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Hierarchical filtration for distributed linear multisensor systems

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In the paper two filtration algorithms for distributed multisensor system are presented. The first one is derived for a linear dynamical system composed of local subsystems described by local state equations. Local estimates are sent to a central station to be fused and formed an optimal global estimate. The second algorithm is derived for a system observed by local nodes that determine estimates of the whole system using local information and periodically aggregated information from other nodes. Periodically local estimates are sent to the central station to be fused. Owing to this a reduced communication can be achieved.

Key words: multisensor system, distributed Kalman filtering, decentralized filtration, hierarchical fusion, aggregated information

1. Introduction

Multisensor systems find applications in many areas such as aerospace, robotics, image processing, military surveillance, medical diagnosis. The advantage of using these systems over a systems with a single sensor results from e.g. improved reliability, robustness, extended coverage, improved resolution e.t.c. In these systems an optimal state estimation problem is one of the critical concerns.

Theoretically, state estimate can be determined by using Kalman filter in a centralized structure. Conventional Kalman filtration requires that all process measurements are sent to a central station which determines an estimate of the state system. The centralized architecture produces an optimal estimate in a minimum mean square error (MMSE) sense, but it may imply low survivability and requires high processing and communication loads.

In order to integrate data from distributed sensors estimation fusion algorithms and appropriate architectures are proposed. The fusion approach has been researched for years and some results are known. In [4, 5, 9] a centralized optimal state estimate is calculated from estimates determined by local nodes. The global estimate is equivalent

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to the optimal centralized one. In [1, 2, 3] are presented fusion algorithms guaranteed local optimality, only.

Fusion algorithms are realized in different structures.

In a hierarchical structure local nodes process its sensor data to form local estimates. These estimates are sent to a central node to be fused. The hierarchical structure has the advantage (as compared with a centralized one) of lower communication, lower processing cost and increased reliability. It is possible that information flows from local nodes to the central node and also from the central node to local nodes (feedback architecture). Owing to this an accuracy of local estimates can be improved.

In a fully decentralized structure the central node is absent. Each sensor node can operate independently of other component and communication between nodes is one to one. From communication constraints point of view it is useful for local nodes to send information as transformed data. It has been discussed e.g. in [4, 9, 10, 11, 12, 13]. Reduced communication can be also achieved by lower rate of communication from local nodes to the central node in comparison with the sensor observation rate. This approach is proposed e.g. in [2, 6, 8].

In the paper two filtration algorithms for large scale multisensor systems are presented. In the section 4 a state estimation for a system with local dynamical models connected with local measurement nodes is considered. In the section 5 an original proposal of a hierarchical fusion architecture is presented. In this structure local nodes determine estimates of the whole system using local information and periodically aggregated information from other nodes. Periodically local estimates are sent to the central node to be combined.

2. Preliminaries

It is well known that a minimum mean square error (MMSE) estimate $\hat{x}$ of a random signal x given information $\vec{i}$ is a conditional expectation $\hat{x} = E(x|\vec{i})$. For dynamical systems a state estimate $\hat{x}_{n+1|j}$ at time $n+1$ given measurement information $\vec{i}_j = [i_0^T, i_1^T, ..., i_j^T]^T$ at time j has the form

$$\hat{x}_{n+1|j} = E(x_{n+1}|\vec{i}_j). \tag{1}$$

Thus, for $j = n+1$ we have

$$\hat{x}_{n+1|n+1} = E(x_{n+1}|\vec{i}_{n+1}) = E(x_{n+1}|\vec{i}_n, i_{n+1}) \tag{2}$$

where $\vec{i}_{n+1} = [\vec{i}_n^T, i_{n+1}^T]^T$. If the random vector $[x_{n+1}^T, \vec{i}_n^T, i_{n+1}^T]^T$ is gaussian, then

$$\hat{x}_{n+1|n+1} = E(x_{n+1}|\vec{i}_{n+1}) = E(x_{n+1}|\vec{i}_n, i_{n+1}) = E(x_{n+1}|\vec{i}_n) + E(x_{n+1}|\widetilde{i}_{n+1|n}) - Ex_{n+1} \tag{3}$$

