

# Overview of Fading Channel Modeling

Krystyna M. Noga and Beata Pałczyńska

**Abstract**—VisSim and LabVIEW based approaches are proposed and implemented to demonstrate fading in the communication systems. The introduction to fading, models for flat fading like Rayleigh, Weibull, Nakagami- $m$  and the results of simulations are presented.

**Keywords**—Communication channel, random variables, simulation, Nakagami- $m$  fading, Rayleigh fading, Weibull fading.

## I. INTRODUCTION

THE CHANNEL CHARACTERIZATION in mobile communication systems is an important and fast growing part of wireless communication. It plays a main role in the transmission of information through a propagation medium from the transmitter to the receiver. The propagation path between the transmitter and receiver can vary from simple line-of-sight to one that is severely affected by buildings, mountains and trees. Radio channels are extremely random and do not offer easy analysis. Modeling radio channels has been one of the most difficult parts of mobile radio system design. It is typically done using statistical analysis. Without knowledge of channel models, a wireless system could never be developed. This knowledge is very important for designing radio communication systems. The mobile radio propagation channel introduces fundamental limitations to the performance of any wireless communication systems. In digital communication theory the frequently assumed model for a transmission channel is the additive white Gaussian noise (AWGN) channel. However, for many communication systems the AWGN isn't an adequate model. A more precise and complicated channel model should be used. In practice, the fading channel model is frequently applied. Typical examples of such channels are the Rayleigh, Nakagami- $m$ , Rice channels [1]–[4]. They are combined by different researchers in order to form a model for describing a particular channel.

In this paper, VisSim and LabVIEW based approaches are proposed and implemented to demonstrate the concept of fading. Models for Rayleigh, Weibull, Nakagami- $m$  fading are presented and simulated.

## II. FADING IN A WIRELESS ENVIRONMENT

Radio waves propagate from a transmitting antenna, and travel through the space, undergoing reflections, absorption, refraction, scattering and diffraction. The transmitted signal arrives at the receiver via several paths with different time delays creating a multi-path situation. At the receiver, these multipath waves with randomly distributed envelopes and

phases combine to give a resultant signal that fluctuates in time and space. The mobile radio channel is characterized by two types of fading. The short-term fluctuation in the signal envelope, caused by the local multipath, is called a small-scale fading. Such a situation is observed over distances of about half of a wavelength. The term small-scale fading denotes rapid variations of the envelope of a received signal. The second type of fading is called a large-scale fading. The term large-scale fading denotes the variations of the mean envelope or mean power of a received signal. It is assumed to be a slow process and is usually modeled as having lognormal statistics. For example, the slow variation of the mean signal level for outdoor and indoor environments can be described by a lognormal statistics. This paper will be limited to the flat multipath fading, as multi-path in the radio channel creates small-scale fading effects. Usually the fading envelope of the signal is modeled by a Rice, Rayleigh and Nakagami- $m$  distribution [1]–[5]. The Rice and Rayleigh distributions describe multipath effects and Nakagami- $m$  can replace them both. Analyses based on Rayleigh or Rice fading is sufficient to model the nature of many different mobile channels. But a lot of recent publications, i.e. [3], [4], [6], [7], have suggested the use of Nakagami- $m$ , Weibull fading models to provide a generalized description of fading in wireless systems.

## III. GENERATION OF UNCORRELATED FADING

The random fluctuation in the received signal due to fading can be modeled by treating  $C[\tau(t), t]$  as a random process in  $t$ , where  $\tau$  is the propagation delay. The multipath channel can be described by a time-varying, complex, lowpass-equivalent impulse response  $C[\tau(t), t]$ . Statistics of the received signal in a mobile radio environment are obtained by using the scattering model and the assumption of large number of randomly phased components. Since the components of the multipath signal arise from a large number of reflections or scattering from rough or surface, according to the central limit theorem,  $C[\tau(t), t]$  can be modeled as a complex Gaussian realization. The probability density function of the real and imaginary parts, i. e. the in-phase and quadrature components, are Gaussian. If  $C[\tau(t), t]$  has a mean equal to zero, then the envelope  $R(t) = |C[\tau(t), t]|$  has a Rayleigh probability density function (pdf) [3], [4].

$$p_r(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad r \geq 0 \quad (1)$$

where:  $2\sigma^2 = E(R^2)$  and  $E(*)$  denotes statistical averaging.

