

An off-line application that determines the maximum accuracy of the realization of reference points from G-code for given parameters of CNC machine dynamics

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Abstract. The paper presents an off-line application that determines the maximum accuracy of the reference points for the given dynamics parameters of a CNC machine. These parameters are maximum speed, acceleration, and JERK. The JERK parameter determines the rate of change of acceleration. These parameters are defined for each working axis of the machine. The main achievement of the algorithm proposed in the article is the determination of the smallest error specified for each reference point resulting from the implemented G-code for the considered dynamic parameters of the CNC machine. The solutions to this problem in industry consider the improvement in the accuracy of hitting the reference points, but they do not provide information on whether the obtained solution is optimal for such parameters of the machine dynamics. The algorithm makes the accuracy dependent on the adopted dynamic parameters of the machine and the parameters of the PLC controller used in the CNC machine.

Keywords: JERK; reference points; CNC machine; feed.

1. INTRODUCTION

The issue related to the rating parameters of PLCs used in multi-axis CNC machines is very topical and important, as it relates to the energy intensity of the technological process, ecology, and quality of execution of the final product. Multi-axis machining centres should ensure that the quality of the manufactured product is exceptionally good and that tool movements are optimized for complex technological tasks. Recently, the scientific and industry literature has been poring over issues related to the interpolation of dynamic parameters [1, 2], kinematics and modelling (minimization) of geometric errors [3–5], efficient generation of tool paths [6–8] and feed planning [9–11]. These include the optimization of feed rates, from which the quality and time of the task are derived [12, 13]. These issues are of great interest to the industry that uses CNC machines in its technological process, and to the scientific community [14, 15].

In order to achieve maximum process efficiency, it is usually required that the feed rates of the individual linear and rotary axes and the spindle speeds be as high as possible [16, 17]. In the selection of feed rates, acceleration and JERK set separately for each axis of the machine play a very important role. The JERK parameter is a very important element shaping the dynamics of

the machine since it controls the rate of change of acceleration as well as the deceleration of individual axes of a numerically controlled machine [18, 19]. It can have a constant value or it can vary in a non-linear manner. The JERK type is also set for each axis of the CNC machine. In the considerations presented in the article, the value of JERK is assumed to be constant. It can be calculated from the formula (1):

$$\vec{a}(t) = \frac{da}{dt} = \dot{\vec{a}}(t) = \frac{d^2v}{dt^2} = \ddot{\vec{v}}(t) = \frac{d^3r}{dt^3} = \ddot{\vec{r}}(t), \quad (1)$$

where: a – acceleration, v – velocity (often used as speed), r – movement (relocation), t – time.

So the problem is to optimize the feed rate on a given path so that the machining time is minimal taking into account the rated parameters of the servo motors used in the machine. Solving this problem, that is, optimally planning the feed rate for a given toolpath, makes it possible to increase the efficiency of CNC machining by using the full capabilities of the machines. Several research papers, e.g. [20, 21], present algorithms for determining the minimum time of movement of a robotic arm along a predetermined motion trajectory with a limitation of the acceleration value on the X -axis. In other publications, the authors [22–25] considered modeling the feed rate values in CNC machining for z -set maximum acceleration values on each axis. However, the considered method has disadvantages, since the acceleration can jump from maximum to minimum, which is impossible to achieve in the actual operation of the machine.

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In the algorithm proposed by the authors, such a situation does not occur.

In industrial solutions, experience and expertise are used to plan the technological process [26,27]. Sometimes the accuracy of technological solutions is obtained experimentally without a precise analysis of the determination of the dynamics parameter values [28]. The existing solutions in controllers concerning the set dynamics parameters do not consider the accuracy of the realisation of subsequent reference points. Sometimes it is necessary to achieve the desired accuracy for the given reference points whose realisation determines the quality of the whole product. The paper proposes a solution for obtaining the maximum possible accuracy of execution of all reference points for the set parameters of the machine dynamics [29]. In the off-line mode, the application determines the control of the CNC machine generating the highest possible accuracy of reaching the reference points [30–32].

2. ALGORITHM FOR MAXIMUM ACCURACY OF REFERENCE POINTS

There are various proposals for solving the problem of determining the control for a CNC machine. The proposed method is based on the set parameters of the dynamics of the CNC machine (Fig. 1).

