

# Resource and Task Allocation Algorithm for WAN-based Distributed Computing Environment

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**Abstract**—In the paper an approximate algorithm for optimizing of distributed computing WAN network is proposed. Distributed computing systems become the common tools in different kind of business, science and even entertainment. In order to minimize processing time of data and utilize spare resources available on remote systems, many companies and institutions decide to build and maintain own wide area networks (WAN) for ensuring reliable and secure distributed processing of data. Design of WANs is concerned with solving different optimization problems, like routing assignment, capacities of channel selection, resource (i.e. servers, management centre) allocation. Due to peculiar structure of wide area networks and nature of protocols, proper optimization methods and algorithms should be constructed for WAN-based distributed computing systems. In the paper the model of the distributed environment, built on WAN infrastructure is presented. Then, the optimization problem for routing assignment, channel capacities assignment and grid management center (data repository) allocation is formulated. Finally, an approximate algorithm is presented for formulated problem. Proposed algorithm, observations and conclusions should effect in improving of distributed computing systems design.

**Keywords**—Capacity and flow assignment problem, distributed computing, resource management, wide area networks.

## I. INTRODUCTION

**H**UGE amount of data which must be processed and analyzed in nowadays computer systems entails high demands for efficiency and computational power. In many cases single machine is not enough, and the cluster solutions are utilized. Meanwhile, a plenty of unused computational power is available on hosts and computers connected to private and public computer networks. It is shown, that usual server (web server, DNS server) utilizes about few percent of its resources during normal work. The conception of distributed computing arose in order to utilize free computational power of machines located in different networks and different geographical areas. Optimization and management problems in distributed computing are well known in literature. The nature and features of distributed

systems are described in [1]. Some common optimization problems and solutions may be found in [12], [22]. An example of distributed computing systems are grid networks [5], [6], [7], [14], [20], [23], [24]. Grid (distributed cluster) is a set of distributed machines or networks – called grid nodes. The Grid concept is coordinated resource sharing and problem solving in dynamic, multi-institutional virtual organizations [8].

One of the important problem that must be solved for grid networks is communication between nodes. Since nodes are located in far locations, then the infrastructure of wide area networks is often used for data transfer between grid nodes. Communication services are one of the main design features required by grid environment [2]. The network connecting grid nodes should provide the grid with important parameters such as latency, bandwidth, reliability, fault-tolerance, jitter control and security [2]. Since it is often hard to ensure those parameters over the public wide area networks, the private WANs may be built for grid usage.

Design and optimization process of wide area networks involves different kinds of problem which must be considered. One of them is the flow assignment (FA) problem [4], [9], strictly connected with routing in the network. FA problem may be solved as a part of design process or may be solved in existing networks (dynamic routing) in order to optimize flow routes and channels utilization. Designing of new WAN requires to solve the capacity and flow assignment (CFA) problem, with two design variables: channel capacities and flow routes (routing) [10], [11], [13], [21]. The task is to select those variables in order to minimize the criterion function, for example the total average delay per packet in wide area network or the capacity leasing cost [9].

Since the important part of network traffic are data exchanged between users and servers, then allocation of servers over the network has a critical impact on the quality of service (QoS) in the whole network. The servers' allocation and replication problems, often called resource allocation (RA) problems, are well known in the literature. RA problems may be divided into two groups: single servers' allocation problems [15] and servers' replica allocation problems [3], [16], [17], [18], [19]. They are considered alone or together with CFA problems. In the grid environment the computational tasks are sent to grid nodes from task management centre and results must be sent to repository. Then, the problem of optimal allocation of the repository and the task management centre should be considered in the designing process of WAN-based distributed computing systems.