There are a number of methods used to generate different (non-uniform) variables. Each method is applicable only to a subset of distribution. Also, for a particular distribution, one method may be more efficient than the others [8].

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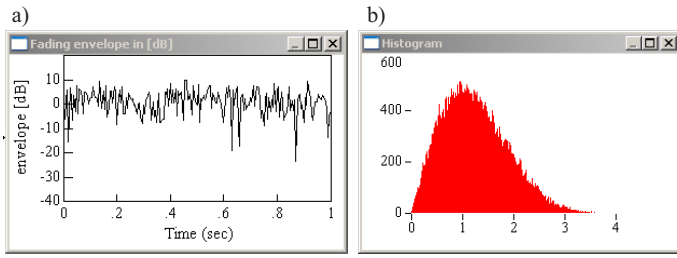


Fig. 1. The Rayleigh fading process – the envelope (a) and its histogram (b).

The generation of Rayleigh fading envelope  $R = \sqrt{X^2 + Y^2}$  follows from the basic fact that the envelope of a Gaussian process with independent quadrature  $Y$  and in-phase  $X$  components, with mean value  $\mu = 0$  and variance  $\sigma^2 = 1$ , has a Rayleigh distribution. Moreover, the phase  $\theta = \arctg(Y/X)$  has a uniform pdf. This fact can be used to generate two samples of a Gaussian variable by transforming a pair of Rayleigh and uniform variables. If  $U_1$  and  $U_2$  are two independent variables uniformly distributed in the unit interval, i.e.  $U_1 := rnd(1)$  and  $U_2 := rnd(1)$  where  $rnd(x)$  returns a uniformly distributed random number between 0 and  $x$ , then

$$\begin{aligned} X &= \mu + \sigma \sqrt{-2 \ln u_1} \cos(2\pi u_2) \\ Y &= \mu + \sigma \sqrt{-2 \ln u_1} \sin(2\pi u_2) \end{aligned} \quad (2)$$

are independent Gaussian random variables. This algorithm is known as the Box-Muller method [1], [8]. There is some concern that if this method is used with mean value  $\mu \neq 0$  from an Linear-Congruential Generators (LCG), the resulting  $X, Y$  may be correlated [8].

Another method for generating random variables is the convolution technique. This technique can be used if the random variable  $X$  can be expressed as a sum of  $n$  random variables  $Z_i$

$$X = \sum_{i=1}^n Z_i \quad (3)$$

If  $X$  is a sum of two random variables, then the pdf of  $X$  can be obtained analytically by a convolution of the pdf's of  $Z_1$  and  $Z_2$ . This is why the technique is called convolution, although no convolution is required in random-number generation. Using this method and, taking into account that the sum of large number of uniform  $U[0, 1]$  variables has a normal distribution, for normal variable with mean value  $\mu$  and variance  $\sigma^2$  it can be written [3]

$$X = \mu + \sigma \frac{\left( \sum_{i=1}^n u_i \right) - \frac{n}{2}}{\sqrt{\frac{n}{12}}} \quad (4)$$

This method is also known as the sum of 12 [1]. The value  $n = 12$  represent some compromise between speed and accuracy. It should be noted that whereas a Gaussian random variable has value ranging from  $-\infty$  to  $+\infty$ , formula (4) produced value of normal variable in the interval  $[-6, +6]$ .

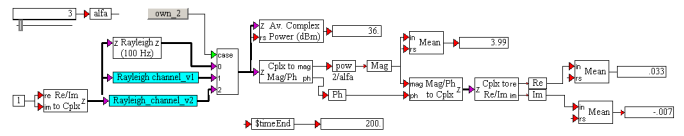


Fig. 2. The generator of the Weibull envelope.

It should be emphasized that in VisSim and LabView environments used for simulation, Gaussian signal generators are available.

Figure 1 shows the Rayleigh envelope waveform and its histogram. The model of fading is simulated using VisSim environment, created by the Regents of the University of California.