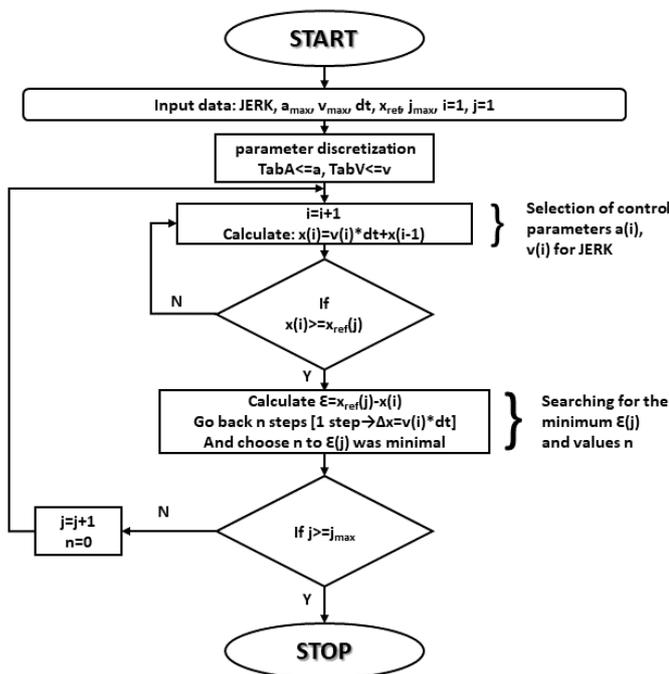


Fig. 1. Algorithm of the off-line error minimisation method for reference points

The solution is generated off-line. The final result is a generated G-code which guarantees the maximum accuracy of hitting the reference points and at the same time eliminates the so-called machine jerks during the realisation of a given detail. In addition, the solution obtained is characterised by a singularity

of accuracy for each reference point. As a result, the accuracy of detail execution is higher at particular reference points in comparison to the programs used so far to control the operation of the CNC machine. The main achievement of the proposed algorithm is to obtain the lowest possible error for the set dynamic parameters. So far, the authors have not encountered such a formulated problem. The solutions used for this kind of problem in the industry consider the improvement of the accuracy of hitting the reference points, but they do not give information on whether the smallest possible error was obtained. The algorithm makes the final accuracy dependent on the adopted dynamic parameters of the machine (Fig. 1). The calculation of the reference time is determined by reaching the first reference point after a certain number of time steps. The number of these steps depends on the position of the reference point. The time step is always the same and equals 0.002 s. The accuracy of reaching the reference points is obtained by correcting the speed and acceleration values accordingly. It should be emphasised that increasing the accuracy of reaching subsequent reference points does not change the number of time steps between them, but only the control values are modified for them. The number of steps with modified control is denoted by n (see Fig. 1). The first encounter of reference points determines the number of time steps. This mechanism is repeated for all other reference points.

3. IMPLEMENTATION AND VERIFICATION OF THE ALGORITHM

Selected test results relate to the constant dynamic values of the machine (JERK, maximum values of acceleration, and speed/feed) for a given product. An important influence on the accuracy of obtaining reference points is the so-called density of distribution of the reference points. In order to take into account the influence of the location of the reference points on the accuracy, the terms rare and dense distribution are used. Dense distribution means such a location of neighbouring reference points for which the machine cannot reach the maximum speed or acceleration values. The so-called sparse points mean that until they are reached, the machine works with the set maximum values of speed or acceleration. During operation, the machine can achieve the maximum values of the control parameters or possibly be reduced by their step value resulting from the discretisation step of the controller. Verification of the presented algorithm was made by comparing the obtained results with the currently used algorithms in CNC machine control.

The studied algorithm was tested for the given dynamics parameters of the CNC machine: $v_{\max} = 5000$ mm/min and $v_{\max} = 10000$ mm/min and $a = 1500$ mm/s² and JERK = 30000 mm/s³. Noteworthy here is the explanation of the JERK magnitude, this is understood as the rate of change of acceleration of the CNC machine. Figure 2 shows the times for reaching the individual reference points for the so-called rare points for the given two different maximum speeds. It is also worth noting that both graphs from about 0.1 s are straight lines, which indicates that the machine is moving at practically constant speed.