where

$$\widetilde{i}_{n+1|n} = i_{n+1} - E(i_{n+1}|\vec{i}_n) \tag{4}$$

If the random vector $[x^T_{n+1}, \widetilde{i}^T_{n+1|n}]^T$ is gaussian then

$$E(x_{n+1}|\widetilde{i}_{n+1|n}) = Ex_{n+1} + P_{x_{n+1}\widetilde{i}_{n+1|n}} P^{-1}_{\widetilde{i}_{n+1|n}\widetilde{i}_{n+1|n}} (\widetilde{i}_{n+1|n} - E\widetilde{i}_{n+1|n}) \tag{5}$$

where $P_{\alpha\beta}$ denotes the covariance matrix of the random vectors α and β.

Under above assumptions, (3) can be written in the form

$$\hat{x}_{n+1|n+1} = \hat{x}_{n+1|n} + K_{n+1}[i_{n+1} - E(i_{n+1}|\vec{i}_n)] \tag{6}$$

where

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{i}_n) \tag{7}$$

and

$$K_{n+1} = P_{x_{n+1}\widetilde{i}_{n+1|n}} P^{-1}_{\widetilde{i}_{n+1|n}\widetilde{i}_{n+1|n}}. \tag{8}$$

The equations (6)-(8) are used for determination of a state estimate $\hat{x}_{n|n}$ given available information $\vec{i}_n$.

3. Model of a system and a problem statement

Consider a linear system described by the equation

$$x_{n+1} = A_n x_n + w_n \tag{9}$$

where x_n is a state, A_n is a system matrix, w_n is a state noise. It is assumed that $x_0 \sim N(\bar{x}_0, X_0)$, $w_n \sim N(\bar{w}_n, W_n)$ and $x_n \in R^k$, $w_n \in R^k$, $A_n \in R^{k\times k}$. Additionally, w_n is a gaussian white noise process independent of the gaussian initial state x_0.

Let there exist M local measurement nodes with measurement equations

$$y^i_n = \bar{C}^i_n x^i_n + r^i_n, \quad i = 1, \dots, M \tag{10}$$

where

$$x^i_n = \bar{D}^i_n x_n \tag{11}$$

is a local state described by a local state equation

$$x^i_{n+1} = A^i_n x^i_n + w^i_n, \;\; i = 1, 2, \dots M. \tag{12}$$

It is assumed that $w^i_n \sim N(\bar{w}^i_n, W^i_n)$, $r^i_n \sim N(0, R^i_n)$, w_n, r^i_m and w^i_n, $r^i_m$are gaussian white noise processes independent of each other and of the gaussian initial state x^i_0; $x^i_n \in R^{k_i}$, $y^i_n \in R^{p^i}$, $\bar{C}^i_n, \bar{D}^i_n, A^i_n$ are known matrices with appropriate dimensions.

Equation (12) may be treated as a local model connected with the local measurement node.

For the above model first, in the section 4, the distributed estimation fusion algorithm is derived. Then, in the section 5, for $\bar{D}^i_n = \mathbf{1}$ in (11), an algorithm of state filtration realized in a hierarchical, partially decentralized, fusion architecture is proposed.

4. Kalman filtration for $\vec{i}_{n+1} = [\vec{y}_n^T, y_{n+1}^T]$

Let us consider a state filtration for the system (9) with the measurements model (10) and (11). Equation (10) can be written in the form

$$y_n^i = C_n^i x_n + r_n^i \tag{13}$$

where

$$C_n^i = \bar{C}_n^i \bar{D}_n^i. \tag{14}$$

Denote by $y_n = [y_n^{1T}, \dots, y_n^{MT}]^T$, $\vec{y}_{n+1} = [y_0^T, \dots, y_n^T, y_{n+1}^T]^T = [\vec{y}_n^T, y_{n+1}^T]^T$, $\widetilde{y}_{n+1|n} = y_{n+1} - E(y_{n+1}|\vec{y}_n)$ and $\vec{i}_n = \vec{y}_n^T$, $i_{n+1} = y_{n+1}^T$. The vector y_n can be written in the form

