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Galileo's Adventures in Wonderland

It turns out that the magical world behind the looking-glass portrayed in Lewis Carroll's novel about Alice is real – it has been discovered and described by physicists. So how do we find a way in?

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I'm sure everyone reading this has experienced travelling by bus, so let's try a little thought experiment. As you wait for at a bus stop, you synchronize two watches; next you leave one on the bench and take the other with you, ride to the end of the line and back again, and then finally compare the watches. You won't be surprised to discover that they both show exactly the same time. These kinds of observations, expressed somewhat differently, are known as Galilean transformations. This can be expressed in simple terms: when we move relative to someone else, our position relative to the other person changes, whereas time continues to flow for both of us at the same, constant rate.

Relativity – Special and General

This intuitive principle remained dominant in physics for centuries – space was regarded as something relative to the observer, while time was seen as an absolute. But then the paradigm was turned upside-down by Albert Einstein, when he proposed his Special Theory of Relativity in 1905. The separate concepts of space and time were replaced by a single notion: spacetime. The theory of relativity sees time and space as equivalent; this means that since motion means a change in location, it follows that time must also flow differently. Time becomes dilated – it turns out that watches which are in motion run more slowly, so the watch we took for a bus ride above should show an earlier time than the one that stayed put. This is in stark denial of the intuition demonstrated by the thought experiment: after all, I promised both watches would show the same time. Then again, we should remember that all measurement devices have a limited accuracy, and so if the time difference caused by a relativistic effect is extremely small, we will not be able to detect it using a simple watch. The scale of a relativistic effect can be gauged by calculating the relationship between the speed of an object (of the bus in our case) and the speed of light in a vacuum (which is 299,792,458 meters per second). The fastest train in the world currently reaches speeds of 167 m/s. While this is, of course, a triumph of technological achievement, in comparison to the speed of light it's barely a snail's pace. Even in such a vehicle, relativistic effects will be minuscule (although greater than zero, and therefore possible to measure – we merely need to build sufficiently accurate clocks).

Einstein's theory of relativity, now over a century old, consistently gets confirmed by all experiments (e.g. using elementary particles) and observations (e.g. detection of gravitational waves from distant regions of space).

While the theory of relativity may seem abstract to our everyday experiences – after all we are talking about absurdly high speeds – it does permeate into our

A black hole shadow is the closest "image" that can be taken of a black hole – an object that light cannot escape from. The black hole's event horizon (which is the source of the name of the Event Horizon Telescope, a global network of synchronized radio observatories) is approx. two and a half times smaller than its shadow, measuring close to 40 billion km in diameter. While that number appears large, the ring covers just 40 micro-arcseconds in the sky, corresponding to the length of a credit card on the moon as seen from the Earth.

Source: https://www.eso.org/public/images/eso1907a/

daily lives. Electrons in copper (the electrical current in a wire) move at a speed of 0.15 mm/s. This seemingly slow speed, together with the attendant relativistic effects, leads to interactions between matter and magnets – something which can be easily observed. The reason why such slow speeds generate such impressive effects is the vast number of free electrons in the wire (on the order of 10^{23} per cubic centimeter) and the fact that their contribution to electromagnetic interactions is compounded.

And that's not all. The theory of relativity (both the Special Theory of 1905 and the General Theory of 1915) is vital for the functioning of the global positioning system (GPS). If we were to naively apply Galilean and Newtonian principles, it would take just fifteen minutes for the satellites to lose their link with a device and the system would no longer be able to navigate. This is a terrific example of how "purely academic" and "impractical" concepts can quickly become crucial for the functioning of the contemporary world.

Faster than light

The first test applied to each new theory in physics is the correspondence principle. How can we explain our daily experience of buses and watches (and all other observations) using the language of Einstein's theory? The key element is the speed of light. When we limit our consideration to far slower objects (or, formally, if we think of the speed of light as approaching infinity), we "reclaim" the classical physics we all know and love. I would like to suggest another thought experiment: what if, instead of being extremely high, the speed of light was actually very low – lower than that of buses? Surely my suggestion sounds preposterous: we are used to the mantra that nothing can ever travel faster than light.