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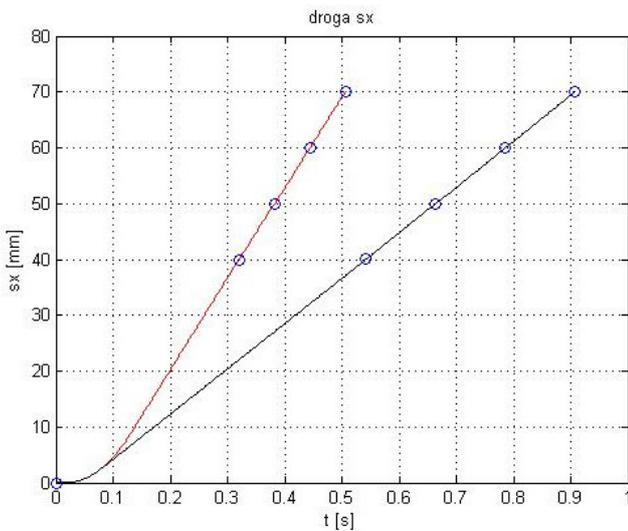


Fig. 2. Times to reach reference points for different maximum speeds (red – $v_{\max} = 10000$ mm/min, black – $v_{\max} = 5000$ mm/min) for sparse points

In the case shown in Fig. 3, the times to reach the dense reference points coincide for a feed rate of 5000 mm/min and 10000 mm/min, since the machine is still in the state of acceleration to the speeds set in G-code. The overlap between the two characteristics is due to the constraints associated with the set parameters of the machine dynamics, i.e. JERK and the resulting values of instantaneous acceleration and instantaneous velocity at each reference point.

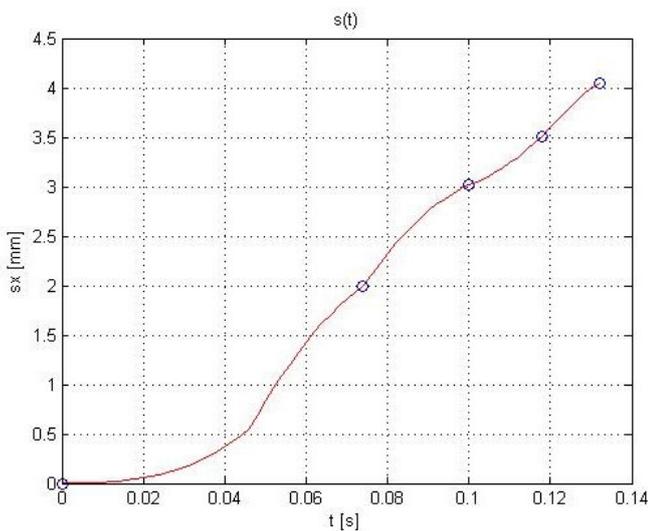


Fig. 3. Times for different maximum speeds (red – $v_{\max} = 10000$ mm/min, black – $v_{\max} = 5000$ mm/min) for dense points

Figure 4 shows the time courses of changes of preset speeds in particular reference points (as in Fig. 2) after the implementation of the algorithm presented in the paper. A natural feature of the solution is that for the set higher speeds shorter task ex-

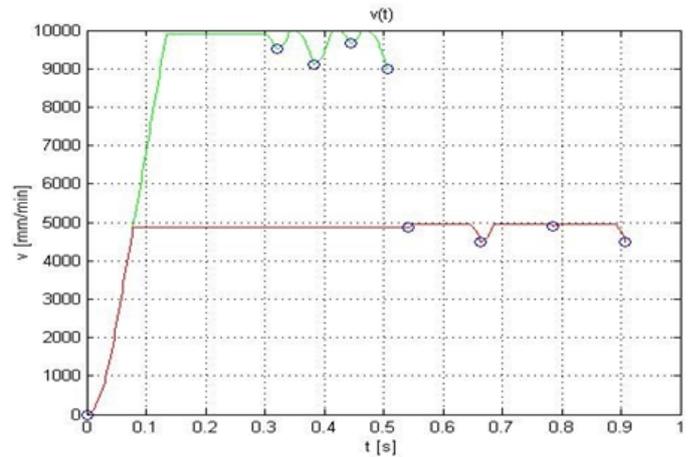


Fig. 4. Instantaneous velocities at given reference points (green – $v_{\max} = 10000$ mm/min, red – $v_{\max} = 5000$ mm/min) for sparsely located points

ecution times were obtained. Instantaneous changes in speed (slowing down of the machine) are caused by the necessity of achieving the maximum accuracy of the set reference points for the assumed parameters of the machine dynamics (JERK, acceleration, feed rate). Reductions in the momentary feed rate are realised gently and in accordance with the dynamic capabilities of the machine (individual axes of the machine) and this does not significantly affect the execution time of the set workpiece, nor the operating parameters of the machine (servo wear).