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## II. PROBLEM FORMULATION

Consider distributed environment system dedicated for proceeding of computational data. In each time period (in example each second) the portion of data is generated and requires to be processed. Denote than computational outlay needed in each time period is similar. Each generated computational task may be divided into separable subtasks (blocks). Then subtasks are sent to nodes of grid network in order to be processed. During processing of particular block there is no necessity to communicate with nodes processing other blocks. When processing is finished, result data must be sent backward to the source.

Data processing in the distributed environment is organized in the following way:

- realization of computational tasks is managed by the computing management centre (CMC). Management centre divides tasks into blocks, transmits blocks to grid nodes, receives and collects result data for each block,
- the grid node, in which CMC is located, also takes part in blocks processing. Blocks and results transferred between this node and CMC are not sent through the network,
- each grid node may communicate only with computational centre node. During proceeding of each block information from other blocks are not necessary,
- results from computations are transmitted and collected in repository located in computing management centre.

Processing block of data is connected with specified processing cost. Costs are defined for each of the blocks. Management centre is located in one of the grid nodes, since it is source or destination of all data transmitted in the network, then the proper allocation of CMC has a critical impact on the quality of service in the network. Maintaining of CMC in the node generates also some maintaining cost in each time period. Minimizing of total processing cost and maintaining cost is one of the objectives of grid optimization.

The computational power resources (also called resource capacity [5]) of grid nodes are limited and determined for each node. It is denoted that the total resources capacity of grid is enough to presses all generated blocks.

For each block that has to be processed the following values are determined:

- size of the block. Data of this size must be sent from CMC to the computing node chosen for the block,
- size of results generated during block processing. Data of this size must be sent from processing node to CMC when processing is finished.

It is assumed that communication network for grid system is the packed switched network. Channel allocation (network topology) is given, capacity of each channel must be assigned in optimization process. Total cost of capacity leasing is limited – given as optimization constraint.

Considered problem is formulated as follows:

Given:

- number of grid nodes, number of channels of wide area network,
- for each node: computational capacity of node,
- for each channel the set of possible capacities and costs (i.e. cost-capacity function),

- list of candidate nodes for Computing Management Centre, for each candidate node the value of maintaining cost,
- number of blocks which must be processed in each time period,
- for each block: size of block, size of result data, computational outlay needed to process block, costs of processing at each computing node,
- budget of the network.

Minimize:

- linear combination of quality of service in the network and the total computing cost (total cost of data processing and maintaining cost of management centre).

Over:

- CMC allocation,
- block allocation at computing nodes,
- channel capacities,
- flow routes (routing).

Subject to:

- channel capacity constraints
- multicommodity flow constraints
- computing capacity of nodes constraints

We assume that channels' capacities can be chosen from discrete sequence defined by international ITU-T (International Telecommunication Union – Telecommunication Sector) recommendations. Then, the formulated above problem is NP-complete, as more general that capacity and flow assignment problem [13], [15].

## III. MODEL OF DISTRIBUTED COMPUTING SYSTEM

Distributed computing system consists of  $n$  computation nodes, located in geographically distant area. Nodes are connected with  $b$  channels generating a wide area network. Channels are realized on communication lines leased from telecoms. For each channel  $i$  there is the set  $C^i = \{c_1^i, \dots, c_{s(i)}^i\}$

of alternative values of capacities from which exactly one must be chosen to build the WAN. Let  $D^i = \{d_1^i, \dots, d_{s(i)}^i\}$  be the set of leasing costs corresponding to channel capacities from the set  $C^i$ . Let  $x_j^i$  be the discrete decision variable, connected with capacity choice for channel  $i$ , defined as follows:

$$x_j^i = \begin{cases} 1, & \text{if the capacity } c_j^i \text{ is assigned to channel } i; \\ 0, & \text{otherwise.} \end{cases}$$

Since exactly one capacity from the set  $C^i$  must be chosen for channel  $i$ , then the following condition must be satisfied:

$$\sum_{j=1}^{s(i)} x_j^i = 1 \quad \text{for } i = 1, \dots, b. \quad (1)$$

Let  $X_r$  be the set of all variables  $x_j^i$  which are equal to one.  $r$  is the number of iteration, since in successive chapters we propose approximate iterative algorithm.

