

Small-signal transmittances of DC-DC step-down PWM converter in various operation modes

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Abstract: Small-signal transmittances: input-to-output and control-to-output of BUCK converter power stage working in CCM or DCM mode are discussed. Ideal converter case and converter with parasitic resistances are considered separately. Derivations of small-signal transmittances, based on different approaches to finding the converter averaged models, are presented and the results are compared. Apart from theoretical considerations, some results of numerical calculations are presented.

Key words: step-down converter, BUCK, CCM, DCM, averaged model, small-signal transmittance, switch-averaging, separation of variables

Selected symbols and abbreviations

A, B (with subscripts) – coefficients in expressions for transmittances H_g and H_d ,
 C – capacitance,
 C_Z – equivalent capacitance,
 D – diode,
 d_A, D_A – duty ratio and its steady-state value,
 f_S – switching frequency,
 G – load conductance,
 G_A – equivalent conductance defined by Eqn (22)
 H_g, H_d (with additional subscripts) – small-signal conductances: input-to-output and control-to-output,
 i (with subscripts in capital letters) – instantaneous values of currents,
 I (with subscripts in capital letters) – quiescent values of currents,
 I (with subscripts in small letters) – small-signal representations of currents in s-domain,
 K – controlled switch (transistor),
 L – inductance,
 M_V, M_I – voltage and current static transmittance,
 R – load resistance,
 R (with subscripts) – parasitic resistances,
 R_Z – equivalent parasitic resistance,
 T – coefficient in several expressions for transmittances,
 T_S – switching period,
 v (with subscripts in capital letters) – instantaneous values of voltages,
 V (with subscripts in capital letters) – quiescent values of voltages,
 V (with subscripts in small letters) – small-signal representations of voltages in s-domain,
 Z_W, Z_{WP} – impedances of the output subcircuit of converter,

θ – small-signal representation of duty ratio in s-domain,
CCM – continuous conduction mode,
DCM – discontinuous conduction mode.

1. Introduction

There is a great amount of applications of switch-mode DC-DC power converters, especially step-down (*BUCK*) converters based on PWM control. Circuit solutions, control methods and component parameters are steadily developed and discussed in many scientific presentations, textbooks, technical reports and application notes. The most known textbooks devoted to switch-mode DC-DC converters are [1] and [2]. The structure of power converter may be considered as proper connection of power stage and control circuit, as shown in Figure 1. Power stage of basic step-down converter discussed in the paper is depicted in Figure 2.

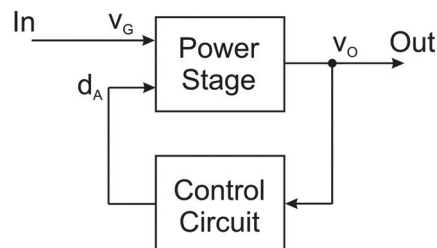


Fig. 1. General structure of switch-mode converter

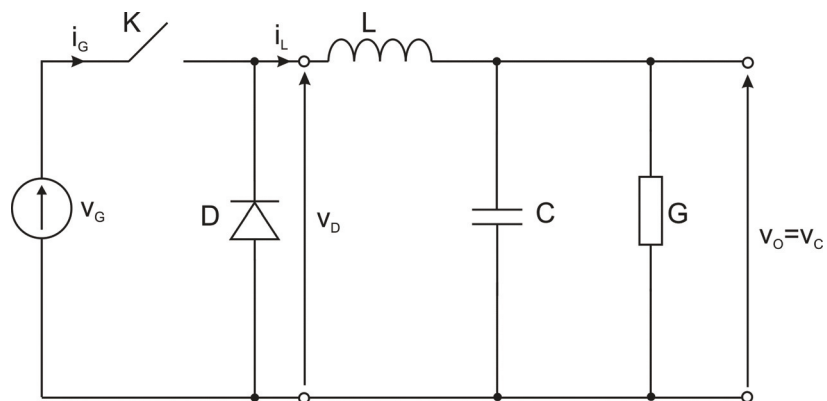


Fig. 2. Power stage of step-down (*BUCK*) converter

The efficient control circuit design is based on the knowledge of power stage dynamic characteristics. The most convenient form of description of power stage dynamics is a set of small-signal transmittances. Small-signal transmittances, being the object of this paper, may be considered as special case of power stage averaged models. Averaged models of switch-

mode power converters are extensively described in the literature and there are several methods of their derivation. Averaged models obtained by different methods are in many cases the same, but in some specific conditions may be different, for example, for discontinuous conduction mode (*DCM*). Differences in averaged models, and related small-signal transmittances, presented in various sources, result also from differences in assumed description of parasitic effects in converter components. It is possible, that some differences in averaged models are only apparent and are the result of different procedures of model derivation.

The main purpose of this paper is to compare small-signal transmittances of step-down (*BUCK*) converter obtained by different ways. In some sources, the averaged and small-signal descriptions have the form of equivalent circuits only, without analytical descriptions of respective transmittances. In such cases the formulas for small-signal transmittances are derived and discussed in the paper.

After general remarks about the derivation of averaged models and small-signal transmittances in Section 2, the presentations of small-signal transmittances of *BUCK* converter for *CCM* and *DCM* mode are given in Secs. 3 and 4 respectively. In both cases, ideal and nonideal converter models are considered separately and the illustrative numerical examples are presented. Descriptions of small-signal transmittances for continuous conduction mode (*CCM*) are more frequently presented in the literature than for *DCM*. In this paper, on the contrary, more attention is paid to discussion of models for *DCM* because the differences in models obtained by different methods are greater for *DCM* than for *CCM*.

2. Methods of averaged models and small-signal transmittances derivation

2.1. Averaged models of switching converters

Averaged models of switch-mode power converters describe relations between values of currents and voltages averaged over single switching period T_S . Averaged models assure sufficient accuracy in description of steady-state and slow transients in power stage corresponding to frequency range much smaller than switching frequency $f_S = 1/T_S$. This frequency range includes characteristic frequencies of power stage and the frequencies of external perturbations (changes of input voltage and load current) [1] – [5]. Averaged models are generally nonlinear and after linearization their small-signal versions are obtained as shown in Section 2.2. Averaged models of switching converters are described in many references and one may find the description of three principal methods of their derivation. The first is based on “state-space averaging” [1, 6-8]. It is mathematically elegant and formal method but it seems to be less convenient to use as compared to other methods [1, 3, 9]. The most popular method is based on “switch-averaging” approach. According to opinions presented in many sources (e.g. [1-3, 10, 11]) this method is simple and clear and leads to models easy to implement in widely known circuit simulators. The idea of switch averaging in application to power converter modeling has been proposed over 20 years ago [9, 12] but its most clear presentation may be found in textbook of Erickson and Maksimowic [1]. The first step to finding averaged model of converter containing two switches is decomposition of power stage of converter into two subcircuits – the first composed of two ideal switches and the second containing other compo-

nents of power stage. Such decomposition of basic *BUCK* power stage is shown in Fig. 3, where i_1, i_2, v_1, v_2 are currents and voltages of switching subcircuit (*A*). The second step is to replace the original switching subcircuit *A* by its averaged model A_1 describing dependencies between values of currents and voltages averaged over single switching period: $i_{1S}, i_{2S}, v_{1S}, v_{2S}$. The form of averaged model of switching subcircuit depends on the operation mode (*CCM* or *DCM*).

