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Dynamic event-triggered sliding-mode fault-tolerant bipartite consensus control for multi-agent systems based on adaptive observer

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Abstract. For a class of nonlinear second-order leader-follower multi-agent systems with actuator faults, an adaptive observer-based dynamic event-triggered sliding-mode fault-tolerant bipartite consensus control strategy is proposed. Firstly, an adaptive fault observer is designed. The position, velocity and actuator fault degree of the agents at the current moment are obtained. Secondly, a dynamic event-triggered mechanism is proposed to save network resources. Then, a dynamic sliding-mode face and a sliding-mode fault-tolerant bipartite consensus control strategy are given based on the output of the fault observer and the dynamic event-triggered mechanism. So that the bipartite consensus of the second-order multi-agent system can still be realized when there is an actuator fault. The conditions for the convergence of the error of fault-tolerant bipartite consensus for multi-agent systems are given. Finally, in a leader-follower multi-agent system connected by an undirected graph, the effectiveness of the designed control strategy is verified through simulations.

Keywords: multi-agent systems; adaptive observer; dynamic event-triggered mechanism; sliding-mode fault-tolerant control; bipartite consensus control.

1. INTRODUCTION

In the last decade, the control problem of multi-agent systems has been widely studied due to the increasing number of applications for multi-agent systems. As the foundation of multi-agent system control, the consistency problem has been a hot topic in the research of multi-agent system control strategies. Various approaches have been proposed for different aspects of the consistency problem of multi-agent system, including input saturation, optimal control, impulse control, and fuzzy adaptive control [1–5]. In [1], a dynamic consistency protocols for nonlinear saturated multi-agent system were introduced, and the limitations of low-gain feedback methods for input-saturated linear systems were emphasized. In [2], a distributed optimal coherent control algorithm for continuous-time multi-agent system was proposed, where the trade-off between the speed of convergence and the energy cost over a finite time horizon was emphasized. In [3], a hybrid protocol to achieve leader-follower consistency in multi-agent systems was proposed, where continuous and impulse control were combined. In [4], the leader-follower consistency problem in nonlinear multi-agent systems with interval time-varying delays using impulse control was studied. In [5], an energy coordination control method based on multi-agent systems and neural networks is proposed and applied to energy management and voltage control of DC microgrids.

In all of the above studies of consistency in multi-agent systems, it is assumed that the agents have only one cooperative relationship with each other. That is, all agents are approaching the same consistency goal. However, in practical situations, agents have not only cooperative, but also competitive relationships with each other. That is, the states of the agents move toward opposite goals, such as the bidirectional flight of a swarm of UAVs, the relative motion of multiple robotic arms, and so on. Bipartite consensus in a multi-agent system is when agents with competing relationships eventually reach a state of opposite sign but equal size. Due to the wide range of applications, the literature on bipartite consensus for multi-agent systems has grown significantly in recent years [6–10]. In [6], a bipartite consensus for nonlinear multi-agent systems under directed signed graphs was focused on, where a new Lipschitz type condition was introduced to handle nonlinear terms. In [7], a bipartite consensus for multi-agent systems with noise on Markov switching topologies was studied. In [8], a differential privacy preserving bipartite consensus for multi-agent systems with opposing information was investigated and the importance of structurally balanced topological graphs was emphasized. In [9], the equivalence between the dichotomous consensus problem and the traditional consensus problem is established through the state feedback and output feedback control methods. The existing state feedback and input feedback consensus algorithms are directly applied to solve the problem of bipartite consensus for multi-agent systems. In [10], a mean-square bipartite consensus was studied for multi-agent systems with measurement noise and communication delay.

Due to equipment aging, communication errors, and other reasons, multi-agent systems may experience a variety of failures in real-world applications. These faults will affect the state of some agents. If faults are not handled or isolated in time, then the

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fault information will be quickly passed to other agents, which may eventually lead to system paralysis or destruction. Generally speaking, fault-tolerant control of multi-agent systems can be categorized into passive fault-tolerant control and active fault-tolerant control. Passive fault-tolerant control mainly relies on the robustness of the system to offset the effects of faults, or to predetermine the types of faults based on experience and compensate for them when they occur. In practice, passive fault-tolerant methods are costly and ineffective. Different from the passive fault-tolerant method, the active fault-tolerant method estimates the faults occurring in the system through adaptive law or fault observer, and then compensates the faults through the active fault-tolerant control strategy. This method has lower cost and better fault-tolerance effect. Therefore, in recent years, the research of active fault-tolerant control algorithms is a hot issue in the fault-tolerant control of multi-agent systems.

Many different observers have been designed in order to estimate the state and the degree of failure of multi-agent systems [11–13]. In [11], a sliding-mode observer was proposed and was used to estimate actuator faults in linear multi-agent systems. In [12], various graph topologies were considered, a proportional-integral observer was proposed, where actuator faults in a leader-follower linear multi-agent system are estimated in a distributed manner. In [13], a class of second-order leader-follower multi-agent systems with actuator faults was considered, where a super-twisted observer was designed and the velocity and actuator faults of each agent were estimated. Various fault-tolerant control methods have been proposed for the multi-agent system failure problem [14–18]. In [14], a class of second-order multi-agent systems with actuator faults is discussed. In [15], a leader state observer is constructed and used to estimate the state of the leader. Meanwhile, a fault-tolerant controller with finite time convergence was designed. A multi-agent system containing multiple leaders and followers was studied. In [16], a neural network based adaptive observer was designed and the unmeasurable states of the system are estimated. Based on the measurements, a fault-tolerant control law was designed. A class of nonlinear multi-agent systems with multiple leaders and followers is also considered. In [17], an improved distributed observer based on which the unmeasurable states of the system are estimated was designed. A finite time fuzzy fault-tolerant controller was designed. A class of multi-agent systems with incipient actuator faults in fixed and switched topologies is considered. In [18], a new distributed fault-tolerant consistency tracking controller is proposed. However, there are not many studies on fault-tolerance for multi-agent systems when failures occur during the realization of bipartite consensus. In fact, when the multi-agent systems experience a fault, it will be difficult for the systems to realize the bipartite consensus.

Sliding-mode control strategies have been widely used in multi-agent systems control methods because of their robustness to disturbances and unmodeled dynamics [19–21]. In [19], the consistency problem of a class of second-order multi-agent systems with unknown disturbances was considered. A sliding-mode control protocol based on the equivalent approximation law and state information between agents was proposed. In [20], a class of high-order uncertain random multi-agent systems was studied. Consistency and tracking error were combined. A new distributed fuzzy sliding-mode controller was designed. In [21], a class of high-order nonlinear multi-agent systems was studied. By using backstepping, a distributed recursive linear sliding-mode control scheme was proposed. Although the sliding-mode control strategy has good robustness, most existing studies have not fully utilized its advantages to address the fault-tolerant bipartite consensus problem in multi-agent systems.

In multi-agent systems, the control information acquired by each agent depends on neighbor agents. When the system contains a large number of agents, each communication is accompanied by a large amount of information transfer. However, not all of this information is useful. Consequently, many scholars have introduced event-triggered mechanisms into multi-agent systems to reduce the waste of network resources [22–26]. In [22], the event-triggered control method was extended to general linear multi-agent systems. In [23], a method was proposed for systems with identical linear dynamic models, where edge-based event-triggered control in a directed communication topology was introduced. In [24], positive lower bounds on non-zeno behaviors and inter-event intervals were ensured, and the problem of event-triggered time-varying formation control for general linear multi-agent systems was studied. In [25], a class of multi-agent systems with input saturation and actuator failure was considered. A fully distributed dynamic event-triggered control was proposed. In [26], the problem of event-triggered control of a class of first-order multi-agent systems with structurally balanced topological graphs was studied. Synergistic criteria for trigger condition parameters and validation cycles were established, and the bipartite consensus of the system was guaranteed. However, in the current research, event-triggered mechanisms are seldom applied to the bipartite fault-tolerant consistency of multi-agent systems. In fact, there is also a large amount of network resource waste in the process of realizing bipartite fault-tolerant consistency for multi-agent systems. The use of event-triggered mechanism or dynamic event-triggered mechanism can well save the network resources of multi-agent systems.

In fact, actuator faults or sensor faults have a significant impact on multi-agent systems. If these faults are not properly addressed, it becomes difficult for the system to achieve bipartite consensus. The strong robustness of the sliding-mode control law allows nonlinear disturbances in multi-agent systems to be effectively handled. Since the execution of control laws in multi-agent systems generally involves substantial network resource consumption, the use of a dynamic event-triggered mechanism can effectively reduce this resource waste. Although several studies have investigated the bipartite consensus problem of multi-agent systems based on event-triggered mechanisms, very little literature has focused on the fault-tolerant bipartite consensus problem of multi-agent systems under such mechanisms. In addition, few studies have applied sliding-mode control laws to the bipartite consensus control of multi-agent systems.

