [t]	26 [t]	30–44 [t]	26–40 [t]	40–44 [t]
Motorways						
Average speed [mph]	70	61	54	54	54	54
Observations [# × 10³]	82884	6369	2863	1716	7260	42830
Dual carriageways						
Average speed [mph]	68	59	53	53	53	53
Observations [# × 10³]	7127	2548	257	210	453	2762
Single carriageways						
Average speed [mph]	48	46	42	43	43	44
Observations [# × 10³]	5816	1957	223	167	275	1164

In order to shape the speed of transportation means as a function of their characteristic loss factor values, statistics available in [56] and [57] have been adopted in order to evaluate characteristic speeds in transport.

Starting from data available in [56] on free flow vehicle speeds observed in UK in 2011 on different road types, an average speed value has been evaluated for each truck category. The average values have been obtained weighing the values observed on the different road types (motorways, dual carriageways, single carriageways) with the corresponding observations number. Results are in Table 4.

Data on ships cruise speeds in [57] have been adopted. For railroads and aircraft transportations means, a cruise speed of 70 [km/h] and 700 [km/h] have been assumed, respectively.

Starting from data on free flow speed of trucks, on cruise speed of different type of ships, and the assumption made in case of rail and aircraft means of transport, a data set has been obtained by relating the loss factor values of different transportation means with their characteristic average speed in transport.

By considering a k value of 0.5, the data set in Table 5 has been obtained. Starting from the data set, a regression analysis has been carried out. Results obtained showed that the relationship between the speed of transportation means and their characteristic loss factor values is best shaped by a quadratic function:

$$v(f) = k_1 \cdot f^2 + k_2 \cdot f + k_3 \qquad (27)$$

with $k_1 = 557.6$ [km/h]; $k_2 = -150.4$ [km/h]; $k_3 = 52.7$ [km/h] ($R^2 = 0.935$).

Problem (26) has been solved for different values of the SL (0.90–0.95–0.99), and of the transportation distance L (200–500–1000 [km]).

Table 5

Loss factor values (f) and transport speed for different means of transport.

Transportation means	f	v [km/h]
Ship – oversee	0.010	24.7
Ship – tank	0.021	18.5
Ship – coast	0.033	17.3
Rail – diesel	0.033	46.7
Rail – electric	0.037	46.7
Ship – inland	0.048	24.7
Truck 34–40 [t]	0.063	56.5
Truck 28–34 [t]	0.069	56.5
Truck 26–28 [t]	0.119	56.5
Truck 20–26 [t]	0.136	56.5
Truck 14–20 [t]	0.127	64.0
Truck 12–14 [t]	0.139	64.0
Truck 7.5–12 [t]	0.149	64.0
Truck 3.5–7.5 [t]	0.165	64.0
Truck 3.5–7.5 [t]*	0.156	64.0
Truck <3.5 [t]**	0.335	72.7
Truck <3.5 [t]***	0.554	72.7
Airplane	0.984	466.7

* – operated without append

** – light good vehicles

*** – vans

EOQ and SOQ comparison in case of deterministic supply lead time

A preliminary comparison between results of the SOQ model proposed with those obtained by the traditional EOQ model [2] has been carried out in case of $cv = 0$.

Results obtained are in Fig. 3–5. In the figures, the lines with squared markers refer to solutions obtained without considering cost of externalities (economic solution in the following, "*econ.*" in the fig-

ures), while solutions depicted by rhombus markers have been obtained also considering external costs in Table 2 (sustainable solution in the following, "*sust.*" in the figures).
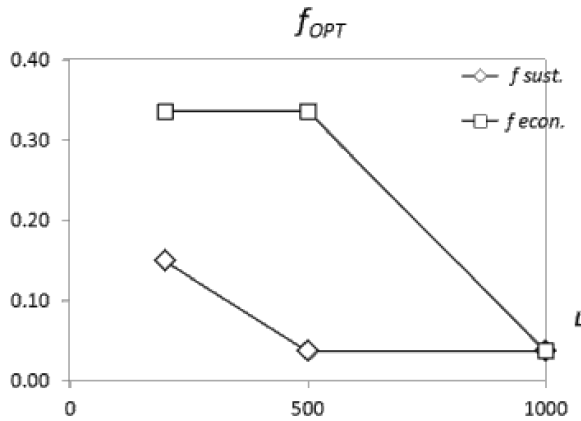

Fig. 3. f_{OPT} values in case of $cv = 0$ for different transportation distances.
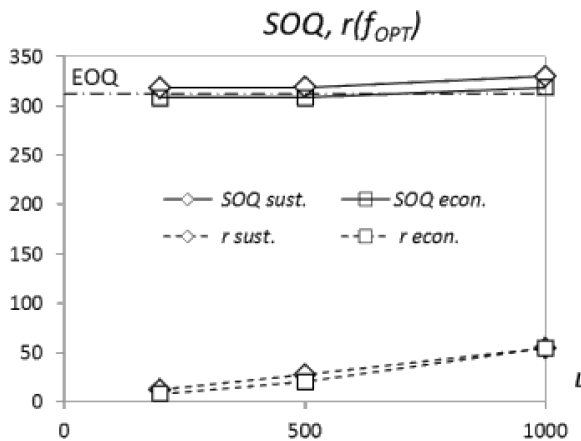