Another distribution used to model fading in a multipath environment is the Weibull distribution [3], [4], [6], [8]. This distribution seems to exhibit good fit to the experimental fading channel measurements, for indoor and outdoor environments. The probability density function of the envelope  $R$  is given by

$$p_r(r) = \frac{\alpha r^{\alpha-1}}{a^\alpha} \exp\left(-\frac{r}{a}\right)^\alpha; \quad r \geq 0, \quad \alpha \geq 0, \quad a > 0 \quad (5)$$

where  $a$  is a scale parameter and  $\alpha$  is a shape parameter.

To generate Weibull variables, the inverse transformation technique can be used [8]. We should generate the uniform random variable  $U[0, 1]$  and then return  $a(\ln u)^{1/\alpha}$  as Weibull variable with parameters  $a, \alpha$ .

The Weibull envelope can also be expressed as a function of the in-phase and quadrature components, so that

$$R = (X^2 + Y^2)^{1/\alpha} \quad (6)$$

where  $X$  and  $Y$  are independent Gaussian processes with mean value equal to zero and  $E(X^2) = E(Y^2) = \sigma^2$ . In

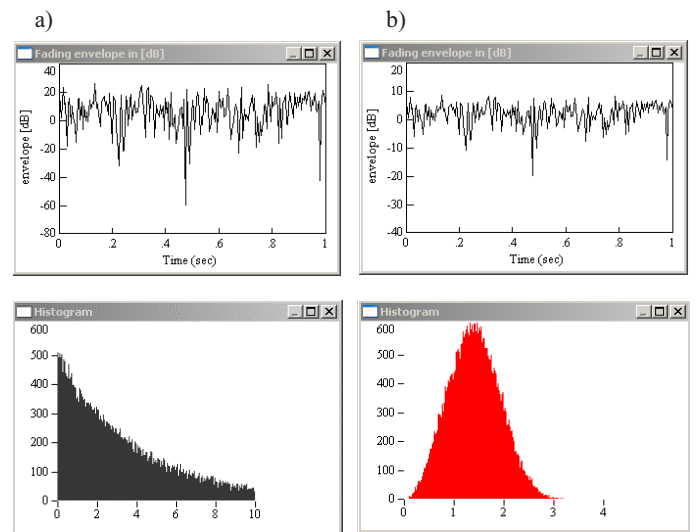


Fig. 3. The envelope and the histogram of Weibull fading process for  $\sigma^2 = 0.5$  and a)  $\alpha = 1$ , b)  $\alpha = 3$ .

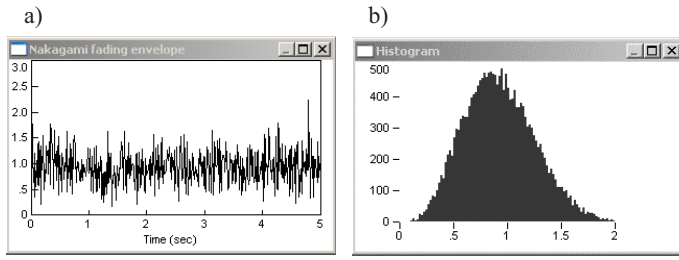


Fig. 4. The Nakagami- $m$  fading envelope (a) and its histogram (b) for  $m = 2$ .

this case the pdf of the envelope  $R$  can be written as

$$p_r(r) = \frac{\alpha r^{\alpha-1}}{r_0} \exp\left(-\frac{r^\alpha}{r_0}\right); \quad r \geq 0, \quad \alpha \geq 0 \quad (7)$$

where  $r_0 = E(R^\alpha) = |E(R^2)/\Gamma(1 + 2/\alpha)|^{\alpha/2}$  and  $\Gamma(m)$  is a gamma function [9]. In a special case, when  $\alpha = 2$  equation (7) describes the Rayleigh distribution and for  $\alpha = 1$ , it becomes an exponential distribution. If  $\alpha = 3.602$ , then the Weibull distribution is close to a normal [8]. Figure 2 shows a block which generates the Weibull fading envelope. Figure 3 shows a waveform and a histogram of the Weibull envelope respectively, using VisSim environment. Another generator of Weibull distributed random variables, in Mathcad environment, has been presented in [10].