$$y_n = C_n x_n + r_n \tag{15}$$

where $C_n = [C_n^{1T}, \dots, C_n^{MT}]^T$, $r_n = [r_n^{1T}, \dots, r_n^{MT}]^T$, $R_n = E r_n r_n^T = blockdiag\{R_n^1, \dots, R_n^m\}$. For the system described by (9) and (15) the random vectors $[x_{n+1}^T, \vec{y}_n^T, y_{n+1}^T]^T$, $[y_{n+1}^T, \vec{y}_n^T]^T$ and $[x_{n+1}^T, \widetilde{y}_{n+1|n}^T]^T$ are gaussians. Thus the estimate

$$\hat{x}_{n+1|n+1} = E(x_{n+1}|\vec{y}_{n+1}) \tag{16}$$

results from (6)–(8) and has the classical form

$$\hat{x}_{n+1|n+1} = \hat{x}_{n+1|n} + K_{n+1}(y_{n+1} - \hat{y}_{n+1|n}) \tag{17}$$

where $\hat{y}_{n+1|n} = E(y_{n+1}|\vec{y}_n) = C_{n+1}\hat{x}_{n+1|n}$ and

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{y}_n) = A_n \hat{x}_{n|n} + \bar{w}_n. \tag{18}$$

The matrix gain K_{n+1} can be found from (8) as

$$K_{n+1} = P_{n+1|n} C_{n+1}^T (C_{n+1} P_{n+1|n} C_{n+1}^T + R_{n+1})^{-1} \tag{19}$$

where the a priori error covariance matrix has the form

$$P_{n+1|n} = E(\widetilde{x}_{n+1|n} \widetilde{x}_{n+1|n}^T) = A_n P_{n|n} A_n^T + W_n. \tag{20}$$

A posteriori error covariance matrix $P_{n|n}$ is given by

$$P_{n|n} = E \widetilde{x}_{n|n} \widetilde{x}_{n|n}^T = (\mathbf{1} - K_n C_n) P_{n|n-1}. \tag{21}$$

An initial condition $\hat{x}_{0|0}$ results from (17)

$$\hat{x}_{0|0} = \hat{x}_{0|-1} + K_0(y_0 - C_0 \hat{x}_{0|-1}) = \bar{x}_0 + K_0(y_0 - C_0 \bar{x}_0). \tag{22}$$

The covariance matrix $P_{0|-1}$ can be determined from (20) as

$$P_{0|-1} = E\widetilde{x}_{0|-1}\widetilde{x}_{0|-1}^T = E(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T = X_0. \tag{23}$$

Kalman filter consists of (17)–(21) with the initial conditions (22)–(23).

The state estimate $\hat{x}_{n|n}$ can be determined in a centralized structure with one central processor using observations $y_n^i, \;\; i = 1,2,...,M$ passed from M sensors. If the number of sensors increases then the inverse of the matrix in (19) increases in proportion to the square of its dimension and this approach becomes limited. In this case an advantage in calculations gives an information filter described in the next section.

4.1. Information Kalman filtration

Let us notice that (17) can be written in the form

$$\hat{x}_{n+1|n+1} \quad = (\mathbf{1} - K_{n+1}C_{n+1})\hat{x}_{n+1|n} + K_{n+1}y_{n+1}. \tag{24}$$

It can be shown that

$$\mathbf{1} - K_{n+1}C_{n+1} = P_{n+1|n+1}P_{n+1|n}^{-1} \tag{25}$$

and

$$K_{n+1} \quad = P_{n+1|n+1}C_{n+1}^T R_{n+1}^{-1}. \tag{26}$$

Inserting (25) and (26) to (24) gives

$$\hat{x}_{n+1|n+1} = P_{n+1|n+1}P_{n+1|n}^{-1}\hat{x}_{n+1|n} + P_{n+1|n+1}C_{n+1}^T R_{n+1}^{-1} y_{n+1}. \tag{27}$$