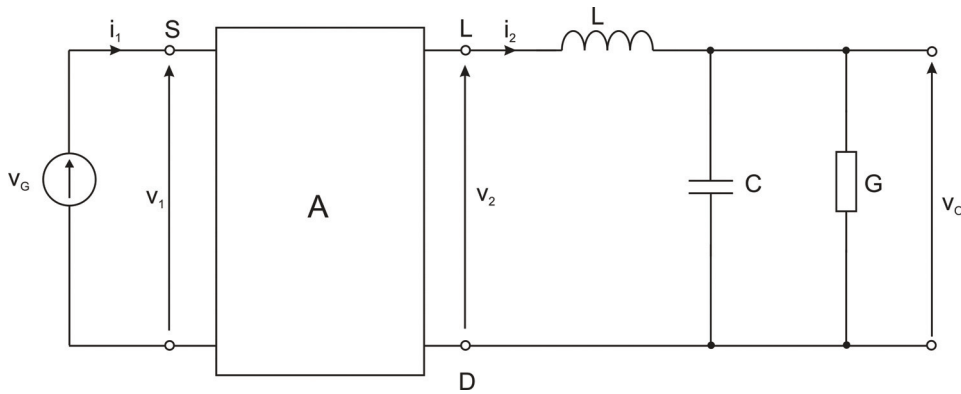


Fig. 3. The decomposition of *BUCK* converter scheme in switch-averaging approach

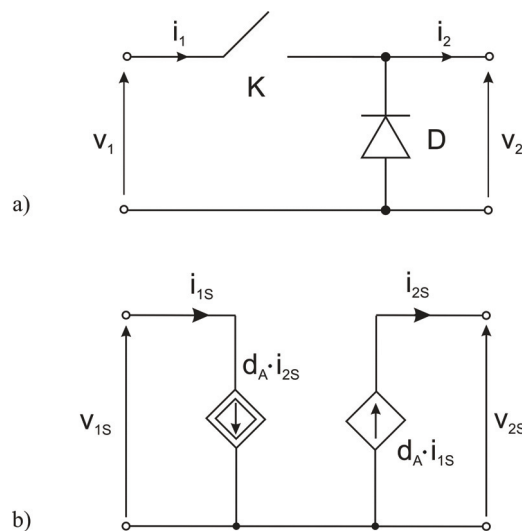


Fig. 4. Switching circuit of *BUCK* converter: a) original scheme; b) equivalent circuit according to [1]

The original form and averaged model of switching subcircuit of ideal *BUCK* converter working in *CCM*, according to [1] and [5] are presented in Figure 4 a) and b). It has been shown however, that the derivation of averaged models of switching converters based on switch averaging approach contains informalities, which may, in some cases, lead to inaccuracies or even errors [6, 13].

Another method of the derivation of averaged models of switching power converters, proposed recently [13, 14] is based on so called separation of variables. The resulting averaged models of ideal *BUCK* converter working in *CCM*, obtained with three methods listed above, are strictly the same and may be represented by equivalent circuit shown in Figure 5. In the case of converter in *DCM* or description including parasitic resistances of converter components, the averaged models obtained by switch averaging and separation of variables approach are different [13-15]. In addition, there are differences in description of parasitic effects in converters, presented in various sources as is discussed in Section 3.2.

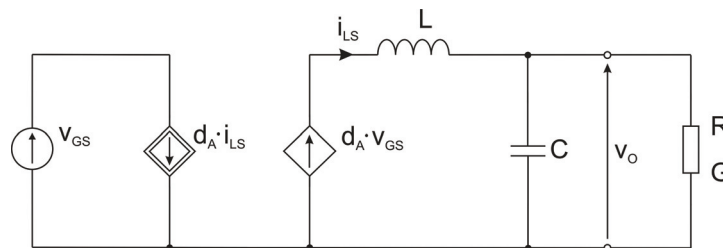


Fig. 5. Averaged model of BUCK converter in CCM in the form of equivalent circuit

The structure of the averaged model for *DCM* obtained by switch averaging approach corresponds to Figure 3 and contains two reactive elements L and C [1, 2, 4]. Averaged models of *BUCK* converter in *DCM*, obtained by separation of variables contain only one reactive element (C). The results of differences in large signal averaged models obtained with different methods are differences in small-signal models and formulas for small-signal transmittances.

2.2. Small-signal transmittances

Small-signal transmittances of switched converters defined in s -domain may be considered as special case of averaged models. These transmittances are used in the process of designing the control subcircuits for converter. The most important small-signal transmittances of typical switching converter are defined as follows [1, 2, 6, 16]:

$$H_g(s) = \left. \frac{V_o}{V_g} \right|_{\theta=0}, \quad (1)$$

$$H_d(s) = \left. \frac{V_o}{\theta} \right|_{V_g=0}, \quad (2)$$

V_o , V_g and θ are s -representations of small-signal variable terms of output and input voltages and duty ratio of switching signal respectively. H_g is known as input-to-output transmittance (or audio susceptibility) of converter and H_d – as control-to-output transmittance. In some applications, other small-signal characteristics may be useful, for example input admittance or output impedance [17-19, 20].

Small-signal version of averaged model is obtained after expressing each quantity in large-signal model as a sum of quiescent value and small variable term [1, 2, 5, 6]. The relations be-

tween small variable terms are obtained by linearization of nonlinear dependencies in the form of linear differential equations and, after transformation into s -domain, in the form of algebraic equations. Small-signal models may be represented in the form of proper equivalent circuits. The example of equivalent circuit form of averaged small-signal model of ideal *BUCK* converter working in *CCM* is shown in Figure 6. The form of this model is independent of the method of averaged model derivation i.e. is the same for switch averaging and separation of variables approach.