Based on the above analysis, a class of second-order nonlinear multi-agent systems with cooperative and competitive relationships is considered, and actuator faults are assumed to occur in the system. An adaptive fault observer is designed to estimate the states and actuator faults of the system, and the necessary conditions for the convergence of the observation error are derived. Based on the output of the fault observer, a sliding-mode fault-tolerant bipartite consensus control is proposed. This ensures that the bipartite consensus of multi-agent systems with actuator faults and nonlinear disturbances can still be achieved, and the conditions for the convergence of the system bipartite consensus error are provided. Finally, to reduce the waste of network resources, a dynamic event-triggered mechanism is designed and incorporated into the multi-agent systems and sliding-mode fault-tolerant bipartite consensus control. The main contributions of this paper are summarized as follows: 1) Considering that the states of multi-agent systems are difficult to obtain directly, distributed adaptive fault observers are designed to estimate both the states and actuator faults of the systems. 2) The effect of actuator faults on the bipartite consensus of multi-agent systems is considered. A sliding-mode fault-tolerant bipartite consensus control is designed, ensuring that bipartite consensus can still be achieved in the presence of actuator faults. 3) A dynamic event-triggered mechanism is introduced into the multi-agent systems and sliding-mode fault-tolerant bipartite consensus control. This significantly reduces the number of communications in the system, thereby minimizing the waste of network resources.

2. PROBLEM STATEMENT AND PREPARATION

In this paper, a class of second-order leader-follower multi-agent systems composed of 1 leader and N followers is considered. Its communication topology can be depicted by graph $\mathbf{\Gamma} = (\boldsymbol{\upsilon}, \boldsymbol{\varepsilon}, \mathbf{A})$, where the node set of graph Γ is denoted by $\upsilon = \{0, 1, \cdots, N\}$ and the edge set is denoted by $\varepsilon \subset \upsilon \times \upsilon$. The adjacency matrix of graph Γ is denoted by $\mathbf{A} = (\mathrm{a}_{ij})_{N\times N}$. If the information of the follower agent j can be received by the follower agent i and the relationship between the follower agent i and the follower agent j is cooperative, then $\mathrm{a}_{ij} = 1$. If the information of the follower agent j can be received by the follower agent i and the relationship between the follower agent i and the follower agent j is competitive, then $\mathrm{a}_{ij} = -1$. If the information of the follower agent j cannot be received by follower agent i, then $\mathrm{a}_{ij} = 0$. The set of edges is described as $\varepsilon = \varepsilon^+ \cup \varepsilon^-$, where the sets the positive and negative edges are denoted by $\varepsilon^+ = \{(j,i) | a_{ij} = 1\}$ and $\varepsilon^- = \{(j,i) | a_{ij} = -1\}$, respectively. The in-degree of node i is defined as $d_i = \sum_{\substack{j=1\\ i\neq j}}^{N} a_{ij}$. For the whole system there are $\mathbf{D} = diag\{d_1, \cdots, d_N\} \in R^{N\times N}$. The Laplace matrix of graph Γ is denoted as $\mathbf{L} = \mathbf{D} - \mathbf{A}$. The graph Γ is said to form a spanning tree with a certain node as the root if there exists a node in Γ that can transmit information to all other nodes.

In this paper, the considered second-order leader-follower multi-agent systems have 1 leader agent and N follower agents. The mathematical model of the navigator agent can be described as follows:

$$\begin{cases} \dot{p}_0(t) = v_0(t), \\ \dot{v}_0(t) = -u_0(t), \end{cases} \tag{1}$$

where the leader position and velocity are denoted by $p_0(t) \in R^n$ and $v_0(t) \in R^n$, respectively. Leader inputs are denoted by $u_0(t) \in R^p$.

The mathematical model of follower agent i can be described as follows:

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = -u_i^F(t) + d_i(t), \end{cases} \tag{2}$$

where $i = 1, 2, \ldots, N$, the position and velocity of follower i are denoted by $p_i(t) \in R^n$ and $v_i(t) \in R^n$, respectively, the fault input of follower i is denoted by $u_i^F(t) \in R^p$, and the nonlinear dynamics of follower i is denoted by $d_i(t)$.

In this paper, follower i is assumed to experience actuator faults, and the mathematical model of the fault inputs can be described as follows:

$$u_i^F(t) = u_i(t) + f_i(t), \tag{3}$$

where the degree of actuator faults is described by $f_i(t) \in R^p$.

The bipartite consensus error of the follower agent i with the leader is described:

$$\begin{cases} \delta_p^i(t) = p_i(t) - m_i p_0(t), \\ \delta_v^i(t) = v_i(t) - m_i v_0(t), \end{cases} \tag{4}$$

where the relationship between the follower agent i and the leader is denoted by m_i. If the relationship between follower i and the leader is cooperative, then $m_i = 1$, and conversely, $m_i = -1$. Therefore, the bipartite consensus error of multi-agent systems is described as

$$\begin{cases} \delta_p(t) = p(t) - M\overline{p}_0(t), \\ \delta_v(t) = v(t) - M\overline{v}_0(t) \end{cases} \tag{5}$$

where
$p = [p_1(t), p_2(t), \cdots, p_N(t)]^T$, $v = [v_1(t), v_2(t), \cdots, v_N(t)]^T$,
$\overline{p}_0(t) = 1_N \otimes p_0(t)$, $\overline{v}_0(t) = 1_N \otimes \overline{v}_0(t)$,
$\mathbf{M} = diag(m_1, m_2, \cdots, m_N)$.

The goal of this paper is to design a fault-tolerant controller such that the bipartite consensus errors of multi-agent systems (1), (2) with actuator faults (3) satisfy the following conditions:

$$\begin{cases} \lim\limits_{t\to\infty} \delta_p(t) \leq \varphi_1, \\ \lim\limits_{t\to\infty} \delta_v(t) \leq \varphi_2, \end{cases} \tag{6}$$

where φ_1 and φ_2 are both positive constants or bounded variables.

In this paper, the following lemmas and assumptions will be used.

Lemma 1. (see [27]) The following inequality holds for a constant μ and a positive definite matrix $\mathbf{P}$:

$$2x^T y \leq \frac{1}{\mu} x^T \mathbf{P} x + \mu y^T \mathbf{P}^{-1} y. \tag{7}$$

Assumption 1. The leader control input is 0 and its state can be acquired.

Assumption 2. There exists a lower bound $\overline{F}$ for the derivative of actuator faults $f(t)$, which can be described as $\dot{f}(t) \geq \overline{F}$. There exists an upper bound $\overline{D}$ for the derivative of nonlinear disturbances $d(t)$, which can be described as $d(t) \leq \overline{D}$.

3. MAIN DESIGN AND ANALYSIS

3.1. Design and analysis of adaptive fault observer

In order to design an active fault-tolerant controller. The state of the multi-agent systems and the degree of actuator faults must be obtained. So, a fault observer is designed and the above information is obtained.

Equation (2) and equation (3) are combined and the state of the system can be rewritten as:

$$\dot{x}_i(t) = \mathbf{A}x_i(t) + \mathbf{B}(u_i(t) + f_i(t)) + \mathbf{D}d_i(t), \tag{8}$$

where $x_i(t) = \begin{bmatrix} p_i(t) \\ v_i(t) \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and the system output is defined:

$$y_i(t) = \mathbf{C}x_i(t), \tag{9}$$

where $\mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, so the following equation is obtained:

$$\begin{cases} \dot{x}_i(t) = \mathbf{A}x_i(t) + \mathbf{B}u_i(t) + B_f f_{di}(t), \\ y_i(t) = \mathbf{C}x_i(t), \end{cases} \tag{10}$$

where

$$\begin{cases} \mathbf{B}_f = \mathbf{B} + \mathbf{D}, \\ f_{di}(t) = f_i(t) + d_i(t). \end{cases} \tag{11}$$