From (27) it results that

$$P_{n+1|n+1}^{-1}\hat{x}_{n+1|n+1} = P_{n+1|n}^{-1}\hat{x}_{n+1|n} + C_{n+1}^T R_{n+1}^{-1} y_{n+1}. \tag{28}$$

Denoting by

$$\hat{x}_{n+1|n+1}^* = P_{n+1|n+1}^{-1}\hat{x}_{n+1|n+1}, \qquad \hat{x}_{n+1|n}^* = P_{n+1|n}^{-1}\hat{x}_{n+1|n} \tag{29}$$

and using the definition of C_{n+1}, R_{n+1} and y_{n+1} we can write (28) in the form

$$\hat{x}_{n+1|n+1}^* = \hat{x}_{n+1|n}^* + C_{n+1}^T R_{n+1}^{-1} y_{n+1} = \hat{x}_{n+1|n}^* + \sum_{i=1}^{M} C_{n+1}^{iT}(R_{n+1}^i)^{-1} y_{n+1}^i. \tag{30}$$

Equation (30) is a measurement – update information filter equation.

In order to determine $\hat{x}_{n+1|n}^*$ multiply the both sides of (18) by $P_{n+1|n}^{-1}$. Using (29) we obtain

$$P_{n+1|n}^{-1}\hat{x}_{n+1|n} = P_{n+1|n}^{-1}A_n P_{n+1|n}P_{n+1|n}^{-1}\hat{x}_{n|n} + P_{n+1|n}^{-1}\bar{w}_n, \tag{31}$$

$$\hat{x}^*_{n+1|n} = P^{-1}_{n+1|n} A_n P_{n+1|n} \hat{x}^*_{n|n} + P^{-1}_{n+1|n} \bar{w}_n. \tag{32}$$

Equation (32) is a time-update information filter equation.

It can be shown that a recursive form of the covariance matrix $P^{-1}_{n+1|n+1}$ has the form

$$P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + C^T_{n+1} R^{-1}_{n+1} C_{n+1} = P^{-1}_{n+1|n} + \sum_{i=1}^{M} C^{iT}_{n+1} (R^i_{n+1})^{-1} C^i_{n+1}. \tag{33}$$

Finally, the Kalman information filter consists from (30) and (32) with (33) and (20).

Let us notice that information Kalman filter results from classical covariance filter. But in the information filter it is possible to increase processing speed.

Summarizing, the measurement-update equations (30) and (33) are computationally simpler than the equations (17) and (21). The time-update equation (32) with an appropriate matrix$P^{-1}_{n+1|n}$ is more complex than (18) and (20) but they do not depend on the observations.

4.2. Hierarchical filtration

Consider a local state estimate of the system (12) based on the measurements y^i_n described by the model (10) i.e.

$$\hat{x}^i_{n+1|n+1} = E(x^i_{n+1} | \vec{y}^i_{n+1}) \tag{34}$$

where $\vec{y}^i_{n+1} = [y^{iT}_0, \ldots, y^{iT}_{n+1}]^T = [\vec{y}^{iT}_n, y^{iT}_{n+1}]^T$. Thus the information estimate $\hat{x}^{i*}_{n+1|n+1}$ results directly from the section 4.1 and has the form

$$\hat{x}^{i*}_{n+1|n+1} = \hat{x}^{i*}_{n+1|n} + \bar{C}^{iT}_{n+1} (R^i_{n+1})^{-1} y^i_{n+1} \tag{35}$$

$$\hat{x}^{i*}_{n+1|n} = (P^i_{n+1|n})^{-1} A^i_n P^i_{n+1|n} \hat{x}^{i*}_{n|n} + (P^i_{n+1|n})^{-1} \bar{w}^i_n \tag{36}$$

with

$$P^i_{n+1|n} = A^i_n P^i_{n|n} A^{iT}_n + W^i_n \tag{37}$$

$$(P^i_{n+1|n+1})^{-1} = (P^i_{n+1|n})^{-1} + \bar{C}^{iT}_{n+1} (R^i_{n+1})^{-1} \bar{C}^i_{n+1}. \tag{38}$$