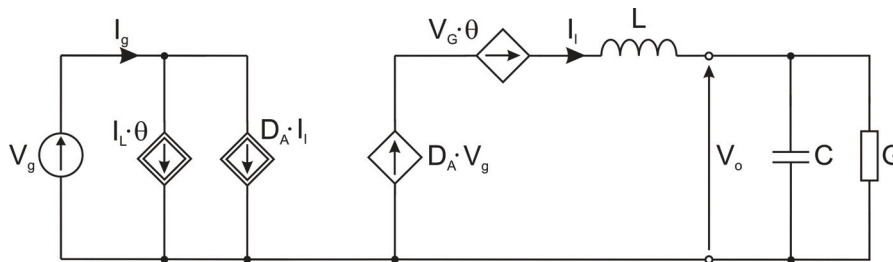


Fig. 6. Equivalent circuit representing small-signal averaged model of ideal *BUCK* converter

3. Small-signal transmittances of *BUCK* converter in *CCM*

3.1. Ideal converter

Small-signal transmittances of ideal *BUCK* converter (without parasitic resistances) working in *CCM*, obtained from definitions (1) and (2), using equivalent circuit shown in Figure 6, are of the form:

$$H_C = \frac{H_{OC}}{A_C \cdot s^2 + B_C \cdot s + 1}. \quad (3)$$

Such (or slightly modified) form is obtained for various transmittances discussed in this paper. Subscript “*C*” refers to *CCM* mode of operation; subscript “*O*” concerns the low-frequency value of H (i.e. for $s = 0$). The denominators of H_g and H_d transmittances (defined in (1) and (2)) are identical. The coefficients in denominator for ideal *BUCK* in *CCM* are:

$$A_C = L \cdot C \quad (4)$$

$$B_C = L \cdot G \quad (5)$$

Particular expressions for H_{gC} and H_{dC} are obtained by substitution H_{gOC} or H_{dOC} for H_{OC} in Equation 3, where:

$$H_{gOC} = D_A, \quad (6)$$

$$H_{dOC} = V_G \quad (7)$$

D_A and V_G are quiescent values of duty ratio and input voltage.

The general structure of transmittance (3) and expressions for coefficients A_C , B_C , H_{gOC} and H_{dOC} for ideal *BUCK* converter in *CCM* are independent of the method of obtaining the averaged model. The above expressions are presented in some references, for example [1, 2, 6, 16].

3.2. The influence of parasitic resistances on small-signal transmittances in *CCM*

Parasitic effects in all components of converter power stage influence its characteristics, especially the frequency dependencies of small-signal transmittances. In many sources discussing the dynamic characteristics of switching converters, the influence of parasitic effects is neglected (for example [1, 4, 5, 16, 21-25]) or considered only to a limited extent [26-34], for example – by including only a part of parasitic resistances. In textbook [2] and in some papers, for example [6, 8, 13, 35], the parasitic effects in power stage are represented by resistances in series with each ideal component of converter. In further part of this paper, parasitic resistances of components are denoted as R_T (transistor), R_D (diode), R_C (capacitor), R_L (inductor coil). The modification of diode model, used in part of further considerations, consists in addition of voltage source V_F in series with R_D .

In derivation of averaged models of several types of converters in [2] (based on earlier works [36, 37]), equivalent averaged resistances of transistor and diode are calculated on “conservation of energy rule” and are described as:

$$R_T(AV) = \frac{R_T}{D_A}, \quad (8)$$

$$R_D(AV) = \frac{R_D}{1 - D_A}. \quad (9)$$

After some network rearrangement described in [2], Ch. 10.4-10.7, the large-signal and next, small-signal averaged models of *BUCK* converter in *CCM* are obtained in the form of equivalent circuits shown in Figure 7. Small-signal transmittances of nonideal *BUCK* are not presented in [2] but can be derived from Figure 7b. The resulting transmittances H_{gCP} and H_{dCP} may be expressed in the form:

$$H_{CP} = H_{OCP} \cdot \frac{s/\omega_Z + 1}{A_{CP} \cdot s^2 + B_{CP} \cdot s + 1}. \quad (10)$$

The additional subscript “P” at some symbols refers to converter with parasitics. Low-frequency values of H_{gCP} and H_{dCP} calculated this way are as follows:

$$H_{gOCP} = \frac{D_A}{G \cdot R_Z + 1}, \quad (11)$$

$$H_{dOCP} = \frac{V_G}{G \cdot R_Z + 1}. \quad (12)$$

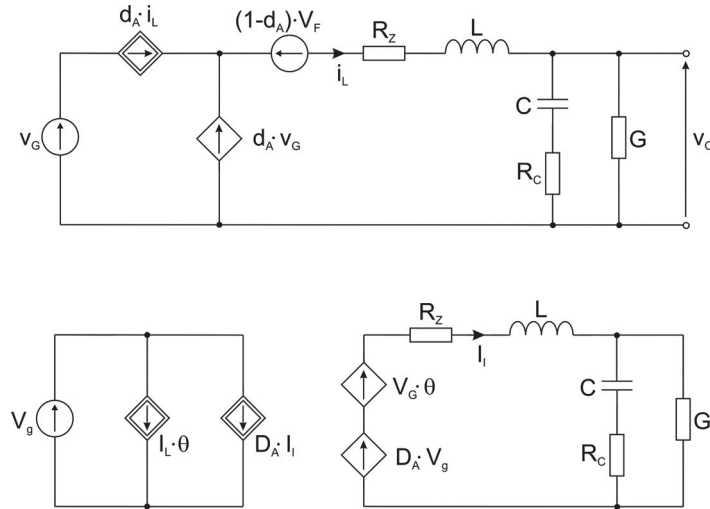


Fig. 7. Averaged models of BUCK converter with parasitic resistances according to [2], Ch. 10:
a) large-signal, b) small-signal

Coefficients ω_z , A_{CP} and B_{CP} are the same in formulas for H_{gCP} and H_{dCP} and are expressed as:

$$A_{CP} = \frac{L \cdot C_Z}{G \cdot R_Z + 1}, \quad (13)$$

$$B_{CP} = \frac{G \cdot L + C_Z \cdot R_Z + C \cdot R_C}{G \cdot R_Z + 1}, \quad (14)$$

$$\omega_z = \frac{1}{C \cdot R_C}, \quad (15)$$

where:

$$R_Z = R_L + D_A \cdot R_T + (1 - D_A) \cdot R_D, \quad (16)$$

$$C_Z = C \cdot (1 + G \cdot R_C). \quad (17)$$

It is worth noting that in Ch. 10 of [2], in derivation of large-signal averaged model of *BUCK* in *CCM*, voltage source V_F is taken into account in description of diode and is present in equivalent circuit in Figure 7 a) but is absent in small signal model.

Averaged small-signal model of *BUCK* converter in *CCM* derived by separation of variables method shown in [13, 14] includes parasitic resistances without voltage source V_F . Parameters of general expression (10) obtained this way are the same as in [2], Ch. 10 with the exception of H_{dOCP} that is described as:

$$H_{dOCP}(J) = \frac{V_G + I_O \cdot (R_D - R_T)}{G \cdot R_Z + 1}, \quad (18)$$

where I_O denotes *D.C.* value of the load current.

The modification of models obtained with separation of variables approach, including voltage V_F leads to next version of parameter H_{dOCP} [Marcin Walczak, private information]:

$$H_{dOCP}(W) = \frac{V_G + I_O \cdot (R_D - R_T) - V_F}{G \cdot R_Z + 1}. \quad (19)$$

In modern *BUCK* converters, diode D is often replaced by second transistor. In this case the term V_F in averaged models should be omitted.

Description of small-signal transmittances of *BUCK* converter with parasitic resistances working in *CCM*, expressed by Equations (10)-(19) differs substantially from description of ideal transmittances. Descriptions of coefficients in small-signal transmittances of nonideal converters obtained by switch-averaging method and by separation of variables are slightly different.