As expected for dense points, the velocity waveforms for a feed rate of 5000 mm/min and 10000 mm/min overlapped (see Fig. 5). This is due to the previously described parameters of the dynamics of the CNC machine. The characteristic slowdown of the machine at the reference points is caused by the application of the presented algorithmic solution, which ensures that the maximum accuracy of the reference points set in the G-code is achieved.

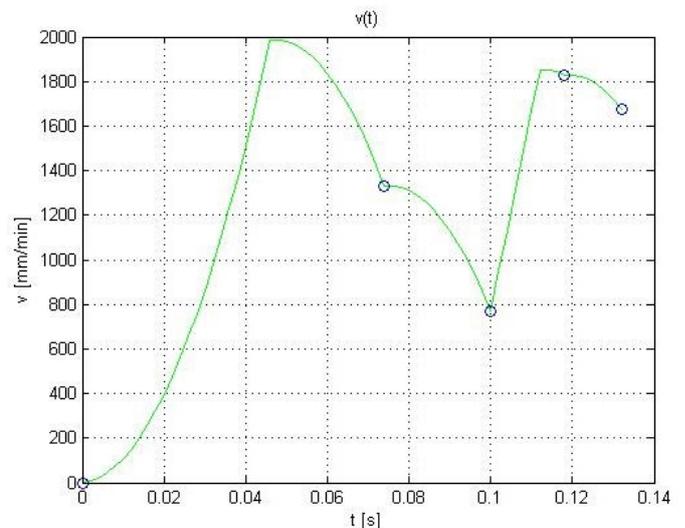


Fig. 5. Instantaneous velocities (green – $v_{\max} = 10000$ mm/min, red – $v_{\max} = 5000$ mm/min) for dense points

Figures 6 and 7 show the speeds reached by the machine for reference points distributed sparsely (Fig. 6) and densely (Fig. 7). For the experiments conducted ($v_{\max} = 10000$ mm/min $v_{\max} = 5000$ mm/min) the waveforms in Fig. 7 coincide.

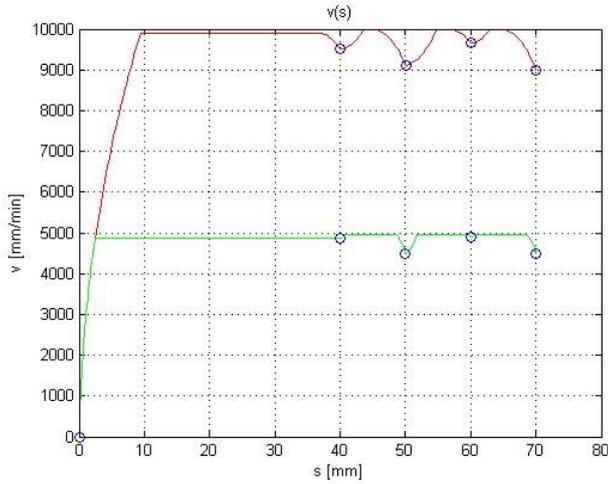


Fig. 6. Machine feed rate (red – $v_{\max} = 10000$ mm/min, green – $v_{\max} = 5000$ mm/min) for sparse points

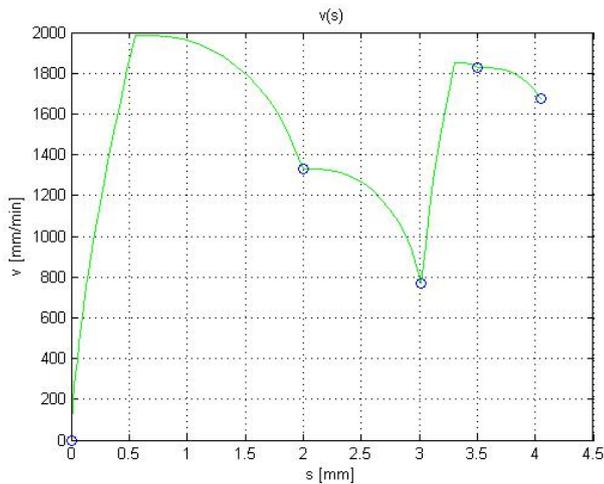


Fig. 7. Machine feed rate (red – $v_{\max} = 10000$ mm/min, green – $v_{\max} = 5000$ mm/min) for dense points