From (35) and (38) it results that

$$\bar{C}^{iT}_{n+1} (R^i_{n+1})^{-1} y^i_{n+1} = \hat{x}^{i*}_{n+1|n+1} - \hat{x}^{i*}_{n+1|n} \tag{39}$$

and

$$\bar{C}^{iT}_{n+1} (R^i_{n+1})^{-1} \bar{C}^i_{n+1} = (P^i_{n+1|n+1})^{-1} - (P^i_{n+1|n})^{-1}. \tag{40}$$

Let us notice that $C^{iT}_{n+1}(R^i_{n+1})^{-1}y^i_{n+1}$ and $C^{iT}_{n+1}(R^i_{n+1})^{-1}C^i_{n+1}$ in (30) and (33), using (39)and (40) can be written in the form

$$\begin{aligned} C^{iT}_{n+1}(R^i_{n+1})^{-1}y^i_{n+1} &= \bar{D}^{iT}_n\bar{C}^{iT}_n(R^i_{n+1})^{-1}y^i_{n+1} = \bar{D}^{iT}_n(\hat{x}^{i*}_{n+1|n+1} - \hat{x}^{i*}_{n+1|n}) \\ C^{iT}_{n+1}(R^i_{n+1})^{-1}C^i_{n+1} &= \bar{D}^{iT}_n\bar{C}^{iT}_n(R^i_{n+1})^{-1}\bar{C}^i_n\bar{D}^i_n = \bar{D}^{iT}_n[(P^i_{n+1|n+1})^{-1} - (P^i_{n+1|n})^{-1}]\bar{D}^i_n \end{aligned} \tag{41}$$

and finally (30) and (33) can be written as

$$\hat{x}^*_{n+1|n+1} = \hat{x}^*_{n+1|n} + \sum_{i=1}^{M}\bar{D}^{iT}_n(\hat{x}^{i*}_{n+1|n+1} - \hat{x}^{i*}_{n+1|n}) \tag{42}$$

$$P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + \sum_{i=1}^{M}\bar{D}^{iT}_n[(P^i_{n+1|n+1})^{-1} - (P^i_{n+1|n})^{-1}]\bar{D}^i_n. \tag{43}$$

Equations (42) , (43) with (32) and (20) summarise partially decentralized filtration.

For $\bar{D}^i_n = \mathbf{1}$ we have that $x^i_n = x_n$, $A^i_n = A_n$, $w^i_n = w_n$. It means that local nodes determine the estimate of the system (9) based on the measurement model (13). This case is generally considered in a literature.

We can treated the global estimate performed by the central processor as overhead. In this case it needs information of the difference $\bar{D}^{iT}_n(\hat{x}^{*i}_{n+1|n+1} - \hat{x}^{*i}_{n+1|n})$ and $[\bar{D}^{iT}_n[(P^i_{n+1|n+1})^{-1} - (P^i_{n+1|n})^{-1}]\bar{D}^i_n]$ from local nodes. There the communication between local nodes is not needed.

5. Aggregated filtration

Consider the system (12)-(10) for $\bar{D}^i_n = \mathbf{1}$. It means that the state x_n described by (9) is measured by local nodes according to a measurement model

$$y^i_n = \bar{C}^i_n x_n + r^i_n. \tag{44}$$

Let

$$m^i_{lk} = D^i_{lk}y^i_{lk}, \quad l = 0,1,\ldots, \quad i = 1,\ldots,M \tag{45}$$

be aggregated, at every k units, information of the ith local sensor. It is assumed that the vector m^i_{lk} is the vector of smaller dimension than y^i_{lk}. The aggregated information of the whole system has the form

$$m_{lk} = D_{lk}y_{lk} = D_{lk}C_{lk}x_{lk} + D_{lk}r_{lk} \tag{46}$$

where $m_{lk} = [m^{1T}_{lk},\ldots,m^{MT}_{lk}]^T, y_{lk} = [y^{1T}_{lk},\ldots,y^{MT}_{lk}]^T, D_{lk} = diag\{D^i_{lk}, i = 1,\ldots,M\}, C_{lk} = [C^{1T}_{lk},\ldots,C^{MT}_{lk}]^T, r_{lk} = [r^{1T}_{lk},\ldots,r^{MT}_{lk}]^T$.

