

## 3-D optimal design of laminated yoke of billet heater for rolling wire rod using ON/OFF method\*

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(Received: 10.07.2011, revised: 22.12.2011)

**Abstract:** The optimization method using the ON/OFF sensitivity analysis has an advantage that an epoch-making construction of magnetic circuit may be obtained. Therefore, it is attractive for designers of magnetic devices. We have already developed the ON/OFF method for the optimization of a static magnetic field problem, and the effectiveness is verified by applying it to the optimization of magnetic recording heads. In this paper, the ON/OFF sensitivity method is extended to the optimization of the eddy current problem using the adjoint variable. The newly developed ON/OFF method is applied to the determination of the optimal topology of the yoke of the billet heater for rolling wire rod. As a result, the optimal shape of yoke, which we could not imagine beforehand can be obtained. It is shown that the local heating of the yoke was reduced without decreasing the heating efficiency.

**Key words:** ON/OFF method, step-by-step method, eddy current problem, billet heater

### 1. Introduction

In the development of an electromagnetic device, the design based on the experience of designers is usually performed. Recently, various optimal design methods using electromagnetic field analysis are developed and applied to the design of actual magnetic devices. In those methods, firstly we must imagine the outline of the optimal shape of the magnetic circuit, and specifies the dimensions of cores etc. as design variables, then the dimensions etc. are determined by the optimization software. Therefore, it may be difficult to get a newly developed magnetic circuit which we could not imagine beforehand.

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\*This is extended version of a paper which was presented at the 8th *International Conference on Computation in Electromagnetics*, Wrocław, Poland, 11-14.04.2011.

If a topology optimization method which can determine the optimal topology by distributing materials in a design domain is used, there is a possibility that a new magnetic circuit can be discovered, because it is not necessary to set design variables in advance [1-4]. The application of the ON/OFF method to the 3-D static magnetic field problem, such as magnetic recording head [5, 6] has already been reported. The topology optimization of a simple linear eddy current problem is reported [7, 8]. But, the topology optimization method which is applicable to the 3-D eddy current problem of actual magnetic device is not yet developed. Although a technique for calculating the design sensitivity by representing it by a finite difference equation is proposed [8], a long CPU time is necessary because an adjoint variable method [7] was not utilized. An efficient calculation method of sensitivity of time harmonic systems is proposed [9] for 2D eddy current problem, but it is not applied to the topology optimization.

In this paper, the ON/OFF method is extended to the optimization of the 3-D eddy current problem by combining the ON/OFF method with the step-by-step method and the adjoint variable method. The ON/OFF method is applied to the optimization of the laminated yoke of a billet heater for rolling wire rod. The laminated yoke is installed in the outside of the coil as the return path of flux which flows in the billet. There is a problem that the heating efficiency of the billet decreases by the eddy current loss generated in the laminated yoke. The overheating of yoke should also be avoided. Then, a 3-D optimal topology of the laminated yoke of the model of an actual billet heater which avoids this problem is obtained using the newly developed ON/OFF optimization method.

## 2. ON/OFF method for eddy current problem using adjoint variable

The step-by-step method was chosen in the eddy current analysis. The nonlinearity of magnetic material is not taken into account, and the linear analysis is carried out. Using the magnetic vector potential  $\mathbf{A}$ , the governing equation can be written as

$$\text{rot}(\nu \text{rot } \mathbf{A}) = \mathbf{J}_0 - \sigma \frac{\partial \mathbf{A}}{\partial t}, \quad (1)$$

where  $\partial \mathbf{A} / \partial t$  is the time differential term of  $\mathbf{A}$ ,  $\nu$  is the reluctivity,  $\sigma$  is the conductivity,  $\mathbf{J}_0$  is the source current vector. By using the step-by-step method,  $\partial \mathbf{A} / \partial t$  is written as

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\mathbf{A} - \mathbf{A}^*}{\Delta t} \quad (2)$$

where  $\mathbf{A}^*$  is the vector potential at one step before, and  $\Delta t$  is the time step size.