Fig. 4. SOQ and $r(f_{OPT})$ values in case of $cv = 0$ for different transportation distances.
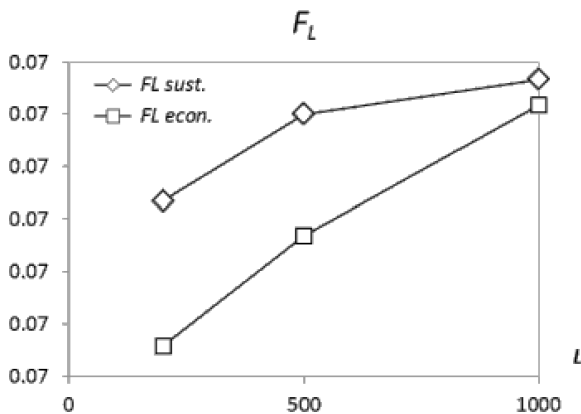

Fig. 5. $F_L(f_{OPT}, SOQ)$ values in case of $cv = 0$ for different transportation distances.

As it can be observed (see Fig. 4), in the sustainable solution a SOQ > EOQ is identified as the optimal choice for each transportation distance considered.

As depicted in Fig. 3, loss factor values minimizing the logistic cost function decreases as the transport distance increases both in case of economic and in case of sustainable solutions. As a consequence, higher reorder levels are required in case of long transportation distance (see Fig. 4). For the sustainable solution, when compared with the economic solution, slower transportations means (and higher reorder levels) are identified as optimal choice.

Finally, as expected, logistic cost function (F_L) increases with the transportation distance and due to external costs assumes higher values in case of the sustainable solutions.

The effects of the supply lead time variability

In case of a variable supply lead time, solution of (26) depends also on the shortage costs, which in turn are affected by the safety stock level. SS level is a function of the lead time variability and of the service level performed by the inventory system (see Eq. (8)). For this reason, in order to investigate the effects of the supply lead time variability on the solutions of (26), different values of the cv and of the SL have been considered. Due to the small value of the unitary shortage cost (c_S), results obtained show a negligible influence of the service level on the optimal f, SOQ, and r values.

As an example, results obtained in case of $SL = 0.95$ are in Table 6 and 7.

Table 6
Optimal loss factor (f_{OPT}) and SOQ values for different L and cv values in case of $SL = 0.95$.

	cv	L [km]		
		200	500	1000
f_{OPT}	0.00	Truck 7.5–12 [t]	Rail – electric	Rail – electric
	0.10	Truck 7.5–12 [t]	Rail – electric	Ship
	0.25	Vans	Rail – electric	Ship
	0.50	Vans	Rail – electric	Ship
	0.75	Vans	Vans	Ship
	1.00	Vans	Vans	Ship
	2.00	Vans	Vans	Vans
SOQ	0.00	318	318	330
	0.10	318	318	318
	0.25	319	318	318
	0.50	319	318	318
	0.75	319	319	318
	1.00	319	319	318
	2.00	319	319	318

Table 7
r, SS, and F_L values for different L and cv values in case of $SL = 0.95$.

	cv	L [km]		
		200	500	1000
	0.00	25	55	109
	0.10	25	55	102
	0.25	7	55	102
$r(f_{OPT})$	0.50	7	55	102
	0.75	7	19	102
	1.00	7	19	102
	2.00	7	19	37
	0.00	0	0	0
	0.10	2	4	8
	0.25	2	11	21
$SS(f_{OPT})$	0.50	3	22	42
	0.75	5	12	63
	1.00	6	15	84
	2.00	12	31	61
	0.00	0.070	0.073	0.074
	0.10	0.071	0.074	0.076
	0.25	0.071	0.076	0.080
$F_L(f_{OPT}, p_{OPT})$	0.50	0.072	0.079	0.085
	0.75	0.072	0.081	0.091
	1.00	0.073	0.082	0.097
	2.00	0.074	0.087	0.107

In case of short distance ($L = 200$ [km]) road transport is the optimal option for each level of supply lead time variability. When a high variability level is considered, faster road transportation means are solution of (26). In case of long transportation distances ($L = 500$–1000 [km]), fast road transportation means are solution of (26) only in case of high supply lead time variability. In the other cases considered, slower transportation means (rail, ship) are the optimal choice (see Table 6). Reorder level observed ($r(f_{OPT})$, see Table 7) are consistent with the optimal loss factor values identified by means of (26): when faster transportation means are adopted, smaller reorder levels can be adopted. In all cases considered, SOQ values higher than EOQ are observed only in case of short transport distance and low variability of supply lead time. A negligible influence of the supply lead time variability on SOQ values is observed. As expected, SS value increases with the supply lead time variability and the transportation distance (see Table 7). Finally, the effect of the supply lead time variability on total logistic costs ($F_L(f_{OPT.})$, see Table 7) proved to be dependent on the transportation distance considered. Comparing the two limit cases considered ($cv = 0.00$ and $cv = 2.00$, $SL = 0.95$) an increase of 5.5%, 18.7%, and 45.7% is observed in case of a transportation distance of 200 [km], 500 [km], and 1000 [km], re-

spectively (see Table 7), corresponding to an increase in total annual costs of 0.3, 1.2, and 3.0 [k€/year], respectively. Finally, in order to evaluate the effects of the order costs on the solution provided by (26), a sensitive analysis has been carried out by considering two additional unitary order cost values: 10 [€/order] and 200 [€/order].