Denote by $\vec{y}_n^i = [y_1^{iT},...,y_j^{iT},...y_n^{iT}]^T$, $j \neq lk$, $l = 0,...,int[\frac{n}{k}]$, $\vec{m}_{lk} = [m_k^T,..,m_{2k}^T,..,m_{lk}^T]^T$. The estimation problem considered in this section is to find the state estimates in the form:

$$\hat{x}_{n|n,i} = E(x_n|\vec{y}_n^i, \vec{m}_{lk}), \quad \text{for} \quad lk < n < (l+1)k, \quad l = int\left[\frac{n}{k}\right] \tag{47}$$

and

$$\hat{x}_{lk|lk,i} = E(x_{lk}|\vec{y}_{lk-1}^i, \vec{m}_{lk}), \quad \text{for} \quad l = 0,1,... \tag{48}$$

Proposed algorithm may be used in a hierarchical partially decentralized structure. The ith local subsystem determines the state estimate using its own local information $\vec{y}_n^i$ and, at every k units, aggregated information $\vec{m}_{lk}$ received from other subsystems. The local estimates$\hat{x}_{lk|lk,i}$, $i = 1,2,...,M$ are transmitted every k units to a central node where global state estimate is reconstructed.

Notice that for D_{lk}^i equal to a unit matrix $(D_{lk}^i = \mathbf{1})$, all global information available at time lk is used for filtration. For $D_{lk}^i = \mathbf{0}$ no information is transmitted from the ith subsystem to other subsystems.

5.1. Derivation of the state estimate $\hat{x}_{n|n,i}$ for $n = lk$

According to (48) we have

$$\begin{aligned} \hat{x}_{n|n,i} &= E(x_n|\vec{y}_{n-1}^i, \vec{m}_n) = E(x_n|\vec{y}_{n-1}^i, \vec{m}_{n-k}, m_n) = \\ &= E(x_n|\vec{y}_{n-1}^i, \vec{m}_{n-k}) + E(x_n|\widetilde{m}_{n,i}) - \bar{x}_n = \\ &= \hat{x}_{n|n-1,i} + P_{x_n\widetilde{m}_{n,i}} P_{\widetilde{m}_{n,i}\widetilde{m}_{n,i}}^{-1} \widetilde{m}_{n,i} \end{aligned} \tag{49}$$

where

$$\begin{aligned} &\hat{x}_{n|n-1,i} = A_{n-1}\hat{x}_{n-1|n-1,i} + \bar{w}_{n-1} \\ &\widetilde{m}_{n,i} = m_n - \hat{m}_{n|n-1,i} \\ &\hat{m}_{n|n-1,i} = E(m_n|\vec{y}_{n-1}^i, \vec{m}_{n-k}) = D_n C_n \hat{x}_{n|n-1,i}. \end{aligned} \tag{50}$$

It can be shown that

$$\hat{x}_{n|n,i} = \hat{x}_{n|n-1,i} + K_{n,i}(m_n - \hat{m}_{n|n-1,i}) \tag{51}$$

where

$$\begin{aligned} &K_{n,i} = P_{n|n-1,i} C_n^T D_n^T [D_n(C_n P_{n|n-1,i} C_n^T + R_n) D_n^T]^{-1} \\ &P_{n|n-1,i} = A_{n-1} P_{n-1|n-1,i} A_{n-1}^T + W_{n-1} \\ &P_{n|n,i} = (\mathbf{1} - K_{n,i} D_n C_n) P_{n|n-1,i}. \end{aligned} \tag{52}$$

The estimate $\hat{x}_{n-1|n-1,i}$ $(\hat{x}_{lk-1|lk-1,i})$ is determined in the next section.