The region (design domain) of which the shape (topology) is determined is subdivided into many design cells (finite elements). The material distribution is updated so that the desired result can be obtained, which allows each design cell to have only one state, that of a void (this state is called as “OFF”) or a solid (“ON”). We call this algorithm as an “ON/OFF method”. To determine the material distribution by judging ON or OFF, the sensitivity  $dW/dp$  ( $p$ : design variable (conductivity or reluctivity),  $W$ : objective function) of each cell is evaluated.

The adjoint variable method [7] is applied for calculating the sensitivity. If the objective function  $W$  is the function of  $\mathbf{A}'$  (time differential of vector potential  $\partial\mathbf{A}/\partial t$ ) and design variable  $p$ , the derivative of  $W$  with respect to  $p$  is given as:

$$\frac{dW}{dp} = \left. \frac{\partial W}{\partial p} \right|_{\mathbf{A}'=\text{const}} + \left. \frac{\partial W}{\partial \mathbf{A}'} \right|_{p=\text{const}} \frac{\partial \mathbf{A}'}{\partial p}. \quad (3)$$

As the objective function  $W$  is not explicit in terms of the design variable  $p$ , the first term of Eq. (3) is zero. The second term  $\partial W / \partial \mathbf{A}'$  can be determined.  $\partial \mathbf{A}' / \partial p$  is not explicit. This can be obtained from the finite element equation as follows:

$$\mathbf{H} \cdot \mathbf{A}' = \mathbf{G}, \quad (4)$$

where  $\mathbf{H}$  is the coefficient matrix and  $\mathbf{G}$  is the right-hand side vector related to a current source. If the residual vector  $\mathbf{r}$  is used, Eq. (4) is given as:

$$\mathbf{r} = \mathbf{H} \cdot \mathbf{A}' - \mathbf{G}. \quad (5)$$

The derivative of the residual vector  $\mathbf{r}$  with respect to the design variable  $p$  is given as:

$$\frac{d\mathbf{r}}{dp} = \left. \frac{\partial \mathbf{r}}{\partial p} \right|_{\mathbf{A}'=\text{const}} + \left. \frac{\partial \mathbf{r}}{\partial \mathbf{A}'} \right|_{p=\text{const}} \frac{\partial \mathbf{A}'}{\partial p}. \quad (6)$$

Substituting Eq. (5) into Eq. (6), the first term of the right-hand side of Eq. (6) is given as:

$$\left. \frac{\partial \mathbf{r}}{\partial p} \right|_{\mathbf{A}'=\text{const}} = \frac{\partial \mathbf{H}}{\partial p} \mathbf{A}' - \frac{\partial \mathbf{G}}{\partial \mathbf{J}} \frac{\partial \mathbf{J}}{\partial p}, \quad (7)$$

where  $\mathbf{J}$  is the current density. Moreover, the second term of Eq. (6) is given as:

$$\left. \frac{\partial \mathbf{r}}{\partial \mathbf{A}'} \right|_{p=\text{const}} = \mathbf{H} + \frac{\partial \mathbf{H}}{\partial \mathbf{A}'} \mathbf{A}' - \frac{\partial \mathbf{G}}{\partial \mathbf{A}'}. \quad (8)$$

As the linear field problem is treated here, the coefficient matrix  $\mathbf{H}$  is not a function of  $\mathbf{A}'$ , then the second term of Eq. (8) is zero. Since  $\mathbf{J}$  is not a function of  $\mathbf{A}'$  in this case, the third term of Eq. (8) is zero. Substituting Eqs (7) and (8) into Eq. (6),  $d\mathbf{r}/dp$  is given as:

$$\frac{d\mathbf{r}}{dp} = \frac{\partial \mathbf{H}}{\partial p} \mathbf{A}' + \mathbf{H} \frac{\partial \mathbf{A}'}{\partial p} = 0. \quad (9)$$

From Eq. (9),  $\partial \mathbf{A}' / \partial p$  is obtained as:

$$\frac{\partial \mathbf{A}'}{\partial p} = -\mathbf{H}^{-1} \frac{\partial \mathbf{H}}{\partial p} \mathbf{A}'. \quad (10)$$

Substituting Eq. (10) into Eq. (3), Eq. (11) can be obtained.