Figures 8 to 10 illustrate the shape of the acceleration that the machine obtains during the execution of reference points distributed sparsely for the speed of 5000 mm/min (Fig. 8) and the speed of 10 000 mm/min (Fig. 9) and reference points distributed densely for the speed of 10 000 mm/min and the speed of 5000 mm/min (Fig. 10). In all cases, in each time step the acceleration value does not exceed the set maximum value, i.e. 1500 mm/s². In selected time steps the acceleration values change linearly. An increasing line means that the acceleration value increases in particular time steps until the maximum value is reached. The linear increase is associated with a constant value of the JERK parameter. A decreasing line means that the machine decelerates according to the previously described values

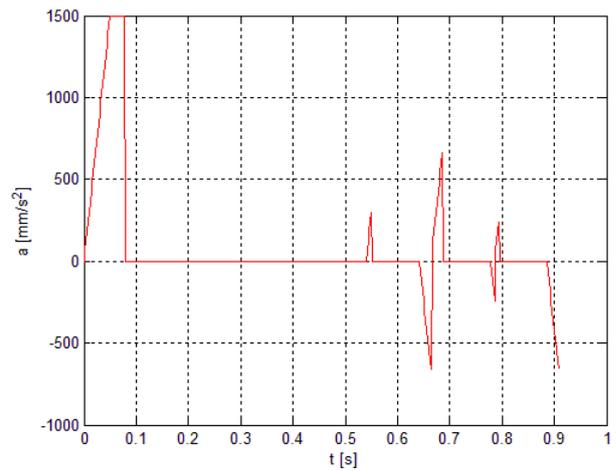


Fig. 8. Acceleration for $v_{\max} = 5000$ mm/min for sparsely located points

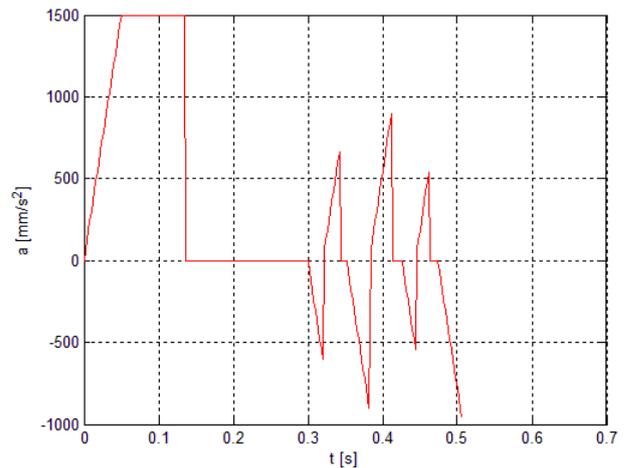


Fig. 9. Acceleration for $v_{\max} = 10000$ mm/min for sparsely located points

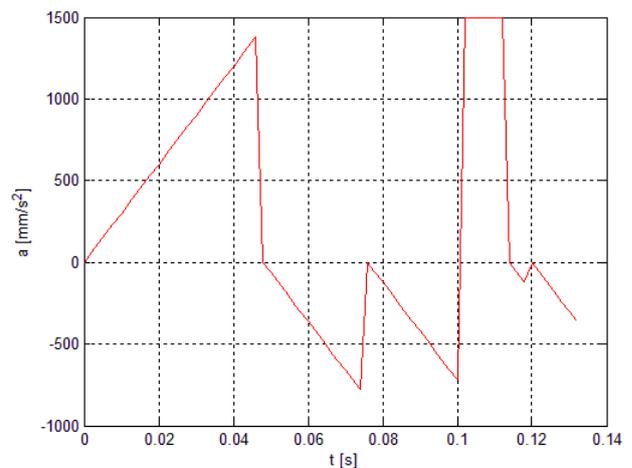


Fig. 10. Acceleration for $v_{\max} = 10000$ mm/min and $v_{\max} = 5000$ mm/min dense points

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of the JERK parameter. For experimental calculations, it was assumed that the value of the JERK parameter responsible for the rate of change of acceleration and the rate of change of deceleration are the same. In industrial conditions, this solution is most often adopted, but it is possible to set different values of the JERK parameter responsible for the acceleration of the machine called aJERK (acceleration JERK) and deceleration of the machine called dJERK (deceleration JERK).