5.2. Derivation of the state estimate $\hat{x}_{n|n,i}$ for $lk < n < (l+1)k$

Using (47) we have

$$\begin{aligned}\hat{x}_{n|n,i} &= E(x_n|\vec{y}_n^i, \vec{m}_{lk}) = E(x_n|\vec{y}_{n-1}^i, \vec{m}_{lk}, y_n^i) = \\ &= E(x_n|\vec{y}_{n-1}^i, \vec{m}_{lk}) + E(x_n|\widetilde{y}_{n,i}) - \bar{x}_n = \\ &= \hat{x}_{n|n-1,i} + P_{x_n\widetilde{y}_{n,i}} P^{-1}_{\widetilde{y}_{n,i}\widetilde{y}_{n,i}} \widetilde{y}_{n,i}\end{aligned} \tag{53}$$

where

$$\begin{aligned}&\hat{x}_{n|n-1,i} = A_{n-1}\hat{x}_{n-1|n-1,i} + \bar{w}_{n-1} \\ &\widetilde{y}_{n,i} = y_n^i - \hat{y}_{n|n-1,i} \\ &\hat{y}_{n|n-1,i} = E(y_n^i|\vec{y}_{n-1}^i, \vec{m}_{lk}) = C_n^i \hat{x}_{n|n-1,i}^i.\end{aligned} \tag{54}$$

It can be shown that

$$\hat{x}_{n|n,i} = A_{n-1}\hat{x}_{n-1|n-1,i} + \bar{w}_{n-1} + K_{n,i}(y_n^i - \hat{y}_{n|n-1}^i) \tag{55}$$

where

$$\begin{aligned}&K_{n,i} = P_{n|n-1,i} C_n^{iT} (C_n^i P_{n|n-1,i} C_n^{iT} + R_n^i)^{-1} \\ &P_{n|n-1,i} = A_{n-1} P_{n-1|n-1,i} A_{n-1}^T + W_{n-1} \\ &P_{n|n,i} = (\mathbf{1} - K_{n,i} C_n^i) P_{n|n-1,i}.\end{aligned} \tag{56}$$

An initial condition $\hat{x}_{0|0,i}$ results from (53)

$$\hat{x}_{0|0,i} = \hat{x}_{0|-1,i} + K_{0,i}(y_0^i - C_0^i \hat{x}_{0|-1,i}) = \bar{x}_0 + K_{0,i}(y_0^i - C_0^i \bar{x}_0). \tag{57}$$

The covariance matrix $P_{0|-1,i}$ can be determined as

$$P_{0|-1}^i = E(\widetilde{x}_{0|-1}^i \widetilde{x}_{0|-1}^{iT}) = E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] = X_0. \tag{58}$$

5.3. Information Kalman filter for $n = lk$

Let us consider (51) with (52). It can be written in the form

$$\hat{x}_{n|n,i} = (\mathbf{1} - K_{n,i} D_n C_n)\hat{x}_{n|n-1,i} + K_{n,i} m_n. \tag{59}$$

Now we transform $(\mathbf{1} - K_{n,i} D_n C_n)$ and $K_{n,i}$ to an appropriate form. We have

$$\mathbf{1} - K_{n,i} D_n C_n = \overbrace{(\mathbf{1} - K_{n,i} D_n C_n) P_{n|n-1,i}}^{P_{n|n,i}(52)} (P_{n|n-1,i})^{-1} = P_{n|n,i}(P_{n|n-1,i})^{-1}. \tag{60}$$

Denote by

$$O_{n,i} = D_n (C_n P_{n|n,i} C_n^T + R_n) D_n^T. \tag{61}$$

Multiplying the both sides of (43) by $O_{n,i}$ gives

$$K_{n,i}(\overbrace{D_n(C_nP_{n|n-1,i}C_n^T+R_n)D_n^T}^{O_{n,i}(61)}) = P_{n|n-1,i}C_n^TD_n^T. \tag{62}$$