$$\frac{dW}{dp} = -\frac{\partial W}{\partial \mathbf{A}'} \mathbf{H}^{-1} \frac{\partial \mathbf{H}}{\partial p} \mathbf{A}'. \quad (11)$$

In order to avoid the calculation of the inverse of  $\mathbf{H}$ , an adjoint vector  $\boldsymbol{\lambda}$  is introduced [3]. The adjoint equation is given by

$$\mathbf{H} \cdot \boldsymbol{\lambda} = \frac{\partial W}{\partial \mathbf{A}'}. \quad (12)$$

$\boldsymbol{\lambda}$  is obtained by solving (12), and  $dW/dp$  is calculated by substituting  $\boldsymbol{\lambda}$  into Equation (13)

$$\frac{dW}{dp} = -\boldsymbol{\lambda} \frac{\partial \mathbf{H}}{\partial p} \mathbf{A}'. \quad (13)$$

As the step-by-step method is used in this case,  $dW/dp$  is the summation of all values of  $dW/dp$  at every steps. If there are four elements in the design region as shown in Fig. 1, four kinds of design variables  $p_1 - p_4$  are considered. In this case, Eq. (13) is given as follows:

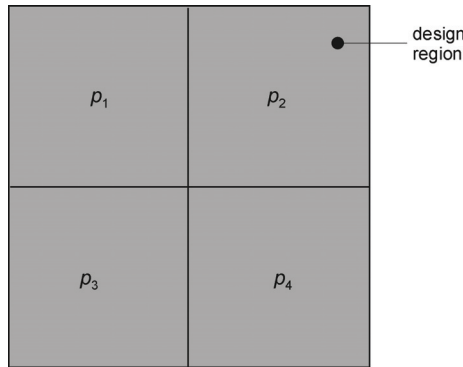


Fig. 1. Design variable in each element

$$\frac{dW}{dp_1} = -[\lambda_1 \dots \lambda_n] \begin{bmatrix} \frac{\partial H_{11}}{\partial p_1} & \dots & \frac{\partial H_{1n}}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial H_{n1}}{\partial p_1} & \dots & \frac{\partial H_{nn}}{\partial p_1} \end{bmatrix} \begin{bmatrix} A'_1 \\ \vdots \\ A'_n \end{bmatrix}, \quad (14)$$

$$\frac{dW}{dp_4} = -[\lambda_1 \dots \lambda_n] \begin{bmatrix} \frac{\partial H_{11}}{\partial p_4} & \dots & \frac{\partial H_{1n}}{\partial p_4} \\ \vdots & \ddots & \vdots \\ \frac{\partial H_{n1}}{\partial p_4} & \dots & \frac{\partial H_{nn}}{\partial p_4} \end{bmatrix} \begin{bmatrix} A'_1 \\ \vdots \\ A'_n \end{bmatrix}, \quad (15)$$

where  $n$  is the number of edges, of which the vector potential is unknown, in the analyzed region. The adjoint variables can be obtained by solving the following equation:

$$\begin{bmatrix} H_{11} & \dots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{n1} & \dots & H_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} \frac{\partial W}{\partial A'_1} \\ \vdots \\ \frac{\partial W}{\partial A'_n} \end{bmatrix} \quad (16)$$

### 3. Optimization of laminated yoke of billet heater

#### Analyzed model

Figure 2 shows the analyzed model of the billet heater. This is a 1/8 model. Although the original billet heater has four yokes around the coil, it is assumed that there are only two yokes in order to reduce the number of elements. Material constants are shown in Table 1. The yoke is composed of the laminated non-oriented electrical steel sheets. As the temperature rise of