Figures 11, 12, and 13 show the absolute error with which individual reference points were achieved for different dynamics parameters of the CNC machine, as well as for the sparse and dense distribution of reference points in the realized G-code. A characteristic feature of the presented graphs is the different accuracy of the realisation of individual reference points for the given dynamics parameters of the CNC machine. Thus, for the workpiece executed on the CNC machine, it is not possible to give unambiguously the accuracy with which it was executed,

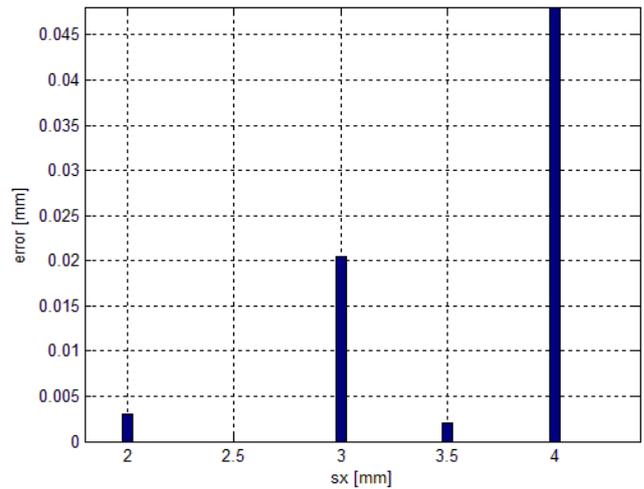


Fig. 13. Absolute error for $v_{max} = 5000$ mm/min for dense points

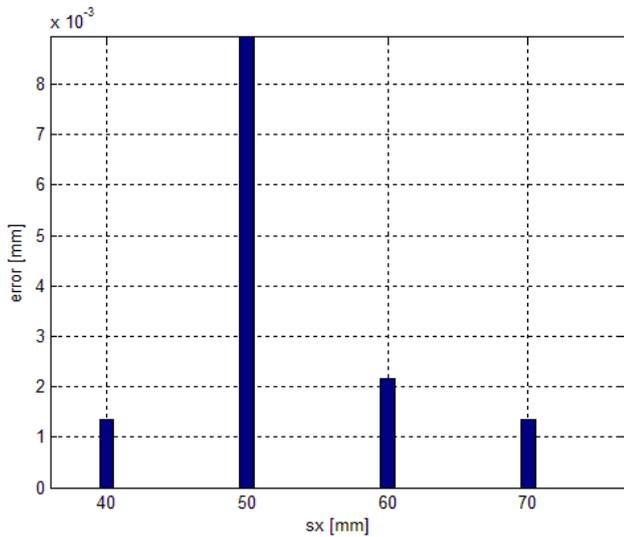


Fig. 11. Absolute error for $v_{max} = 10000$ mm/min for sparsely located points

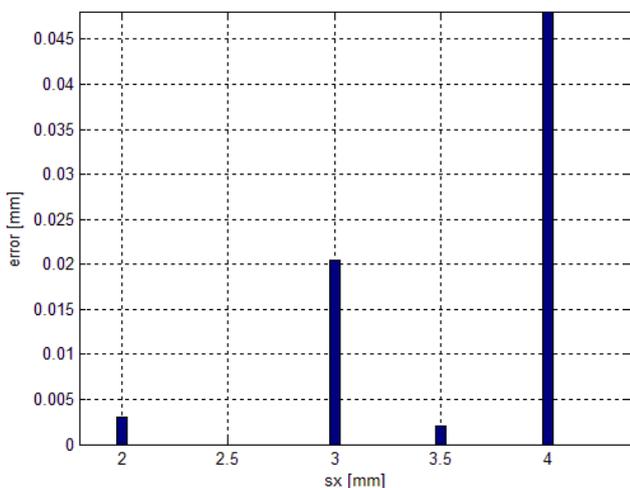


Fig. 12. Absolute error for $v_{max} = 10000$ mm/min for dense points

because it is different for each point. It is possible to give the average accuracy for the whole element or the local accuracy related to the realisation of critical reference points that have a significant impact on the technological purpose of the manufactured element. The error of the realisation of reference points is also significantly influenced by their distribution (dense, rare) and the preset parameters of the dynamics of the CNC machine, i.e. feed rate, acceleration, and JERK.