Thus

$$K_{n,i}D_nR_nD_n^T = (\mathbf{1}-K_{n,i}D_nC_n)P_{n|n-1,i}C_n^TD_n^T \tag{63}$$

and

$$K_{n,i} = \overbrace{(\mathbf{1}-K_{n,i}D_nC_n)}^{P_{n|n,i}(P_{n|n-1,i})^{-1}(60)}P_{n|n-1,i}C_n^TD_n^T(D_nR_nD_n^T)^{-1} = P_{n|n,i}C_n^TD_n^T(D_nR_nD_n^T)^{-1}. \tag{64}$$

Inserting (60) and (64) to (59) gives

$$\hat{x}_{n|n,i} = P_{n|n,i}(P_{n|n-1,i})^{-1}\hat{x}_{n|n-1,i} + P_{n|n,i}C_n^TD_n^T(D_nR_nD_n^T)^{-1}m_n. \tag{65}$$

Multiplying the both sides of (65) by $(P_{n|n,i})^{-1}$ gives

$$(P_{n|n,i})^{-1}\hat{x}_{n|n,i} = (P_{n|n-1,i})^{-1}\hat{x}_{n|n-1,i} + C_n^TD_n^T(D_nR_nD_n^T)^{-1}m_n. \tag{66}$$

Denoting by

$$\hat{x}^*_{n|n,i} = (P_{n|n,i})^{-1}\hat{x}_{n|n,i}, \tag{67}$$

and using the definition of C_n, D_n, R_n and m_n we can write (66) in the form

$$\hat{x}^*_{n|n,i} = (P_{n|n-1,i})^{-1}\hat{x}_{n|n-1,i} + \sum_{j=1}^{M} i_n^j \tag{68}$$

where

$$i_n^j = C_n^{jT}D_n^{jT}(D_n^jR_n^jD_n^{jT})^{-1}m_n^j. \tag{69}$$

At time $n = lk$ an information vector i_{lk}^j is computed by jth ($j = 1,2,\ldots,M$) local node and sent to other local nodes. The ith ($i = 1,2,\ldots,M$) local node computes the estimate $\hat{x}^*_{lk|lk,i}$ according to (68), covariance matrix $P_{lk|lk,i}$ according to (52) and the state estimate $\hat{x}_{lk|lk,i} = P_{lk|lk,i}\hat{x}^*_{lk|lk,i}$.

The information of $\hat{x}_{lk|lk,i}$ and $P_{lk|lk,i}$, $i = 1,2,\ldots,M$ is sent to the central node, where a global estimate is calculated using e.g. covariance intersection method [7]. According to [7] the global estimate $\hat{x}_{lk|lk}$ is calculated as

$$\hat{x}_{lk|lk} = \sum_{i=1}^{M} \alpha_{lk,i}P_{lk}P^{-1}_{lk|lk,i}\hat{x}_{lk|lk,i} \tag{70}$$

where

$$\alpha_{lk,i} = \frac{(trP_{lk|lk,i})^{-1}}{\sum_{i=1}^{M}(trP_{lk|lk,i})^{-1}}, \quad P_{lk}^{-1} = \sum_{i=1}^{M}\alpha_{lk,i}(P_{lk|lk,i})^{-1}. \tag{71}$$

6. Conclusions

In the paper are presented two partially decentralized Kalman filters realized in the hierarchical structures. The first one may be used for large scale distributed multisensor systems described by global and local states models. This approach can be applied for a multisensor system in which local state models are equivalent to the global model. In order to reduce communication and calculation requirements periodically fused formula is proposed. Local nodes produce optimal estimates of the global state in MMSE sense using its own detailed information and periodically aggregated information from other local nodes. Local state estimates and appropriate covariance matrices are sent to the central node to be fused. Fusion may be performed in local nodes without central node, but it requires full communication between nodes.

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