In this paper, adaptive laws are combined. By using the residuals of the system output, actuator faults and nonlinear disturbances are estimated. The observer are described as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = \mathbf{A}\hat{x}_i(t) + \mathbf{B}u_i(t) + \mathbf{B}_f \hat{f}_{di}(t) + \mathbf{H}e_{yi}(t), \\ \hat{y}_i(t) = \mathbf{C}\hat{x}_i(t), \\ \hat{f}_{di}(t) = FRe_{yi}(t), \end{cases} \tag{12}$$

where the state observation matrix $\mathbf{H} \in \mathbb{R}^{n\times 1}$ is denoted, $R > 0$ is the fault observation gain, and $F > 0$ is the adaptive learning rate gain. Both R and F are design parameters introduced in the observer to estimate the actuator fault values. The observation error of the observer is obtained from equation (9) and equation (12):

$$\begin{cases} e_{xi}(t) = x_i(t) - \hat{x}_i(t), \\ e_{fi}(t) = f_{di}(t) - \hat{f}_{di}(t), \\ e_y(t) = y_i(t) - \hat{y}_i(t). \end{cases} \tag{13}$$

Theorem 1. An adaptive fault observer (11) that satisfies the following conditions can observe the states $x_i(t)$ and $f_{di}(t)$ of the leader-follower multi-agent systems (1) and (2) with actuator faults (3), and the observation error will eventually converge to zero.

$$\begin{bmatrix} (\mathbf{A}-\mathbf{HC})^T\mathbf{P}_1 + \mathbf{P}_1(\mathbf{A}-\mathbf{HC}) & 0 & 0 \\ * & -G_1 & 0 \\ * & * & -F^{-1}G_1F^{-1} \end{bmatrix} < 0, \tag{14}$$

$$(\mathbf{P}_1\mathbf{B}_f)^T = \mathbf{RC}, \tag{15}$$

where $G_1 > 0$ is the coefficient to be determined, and $\mathbf{P}_1 \in \mathbb{R}^{n\times n}$ is a positive definite matrix to be designed.

Proof. Taking the derivative of the observation error:

$$\begin{aligned} \dot{e}_{xi}(t) &= \mathbf{A}e_{xi}(t) + \mathbf{B}_f e_{fi}(t) - \mathbf{HC}e_{xi}(t) \\ &= (\mathbf{A}-\mathbf{HC})e_{xi}(t) + \mathbf{B}_f e_{fi}(t), \end{aligned} \tag{16}$$

$$\dot{e}_{fi}(t) = \dot{f}_{di}(t) - FR\mathbf{C}e_{xi}(t). \tag{17}$$

Define the Lyapunov function

$$V_1(t) = e_{xi}^T(t)\mathbf{P}_1 e_{xi}(t) + e_{fi}^T(t)F^{-1}e_{fi}(t). \tag{18}$$

Taking the derivative for $V_1(t)$ gets:

$$\begin{aligned} \dot{V}_1(t) &= 2e_{xi}^T(t)\mathbf{P}_1\dot{e}_{xi}(t) + 2e_{fi}^T(t)F^{-1}\dot{e}_{fi}(t) \\ &= e_{xi}^T(t)\left[(\mathbf{A}-\mathbf{HC})^T\mathbf{P}_1 + \mathbf{P}_1(\mathbf{A}-\mathbf{HC})\right]e_{xi}(t) \\ &+ 2e_{xi}^T(t)\mathbf{P}_1\mathbf{B}_f e_{fi}(t) + 2e_{fi}^T(t)\mathbf{F}^{-1}\dot{f}_{di}(t) \\ &- 2e_{fi}^T(t)R\mathbf{C}e_{xi}(t). \end{aligned}$$

From equation (14):

$$\begin{aligned} \dot{V}_1(t) &= e_{xi}^T(t)\left[(\mathbf{A}-\mathbf{HC})^T\mathbf{P}_1 + \mathbf{P}_1(\mathbf{A}-\mathbf{HC})\right]e_{xi}(t) \\ &+ 2e_{fi}^T(t)F^{-1}\dot{f}_{di}(t). \end{aligned} \tag{19}$$

It is obtained from Lemma 1:

$$\begin{aligned} 2e_{fi}^T(t)F^{-1}\dot{f}_{di}(t) &\leq -e_{fi}^T(t)\mathbf{G}_1\dot{e}_{fi}(t) \\ &- \dot{f}_{di}^T(t)F^{-1}\mathbf{G}_1F^{-1}\dot{f}_{di}(t). \end{aligned} \tag{20}$$

Therefore, equation (19) can be rewritten as:

$$\begin{aligned} \dot{V}_1(t) &\leq e_{xi}^T(t)\left[(\mathbf{A}-\mathbf{HC})^T\mathbf{P}_1 + \mathbf{P}_1(\mathbf{A}-\mathbf{HC})\right]e_{xi}(t) \\ &- e_{fi}^T(t)\mathbf{G}_1 e_{fi}(t) - \dot{f}_{di}^T(t)\mathbf{F}^{-1}\mathbf{G}_1\mathbf{F}^{-1}\dot{f}_{di}(t) \\ &\leq \begin{bmatrix} e_{xi}(t) \\ e_{fi}(t) \\ \dot{f}_{di}(t) \end{bmatrix}^T * \Omega * \begin{bmatrix} e_{xi}(t) \\ e_{fi}(t) \\ \dot{f}_{di}(t) \end{bmatrix}, \end{aligned} \tag{21}$$

where

$$\Omega = \begin{bmatrix} (\mathbf{A}-\mathbf{HC})^T\mathbf{P}_1 + \mathbf{P}_1(\mathbf{A}-\mathbf{HC}) & 0 & 0 \\ * & -\mathbf{G}_1 & 0 \\ * & * & -F^{-1}\mathbf{G}_1F^{-1} \end{bmatrix}.$$

From equation (14): $\dot{V}_1(t) < 0$. According to the Lyapunov stability criterion, the observation error of the adaptive fault observer will eventually converge to zero. Therefore, the following equation holds:

$$\begin{cases} \lim\limits_{t\to\infty} e_{xi}(t) = 0, \\ \lim\limits_{t\to\infty} e_{fi}(t) = 0. \end{cases} \tag{22}$$

Proof complete. □

Remark 1. In practice, equation (14) and equation (15) are difficult to satisfy at the same time. According to the literature [24], solving equation (15) can be transformed into solving the following inequality:

$$\begin{vmatrix} I & \mathbf{B}_f^T\mathbf{P}_1 - R\mathbf{C} \\ (\mathbf{B}_f^T\mathbf{P}_1 - R\mathbf{C})^T & I \end{vmatrix} > 0. \tag{23}$$

3.2. Design and analysis of dynamic event-triggered bipartite fault-tolerant consistency controller

To save the communication resources of multi-agent systems in executing a bipartite fault-tolerant consensus controller, a dynamic event-triggered mechanism is designed. Sampling is performed when the system meets the trigger condition and the controller inputs are updated at the same time. Define the k-th sampling moment of the system as t_k, when the state of agent i at the k-th sampling moment is $x_i(t_k)$. The state of the system will only be updated when the trigger condition is satisfied, otherwise the state will remain the same as the last updated value. Since the state information of the system is obtained through the observer, the state of the multi-agent systems is $\hat{x}_i(t_k)$ at the k-th sampling moment. Define the sampling error of agent i at t_k as:

$$\xi_i(t) = \hat{x}_i(t) - \hat{x}_i(t_k), \tag{24}$$

where the dynamic event-triggered mechanism is designed as:

$$t_{k+1} \triangleq \min\left\{t > t_k \,|\, \eta_i + \theta(\rho\,\|\hat{x}_i(t)\| + \varpi - \|\xi_i(t)\|) < 0\right\}, \tag{25}$$

where $\theta > 0$ and $\rho > 0$ are normal numbers and $\varpi > 0$ is a fixed threshold. The dynamic variable of agent i is defined as η_i and is used to dynamically adjust the trigger interval. Its update law is designed as follows:

$$\dot{\eta}_i = -\eta_i + \varpi - \|\xi_i(t)\|. \tag{26}$$

Based on equation (4), equation (8), and equation (12), the bipartite consensus error of multi-agent systems can be written as:

$$\hat{\delta}_i(t) = \hat{x}_i(t) - m_i\hat{x}_0(t), \tag{27}$$

where the observation of $\delta_i(t)$ is defined as $\hat{\delta}_i(t)$, and $\delta_i(t) = \begin{bmatrix} \delta_p^i(t) \\ \delta_v^i(t) \end{bmatrix}$. With the event-triggered mechanism (25), equation (27) can also be written as:

$$\hat{\delta}_i(t_k) = \hat{x}_i(t_k) - m_i\hat{x}_0(t_k). \tag{28}$$

Therefore, the control objective of the system can be expressed as

$$\lim_{t_k\to\infty} \delta(t_k) \le \varphi, \tag{29}$$

where φ is a constant or bounded variable. Assume that each agent in the multi-agent systems can only communicate with its neighbor agents and the set of neighbor agents of agent i is N_i. Based on the fault observer (12), the state error between agent i and its neighbors can be expressed as:

$$w_i(t) = \sum_{j\in N_i} a_{ij}\left(\hat{x}_i(t) - \operatorname{sgn}(a_{ij})\hat{x}_j(t)\right) + g_i\left(\hat{x}_i(t) - m_i\hat{x}_0(t)\right) \tag{30}$$

or

$$\begin{cases} w_p^i(t) = \sum\limits_{j\in N_i} a_{ij}\left(\hat{p}_i(t) - \operatorname{sgn}(a_{ij})\hat{p}_j(t)\right) \\ \qquad + g_i(\hat{p}_i(t) - m_i\hat{p}_0(t)), \\ w_v^i(t) = \sum\limits_{j\in N_i} a_{ij}\left(\hat{v}_i(t) - \operatorname{sgn}(a_{ij})\hat{v}_j(t)\right) \\ \qquad + g_i(\hat{v}_i(t) - m_i\hat{v}_0(t)), \end{cases} \tag{31}$$

where the observations of $p_i(t)$, $p_j(t)$, $p_0(t)$, $v_i(t)$, $v_j(t)$ and $v_0(t)$ are denoted by $\hat{p}_i(t)$, $\hat{p}_j(t)$, $\hat{p}_0(t)$, $\hat{v}_i(t)$, $\hat{v}_j(t)$ and $\hat{v}_0(t)$, respectively. The position and velocity errors of agent i based on neighbor information are denoted by $w_p^i(t)$ and $w_v^i(t)$, respectively.

Combined with the graph theory, the position error and velocity error of multi-agent systems can be represented as:

$$\begin{cases} w_p(t) = (L+G)\,\hat{p}(t) - G\overline{p}_0(t), \\ w_v(t) = (L+G)\,\hat{v}(t) - G\overline{v}_0(t), \end{cases} \tag{32}$$

where
$w_p(t) = \left[w_p^1(t), w_p^2(t), \cdots, w_p^N(t)\right]^T$,
$w_v(t) = \left[w_v^1(t), w_v^2(t), \cdots, w_v^N(t)\right]^T$,
$\overline{p}_0(t) = 1_N \otimes p_0(t)$, $\overline{v}_0(t) = 1_N \otimes v_0(t)$,

$\mathbf{G} = diag\{g_1, g_2, \cdots, g_N\}$. The Laplace matrix of the multi-agent systems (1), (2) is assumed to be $\mathbf{L}$ and $\mathbf{L} = [l_{ij}] \in R^{N\times N}$. The following equation is satisfied:

$$\begin{cases} l_{ii} = \sum\limits_{j\in N_i} a_{ij}, \\ l_{ij} = -a_{ij}, \quad i \ne j. \end{cases} \tag{33}$$

Combining Assumption 1 and taking the derivative of equation (32), the following equation can be obtained:

$$\begin{cases} \dot{w}_p(t) = (L+G)\,\dot{p}(t) - G\dot{\overline{p}}_0(t) \\ \qquad = (L+G)\,v(t) - G\overline{v}_0(t), \\ \dot{w}_v(t) = (L+G)\,\dot{v}(t) - G\dot{\overline{v}}_0(t) \\ \qquad = (L+G)\,(-u(t) - f(t) + d(t)). \end{cases} \tag{34}$$

In order to make multi-agent systems (1), (2) bipartite consensus can still be realized under the action of actuator faults (3). A sliding-mode fault-tolerant bipartite consensus control is designed. The sliding-mode variable $\sigma_i(t)$ for agent i is designed as follows:

$$\sigma_i(t) = c_1 w_p^i(t) + w_v^i(t), \tag{35}$$

where c_1 is the normal number.

At the trigger moment t_k, the sliding-mode variable of multi-agent systems can be expressed as:

$$\sigma(t_k) = c_1 w_p(t_k) + w_v(t_k), \tag{36}$$

where $\sigma(t_k) = [\sigma_1(t_k), \sigma_2(t_k), \cdots, \sigma_N(t_k)]^T$. Taking the derivative of equation (36) yields the following equation:

$$\begin{aligned}\dot{\sigma}(t_k) &= c_1 \dot{w}_p(t_k) + \dot{w}_v(t_k)\\ &= c_1 w_v(t_k) + (\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right).\end{aligned} \tag{37}$$

Chattering is an unavoidable problem for sliding-mode controllers. To reduce the amplitude of chattering, a sliding-mode variable $\sigma(t_k)$ is designed based on $s(t_k)$ and is expressed as follows:

$$\begin{aligned}s(t_k) &= c_2\sigma(t_k) + \dot{\sigma}(t_k)\\ &= c_2\sigma(t_k) + c_1 w_v(t_k)\\ &\quad + (\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right),\end{aligned} \tag{38}$$

where $c_2 > 0$ is the sliding-mode gain, which determines the convergence rate of the sliding surface (38). Under the assumption $\dot{u}(t_k) = r(t_k)$ and with Assumption 2, the derivative of equation (38) is derived as follows:

$$\begin{aligned}\dot{s}(t_k) &= c_2\dot{\sigma}(t_k) + c_1\dot{w}_v(t_k) + (\mathbf{L}+\mathbf{G})\left(-\dot{u}(t_k) - \dot{f}(t) + \dot{d}(t)\right)\\ &= c_2\left(c_1 w_v(t_k) + (\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right)\right)\\ &\quad + c_1(\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right)\\ &\quad + (\mathbf{L}+\mathbf{G})\left(-r(t_k) - \dot{f}(t) + \dot{d}(t)\right)\\ &= c_1c_2 w_v(t_k) + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right)\\ &\quad + (\mathbf{L}+\mathbf{G})\left(-r(t_k) - \dot{f}(t) + \dot{d}(t)\right).\end{aligned} \tag{39}$$

Assume that the sliding-mode variable $s(t)$ has the following exponential convergence law:

$$\dot{s}(t_k) = -\varepsilon\,\mathrm{sgn}(s(t_k)) - k s(t_k), \tag{40}$$

where $\varepsilon > 0$ and $k > 0$ are positive control gains that determine the convergence rate of the sliding-mode surface. Combining equation (39) and equation (40) yields the following equation:

$$\begin{aligned}&-\varepsilon\,\mathrm{sgn}(s(t_k)) - k s(t_k)\\ &\quad = c_1c_2 w_v(t_k) + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right)\\ &\qquad + (\mathbf{L}+\mathbf{G})\left(-r(t_k) - \dot{f}(t) + \dot{d}(t)\right).\end{aligned} \tag{41}$$

Therefore, $r(t_k)$ can be designed:

$$\begin{aligned}r(t_k) &= (\mathbf{L}+\mathbf{G})^{-1}\Big[\varepsilon\,\mathrm{sgn}(s(t_k)) + k s(t_k) + c_1c_2 w_v(t_k)\\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(-u(t_k) - \hat{f}(t) + d_c(t)\right)\\ &\quad + k_s\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau - (\mathbf{L}+\mathbf{G})\left(\overline{F} - \overline{D}\right)\Big],\end{aligned} \tag{42}$$

where d_c is a positive real number related to the bounds of the disturbance, $k_s > 0$ is the constant of integration. The values of $\overline{F}$ and $\overline{D}$ are the upper bounds specified in Assumption 2.

Therefore, the sliding-mode fault-tolerant bipartite consensus control law $u(t_k)$ can be described as:

$$u(t_k) = \int r(t_k)\,\mathrm{d}t_k\,. \tag{43}$$

Substituting equation (42) into equation (39), the following equation can be obtained:

$$\begin{aligned}\dot{s}(t_k) &= c_1c_2 w_v(t_k) + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(-u(t_k) - f(t) + d(t)\right)\\ &\quad + (\mathbf{L}+\mathbf{G})\left(-\dot{f}(t) + \dot{d}(t)\right) - (\mathbf{L}+\mathbf{G})\,r(t_k)\\ &= -\varepsilon\,\mathrm{sgn}(s(t_k)) - k s(t_k) - k_s\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau\\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(\hat{f}(t) - f(t) + d(t) - d_c(t)\right)\\ &\quad + (\mathbf{L}+\mathbf{G})\left(\overline{F} - \dot{f}(t)\right) + (\mathbf{L}+\mathbf{G})\left(\dot{d}(t) - \overline{D}\right).\end{aligned} \tag{44}$$

It follows from Theorem 1 and Assumption 2:

$$\begin{aligned}\dot{s}(t_k) &\le -\varepsilon\,\mathrm{sgn}(s(t_k)) - k s(t_k)\\ &\quad - k_s\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_c(t)).\end{aligned} \tag{45}$$

Assume that the lower and upper bounds of the nonlinear disturbance $d(t)$ are denoted as d_l and d_u, respectively, that is, $d_l \le d(t) \le d_u$. Defining $d_1 = \dfrac{d_u - d_l}{2}$ and $d_2 = \dfrac{d_u + d_l}{2}$. Then d_c can be designed as:

$$d_c = d_2 + d_1\,\mathrm{sgn}(s(t_k)). \tag{46}$$

In order to prove the effectiveness of the designed sliding-mode fault-tolerant bipartite consensus control (42). The following theorem is given.