3.3. Numerical examples

Some numerical experiments have been performed for quantitative evaluation of the influence of component parameters and the method of averaged model derivation on small-signal transmittances of *BUCK* converter in *CCM*. A basic set of converter parameters is: $V_G = 12$ V; $D_A = 0.5$; $f_S = 200$ kHz; $L = 20$ μ H; $C = 100$ μ F; $G = 1$ S. Parasitic resistances are: $R_L = R_C = R_T = 50$ m Ω ; $R_D = 100$ m Ω and $V_F = 0.7$ V.

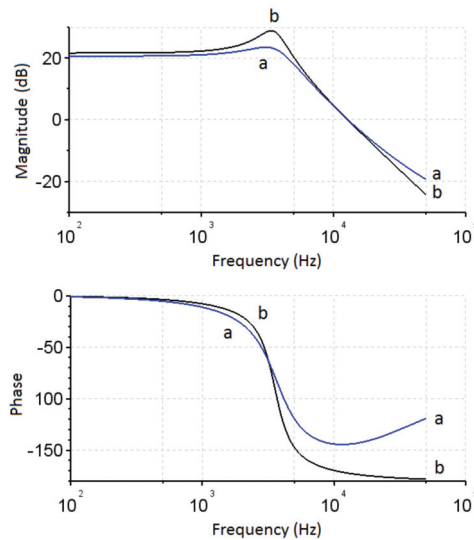


Fig. 8. Exemplary frequency dependencies of magnitude (*A*) and phase (*B*) of H_d transmittance: curve a) for ideal converter, b) for nonideal converter

Coefficients in Equation (3) for ideal converter calculated from Equations (4) – (7) are: $A_C = 2 \cdot 10^{-9} s^2$; $B_C = 2 \cdot 10^{-5} s$; $H_{gOC} = 0.5$; $H_{dOC} = 12$ V. Coefficients in Equation (10) for non-

ideal converter calculated from Equations (11)-(15) are: $A_{CP} = 1.867 \cdot 10^{-9} \text{ s}^2$; $B_{CP} = 3.389 \cdot 10^{-5} \text{ s}$; $H_{gOCP} = 0.444$; $H_{dOCP} = 10.67 \text{ V}$; $\omega_Z = 200 \text{ krd/s}$. Values of H_{dOCP} calculated from Equations (18) and (19) are 10.904 V and 10.281 V respectively. The magnitude and phase of transmittances H_{dC} and H_{dCP} versus frequency for ideal and nonideal converter obtained for the above values of coefficients are presented graphically in Figure 8 a) and b). The substantial differences between frequency dependencies of ideal and nonideal converter transmittances are observed. Differences in transmittances of nonideal converter obtained with different methods (switch averaging or separation of variables) are relatively small and express themselves in differences of coefficients H_{dOCP} .

4. Small-signal transmittances of *BUCK* converter working in *DCM*

4.1. Introduction

Ideal *BUCK* converter works in discontinuous conduction mode (*DCM*) if the following condition holds:

$$G < \frac{T_S}{2L} \cdot (1 - D_A). \quad (20)$$

Averaged models and particularly small-signal transmittances obtained for converter in *DCM* with different methods are different, even in the case of ideal converter. Averaged models for *DCM* presented in the literature are, most frequently, obtained using switch averaging approach. There are two variants of averaged models derived by switch averaging and they are discussed in Sections 4.2 and 4.3. These models are presented in original works only in the form of equivalent circuits therefore, the respective small-signal transmittances are derived in 4.2 and 4.3. Models and transmittances for *DCM* obtained by separation of variables are discussed in Sec. 4.4. Considerations in Sections 4.2, 4.3 and 4.4 are restricted to ideal *BUCK* converter. The influence of parasitics on models and transmittances for *DCM* is presented in Section 4.5. Numerical examples are given in Section 4.6.

In further considerations the *D.C.* voltage transmittance M_V is frequently used. This transmittance for ideal *BUCK* in *DCM* is described by formula [1, 2, 13]:

$$M_V = \frac{V_O}{V_G} = \frac{G_A}{2 \cdot G} \cdot \left(\sqrt{1 + \frac{4G}{G_A}} - 1 \right), \quad (21)$$

where:

$$G_A = D_A^2 \cdot \frac{T_S}{2L}. \quad (22)$$

4.2. Small-signal transmittances resulting from models given in [1]

Small-signal model of ideal *BUCK* converter working in *DCM*, presented in [1] and [5], is based on earlier works, e.g. [12]. This model has been next discussed and modified in some

papers, for example [38-44]. The basic structure of the model shown in Figure 9 corresponds to Figure 3, in accordance with general method of switch averaging. The quantities g_1 , r_1 , etc., according to [1] are expressed by following equations:

$$r_1 = R_e, \tag{23}$$

$$g_1 = \frac{1}{R_e}, \tag{24}$$

$$j_1 = \frac{2 \cdot (1 - M_V) \cdot V_G}{D_A \cdot R_e}, \tag{25}$$

$$g_2 = \frac{2 - M_V}{M_V^2 \cdot R_e}, \tag{26}$$

$$r_2 = M_V^2 \cdot R_e, \tag{27}$$

$$j_2 = \frac{2 \cdot (1 - M_V) \cdot V_G}{D_A \cdot M_V \cdot R_e}, \tag{28}$$

$$R_e = \frac{2L}{D_A^2 \cdot T_s}. \tag{29}$$

From the above equations and equivalent circuit in Figure 9, various small-signal characteristics may be obtained. Transmittances H_g and H_d defined in Section 2 are derived below. For that, only right-hand part of the scheme in Figure 9 is used. Voltage V_o depends only on V_g and θ which may be taken as external, independent signals.

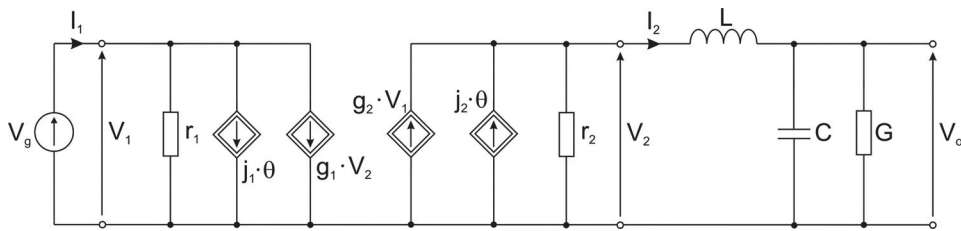


Fig. 9. Small-signal model of ideal BUCK converter in DCM, according to [1], obtained with switch-averaging approach

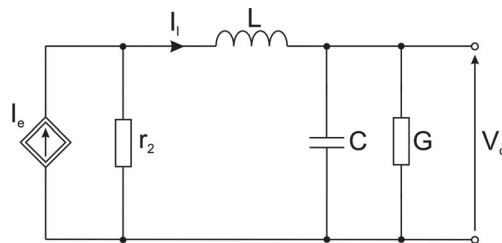


Fig. 10. Subcircuit of the model of Fig. 9 for calculations of H_g and H_d

Therefore, from Figure 9, one obtains circuit shown in Figure 10, where I_e is a sum of two terms generated by current sources.