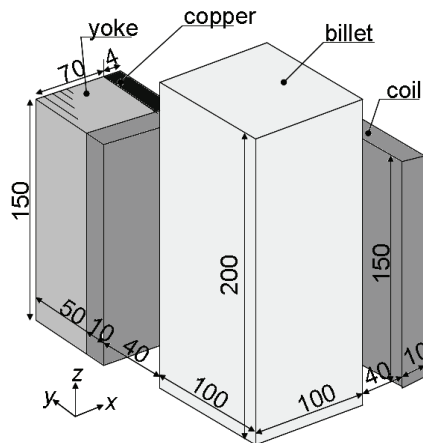


Fig. 2. Model of billet heater

Table 1. Material constants

	$\mu_s$	$\sigma_x$ (S/m)	$\sigma_y$ (S/m)	$\sigma_z$ (S/m)
Coil	1	0	0	0
Billet (1200°C)	1	$8.26 \times 10^5$	$8.26 \times 10^5$	$8.26 \times 10^5$
Yoke (ON)	1000	0	$0.7 \times 10^7$	$0.7 \times 10^7$
Yoke (OFF)	1	0	$1.0 \times 10^3$	$1.0 \times 10^3$
Copper	1	$5.8 \times 10^7$	$5.8 \times 10^7$	$5.8 \times 10^7$

the yoke is not remarkable and limited below about 200°C, the conductivity is chosen as  $0.7 \times 10^7$  S/m. Since the yoke is laminated, the conductivity in the  $x$  direction (thickness direction) of the yoke is set to zero. We have already examined the applicability of such an anisotropic conductivity in modeling the steel lamination [10]. The relative magnetic permeability was set as 1000, and the linear analysis is carried out. The current of the coil is set as 50 000 AT (700 Hz). The conductivity of the billet at 1200°C is used. The copper shield was installed on both sides of yoke in order to reduce the perpendicular flux which invades into a side surface of the yoke.

### Objective function

The following objective function is defined as follows in order to maximize the eddy current loss in the billet (this is equal to minimize  $W_1$ ), and minimize the eddy current loss  $W_2$  in the laminated yoke:

$$W = k_1 \cdot W_1 + k_2 \cdot W_2, \quad (17)$$

$$W_1 = \sum_{ie} V^{(ie)} \left\{ \frac{\left( J_{e_x}^{(ie)} \right)^2}{\sigma_x} + \frac{\left( J_{e_y}^{(ie)} \right)^2}{\sigma_y} + \frac{\left( J_{e_z}^{(ie)} \right)^2}{\sigma_z} \right\}^{-1}, \quad (18)$$

$$W_2 = \sum_{ie} V^{(ie)} \left\{ \frac{\left( J_{e_y}^{(ie)} \right)^2}{\sigma_y} + \frac{\left( J_{e_z}^{(ie)} \right)^2}{\sigma_z} \right\}, \quad (19)$$

$$k_1 = 1, \quad k_2 = \frac{W_1}{W_2}, \quad (20)$$

where  $ie$  is the element number in the target region. The target region is the billet in the case of  $W_1$ , and is the yoke in the case of  $W_2$ .  $J_{e_x}$ ,  $J_{e_y}$  and  $J_{e_z}$  are the  $x$ -,  $y$ - and  $z$ -components of the eddy current vector,  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$  are the conductivities,  $V$  is the volume of the element.  $k_1$  and  $k_2$  are the weighting factors. Equation (20) means that  $k_2$  is determined using the previous values of  $W_1$  and  $W_2$  in order to arrange the amplitudes of first and second terms of Eq. (17) as nearly the same values. In the optimal design, the conductivity  $\sigma$  is chosen as the design variable  $p$  in Eq. (3).

### Results of optimization

Figure 3 (a) shows the initial shape. In the initial shape, the material was placed in all elements of design region. The optimized shape using the ON/OFF method is shown in Fig. 3 (b). The yoke became thin toward the center. The flux flows into the wide entrance of yoke. As the optimal yoke is thinner than the initial one, the flux density in the optimal yoke is increased as shown in Fig. 4. Since the volume of a yoke is decreased compared with the initial shape, the eddy current loss in the yoke is also decreased. The leakage flux from the coil which penetrates to the yoke in the  $y$ -direction is reduced because the distance between the

center part of the yoke and coil is increased. Then, the eddy current induced in the yoke due to the flux which is perpendicular to the yoke surface is reduced.