Theorem 2. Under the action of a sliding-mode fault-tolerant bipartite consensus control (43) that satisfies the following conditions. The sliding-mode variables $\sigma(t_k)$ of the multi-agent systems (1), (2) with actuator faults (3) can be driven to the equilibrium point $\sigma(t_k) = 0$.

$$\begin{cases} \psi_1 = \dfrac{1}{2k_s}(-k_sc_1 - k_sc_2 - 1 \\ \quad + \sqrt{c_1{}^2k_s{}^2 - 2k_s{}^2c_1c_2 + c_2{}^2k_s{}^2 + 2k_sc_1 + 2k_sc_2 + 1}) \\ \quad < 0, \\ \psi_1 = \dfrac{1}{-2k_s}(k_sc_1 + k_sc_2 + 1 \\ \quad + \sqrt{c_1{}^2k_s{}^2 - 2k_s{}^2c_1c_2 + c_2{}^2k_s{}^2 + 2k_sc_1 + 2k_sc_2 + 1}) \\ \quad < 0. \end{cases} \tag{47}$$

Proof. Define the Lyapunov function $V_2(t_k) = \frac{1}{2}s(t_k)^2 + \frac{1}{2}I(t_k)^2$ where $I(t_k) = \int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau$, and the derivative of $V_2(t)$ is obtained:

$$\begin{aligned} \dot{V}_2(t_k) &= s(t_k)\dot{s}(t_k) + I(t_k)\dot{I}(t_k) \\ &\le s(t_k)[-\varepsilon\mathrm{sgn}(s(t_k)) - ks(t_k) - k_s\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_c(t))] + I(t_k)w_v(t_k) \\ &\le s(t_k)[-\varepsilon\mathrm{sgn}(s(t_k)) - ks(t_k) \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_c(t))] \\ &\quad + w_v(t_k)\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau - k_ss(t_k)\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau. \end{aligned} \tag{48}$$

Since $\int_0^{t_k} w_v(\tau)\,\mathrm{d}\tau = w_p(t_k)$ and $\dot{w}_p(t_k) = w_v(t_k)$, the last two terms of equation (48) can be reduced to:

$$\begin{aligned} &\dot{w}_p(t_k)w_p(t_k) - k_ss(t_k)w_p(t_k) \\ &= \dot{w}_p(t_k)w_p(t_k) - k_s[c_1c_2w_p(t_k)^2 \\ &\quad + (c_1+c_2)w_p(t_k)\dot{w}_p(t_k) + w_p(t_k)\ddot{w}_p(t_k)] \\ &= -k_sc_1c_2w_p(t_k)^2 - (k_s(c_1+c_2)+1)w_p(t_k)\dot{w}_p(t_k) \\ &\quad - k_sw_p(t_k)\ddot{w}_p(t_k). \end{aligned} \tag{49}$$

Let the above equation be equal to zero, so the following equation can be obtained:

$$\begin{aligned} 0 &= -k_sc_1c_2w_p(t_k)^2 - (k_s(c_1+c_2)+1)w_p(t_k)\dot{w}_p(t_k) \\ &\quad - k_sw_p(t_k)\ddot{w}_p(t_k). \end{aligned} \tag{50}$$

The differential equation (50) is solved to obtain:

$$w_p(t_k) = 0 \tag{51}$$

or

$$w_p(t_k) = c_1e^{\psi_1 t_k} + c_2e^{\psi_2 t_k}, \tag{52}$$

where ψ_1 and ψ_2 are the two constants to be solved in equation (47). The set of inequalities (47) can be substituted into equation (52) to obtain that it will eventually converge to zero. Therefore, equation (50) will hold whether equation (51) or equation (52) is used. Consequently, equation (48) can be reduced to:

$$\begin{aligned} \dot{V}_2(t_k) \le s(t_k)\big[&-\varepsilon\mathrm{sgn}(s(t_k)) - ks(t_k) \\ &+ (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_c(t))\big]. \end{aligned} \tag{53}$$

If $s(t) > 0$. By substituting equation (46) into equation (47), equation (53) can be simplified as follows:

$$\begin{aligned} \dot{V}_2(t_k) &\le s(t_k)[-\varepsilon - ks(t_k) \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_2(t) - d_1(t))] \\ &\le s(t_k)[-\varepsilon - ks(t_k) \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(d(t) - \frac{d_u+d_l}{2} - \frac{d_u-d_l}{2}\right)] \\ &\le s(t_k)[-\varepsilon - ks(t_k) \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_u)] \\ &< 0. \end{aligned} \tag{54}$$

Similarly, if $s(t) < 0$, equation (53) can be reduced to:

$$\begin{aligned} \dot{V}_2(t) &\le s(t_k)[\varepsilon - ks(t_k) \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_2(t) + d_1(t))] \\ &\le s(t_k)[\varepsilon - ks(t_k) \\ &\quad + (c_1+c_2)(\mathbf{L}+\mathbf{G})\left(d(t) - \frac{d_u+d_l}{2} + \frac{d_u-d_l}{2}\right)] \\ &\le s(t_k)[\varepsilon - ks(t_k) + (c_1+c_2)(\mathbf{L}+\mathbf{G})(d(t) - d_l)] \\ &< 0 \end{aligned} \tag{55}$$

According to the Lyapunov stability condition, $s(t_k)$ can be converged to zero eventually. From equation (36) and equation (38), the sliding-mode variable $\sigma(t_k)$ can be driven to the equilibrium point $\sigma(t_k) = 0$. The proof is completed. □

Remark 2. From the dynamic event-triggered mechanism (25), the triggering interval is non-negative due to the existence of the fixed threshold ϖ. Therefore, the multi-agent systems (1) and (2) that have applied the dynamic event-triggered mechanism (25) will not have zeno behavior.

Theorem 3. Multi-agent systems (1), (2) with actuator faults (3) can have their bipartite consensus errors $w_p(t_k)$ and $w_v(t_k)$ converged to zero by the action of the sliding-mode fault-tolerant bipartite consensus control (42), that is, the control objective (6) can be achieved.

Proof. By Theorem 2, $\sigma(t_k) = 0$. From the sliding-mode variable (36):

$$c_1w_p(t_k) + w_v(t_k) = 0. \tag{56}$$

Because of $\dot{w}_p(t_k) = w_v(t_k)$, the following equation holds:

$$c_1w_p(t_k) + \dot{w}_p(t_k) = 0. \tag{57}$$

By solving (57), $w_p(t_k) = c_1 e^{-c_1 t_k}$, so there is $\lim_{t_k\to\infty} w_p(t_k) = 0$. Substituting into equation (56), one gets $\lim_{t_k\to\infty} w_v(t_k) = 0$, so the control objective (6) can be realized. The proof is completed. □

3.3. Application simulation

In order to verify the effectiveness of the design. The designed fault observer, sliding-mode fault-tolerant controller and dynamic event-triggered mechanism are acted on faulty multi-agent systems.

Multi-agent systems consisting of 1 leader and 4 followers are considered. Its communication topology is shown in Fig. 1, where the leader is agent 0 and the follower agents are numbered 1 to 4. Follower agent 1 and follower agent 2 are in a cooperative relationship with the leader. Follower agent 3 and follower agent 4 are in competitive relationship with the leader. The weight values of the edges are all 1. From Fig. 1, the Laplace matrix $\mathbf{L}$ and connection matrix $\mathbf{G}$ of this multi-agent systems can be represented as:

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & 3 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$
$$\mathbf{G} = diag(1,1,0,0).$$

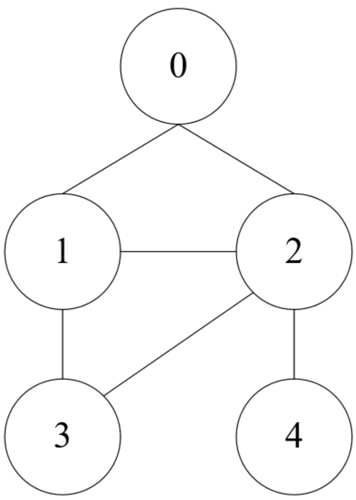

Fig. 1. Communication topology between agents

The dynamics of multi-agent systems are described by equation (1) and equation (2). After describing the system state as equation (8), there are $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & -1 \end{bmatrix}^T$, $\mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T$. Therefore, $(\mathbf{A},\mathbf{B})$ is controllable and $(\mathbf{A},\mathbf{C})$ is observable. By solving the inequality in Theorem 1, there are $\mathbf{P}_1 = \begin{bmatrix} 1.0976 & 0 \\ 0 & 1.0976 \end{bmatrix}$, $\mathbf{G}_1 = 0.0766$, $\mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T$.