Transmittance H_g , according to its definition is obtained after putting $\theta = 0$, therefore $I_e = g_2 \cdot V_g$; H_d is obtained by putting $V_g = 0$, so $I_e = j_2 \cdot \theta$. From Figure 10 we get:

$$I_l \cdot Z_W = (I_e - I_l) \cdot r_2, \quad (30)$$

where:

$$Z_W = s \cdot L + \frac{1}{s \cdot C + G}, \quad (31)$$

therefore:

$$I_l = I_e \cdot \frac{r_2}{s \cdot L + \frac{1}{s \cdot C + G} + r_2}, \quad (32)$$

and,

$$V_o = I_l \cdot \frac{1}{s \cdot C + G}. \quad (33)$$

From (32) and (33):

$$V_o = \frac{I_e \cdot r_2}{G \cdot r_2 + 1} \cdot \frac{1}{s^2 \cdot \frac{LC}{G \cdot r_2 + 1} + s \cdot \frac{G \cdot L + r_2 \cdot C}{G \cdot r_2 + 1} + 1}. \quad (34)$$

By substituting $g_2 \cdot V_g$ or $j_2 \cdot \theta$ for I_e in Equation (34), according to definitions of transmittances, one obtains:

$$H_{gD}(E) = \frac{H_{gDO}(E)}{A_E \cdot s^2 + B_E \cdot s + 1}, \quad (35)$$

$$H_{dD}(E) = \frac{H_{dDO}(E)}{A_E \cdot s^2 + B_E \cdot s + 1} \quad (36)$$

Additional subscript “D” refers to DCM mode; symbol “E” in brackets, to the name of first author of book [1]. Quantities in (35) and (36) may be obtained from Equations (23)-(29) and (34). The results are:

$$H_{gDO}(E) = \frac{g_2 \cdot r_2}{G \cdot r_2 + 1} = M_V, \quad (37)$$

$$H_{dDO}(E) = \frac{j_2 \cdot r_2}{G \cdot r_2 + 1} = \frac{2V_o}{D_A} \cdot \frac{1 - M_V}{2 - M_V}, \quad (38)$$

$$A_E = \frac{L \cdot C}{G \cdot r_2 + 1} = \frac{LC}{2 - M_V}, \quad (39)$$

$$B_E = \frac{G \cdot L + (1 - M_V) \cdot R \cdot C}{2 - M_V}. \quad (40)$$

4.3. Small-signal transmittances obtained from model in [2]

Averaged small-signal model of ideal BUCK converter in DCM derived by switch averaging approach in book [2], Ch. 10, is represented by equivalent circuit shown in Figure 11. Symbol Z_W has been described previously (Eqn. 31), other quantities, according to [2] are given by formulas:

$$r_i = \frac{V_{SL}}{I_D \cdot V_{LD}}, \quad (41)$$

$$k_i = \frac{2I_D \cdot V_{LD}}{D_A \cdot V_{SL}}, \quad (42)$$

$$g_m = \frac{2I_S}{V_{LD}}, \quad (43)$$

$$k_o = \frac{2I_S \cdot V_{SL}}{D_A \cdot V_{LD}}, \quad (44)$$

$$r_o = \frac{1}{g_o} = \frac{V_{LD}^2}{I_S \cdot V_{SL}} \quad (45)$$

V_{SL} , V_{LD} , I_D , I_S , according to [2] denote D.C. terms of voltages and currents of switching sub-circuit.

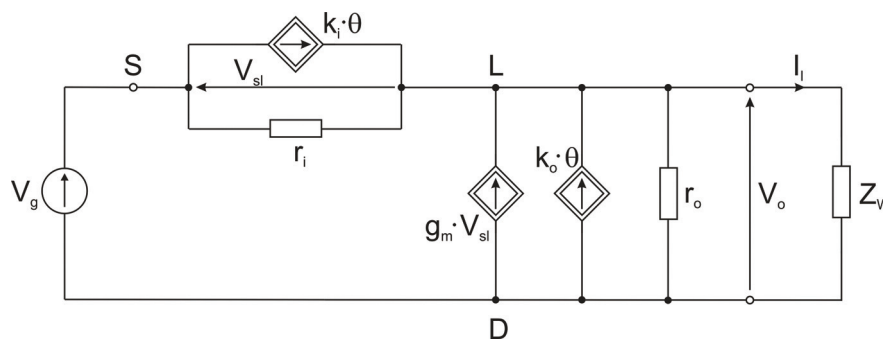


Fig. 11. Small-signal model of ideal BUCK converter in DCM, according to [2], Ch. 10, obtained with switch-averaging approach

According to definition of H_g transmittance, the circuit of Figure 11 is replaced by Figure 12 and the resulting equations are:

$$I_g \cdot (1 + g_m \cdot r_i) = I_l \left(1 + \frac{Z_W}{r_o} \right), \quad (46)$$

$$V_g = I_i \cdot Z_W + I_g \cdot r_i. \quad (47)$$

From (46) and (47):

$$H_{gD}(K) = \frac{1 + g_m \cdot r_i}{s^2 \cdot L \cdot C \cdot [1 + r_i(g_m + g_o)] + s \cdot [r_i \cdot C + L \cdot G \cdot (1 + g_i \cdot r_i + g_o \cdot r_i)] + r_i \cdot (g_m + g_o + G) + 1}. \quad (48)$$

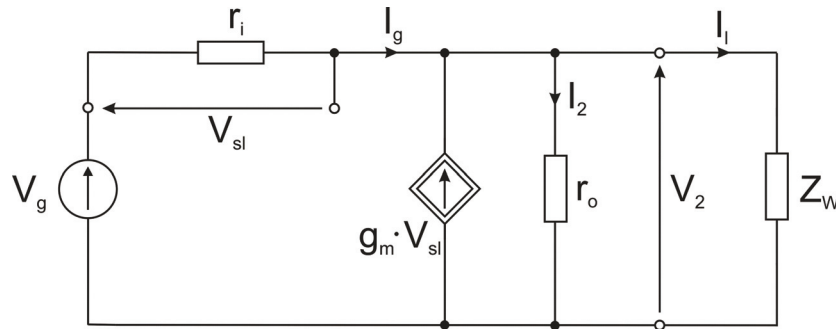


Fig. 12. Subcircuit of the model of Fig. 11, for calculations of H_g

Equation (48) can be expressed in the form similar to Equation (35):

$$H_{gD}(K) = \frac{H_{gDO}(K)}{A_K \cdot s^2 + B_K \cdot s + 1}, \quad (49)$$

where:

$$H_{gDO}(K) = \frac{1 + g_m \cdot r_i}{1 + r_i \cdot (g_m + g_o + G)}, \quad (50)$$

$$A_K = L \cdot C \cdot \frac{1 + r_i(g_m + g_o)}{1 + r_i \cdot (g_m + g_o + G)}, \quad (51)$$

$$B_K = \frac{r_i \cdot C + L \cdot G [1 + r_i(g_m + g_o)]}{1 + r_i \cdot (g_m + g_o + G)}. \quad (52)$$

Additional letter K in brackets or subscripts refers to the name of author of book [2].