One of the most versatile fading models is described by the Nakagami- $m$  distribution. It has a greater flexibility and accuracy in matching some experimental data than Rayleigh, log-normal, Weibull or Rician distributions. The Nakagami- $m$  fading distribution was first used for modeling ionospheric and tropospheric fast fading channels and has been widely adopted for multipath modeling in wireless communications. The Nakagami- $m$  distribution gave the best fit to some urban multi-path data. This distribution is a purely empirical model, not phenomenological. The probability density function of Nakagami- $m$  distributed random variables  $R$  is [4]

$$p_r(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} \exp\left(-\frac{m}{\Omega} r^2\right) \quad (8)$$

where  $\Omega = E(R^2)$  and the parameter  $m$  denotes the reciprocal value of the variance of  $R^2$  normalized to  $\Omega^2$ , i.e.

$$m = \Omega^2 / E(R^2 - \Omega^2) \geq 1/2$$

This is called central  $x^2$  distribution with  $m$  degrees of freedom. The Nakagami- $m$  distribution is often used to model multipath fading because it can model fading conditions that are either more or less severe than Rayleigh fading. When  $m = 1$ , the Nakagami- $m$  distribution becomes the Rayleigh distribution. When  $m = 0.5$ , it becomes a one-sided Gaussian distribution, and when  $m = \infty$ , it becomes a no fading channel. For integer  $m$ , we can write the envelope of received signal as

$$R = \sqrt{\sum_{i=1}^m X_i^2 + Y_i^2} \quad (9)$$

where  $X_i$  and  $Y_i$  are independent Gaussian processes with the mean value equal to zero and  $E(X_i^2) = E(Y_i^2) = \sigma^2$ . Figure 4 shows the Nakagami- $m$  fading envelope and its histogram.

Another model of Rayleigh fading are Jake's and modified Jake's models [3], [11], that are based on summing of sinusoids. The Jake's model produces a signal which has the Doppler power spectrum. It is a deterministic method, which simulates time-correlated Rayleigh fading waveforms. The model assumes that a signal propagates through  $n$  equal-strength paths and arrives at a moving receiver with uniformly distributed arrival angles. Therefore this signal experiences a Doppler shift.

The fading channel simulator, created using this model, was built in LabVIEW environment. Its front panel is shown in Figure 5. It contains several functional blocks for specification of simulation, determination of fading channel model and visualization of the results (waveform graphs, output data). The presented Virtual Instrument (VI) generates the Rayleigh and Weibull distributions fading profiles. The user specifies input data like fading variance and shape parameter for Weibull fading. The VI applies a user-defined profile to fade the complex baseband modulated waveform. The simulator performs QAM modulation. The user has to specify the QAM system parameters, i.e. the  $M$ -ary number of modulation, which is the number of distinct states that represent symbols in the complex baseband modulated waveform, number of message symbols and type of pulse shaping filters applied by the QAM. Simulations can be carried out for determined values of a symbol rate and the specific Doppler spread. Results of the simulation are shown in form of waveform graphs. The VIs, contained in the area of graphs, show a fading envelope, histograms of a fading envelope and a instantaneous phase, In-phase and Quadrature-phase (IQ) profile, waveforms of real and imaginary parts of faded signal, constellations of transmitter and receiver, power spectral densities of transmitter, receiver and faded profile, instantaneous power for both transmitter and receiver.

#### IV. GENERATION OF CORRELATED RANDOM VARIABLE

The time-varying nature of the channel  $C[\tau(t), t]$  is modeled as a wide-sense stationary random process in  $t$  with an autocorrelation function [1]

$$R_c(\tau_1, \tau_2, \Delta t) = E[C^*(\tau_1, t)C(\tau_2, t + \Delta t)] \quad (10)$$

where  $C^*(\tau_1, t)$  denotes the conjugate value.

A correlated Gaussian variable can be generated from an uncorrelated Gaussian variable using a linear transformation or filtering [1], [12], [13]. The coefficient of the linear transformation can be obtained from the specified correlation function of the output.