According to equation (42). The parameters of the sliding-mode fault-tolerant bipartite consensus control law for multi-agent systems are chosen as: $\varepsilon = 0.5$, $k = 1$, $c_1 = 20$, $c_2 = 20$, $k_s = 0.1$. Substituting into equation (47) gives:

$$\begin{cases} \psi_1 = \dfrac{1}{2k_s}\Big(-k_s c_1 - k_s c_2 - 1 \\ \quad + \sqrt{{c_1}^2{k_s}^2 - 2{k_s}^2 c_1 c_2 + {c_2}^2{k_s}^2 + 2k_s c_1 + 2k_s c_2 + 1}\Big) \\ \quad = -10, \\ \psi_1 = \dfrac{1}{-2k_s}\Big(k_s c_1 + k_s c_2 + 1 \\ \quad + \sqrt{{c_1}^2{k_s}^2 - 2{k_s}^2 c_1 c_2 + {c_2}^2{k_s}^2 + 2k_s c_1 + 2k_s c_2 + 1}\Big) \\ \quad = -40. \end{cases} \tag{58}$$

Therefore, the conditions of Theorem 2 can be satisfied.

Note 3: In order to avoid steady state errors in the system, the integral of the velocity error is utilized in the design of the derivative $r(t_k)$ of the sliding-mode fault-tolerant bipartite consensus control. In order to prevent the system from integral saturation, the integration coefficient k_s is replaced by a slowly decaying variable k_i and $k_i = k_s e^{-\vartheta t_k}$, where the decay coefficient ϑ is taken as 0.01.

According to Assumption 1, the control input of the Pilot is set to $u_0(t) = 0$. Nonlinear disturbances are defined as $d(t) = 0.1\sin(t)$, and the multi-agent systems actuator faults input is defined as $u^F(t) = u(t) + f(t)$. Assume that the actuator faults $f(t)$ are of the following form from the 5th second onwards:

$$f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \\ f_4(t) \end{bmatrix} = \begin{bmatrix} 2\sin(t) \\ 2\cos(t) \\ 2\sin(t)\cos(5t) \\ 0 \end{bmatrix}.$$

For the dynamic event-triggered mechanism, $\theta = 0.1$, $\rho = 0.005$, $\varpi = 0.5$ are chosen. The initial state of the leader agent is taken as $x_0(t) = \begin{bmatrix} 10 & 10 \end{bmatrix}^T$. The initial states of the follower agent are defined as: $x_1(t) = \begin{bmatrix} -3 & 3 \end{bmatrix}^T$, $x_2(t) = \begin{bmatrix} -4 & 1 \end{bmatrix}^T$, $x_3(t) = \begin{bmatrix} 0 & 2 \end{bmatrix}^T$, $x_4(t) = \begin{bmatrix} 4 & 8 \end{bmatrix}^T$, respectively.

Remark 3. For the selection of the parameters of the dynamic event-triggered mechanism, a large amount of experimental experience is used in this paper to select each parameter. The value of θ has little effect on the number of samples, but a low value of θ will exacerbate the chattering of the system. ρ also has little effect on the number of samples, but a high value will exacerbate the chattering of the system, and it is appropriate to take a smaller value in order to inhibit the amplitude of the chattering. The fixed threshold ϖ is directly related to the sampling frequency, too high will lead to system instability, too low will lead to more unnecessary sampling.

As shown in Fig. 2 and Fig. 3, the position state of follower agent 1 and follower agent 3 and their observations are represented, respectively.

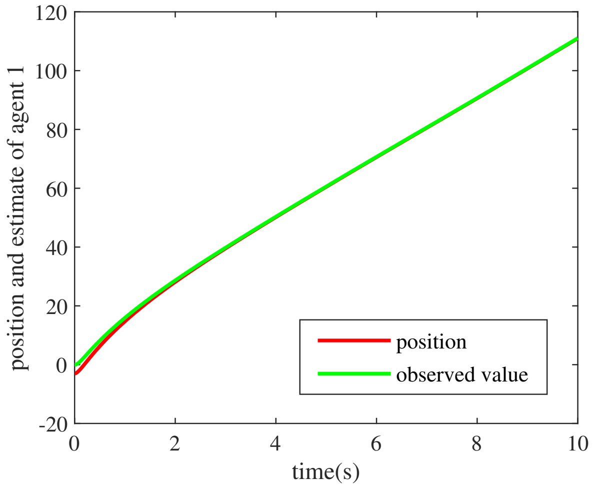

Fig. 2. Position state of follower agent 1 and its observation

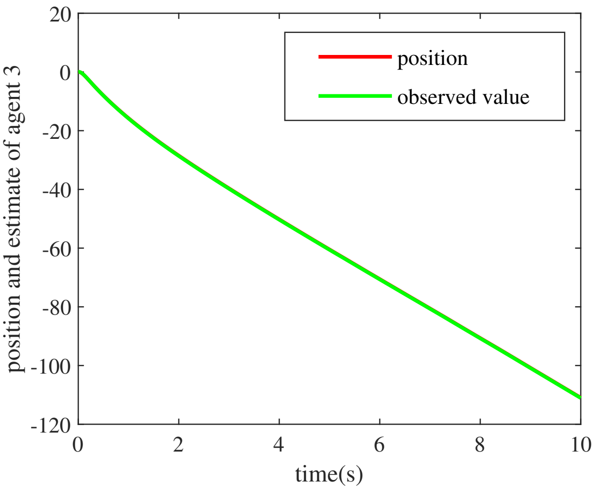

Fig. 3. Position state of follower agent 3 and its observation

It can be seen that the adaptive observer is more effective in observing the position state of follower agent 1 and follower agent 3.

The four follower agent actuator faults and their observations are represented in Figs. 4–7, respectively.

It can be seen that the observations of the faults are chattering due to the sliding-mode fault-tolerant controller. However, the observations still track the actuator faults better and faster.

The state values of the multi-agent systems are represented in Fig. 8 and Fig. 9, respectively.

It can be seen that the states of the multi-agent systems (1) and (2) can achieve bipartite consensus, both in the position state and the velocity state. Since there is a cooperative relationship between follower agent 1 and follower agent 2 and the leader agent, follower 1 and follower 2 can always track the state of the leader agent. Moreover, due to the competitive relationship between follower agent 3 and follower agent 4 with the leader agent, the states of follower agent 3 and follower agent 4 are completely opposite to the state of the leader agent. This is consistent with the bipartite consensus of multi-agent systems.