Let  $y_h$  be the discrete decision variable, connected with allocation of management node, defined as follows:

$$y_h = \begin{cases} 1, & \text{if the management centre is located in node } h; \\ 0, & \text{otherwise.} \end{cases}$$

Let  $Y_r$  be the set of all variables  $y_h$  which are equal to one. Since the management centre is located in one node only, the following condition must be satisfied:

$$\sum_{h=1}^n y_h = 1. \quad (2)$$

Let  $u_h$  be the cost of maintaining of the management centre in the node  $h$ .

Let  $t$  denote the number of blocks, which must be proceeded in the considered time period. The following given data are connected with each block:

- $v_l$  is the size of block  $l$ . It means that in each time period the data of size  $v_l$  must be sent from management node to the proper computing node;
- $w_l$  is the size of data generated as result of proceeding block  $l$ . Data of size  $w_l$  must be sent from computing node to the management node at each time period;
- $p_l$  is the computational outlay needed to proceed block  $l$  – measured in [instructions];
- $q_m^l$  is the cost of proceeding block  $l$  in the computing node  $m$ ,  $Q^l = \{q_1^l, \dots, q_n^l\}$  is the set of proceeding costs of block  $l$  in all computing nodes.

Let  $z_m^l$  be the discrete decision variable, connected with computing node choice for proceeding block  $l$ :

$$z_m^l = \begin{cases} 1, & \text{if block } l \text{ is proceeded in node } m \\ 0, & \text{otherwise} \end{cases}$$

Let  $Z_r$  be the set of all variables  $z_m^l$  which are equal to one. Since each of the blocks must be proceeded exactly in one node, then the following condition must be satisfied:

$$\sum_{l=1}^t \sum_{m=1}^n z_m^l = t. \quad (3)$$

Each of the computing nodes has a limited computing power. Let  $e_m$  be the computing power (also called computational capacity) of computing node  $m$ , given in [instructions per second] (IPS). In order to guarantee that the optimization problem has a solution, the following condition must be satisfied for given data:

$$\sum_{m=1}^n e_m > \sum_{l=1}^t p_l. \quad (4)$$

Let  $r_{mk}$  be the average traffic rate sent from node  $m$  to node  $k$  in each time period. In packet switched networks the flow between nodes is realized as a multicommodity flow. Since we denote, that in the considered networks only proceeding block and results of proceedings block are sent (there in no other network traffic) then  $r_{mk}$  consists of packed exchanging between computing nodes and management node only.  $r_{mk}$  may be calculated as follows:

$$r_{mk} = \sum_{l=1}^t (z_m^l y_k w_l + z_k^l y_m v_l).$$

The triple of sets  $(X_r, Y_r, Z_r)$  is called a selection. Let  $\mathfrak{R}$  be the family of all selections. The selection  $(X_r, Y_r, Z_r)$  defines the unique wide area network and distributed computer system, because:

- $X_r$  determines the values of capacities for channels of the WAN,
- $Y_r$  determines the allocation of CMC at the node of WAN.
- $Z_r$  determines the blocks' allocation at the computing nodes of distributed grid.

Let  $T(X_r, Y_r, Z_r)$  be the minimal average delay per packet in the wide area network in which values of channel capacities are given by set  $X_r$  and traffic requirements are given by sets  $Y_r$  and  $Z_r$  (depend on management node allocation and assigning of block to computing nodes).  $T(X_r, Y_r, Z_r)$  can be obtained solving a multicommodity flow problem in the network [13], [21]:

$$T(X_r, Y_r, Z_r) = \min_{\underline{f}} \frac{1}{\gamma} \sum_{x_j^i \in X_r} \frac{f_i}{x_j^i c_j^i - f_i}$$

subject to:

- $\underline{f}$  is a multicommodity flow satisfying the traffic requirements  $r_{mk}$  given by  $Y_r$  and  $Z_r$ ,
- $f_i \leq x_j^i c_j^i$  for every  $x_j^i \in X_r$ ,
- $f = [f_1, \dots, f_b]$  is the vector of multicommodity flow,  $f_i$  is the total average bit rate on channel  $i$ , and  $\gamma$  is the total packet rate generated and sent through the network by computational nodes and management node.