According to definition of H_{ds} , the circuit in Figure 11 should be replaced by Figure 13 a) and, after simple rearrangement, by Figure 13 b), where g_x is:

$$g_x = \frac{1}{r_x} = g_m + \frac{1}{r_i} + g_o. \quad (53)$$

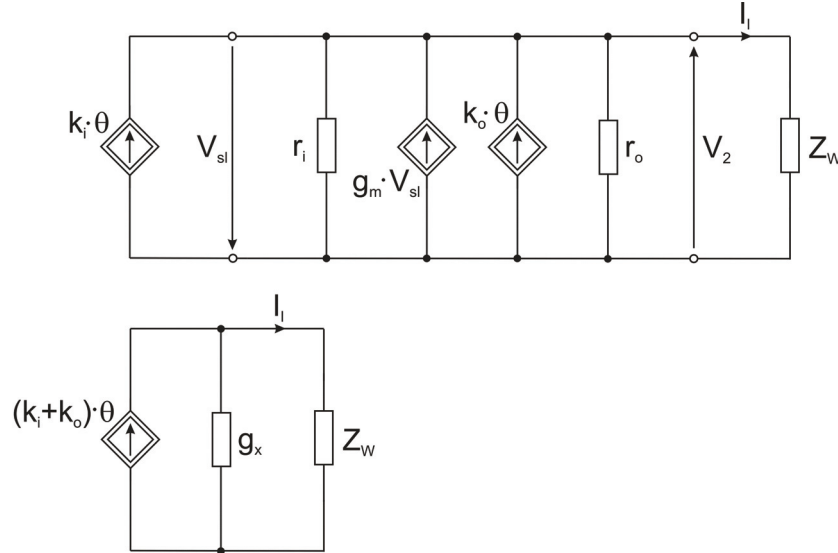


Fig. 13. Subcircuits of the model of Fig. 11 for calculations of H_d : a) general, b) simplified

From Figure 13 we get:

$$r_x \cdot [(k_i + k_o) \cdot \theta - I_l] = Z_w \cdot I_l. \quad (54)$$

From Equations (31), (33), (54) and definition of H_d it may be obtained:

$$H_{dD}(K) = \frac{H_{dDO}(K)}{A_{Kd} \cdot s^2 + B_{Kd} \cdot s + 1}, \quad (55)$$

where:

$$H_{dDO}(K) = \frac{k_i + k_o}{G + g_x}, \quad (56)$$

$$A_{Kd} = \frac{LC \cdot g_x}{G + g_x}, \quad (57)$$

$$B_{Kd} = \frac{C + LC \cdot g_x}{G + g_x}. \quad (58)$$

After simple algebraic manipulations, taking into account Equation (53), one obtains $A_{Kd} = A_K$ and $B_{Kd} = B_K$, therefore transmittances $H_{gD}(K)$ and $H_{dD}(K)$ have identical denominators. A comparison of transmittances $H_{gD}(K)$ and $H_{dD}(K)$ with transmittances $H_{gD}(E)$ and $H_{dD}(E)$ is possible by expressing currents and voltages I_D , I_S , V_{LD} , V_{SL} by DC output voltage V_O and transmittance M_V and by using Equations (41)-(45) for r_b , k_b , g_m , r_o , k_o . It is obtained:

$$V_{SL} = V_G - V_O = \frac{1 - M_V}{M_V} \cdot V_O, \quad (59)$$

$$V_{LD} = V_O, \quad (60)$$

$$I_D = I_L - I_G = G \cdot V_O \cdot (1 - M_V), \quad (61)$$

$$I_S = I_G = G \cdot V_O \cdot M_V \quad (62)$$

and

$$r_i = \frac{1}{g_i} = \frac{1 - M_V}{G \cdot M_V^2}, \quad (63)$$

$$k_i = \frac{2G \cdot M_V \cdot V_O}{D_A}, \quad (64)$$

$$g_m = 2G \cdot M_V, \quad (65)$$

$$k_o = \frac{2G \cdot V_O \cdot (1 - M_V)}{D_A}, \quad (66)$$

$$r_o = \frac{1}{g_o} = \frac{1}{G \cdot (1 - M_V)}, \quad (67)$$

and, finally

$$H_{gDO}(K) = M_V, \quad (68)$$

$$H_{dDO}(K) = \frac{2V_O}{D_A} \cdot \frac{1 - M_V}{2 - M_V}, \quad (69)$$

$$A_K = \frac{LC}{2 - M_V}, \quad (70)$$

$$B_K = \frac{G \cdot L + R \cdot C \cdot (1 - M_V)}{2 - M_V}. \quad (71)$$

The final expressions (68)-(71) are identical as description of transmittances $H_{gD}(E)$ and $H_{dD}(E)$ given in Section 4.2. In spite of differences in the form of equivalent circuits and in description of particular quantities used in [1] and [2], the pairs of transmittances described in Sections 4.2 and 4.3 are identical.

4.4. Small-signal transmittances obtained by separation of variables

Small-signal transmittances of ideal *BUCK* converter in *DCM* obtained with separation of variables approach [13, 14] differ qualitatively from transmittances presented in Sections 4.2 and 4.3, namely, they are one-pole functions of s . This feature is a result of observation, that in *DCM*, the average inductor voltage for one switching period is zero not only in steady-state

conditions but also in transient states [13, 14]. As a consequence the averaged model of converter contains only one reactive component – capacitor (inductor is absent). Small signal transmittances of *BUCK* in *DCM*, according to description in [13], are:

$$H_{gd}(J) = \frac{G_A \cdot (2M_I - 1)}{s \cdot C + G + G_A \cdot M_I^2} = \frac{H_{gDO}(J)}{1 + T \cdot s}, \quad (72)$$

$$H_{dD}(J) = \frac{2G_A \cdot V_G \cdot (M_I - 1) / D_A}{s \cdot C + G + G_A \cdot M_I^2} = \frac{H_{dDO}(J)}{1 + T \cdot s}, \quad (73)$$

where

$$H_{gDO}(J) = M_V, \quad (74)$$

$$H_{dDO}(J) = \frac{2V_O}{D_A} \cdot \frac{1 - M_V}{2 - M_V}, \quad (75)$$

$$T = R \cdot C \cdot \frac{1 - M_V}{2 - M_V}. \quad (76)$$

Symbol M_I used in (72), (73) denotes $1/M_V$.

Quantities $H_{gDO}(J)$ and $H_{dDO}(J)$ are the same as corresponding quantities in Sections 4.2 and 4.3. Formulas for B_K and T are similar and give the same numerical values if:

$$G \cdot L \ll R \cdot C \cdot (1 - M_V). \quad (77)$$

4.5. Influence of parasitic effects on small-signal transmittances in DCM

Presentation of small-signal transmittances of converters in *DCM* (similarly as in *CCM*) in some references are restricted to ideal case – without parasitic components. The example is textbook [1] and related papers, for example [5]. The considerations of this subsection is based on the description of small-signal models of nonideal *BUCK* converter in *DCM* obtained by switch-averaging procedure according to [2] and by separation of variables approach [13].