The correlated complex Gaussian variables  $W$  with a corresponding normalized covariance matrix given by

$$K_g = \begin{pmatrix} 1 & \rho_{g1,2} & \dots & \rho_{g1,N} \\ \rho_{g2,1} & 1 & \dots & \rho_{g2,N} \\ \vdots & \vdots & \dots & \vdots \\ \rho_{gN,1} & \rho_{gN,2} & \dots & 1 \end{pmatrix} \quad (11)$$

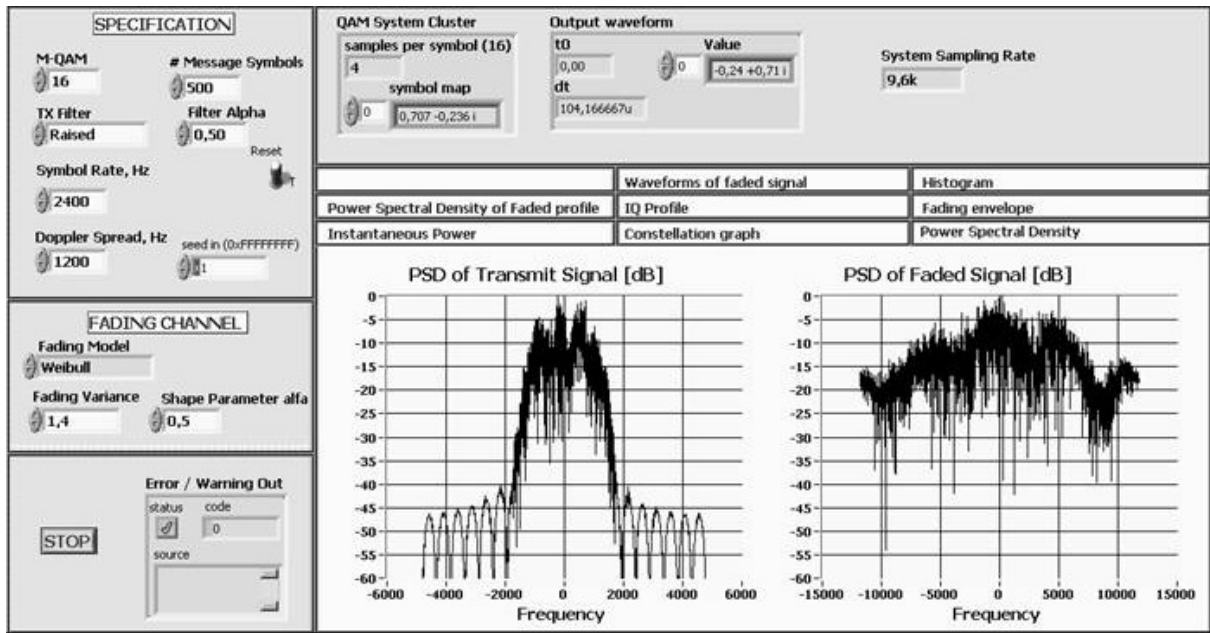


Fig. 5. The simulator of fading channel applied a QAM signal – the front panel of virtual instrument in LabVIEW.

can be generated from uncorrelated complex Gaussian variables  $V$  by employing Cholesky decomposition.

The cross-correlation coefficient  $\rho_{gi,j}$  between the complex Gaussian samples  $v_i$  and  $v_j$  is given by

$$\rho_{gi,j}^2 = \frac{u_1^2 + u_2^2}{u^2} \quad (12)$$

where  $E(X_i^2) = E(Y_i^2) = E(X_j^2) = E(Y_j^2) = u$ ,  $E(X_i X_j) = E(Y_i Y_j) = u_1$ ,  $E(X_i Y_j) = -E(Y_i X_j) = u_2$  and  $Y_i$ ,  $X_i$  are the quadrature and in-phase components of the complex Gaussian sample respectively.

The procedure for generating the  $N$  correlated Gaussian samples is summarized as follows [12], [13]:

- Determine the Cholesky matrix  $S$  corresponding to  $K_g$  (the colouring matrix  $S$  is the lower triangular matrix such that  $SS^T = K_g$ , where  $S^T$  represents the transpose of  $S$ ).
- Generate  $N$  uncorrelated complex Gaussian vector  $V = \{v_1, v_2, \dots, v_N\}$  each with variance  $\sigma_g^2$ .
- Calculate correlated complex Gaussian vector using  $W = SV$ .