But now is a good time to remind ourselves that this is just flat-out false. First of all, when light flows through water its speed is significantly reduced and there's nothing to stop charged particles from overtaking it – in fact this is the mechanism behind the phenomenon known as Cherenkov radiation. The mantra should, in fact, say that nothing can ever travel faster than light *in a vacuum*. Second of all, we must first define what we mean by "something" or "nothing." It's relatively easy to construct a phenomenon moving faster than light (I'll leave it as a puzzle for the reader – my only hint is that all you need is a flashlight and a distant wall), as long as it does not carry any information. Only when the movement carries information does the relativistic limit appear.

Returning to the subject at hand, I'd like to stress that I am not talking about any of these exceptions. I will briefly abandon our observable reality and focus on a mathematical curiosity. We will see that it will lead us back to our own beloved universe. Without getting into the algebraic details, we obtain laws of physics which are the exact opposite to those familiar to Galileo. Absolute time is replaced by relative time (each clock keeps a different time!), but now space is absolute – we have speed but we are not moving. Lewis Carroll was one of the first to consider this possibility:

Illustration by John Tenniel for Lewis Carroll's *Through the Looking-Glass* (1871)

"*Well, in our country,*" *said Alice, still panting a little,* "*you'd generally get to somewhere else – if you run very fast for a long time, as we've been doing.*"
"*A slow sort of country!*" *said the Queen.* "*Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!*"

L. Carroll,
Through the Looking-Glass

Carroll groups

The French mathematician Jean-Marc Lévy-Leblond wrote about the concept described above, coining the name "Carroll group" in honor of the author of the Alice books – a mathematician himself. It is a truly a world full of wonder, in which fragments of time can become dramatically shorter or longer even though we are standing still.

This is all the more fascinating given that places like this, which would seem to be as if on the other side of a mirror, do actually exist. Perhaps the best-known example are black holes – regions where gravitation is so strong that absolutely nothing – not even light travelling at the fastest speed – can escape. The boundary of a black hole is known as an event horizon. For decades, this topic has been fascinating physicists, who have devoted a great deal of effort to understanding the structure and dynamics of event horizons and the mechanisms that underlie them. Theoretical considerations and observations confirm that black holes do exist – the discovery received the Nobel Prize in 2020. We even have photos of black holes! That's right – we have left the world of fantasy behind, and are now discussing cold, hard facts. Just a few years ago it became clear that a black hole event horizon is an example of a Carroll group. The complicated equations of the General Theory of Relativity, describing changes affecting the event horizon, are in fact fundamental laws of nature in a universe with absolute space. Does this mean that Alice actually found herself inside a black hole, and that if we ventured inside the supermassive black hole at the galactic center of the Milky Way, named Sagittarius A*, we might encounter the Mad Hatter there? Probably not, but we should certainly pay due respect to the prescient inventiveness of the 19th-century authors and mathematicians.

Another "place" I should mention is the infinity of spacetime. It is, of course, not an actual place, but from the mathematical perspective it is simpler to treat it this way. Every time we conduct astrophysical observations, for example searching for gravitational waves, and attempt to simulate them, we start from the assumption that we are at a great distance from the source. This simplifies the mechanics and enables us to make predictions. It turns out that this kind of infinity – known as null infinity and denoted by the symbol $\mathscr{I}$ (pronounced skr-EYE, short for "script i") – is another example of a Carroll group. The two examples have nothing in common. Yet less than a year ago, physicists realized that the structures describing event horizons and null infinity are almost identical. However, this is not simply a mathematical observation – it could serve as a starting point for performing "gravitational wave tomography" of black holes. The aim here would be to describe the evolution of an event horizon formed by the collision of two black holes, on the basis of observations of the gravitational waves emitted during the process. The coming years will reveal how close we can get to reaching this ambitious goal.

This outline of the evolution of the theory of relativity – although very brief by necessity – has nevertheless brought us a long way. Whereas relativity was once limited to hypothetical considerations of vast speeds, it now has practical applications even in most automobiles. Black holes were once believed to be abstract mathematical artefacts (errors or shortcomings) of relativity, and yet now we can actually hear them, see photos and even watch videos of them. Rapid improvements being made in observation techniques and detection, in tandem with discoveries of growing numbers of mathematical structures, are allowing us to gain a better understanding of these fascinating objects and to interpret their images. I cannot predict what our research will reveal, but I do know that the future of black hole physics is – so to speak – stunningly bright. ■

Further reading:

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