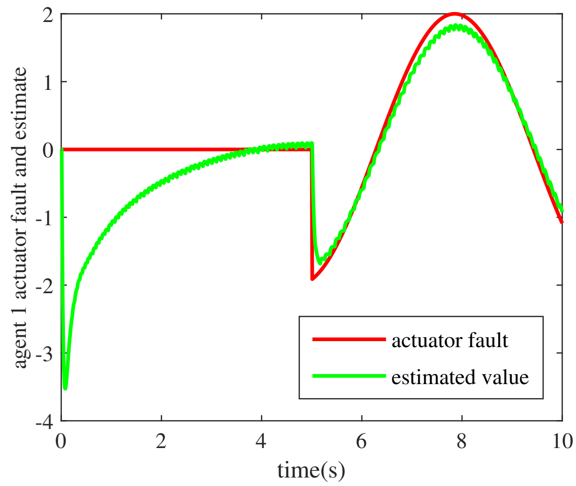

Fig. 4. Actuator fault of agent 1 and its estimate

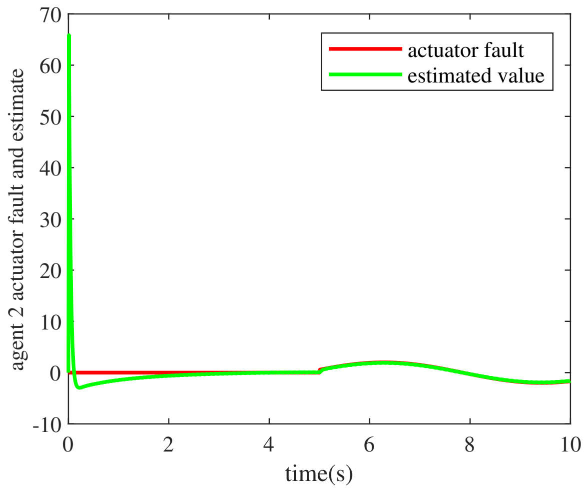

Fig. 5. Actuator fault of agent 2 and its estimate

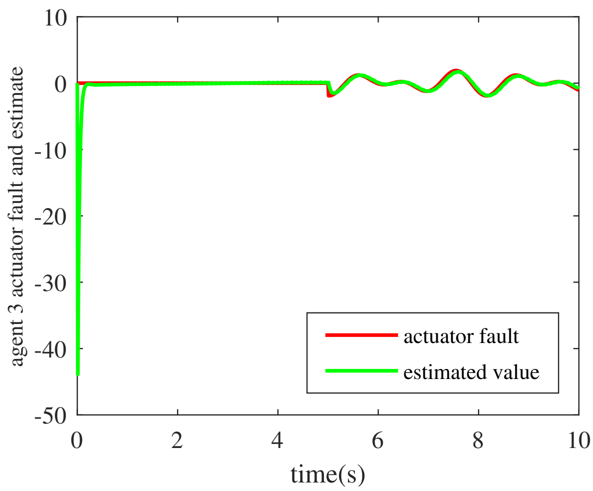

Fig. 6. Actuator fault of agent 3 and its estimate

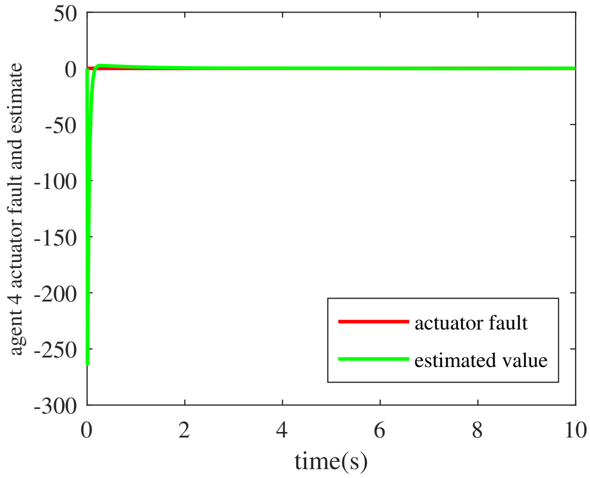

Fig. 7. Actuator fault of agent 4 and its estimate

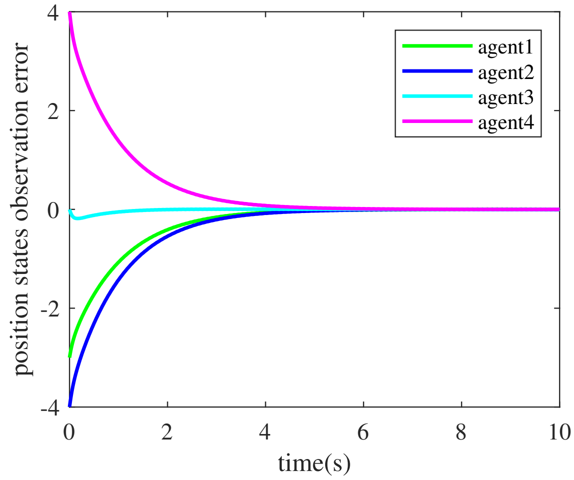

Fig. 10. Position state observation error of multi-agent system

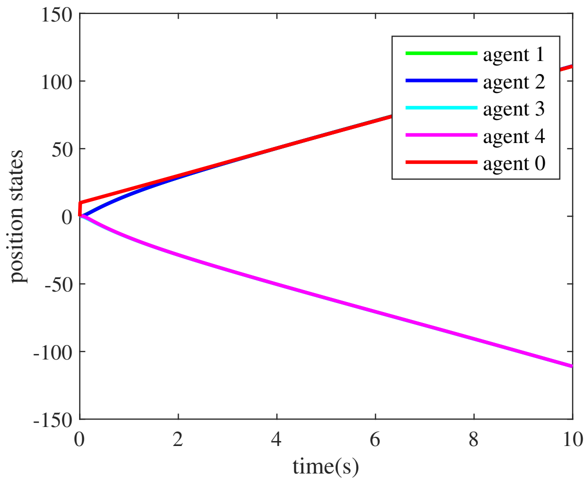

Fig. 8. Position state of multi-agent system

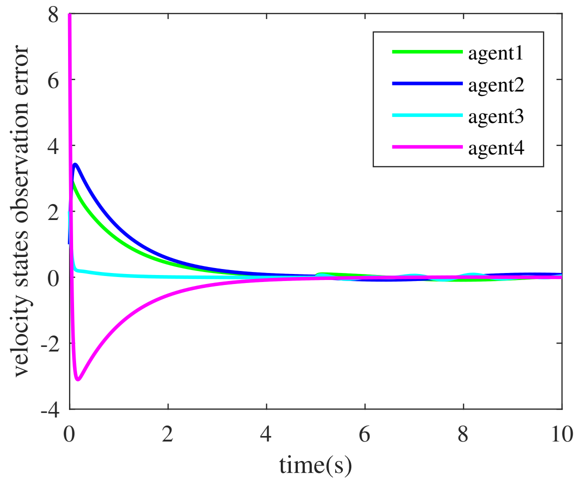

Fig. 11. Velocity state observation error for multi-agent system

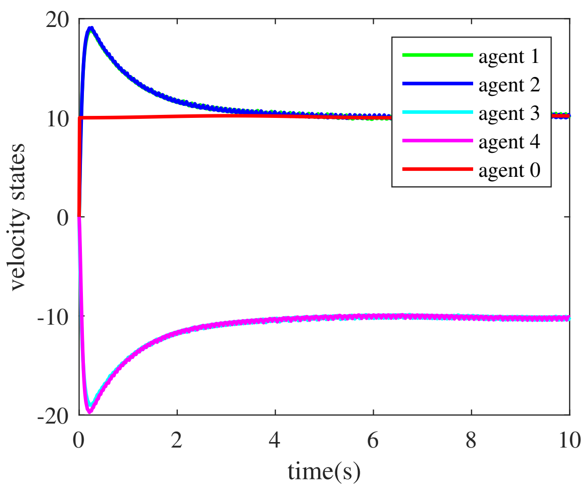

Fig. 9. Velocity state of multi-agent system

The observation errors of the states of multi-agent systems are represented in Fig. 10 and Fig. 11, respectively.

From Fig. 10 and Fig. 11, it can be seen that the fault observer is effective in observing the overall system state.

The state errors of the multi-agent systems are represented in Fig. 12 and Fig. 13, respectively.

It can be seen that the bipartite consensus errors for both the position state and velocity state of the multi-agent systems con-

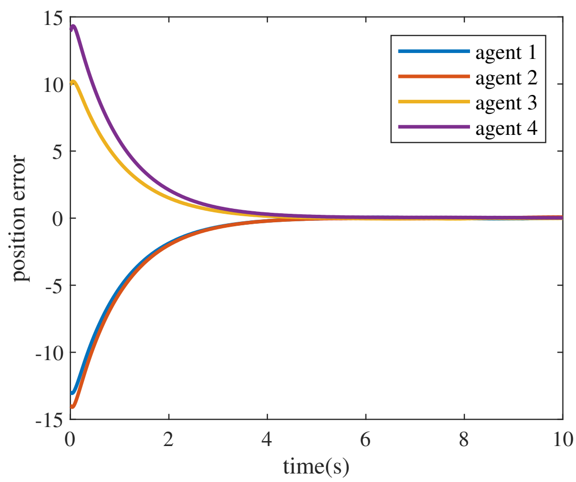

Fig. 12. Bipartite consensus error in the position of multi-agent system

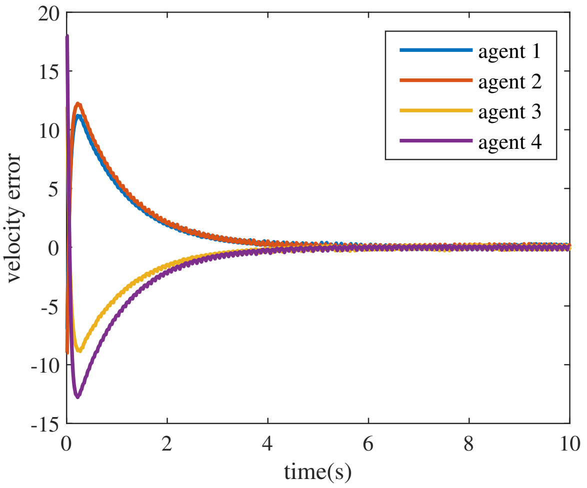

Fig. 13. Bipartite consensus error in the velocity of multi-agent system

verge to near zero. The effectiveness of the bipartite consensus controller is proved.