Let  $A(Y_r, Z_r)$  be the computing cost, composed of proceeding costs of block at computational nodes and cost of maintaining of management node:

$$A(Y_r, Z_r) = \sum_{h=1}^n y_h u_h + \sum_{l=1}^t \sum_{m=1}^n q_m^l.$$

Then, the objective function for the channel capacities, routing assignment and location of management centre and task scheduling problem is defined as follows:

$$OBJ(X_r, Y_r, Z_r) = \sigma T(X_r, Y_r, Z_r) + A(Y_r, Z_r).$$

Let  $B(X_r)$  be the regular leasing cost of channel capacities, given with the formula:

$$B(X_r) = \sum_{x_j^i \in X} x_j^i d_j^i.$$

Let  $\beta$  be the network budget (maximal cost of channel capacity leasing).

Taking into account the above assumptions, the optimization problem is formulated in the following way:

$$\min_{(X_r, Y_r, Z_r)} OBJ(X_r, Y_r, Z_r) \quad (5)$$

Subject to:

$$(X_r, Y_r) \in \mathfrak{R} \quad (6)$$

$$B(X_r) \leq \beta \quad (7)$$

$$\sum_{l=1}^t z_m^l p_l < e_m \text{ for each } m = 1, \dots, n. \quad (8)$$

#### IV. ALGORITHM

The considered problem is NP-complete, as more general than classical CFA problem, which is NP-complete [23]. Optimal solution of NP-complete problem may be found using exact algorithms. In the papers [15], [18] exact algorithms, based on branch-and-bound method, for different WAN optimization problems were proposed. Since considered grid optimization problem includes aspects of WAN network optimization, then exact algorithm for problem (5)-(8) may be based on those algorithms. Exact algorithms give optimal solutions of the problem, but they are useless for bigger wide area networks (over 25 nodes). Approximate algorithm gives near-optimal solution and the computational time is significantly shorter. Approximate algorithms for problem (5)-(8) may be based on top-down method or on intelligent computational methods (i.e. evolutionary method).

Proposed algorithm consists of two phases. In the first one, called initial phase, an acceptable solution of the problem is found. Acceptable solution is the selection  $(X_r, Y_r, Z_r)$  and flow vector  $\underline{f}$ , satisfying conditions (6) - (8) and it is the starting point for optimization process. In case that the problem has no solution (i.e. it is impossible to build the network satisfying flow demands with given budget restriction), it is discovered during this stage and algorithm finishes. The second phase of an algorithm is optimization phase, while sub-optimal solution is being found.

##### A. Initial Phase

The goal on this stage is to verify whether the considered problem with given constraints has the solution. If it has, the initial solution is being found, as the starting point for optimization.

Three main tasks appear during initial phase: choosing allocation for computing management centre, tasks allocation over the computing nodes, capacities of channels and flow assignment.

We propose few criteria (strategies) for choosing the node for CMC allocation:

- Maximal computing power of candidate node;

The grid node maintaining CMC also takes part in blocks processing. Moreover, block processed in CMC node and results of them are not sent through the network. Locating CMC in the node of maximal computing power allows to minimize the data transfer in the network. To valuate quality of

node, according to this criterion, we use the value of computational capacity  $e_m$ .

- Maximal capacity of node;

Since all blocks (except those processed by CMC node) must be sent to grid nodes and results must be sent back, links adjacent to CMC node must ensure enough capacity.