Figures 14 and 15 compare the changes in speed over time during the machining task for sparsely spaced reference points and a feed rate of 5000 mm/min (Fig. 14) and 10 000 mm/min (Fig. 15) obtained on the machine before and after the application of the presented algorithm. In both cases of different speeds, a reduction in the execution time of the task was obtained, with simultaneous elimination of large jumps in instantaneous speed. Such changes (jerks) have an extremely negative effect on the operating elements of the CNC machine. The obtained solution brings the user closer to the knowledge of the quality of the

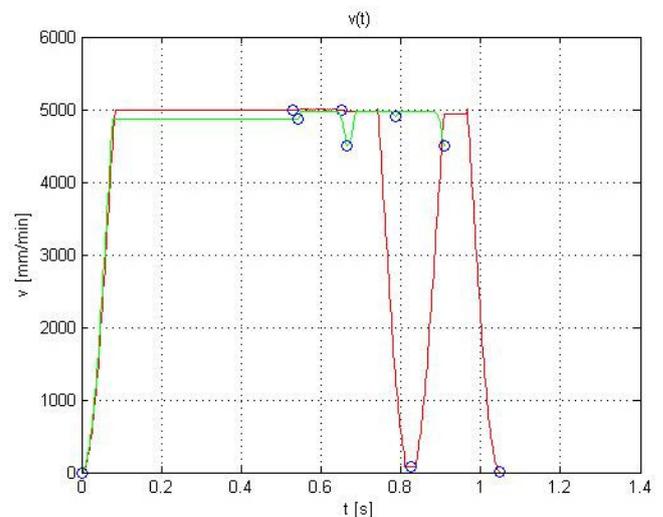


Fig. 14. Velocities for $v_{max} = 5000$ mm/min for sparsely located points (green algorithm, machine red-test)

machine work while performing the given detail and reduces errors by optimal selection of the dynamics parameters of the CNC machine.

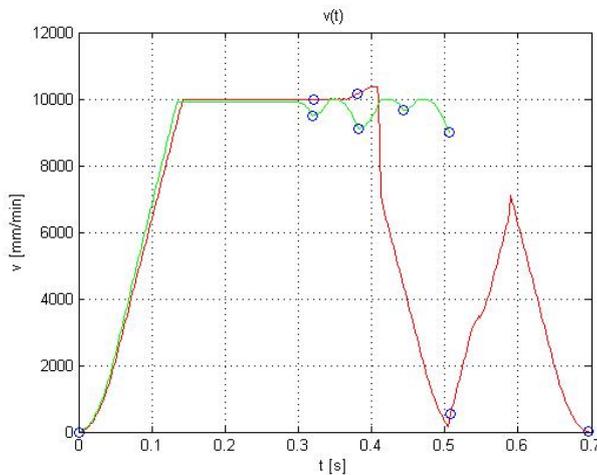


Fig. 15. Speeds for $v_{\max} = 10000$ mm/min for sparsely located points (green algorithm, machine red-test)

For the case when the machine realizes reference points located in a dense area (Fig. 16), the effectiveness of the presented algorithm is even more evident. With the solution used so far, a large variation in speeds and a longer task execution time was obtained. Instantaneous speeds were twice as high.

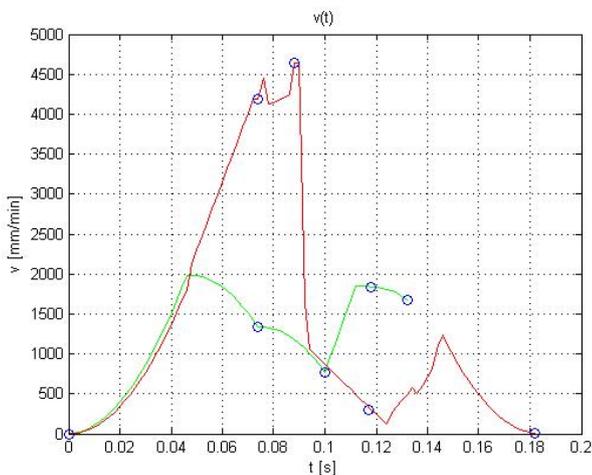


Fig. 16. Velocities for $v_{\max} = 5000$ mm/min and $v_{\max} = 10000$ mm/min for dense points (green algorithm, machine red-test)