Figure 5 shows the eddy current loss distribution in the billet. The eddy current loss of the billet is almost the same for both initial and optimal shapes. Table 2 shows the eddy current losses at each part.  $We_{yoke}$ ,  $We_{shield}$  and  $We_{billet}$  are the eddy current loss in the yoke, shield

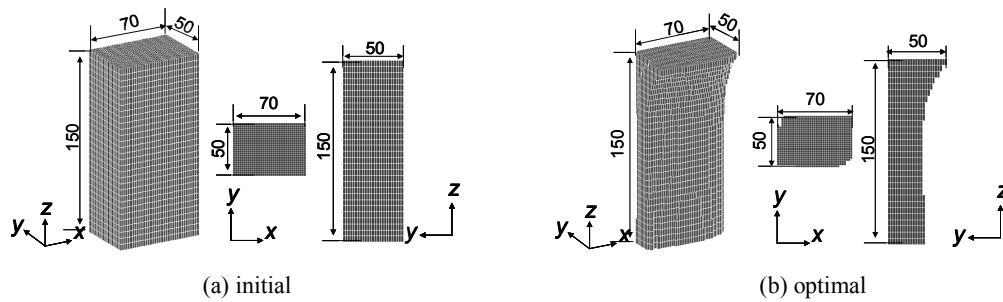


Fig. 3. Shape of laminated yoke

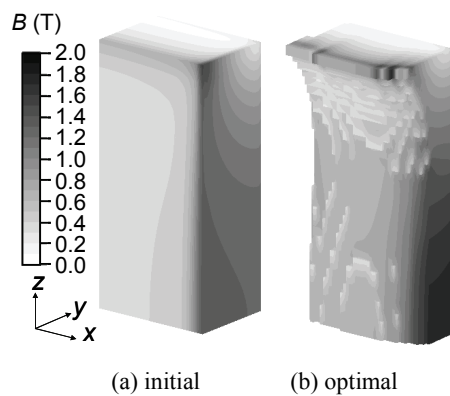


Fig. 4. Flux density distribution in yoke

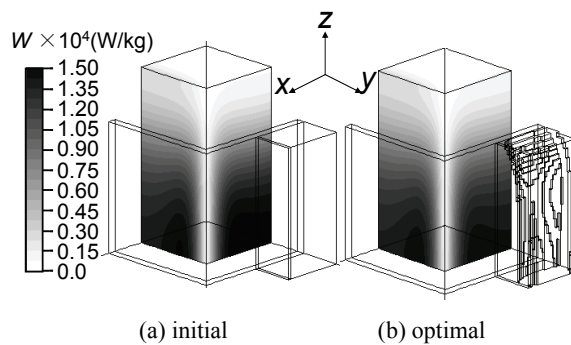


Fig. 5. Eddy current loss in billet

Table 2. Eddy current losses (1/8 region)

	Initial	Optimal
$We_{\text{yoke}}$ (W)	11.9	5.7
$We_{\text{shield}}$ (W)	165.7	141.1
$We_{\text{billet}}$ (W)	25 797	251 146

and billet in the 1/8 region, respectively.  $We_{\text{yoke}}$  and  $We_{\text{shield}}$  are reduced (-52% and -15%) by the optimization, but a  $We_{\text{billet}}$  is almost the same (-2.5%). Although the improvement of the efficiency ( $25146/25292.8 = 99.4\%$ ) of the optimal shape is a little compared with that ( $25797/25974.6 = 99.3\%$ ) of the initial shape, the overheating of yoke and shield can be considerably reduced. The number of FEM calculations is 1280, and the CPU time is 607 hours using 2.67 GHz PC.

#### 4. Conclusion

The ON/OFF method for topology optimization is extended to the 3-D linear eddy current problem by combining the ON/OFF method with the step-by-step method. The newly developed ON/OFF method is applied to the optimization of the model of a 3-D actual billet heater. As a result of optimization of the laminated yoke of billet heater, the local overheating of the yoke was decreased without reducing the heating efficiency. The speed up of calculation, the nonlinear analysis etc. are the future subjects.

#### Acknowledgements

This research is partly supported by “The Subsidy to Promote Science and Technology in the Prefectures where Nuclear Power Plants and Other Power Plants are Located”.

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