Results obtained showed that solution of (26) is slightly affected by order cost values, except for SOQ values. As expected, when fixed order cost growths, higher values of SOQ minimize total logistic costs (see Table 8 and 6).

Table 8
SOQ values for different L, cv, and c_O values in case of $SL = 0.95$.

	cv	L [km]		
		200	500	1000
$c_0 = 10$ [€/ order]				
	0.00	93	98	100
	0.10	93	99	100
	0.25	93	99	98
SOQ	0.50	93	98	109
	0.75	93	98	109
	1.00	93	104	104
	2.00	93	104	98
$c_0 = 200$ [€/ order]				
	0.00	103	420	420
	0.10	403	420	420
	0.25	403	420	464
SOQ	0.50	374	420	464
	0.75	374	467	464
	1.00	374	467	464
	2.00	374	467	467

Table 9
Optimal loss factor (f_{OPT}) values of the sustainable and of the economic ($\Phi_{Ex} = 0$) solution.

	cv	L [km]		
		200	500	1000
	0.00	0.149	0.037	0.037
	0.10	0.149	0.037	0.010
	0.25	0.554	0.037	0.010
f_{OPT}^{sust}	0.50	0.554	0.037	0.010
	0.75	0.554	0.554	0.010
	1.00	0.554	0.554	0.010
	2.00	0.554	0.554	0.554
	0.00	0.335	0.335	0.037
	0.10	0.335	0.335	0.010
	0.25	0.554	0.554	0.554
$f_{OPT}^{econ.}$	0.50	0.554	0.554	0.554
	0.75	0.554	0.554	0.8554
	1.00	0.554	0.554	0.554
	2.00	0.554	0.554	0.984

In order to investigate the effect of the external costs, problem (26) has been solved in case no external costs are computed. Results obtained in case of a $SL = 0.95$ are compared with ones previously obtained in Table 9.

As showed in Table 9, when external costs are computed in (22), slower transportation means are solutions of (26). At the same time, a negligible influence on SOQ values has been observed.

The benefits of the adoption of the sustainable solution have been measured by means of the evaluation of the reduction obtained in terms of external costs of transport. As an example, in Table 10 external costs values of the sustainable (Table 10A) and of the corresponding economic (Table 10B) solution are listed for different values of the lead time variability (cv) and transportation distances (L).

Table 10
External costs of transport (Φ_{Ex})
in the sustainable (A) and economic (B) solution.

	cv	L [km]		
		200	500	1000
(A)				
	0.00	61	17	34
	0.10	61	17	57
	0.25	248	17	57
Φ_{Ex} [€/year]	0.50	248	17	57
	0.75	248	619	57
	1.00	248	619	57
	2.00	248	619	1238
(B)				
	0.00	248	619	34
	0.10	248	619	57
	0.25	248	619	1238
Φ_{Ex} [€/year]	0.50	248	619	1238
	0.75	248	619	1238
	1.00	248	619	1238
	2.00	248	619	1723

Results showed significant savings in external costs mainly in case of low lead time variability and short distances ($L = 200$–500 [km]), and high lead time variability and long transportation distance ($L = 1000$ [km]).

Conclusions

In this paper, an extension of the sustainable order quantity model has been developed by introducing the supply lead time variability; the model also includes external costs of the freight transport.

The model has been applied to an industrial case study. Results obtained have been interpreted under both classical logistic goals as well as under a more comprehensive 'sustainability' perspective which is due to the internalization of external costs of transport.

With reference to pure logistic concerns, the case study revealed, as major effects of the supply lead time variability, the increase in both the order quantity and in the speed of transport required to minimize the total logistic costs. Magnitude of the effects depends on the transportation distance, the service level, and on the unit shortage cost value. Fixed ordering cost value proved to strongly influence solutions in terms of the order quantity size while a negligible influence on transportation means selection was observed.

Interpretation of results under a 'sustainability perspective' shows as the main effect of the internalization of the external cost in the logistic cost function is the shifting of the solution towards slower transportation means. The adoption of transportation means characterized by lower values of the optimal loss factor leads to significant savings in external costs, even in case of high lead time variability.

Results obtained lead to conclude that a general re-thinking of 'just-in-time' logistic solutions is required. More relaxed logistics could be effective from both economic and environmental points of view mainly in a market characterized by lead time uncertainty.

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