Plots 17 and 18 show the realisation of individual reference points with the existing algorithm and with the implemented off-line method minimising the task error. Each deviation from the linear course indicates a large jump in instantaneous velocity. On the obtained characteristics one can also notice the time gain after applying the presented solution. These are gains in the execution time of the given detail of the order of 30% for the entire cycle of the machining task.

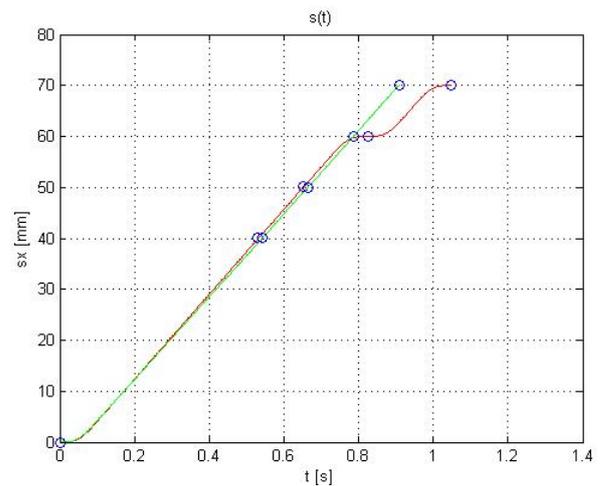


Fig. 17. Path as a function of time for $v_{\max} = 5000$ mm/min for sparsely located points (green algorithm, machine red-test)

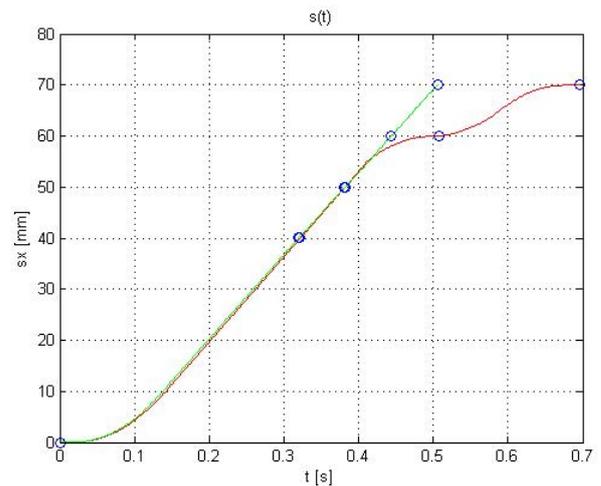


Fig. 18. Path as a function of time for $v_{\max} = 10000$ mm/min for sparsely located points (green algorithm, machine red-test)

4. CONCLUSIONS

The method proposed in the paper makes it possible to determine the maximum accuracy of hitting the reference points. It should be added here that different accuracy was achieved for each reference point. This accuracy is closely related to the values of the CNC machine dynamics parameters assigned to each machining axis. Experiments conducted in industrial conditions allow us to state that the algorithm presented in this paper reduces the execution time of the machining task. It facilitates determining the size of the error committed at particular reference points, which is of significant importance in planning the technological process in an enterprise. The proposed approach considers the global solution for the assumed configuration and the system of reference points. It also ensures that reference points are reached with maximum accuracy during acceleration and deceleration of tool movement in the course of the technological process. Such an approach, determining the control

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in the off-line mode, makes it possible to observe the effects of modifications for different values of the machine dynamics parameters, i.e. JERK, acceleration, or speed. Changes in the above-mentioned parameters for a given technological cycle (technological operation) will cause different final effects in the form of changes in the time of the detail execution, changes in the electric energy (power) consumption, and, in consequence, changes in the failure-free time of the CNC machine operation. Further research work is aimed at investigating the influence of the presented algorithm on the quality and quantity of electric power consumed and on the number of machine failures. Industrial research requires long-term observations and continuous monitoring of many parameters related to energy consumption and parameters related to the quality of CNC machine operation, e.g. vibrations of moving elements of the machine or noise.

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