Let  $P_m$  be the sum of capacities of all channels adjacent to node  $m$ :

$$P_m = \sum_{x_j \in X_r} x_j^i c_j^i p_m(i) \quad (9)$$

where

$$p_m(i) = \begin{cases} 1, & \text{if } i\text{-th channel is adjacent to } m\text{-th node} \\ 0, & \text{otherwise} \end{cases}$$

To valuate quality of node, according to this criterion, we use the value of  $P_m$ .

- Location of node;

In order to minimize the traffic in the whole network, it is beneficial to locate the CMC in the centre of the grid network.

Let  $v_{gh}$  be the distance, in hops, between node  $g$  and node  $h$ . It means that  $v_{gh}$  is the minimal number of channels between nodes  $g$  and  $h$ . Let  $V_g$  be the distance between node  $g$  and all other nodes of distributed grid, defined as follows:

$$V_g = \sum_{h=1}^n v_{gh}. \quad (10)$$

Choosing the node for allocating CMC we should choose such nodes  $g$ , for which the value of  $V_g$  is minimal.

- Random method

Initial allocation of the CMC may be chosen randomly.

Another goal of an initial phase of an algorithm is task allocation to the computing nodes. We propose two strategies for initial task allocation:

- Regular tasks distribution approach.

In this strategy, tasks are allocated to the grid nodes in proportion to the computational power of each grid node.

- Regular traffic distribution approach.

Task are being allocated to grid nodes in proportion to the capacity of node (the sum of capacities of all channels adjacent to the node). The constraint of the node computational power (4) must be satisfied.

Finally, capacity and flow (CFA) assignment problem for the initial phase may be solved using one of the approximate or exact algorithms [10], [16], [18]. CFA problem is well known in the literature and also was the subject of previous work.

##### B. Optimization Phase

We start from the initial selection obtained in the Initial Phase. Then, in consecutive iterations we try to improve the solution by changing CMC allocation node, tasks allocation at grid nodes, routing and channel capacities. Algorithm finishes when there is no possibility of improving present solution.

To get the best choice we have to test all possible pairs of variables  $y_h \in Y_r, y_m$  or  $x_j^i \in X_r, x_s^i$  or  $z_m^l \in Z_r, z_h^l$  using a local optimization criterion. Because of the different nature of the variables denoting channel capacity choice and the host allocation choice, we have to formulate three different criteria.

*Proposition 1.* If the selection  $(X_t, Y_t)$  is obtained from the selection  $(X_r, Y_r)$  by complementing the variable  $x_j^i \in X_r$

where  $j < s(i)$  by another variable  $x_s^i \in X_t$  then only the channel capacity change is being considered. We do not change the CMC allocation, and the traffic requirements between nodes do not change. We can use the following local optimization criterion on variables  $x_j^i$  where  $j < s(i)$ :

$$\Delta_{js}^i = \begin{cases} Q(X_r, Y_r, Z_r) - \frac{\sigma}{\gamma} \left( \frac{f_i}{c_j^i - f_i} - \frac{f_i}{c_s^i - f_i} \right) & \text{for } c_s^i > f_i \\ \infty & \text{for } c_s^i \leq f_i \end{cases}$$

*Proposition 2.* If the selection  $(X_t, Y_t)$  is obtained from the selection  $(X_r, Y_r)$  by complementing variable  $y_h \in Y_r$  by another variable  $y_m \in Y_t$  then only allocation of CMC is being changed. So, the traffic requirements between nodes change, channels' capacities do not change and blocks' allocation at computing nodes do not change. Then, to evaluate the pair  $(y_h, y_m)$  we can use the following criterion:

$$\delta_{hm}^{CMC} = \begin{cases} \frac{\sigma}{\gamma} \sum_{x_j^i \in X_r} \frac{\tilde{f}_i}{x_j^i c_j^i - \tilde{f}_i} + (A(Y_r, Z_r) - u_h + u_m) & \text{if } \tilde{f}_i < x_j^i c_j^i \text{ for } x_j^i \in X_r \\ & \text{and } P_m \geq \sum_{l=1}^l (w_l + v_l) \\ \infty & \text{otherwise} \end{cases}$$

where

$$\tilde{f}_i = \sum_{h=1}^n \sum_{\pi_{hm}^a \in \Pi_{hm}} V_{hm}^a(i) \tilde{f}_{hm}^a$$

$$\tilde{f}_{hm}^a = \frac{m_{hm}^a}{m_{hm}} r_{mk}, \quad m_{ej} = \sum_{\pi_{ej}^a \in \Pi_{ej}} m_{ej}^a$$

$$m_{hm}^a = \min_{i \in \pi_{hm}^a} (x_j^i c_j^i) \text{ for } x_j^i \in X_r$$

$\Pi_{hm}^a$  denotes the  $a$ -th path from node  $h$  to node  $m$ ,

$$V_{hm}^a(i) = \begin{cases} 1 & \text{if } i\text{-th channel belong to path } \Pi_{hm}^a; \\ 0 & \text{otherwise,} \end{cases}$$

$\Pi_{hm}$  denotes the set of all paths from node  $h$  to node  $m$ .

$\tilde{f}$  is the 'new' traffic flow in the network, after reallocating the CMC. After reallocation all routes in the network must be redirected from paths [old CMC node; other nodes] to paths [new CMC node; other nodes]. It is simply calculated as follows. For each node we find all possible routes (paths) from that node to new CMC node. They are denoted by the set  $\Pi_{hm}$ . Then we allocate the traffic along all found routes, proportionally to they residual capacities.

*Proposition 3.* If the selection  $(X_t, Y_t)$  is obtained from the selection  $(X_r, Y_r)$  by complementing variable  $z_m^l \in Z_r$  by another variable  $z_h^l \in Z_t$  then the allocation of  $l$ -th block is being changed. So, the traffic requirements between nodes change, channels' capacities and CMC location do not change. To evaluate the pair  $(z_m^l, z_h^l)$  we propose the following criterion:

$$\delta_{hm}^l = \begin{cases} \frac{\sigma}{\gamma} \sum_{x_j^i \in X_r} \frac{\tilde{f}_i}{x_j^i c_j^i - \tilde{f}_i} + (A(Y_r, Z_r) - q_m^l + q_h^l) & \text{if } \tilde{f}_i < x_j^i c_j^i \text{ for } x_j^i \in X_r \\ & \text{and } \sum_{l=1}^l z_m^l p_l < e_m \\ \infty & \text{otherwise} \end{cases}$$

where

$$\tilde{f}_i = f_i - f_{il}'' + \sum_{e=1}^n \sum_{\pi_{hm}^a \in \Pi_{hm}} V_{hm}^a(i) \tilde{f}_{hm}^a$$

$$\tilde{f}_{hm}^a = \frac{m_{hm}^a}{m_{hm}} r_{mk}, \quad m_{ej} = \sum_{\pi_{ej}^a \in \Pi_{ej}} m_{ej}^a$$

$$m_{hm}^a = \min_{i \in \pi_{hm}^a} (x_j^i c_j^i - f_i + f_{il}'') \text{ for } x_j^i \in X_r$$

$f_{il}''$  is the part of the flow at  $i$ -th channel. It corresponds only to the packets connected with  $l$ -th block.  $\Pi_{hm}^a$ ,  $\Pi_{hm}$  and  $V_{hm}^a(i)$  are defined like previously.