Averaged model of *BUCK* in *DCM* presented in Ch. 10 of textbook [2] may be considered as modification of the model shown here in Fig. 11 accounting for parasitic resistance R_C and equivalent resistance r . The subcircuit containing parasitics mentioned above is shown in Figure 14. Instead of impedance Z_W described by Eqn. (31) we obtain modified impedance:

$$Z_{WP} = r + sL + Z_{CP}, \quad (78)$$

where,

$$\frac{1}{Z_{CP}} = Y_{CP} = G + \frac{1}{R_C + 1/SC} \quad (79)$$

or

$$Z_{CP} = \frac{s \cdot C \cdot R_C + 1}{s \cdot C_Z + G}. \quad (80)$$

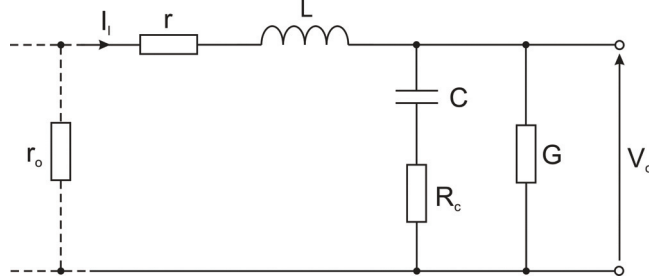


Fig. 14. Subcircuit of the model of BUCK converter in DCM (according to [2], Ch. 10) containing parasitic resistances

Equivalent capacitance C_Z is described by Equation (17). Equivalent resistance in DCM, according to [2], Ch. 10 is:

$$r = \frac{4}{3 \cdot D_A \cdot (1 + V_{SL}/V_{LD})} \cdot \left(R_L + \frac{R_T + R_D \cdot V_{SL}/V_{LD}}{1 + V_{SL}/V_{LD}} \right). \quad (81)$$

Symbols V_{SL} , V_{LD} are explained by Equations (59), (60).

Further derivations and resulting formulas for small-signal transmittances are not presented in [2], therefore they are shown below. From Equations (52), (53) and (81), the final formula for equivalent resistance r is obtained:

$$r = \frac{4M_V}{3 \cdot D_A} \cdot R_{ZD}, \quad (82)$$

where:

$$R_{ZD} = R_L + R_T \cdot M_V + R_D \cdot (1 - M_V). \quad (83)$$

The derivation of r in (82) is slightly simplified because the dependencies (59) and (60) of V_{SL} and V_{LD} on external voltage and M_V are assumed the same as for ideal converter.

Transmittances H_g and H_a of nonideal BUCK converter may be obtained in similar way as in Section 4.3, but modified impedance Z_{WP} given by (78) – (80) should be used instead of Z_W . The result for H_g is:

$$H_{gDP}(K) = \frac{1 + g_m \cdot r_i}{r_i + Z_{WP} \cdot [1 + r_i(g_m + g_o)]}, \quad (84)$$

or,

$$H_{gDP}(K) = H_{gDPO}(K) \cdot \frac{s/\omega_Z + 1}{A_p \cdot s^2 + B_p \cdot s + 1}, \quad (85)$$

where

$$H_{gDPO}(K) = \frac{g_i + g_m}{G + g_x \cdot (r \cdot G + 1)}, \quad (86)$$

$$\omega_z = \frac{1}{C \cdot R_C}, \quad (87)$$

$$A_P = \frac{L \cdot C_Z \cdot g_x}{G + g_x(r \cdot G + 1)}, \quad (88)$$

$$B_P = \frac{C_Z + g_x \cdot (r \cdot C_Z + L \cdot G + C \cdot R_C)}{G + g_x(r \cdot G + 1)}. \quad (89)$$

Additional subscript “P” refer to converter with parasitic resistances. Using Equations (17), (53), (63), (65), (67) and (82), the expressions for $H_{gdPO}(K)$, A_P , B_P are obtained in the form:

$$H_{gdPO}(K) = \frac{M_{VP} \cdot (2 - M_{VP})}{2 - M_{VP} + r \cdot G}, \quad (90)$$

$$A_P = \frac{L \cdot C_Z}{2 - M_{VP} + r \cdot G}, \quad (91)$$

$$B_P = \frac{R \cdot C_Z \cdot (1 - M_{VP}) + r \cdot C_Z + L \cdot G + C \cdot R_C}{2 - M_{VP} + r \cdot G}. \quad (92)$$

The expression for transmittance $H_{dDP}(K)$ may be obtained similarly and has the form similar to Equation (85), but quantity H_{dDPO} is present in the place of H_{gdPO} , where:

$$H_{dDPO}(K) = \frac{2V_O}{D_A} \cdot \frac{1 - M_{VP}}{2 - M_{VP} + r \cdot G} \quad (93)$$

(ω_z , A_P , B_P are the same as previously).

D.C. voltage transmittance M_V in DCM is also influenced by parasitic effects. The formula for M_{VP} (with parasitics) is not presented in [2] and is derived below from D.C. averaged model of switching subcircuit shown in Fig. 15a) according to [2] (p. 428, Fig. 10.20 c). After including external components and some rearrangements, D.C. model shown in Fig. 15b) is obtained. Equations (59) and (60) are not true in the presence of parasitic effects and V_{SL} and V_{LD} should be expressed as:

$$V_{SL} = \frac{I_G}{G_A}, \quad (94)$$

$$V_{LD} = V_G - \frac{I_G}{G_A}. \quad (95)$$

From Figure 15 it is obtained:

$$I_G = G_A \cdot (V_G - G \cdot V_O \cdot r - V_F' - V_O). \quad (96)$$

Formula for V_F' given in [2], (Ch. 10) is:

$$V_F' = \frac{V_F}{1 + V_{LD}/V_{SL}} \tag{97}$$

In approximate calculations of V_F' , quantities V_{SL} and V_{LD} in (97) are described by ideal Equations (59) and (60). From Figure 15, using Equations (94)-(96), the expressions for non-ideal D.C. voltage transmittance is obtained:

$$M_{VP}(K) = \frac{1}{2} \left(\sqrt{X \cdot \frac{r \cdot G_A + X + 4}{1 + r \cdot G} - \frac{2V_{F1} \cdot X}{V_G} + \left(\frac{V_{F1}}{V_G} \right)^2} - X - \frac{V_{F1}}{V_G} \right), \tag{98}$$

where:

$$X = \frac{G_A}{G}, \tag{99}$$

$$V_{F1} = \frac{V_F'}{1 + r \cdot G}. \tag{100}$$

Small-signal transmittances of *BUCK* in *DCM* in the presence of parasitic resistances have been obtained in [13], [14] using separation of variables approach. Similarly as in the case of ideal converter, the small-signal model obtained this way (see Sec. 4.4) contains only one reactive component.