Using the desired normalized covariance matrix of the Rayleigh envelopes expressed as

$$K_r = \begin{pmatrix} 1 & \rho_{r1,2} & \dots & \rho_{r1,N} \\ \rho_{r2,1} & 1 & \dots & \rho_{r2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{rN,1} & \rho_{rN,2} & \dots & 1 \end{pmatrix} \quad (13)$$

where  $\rho_{ri,j}$  is the cross-correlation coefficient between the Rayleigh samples, it is possible to determine the corresponding correlation matrix of the complex Gaussian samples.

The exact analytical relationship between  $\rho_{ri,j}$  and  $\rho_{gi,j}$  is

given by [12], [13]

$$\rho_{ri,j} = \frac{(1 + |\rho_{gi,j}|) E_i \left( \frac{2\sqrt{|\rho_{gi,j}|}}{1 + |\rho_{gi,j}|}, \frac{\pi}{2} \right) - \frac{\pi}{2}}{2 - \frac{\pi}{2}} \quad (14)$$

where  $E_i(\eta, \frac{\pi}{2})$  denotes the complete elliptic integral of the second kind with modulus  $\eta$  [12].

Equality (14) can be calculated by the use of numerical methods, namely polynomial approximation (to evaluate the elliptical integrals). In this way, all elements of  $K_r$  can be mapped to corresponding elements in  $K_g$ . The  $N$  envelopes of the Gaussian samples in  $W$  correspond to Rayleigh random variables  $(R'_1, R'_2, \dots, R'_N)$  with normalized covariance matrix  $K_r$  and equal variance

$$\sigma_r^2 = \left(2 - \frac{\pi}{2}\right) \frac{1}{2} \sigma_g^2 \quad (15)$$

The desired Rayleigh envelopes  $(R_1, R_2, \dots, R_N)$  are created from the samples  $(R'_1, R'_2, \dots, R'_N)$  by evaluating  $R_i = A_i R'_i$  where  $A_i = \frac{\sigma_{ri}}{\sigma_r}$ . The correlated Rayleigh random variables generated via the presented method can be used to generate the multivariate Weibull random variables with arbitrary correlation matrix [7]. The desired Weibull correlation coefficient matrix is expressed as

$$K_w = \begin{pmatrix} 1 & \rho_{w1,2} & \dots & \rho_{w1,N} \\ \rho_{w2,1} & 1 & \dots & \rho_{w2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{wN,1} & \rho_{wN,2} & \dots & 1 \end{pmatrix} \quad (16)$$

Therefore, the correlation coefficient of any pair of Rayleigh random variables in terms of the desired Weibull coefficient is [7]

$$\rho_{ri,j} = \frac{\pi}{4 - \pi} \left[ (1 - \rho_{wi,j})^2 F \left( \left[ \frac{3}{2}, \frac{3}{2} \right], 1, \rho_{wi,j} \right) - 1 \right] \quad (17)$$