$\tilde{f}$  is the 'new' traffic flow in the network, after reallocating the  $l$ -th block. After reallocation packet connected with  $l$ -th block must be redirected from paths [old processing node for  $l$ -th block; CMC node] to paths [new processing node for  $l$ -th block; CMC node]. First we remove from the network traffic [old processing node for  $l$ -th block; CMC node]. It is given with  $f_{il}''$  value. Then, we find all possible routes (paths) from new processing node to CMC node. Paths are denoted by the set  $\Pi_{hm}$ . After that we allocate the traffic along all found routes, proportionally to they residual capacities.

Replacements of decision variables are made in order to obtain the distributed computing network with the possible least value of criterion function  $OBJ$ . We should choose such

pairs  $y_h \in Y_r, y_m$  or  $x_j^i \in X_r, x_s^i$  or  $z_m^l \in Z_r, z_h^l$ , for which the value of the criterion  $\delta_{hm}^{CMC}$ ,  $\Delta_{js}^i$  or  $\delta_{hm}^l$  is minimal.

### C. Calculation Scheme of an Algorithm

#### Phase I. Initial

**Step 1.1.** Choose node for computing management centre. Evaluate nodes using one of criteria defined in section IV.A. Choose node with maximal computing power  $e_m$ , maximal capacity of node  $P_m$  or the node with the best location according to criterion (10). Also combined criteria may be used, in example  $e_m/V_m$  or  $(e_m P_m)/V_m$ .

**Step 1.2.** Allocate tasks to the computing nodes, according to regular tasks distribution strategy or regular traffic distribution strategy.

**Step 1.3.** Solve the capacity and flow assignment problem in the WAN network with traffic requirements gives with CMC allocation and blocks' allocation at nodes. If CFA problem has no solution then we decide that problem (5)-(8) has no solution and algorithm finishes. Otherwise calculate value of objective function and remember it as  $OBJ_{\min}$ . Remember actual sets of decision variables as  $(X_{\min}, Y_{\min}, Z_{\min})$ . Go to phase II.

#### Phase II. Optimization

**Step 2.1.** Perform  $r = 0$ .  $(X_r, Y_r, Z_r) = (X_{\min}, Y_{\min}, Z_{\min})$ .

**Step 2.2.** Perform  $r = r + 1$ . Choose pair  $y_h \in Y_{r-1}, y_m$  or  $x_j^i \in X_{r-1}, x_s^i$  or  $z_m^l \in Z_{r-1}, z_h^l$ , for which the value of the criterion  $\delta_{hm}^{CMC}$ ,  $\Delta_{js}^i$  or  $\delta_{hm}^l$  is minimal. If there is no such pair for which  $\delta_{hm}^{CMC}$ ,  $\Delta_{js}^i$  or  $\delta_{hm}^l$  is less than  $OBJ_{\min}$  then stop,  $(X_{\min}, Y_{\min}, Z_{\min})$  is sub-optimal solution of problem (5)-(8). Otherwise swap values of variables of chosen pair.

**Step 2.3.** Solve the capacity and flow assignment problem in WAN, where traffic requirements are given by CMC allocation and blocks allocation at nodes. Calculate  $OBJ_r(X_r, Y_r, Z_r)$ . If  $OBJ_r < OBJ_{\min}$  (better solution is found), then assign  $(X_r, Y_r, Z_r) = (X_{\min}, Y_{\min}, Z_{\min})$  and  $OBJ_{\min} = OBJ(X_{\min}, Y_{\min}, Z_{\min})$ . Go to step 2.2.

## V. CONCLUSION

In the paper the conception of WAN-based distributed computing system model was presented. An optimization problem, consists in assignment of grid management centre allocation, network channels' capacities, flow routes and task assignment, was formulated. Then, an approximate two-phased algorithm for considered problem was proposed. The future work includes computational experiments with developed algorithm. An interesting goal of experiments is to investigate how the strategies chosen in steps 1.1 and 1.2 influence on the quality of approximate solution and computational time of an algorithm. Another direction of future work is to formulate problems with different criterion functions and constraints (i.e. with criterion composed of WAN maintain cost and computation cost of blocks).

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