Figure 14 represents the trigger moment for dynamic event-trigger of multi-agent systems.

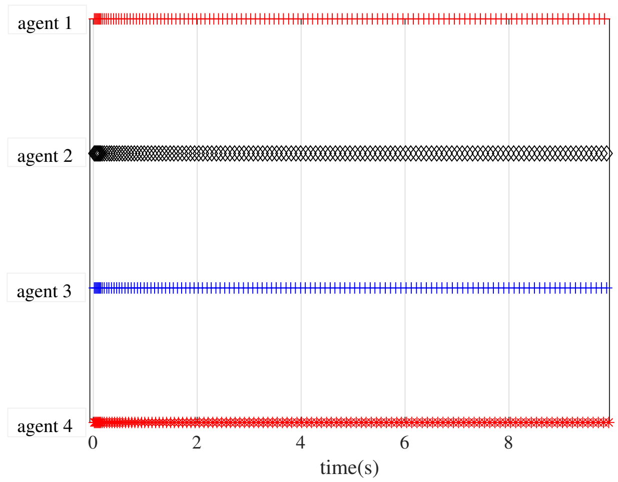

Fig. 14. Release instant and release interval

As can be seen in Fig. 14, the sampling of the system under the dynamic event-triggered mechanism is adopted. As can be seen in Fig. 14, the sampling times of the system are well reduced after the dynamic event-triggered mechanism is adopted, and the waste of the system network resources is also reduced. If periodic sampling is adopted and the sampling period of the system is set to 0.01 s, the number of state updates of the multi-agent systems in 10 s will be in 1000 times. As shown in Fig. 14, when the dynamic event-triggered mechanism is used, the number of state updates from agent 1 to agent 4 are 119, 120, 120 and 120 times, respectively. The number of communications was reduced by 88 percent. It can be seen that after the introduction of the dynamic event-triggered mechanism, the number of updates to the state of multi-agent systems is obviously reduced, and the network resources of multi-agent systems are greatly saved.

To further illustrate the excellence of the sliding-mode fault-tolerant bipartite consensus control designed in this paper. For second-order multi-agent systems (1), (2) with actuator faults (3). A general robust fault-tolerant controller based on a fault observer (12) and a dynamic event-triggered mechanism (25) is designed

$$u(t_k) = kw(t_k) - \hat{f}(t). \tag{59}$$

The robust fault-tolerant controller (59) is applied to multi-agent systems (1) and (2) and numerically simulated. The velocity state, position state, velocity state error, and position state error of the system are shown in Figs. 15–18, respectively.

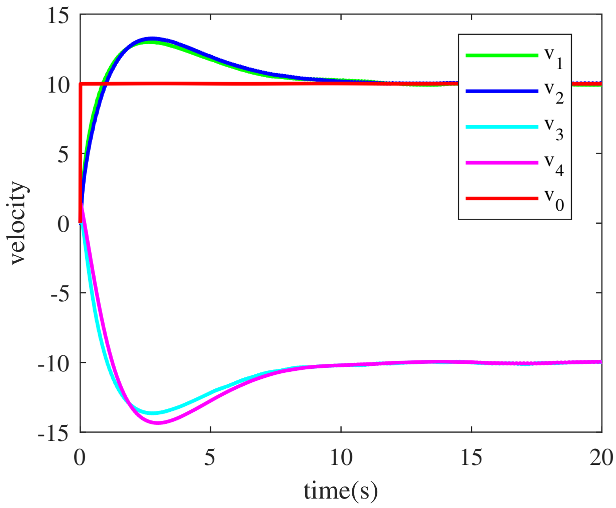

Fig. 15. Velocity state under the action of control law (59)

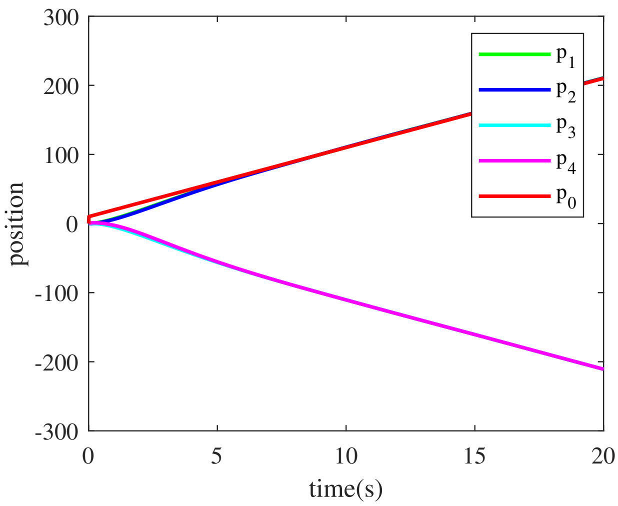

Fig. 16. Position state under the action of control law (59)

From Fig. 15 and Fig. 16, it can be seen that the fault-tolerant controller (59) can also realize the bipartite consensus of multi-agent systems. Comparing Fig. 17 with Fig. 14 and Fig. 18 with Fig. 13, it can be seen that the system error under the action of the sliding-mode fault-tolerant bipartite consensus control (43)

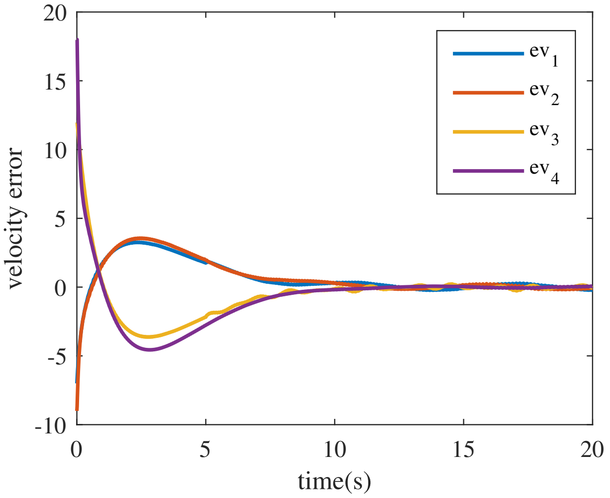

Fig. 17. Velocity error under the action of control law (59)

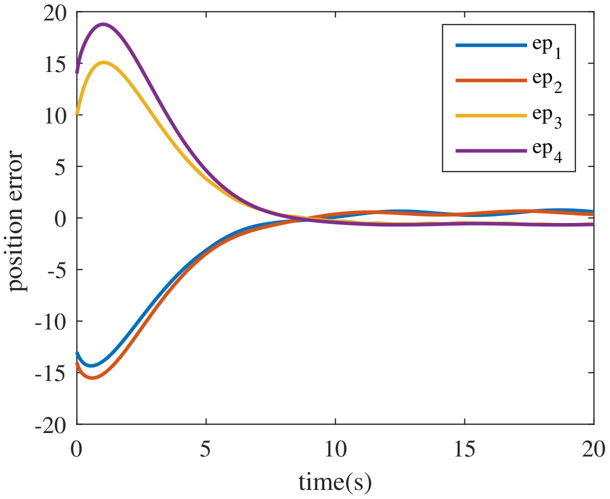

Fig. 18. Position error under the action of control law (59)

designed in this paper has a faster convergence rate. The convergence of the error is better, which further illustrates the excellence of the control law (43).

4. CONCLUSIONS AND FUTURE WORK

In this paper, a new sliding mode bipartite fault-tolerant consensus control method for leader-follower multi-agent systems applicable to undirected topology communication structures is proposed. Techniques such as adaptive strategies, dynamic event triggered mechanisms and sliding mode control are combined and utilized, and fault observers and sliding mode bisection fault-tolerant controllers are designed. Based on the Lyapunov stability theory, the conditions under which the final consistency of the bipartite consensus error is bounded are given. The parameters of the fault observer and fault-tolerant controller proposed in this paper do not require the solution of complex linear matrix inequalities. The fault tolerance and robustness of the designed control method for leader-follower multi-agent systems with actuator faults are verified through numerical simulations. The excellence of the design is further illustrated by comparison. Future research could explore extending the proposed sliding-mode fault-tolerant consensus control to directed topologies, or considering more complex actuator fault models.

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DECLARATIONS

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DATA AVAILABILITY

No data was used for the research described in the article.

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