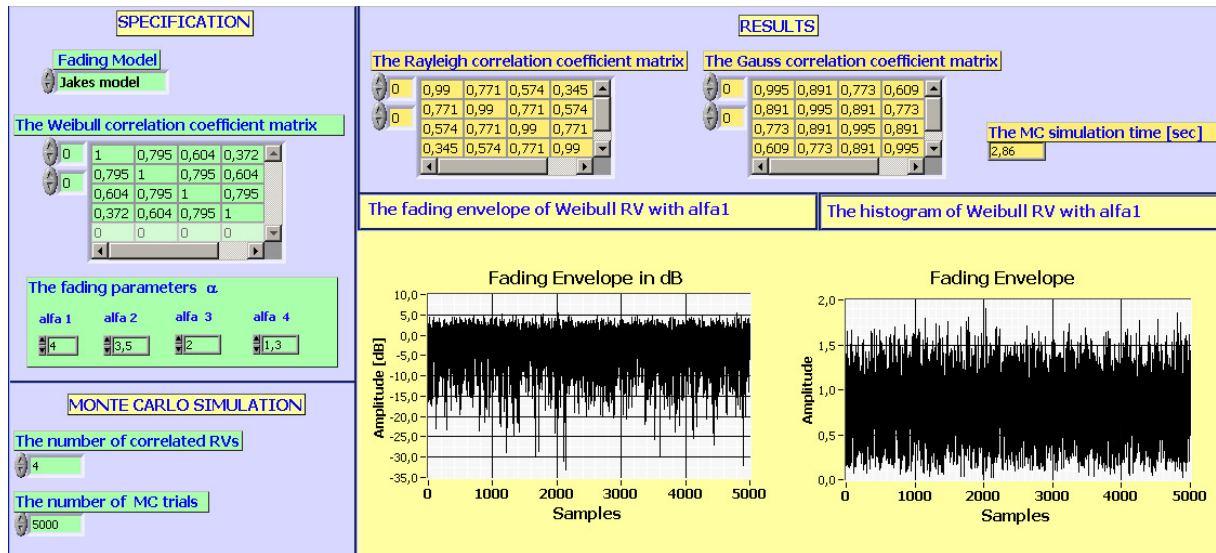


Fig. 6. The generator of correlated Weibull fading envelopes – the front panel of virtual instrument in LabVIEW.

where  $F(a, d, z)$  is the ordinary Gaussian hypergeometric function [9].

A simpler approximate relationship between  $\rho_{ri,j}$  and  $\rho_{wi,j}$  can be obtained using a least-squares polynomial fit to (16), expressed as

$$\rho_{ri,j} \cong 0.10(\rho_{wi,j})^2 + 0.89\rho_{wi,j} \quad (18)$$

The Weibull random variables are obtained from the correlated Rayleigh random variables with desired fading parameter  $\alpha$  using the transformation  $W = R^{2/\alpha}$ .

The front panel of the virtual fading generator, obtained using above procedures is shown in Fig. 6. This VI generates the correlated Rayleigh and Weibull random variables, applying the Monte Carlo Simulation (MCS). The simulation was carried out for the arbitrary Weibull correlation coefficient matrix specified by

$$K_w = \begin{pmatrix} 1 & 0.795 & 0.604 & 0.372 \\ 0.795 & 1 & 0.795 & 0.604 \\ 0.604 & 0.795 & 1 & 0.795 \\ 0.372 & 0.604 & 0.795 & 1 \end{pmatrix} \quad (19)$$

This correlation matrix is quite similar to that obtained from empirical data [2]. The fading parameter vector is given as  $\alpha = [4, 3, 5, 2, 1.3]$  and  $r_0 = [4, 3, 2, 1]$ . The Figure 7 presents

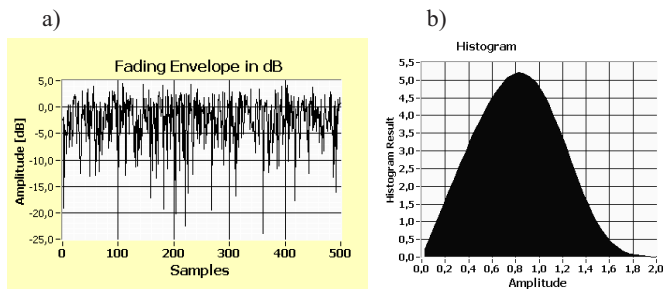


Fig. 7. The Weibull fading envelope (a) and its histogram (b) for  $\alpha = 4$ .

the correlated Weibull fading envelope and its histogram for the chosen value of fading parameters.

## V. CONCLUSION

Radio propagation characterization and modeling are very important for the communications engineers. Without proper knowledge of channel models, a wireless system would never be developed. The mobile radio propagation channel introduces fundamental limitations of the performance of any wireless communication systems.

In this paper an overview of modeling the fading in communication channel is presented. Especially, a simple and efficient procedure for the generation of uncorrelated and correlated Rayleigh, Weibull, Nakagami- $m$  fading channels are described. VisSim and LabVIEW software are simple and straightforward tools used for investigation of the concept of fading. Mathematical models make it easier to understand the relationships between the different parameters involved in fading process. The presented results can be applied to analyze the performance of digital transmission over fading channels.

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