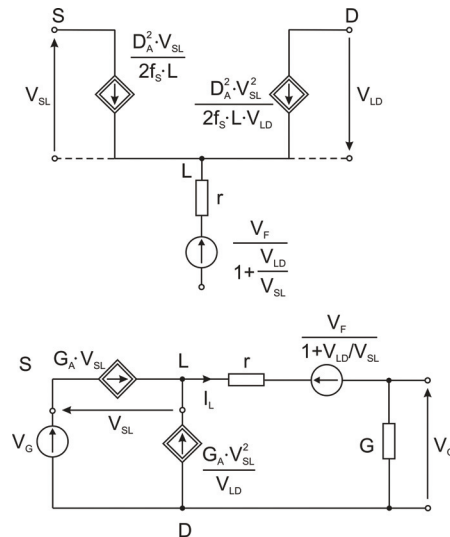


Fig. 15. D.C. averaged model of nonideal *BUCK* converter in *DCM* (according to [2], Ch. 10):
a) direct form, b) modified form

From equations (86) and (90) in paper [13], small-signal transmittances of nonideal *BUCK* converter in *DCM* are of the form:

$$H_{gDP}(J) = H_{gDPO}(J) \cdot \frac{s/\omega_Z + 1}{T_P \cdot s + 1}, \quad (101)$$

$$H_{dDP}(J) = H_{dDPO}(J) \cdot \frac{s/\omega_Z + 1}{T_P \cdot s + 1}, \quad (102)$$

where:

$$H_{gDPO}(J) = \frac{M_V \cdot R_G \cdot (2 - M_V) + D_A \cdot R_P}{R_{PG} + G \cdot R_Y^2}, \quad (103)$$

$$H_{dDPO}(J) = \frac{(V_G - V_O) \cdot (2R_G \cdot M_V / D_A + R_P)}{R_{PG} + G \cdot R_Y^2}, \quad (104)$$

$$T_P = \frac{C \cdot R_C \cdot R_{PG} + C_Z \cdot R_Y^2}{R_{PG} + G \cdot R_Y^2}, \quad (105)$$

$$R_{PG} = D_A \cdot R_P + R_G, \quad (106)$$

$$R_Y = R_G \cdot M_V / D_A + R_P, \quad (107)$$

$$R_G = \frac{2L}{T_S}, \quad (108)$$

$$R_P = R_T + R_L = R_D + R_L \quad (109)$$

and ω_Z is given by Equation (87).

Nonideal D.C. voltage transmittance obtained by separation of variables is [13]:

$$M_{VP}(J) = \frac{D_A \cdot G_Z}{2G} \cdot \left[\sqrt{(D_A + R_P \cdot G)^2 + \frac{4G}{G_Z}} - D_A - R_P \cdot G \right]. \quad (110)$$

4.6. Numerical examples for DCM

Differences in formulas for small-signal transmittances presented in Sections 4.2-4.5 come from differences in the methods of the averaged model derivation and from including or neglecting parasitic resistances. These differences may be evaluated quantitatively by series of numerical calculations. The basic set of data is the same as in Section 3.4 except of load conductance assumed to be $G_l = 0.05$ S to fulfill the condition (20) for DCM.

The first group of calculations has been performed for ideal converter. Quantities independent of the method of transmittance derivation, according to Equations (68), (69), are $H_{gDO} = M_V = 0.5376$; $H_{dDO} = 8.159$ V. Calculations based on switch averaging, according to Equations (70), (71) give: $A_K = 1.368 \cdot 10^{-9}$ s²; $B_K = 0.6331$ ms. Quantity T obtained from Equations (76) equals 0.6324 ms. Calculations for converter with parasitic resistances: from Equation (98), $M_{VP}(K) = 0.517$; from Equation (110), $M_{VP}(J) = 0.535$. Further parameters obtained on the base of switch averaging method (Eqns. 90 – 93) are: $H_{gDPO}(K) = 0.513$; $H_{dDPO}(K) = 8.03$

V; $A_p = 1.343 \cdot 10^{-9} \text{ s}^2$; $B_p = 0.666 \text{ ms}$. Parameters obtained from Equations (103), (104), (105) based on separation of variables are: $H_{gDPO}(J) = 0.535$; $H_{dDPO}(J) = 8.13 \text{ V}$; $T_p = 0.643 \text{ ms}$.

Frequency dependencies of the magnitude and phase of H_{gD} transmittances are presented in Figure 16 A) and B).

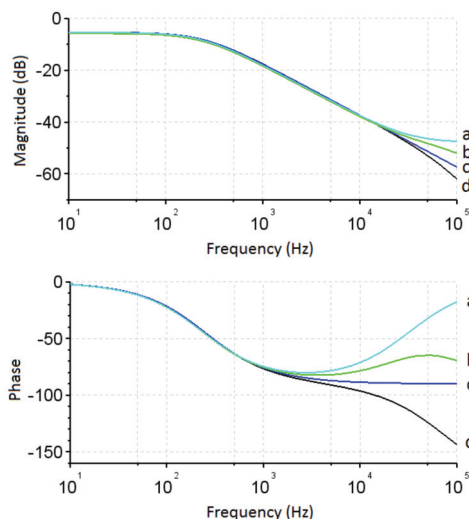


Fig. 16. Exemplary frequency dependencies of magnitude (A) and phase (B) of H_g transmittance for DCM; description of curves in the text

Curves marked a) and b) are obtained for converter with parasitics: a) from equations according to separation of variables, b) according to switch averaging approach. Curves c) and d) correspond to ideal converter: c) obtained according to separation of variables, d) – to switch averaging approach. Differences between numerical values of the magnitude and phase of transmittances become substantial only for relatively high frequencies.

5. Conclusions

Small-signal transmittances of switch-mode step-down power converters are important characteristics useful in design of control circuits for converters. Small-signal transmittances are particular form of converter averaged models. There are several approaches to the derivation of averaged models presented in the literature. In addition, parasitic effects in converter components are in many cases omitted or included only partially in averaged models derivation. In some sources, averaged models are presented only in the form of equivalent circuits, without analytical formulas for small-signal transmittances.

The main purpose of this paper is to derive analytical descriptions of small-signal transmittances of step-down converter in unified, systematic way and to compare results obtained with different methods of averaged models creation. In particular, the influence of parasitic re-

sistances of components on the final form of transmittances and their frequency dependencies is analyzed and evaluated.

Some expressions for small-signal transmittances, especially for converter working in DCM, may be considered as original results of the paper. The examples are Equations (35)-(40), (85), (90)-(93), (98), (101)-(105). It is shown, that two approaches to deriving small-signal averaged models, presented in [1] and [2] lead to identical expressions for transmittances, in the form of Equations (35)-(40) and (68)-(71). Analytical formulas for transmittances obtained by separation of variables approach are different from those obtained by switch-averaging, but differences in numerical values of magnitude and phase are small in wide frequency range and become greater only for high frequencies, as shown in Section 4.6.

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