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ANALYSIS 0F THE PRESSURE PULSATION IN THE STOCK SUPLLIED TO THE PAPER MACHINE HEADBOX

The work presents characteristics of the pressure pulsation of the paper stock flowing through its approach system to the paper machine headbox and through the headbox. Theoretical model of the pulsating flow of the stock in the pipelines is described and conditions are suggested for the pipeline system to avoid creation of the wavy resonance. The relationships have been developed determining the effect of the pressure pulsation of the stock on its distribution on a surface of paper web being formed on the paper machine wire. Analysis of these relationships enables proper selection of the damping means of the pressure pulsation of the stock being delivered on the paper machine wire.

1. Introduction

The stock approach systems supplying the paper stock (the suspension of the fibers and chemical agents in water) to the headbox of the paper machines are considerably diversified. Almost always they are equipped with impeller pumps, overflow chests, stock and water tanks, centrifugal cleaners, feed valves and others, as well as deculators, all of them connected through the pipeline system. All this equipment, especially the impeller pumps, causes pressure pulsation in a stream of the flowing paper stock. Unstable stream of the paper stock, with an excessive pulsation of its pressure, leads to unacceptable variations of the basis weight (weight of the $1m^2$ of paper area measured in g/m^2) in the length of paper web being made on the paper machine. Paper quality depends closely on stability of the basis weight, thus

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such pressure pulsations should be limited to a level determined in the standards.

Various types of damping devices are used to control pressure pulsation of the flowing stock, i.e. absorptive, interferential, and hydraulic-pneumatic ones, perforated discs of various types and others. However, to ensure a proper selection of the pulsation damper, with known characteristics, the exact character of the pressure pulsation in the existing flow of the stock has to be determined first. Theoretical analysis of the stock pressure pulsation enables determination of a correlation between amplitude of these pulsations and the variations of the paper basis weight.

Alternatively, a proper design of the pipeline systems and a proper selection of the modern equipment with low pulsation characteristics should be used to minimize the above pressure pulsations in the flow of the paper stock.

2. Characteristic of the pressure pulsation

The pressure pulsations appear in the stock approach system as the pressure waves (Fig. 1) which are characterized with the following properties [1], [6], [7]:

- *Frequency* – measured in Hz;
- *Amplitude* – measured mainly in Pa or mbar
  Degree of the amplitude may be expressed as:
  – rms, thus as a root-mean – square value of a periodic quantity,
  – peak, that is a maximum value of a cycle or,
  – P – P (peak to peak) – a double peak value during one cycle;
- *Direction* – the pressure wave proceeds through the stock approach system in many directions from the place of its creation just as it is a case with a sound wave in the air;
- *Velocity* – the pressure wave travels approximately with an acoustic velocity (speed of sound). Velocity of the pressure wave propagation depends on its frequency, as well as on air content, pressure, stock consistency and stock conduit properties.
  Velocity of the longitudinal pressure wave in a liquid is determined by the Newton's formula [7]:

$$v = \sqrt{\frac{E^1}{\rho}} \tag{1}$$

where:

E^1 – volumetric model of elasticity,
ρ – density of the medium.
In a typical approach system supplying the stock to a paper machine, values of E^1 and ρ vary with changes of the quantity of air being introduced, average static pressure, concentration of the stock and a mutual interaction between design of the pipeline system and the liquid.
Previous studies [8] showed that velocity of the pressure wave propagation in the liquid depends significantly on the volume of the air fraction being introduced and on the static pressure and, in smaller degree, on the type of material used for the pipeline, wall thickness and concentration of the stock.

- *Interference (wave interference)* is occurring when two or more sources (e.g. pump supplying the stock to a headbox and the screen) initiate the pressure pulsation of the same magnitude. During interference, as a certain point of a medium is being approached at the same time by several waves, the given point is subjected to a combined deflection from a sum of deflections caused by the individual waves.

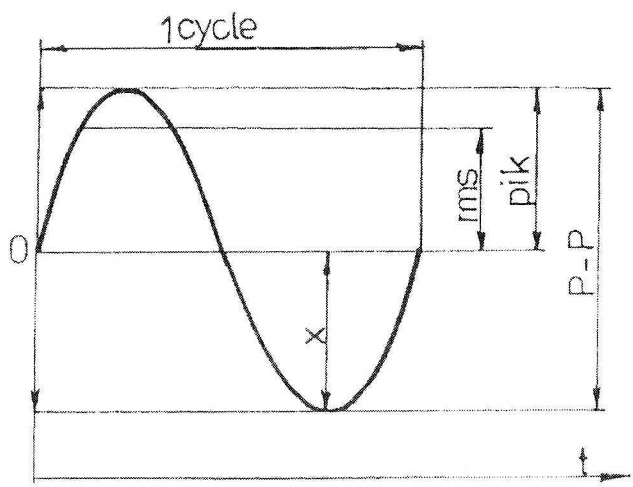

Fig. 1. Graphical and mathematical relationships between properties of the pressure waves

In some stock approach systems various sources may initiate either the pressure waves of the same amplitude that creates a resultant pressure wave with a higher amplitude (Fig. 2a and b) or the pressure waves of the same frequency resulting in the combined wave with a lower amplitude (Fig. 3). Fig. 4 shows three pressure waves of various amplitudes and frequencies as well as their resultant pulsation of higher amplitude. Such a situation is typical in the existing stock approach systems.

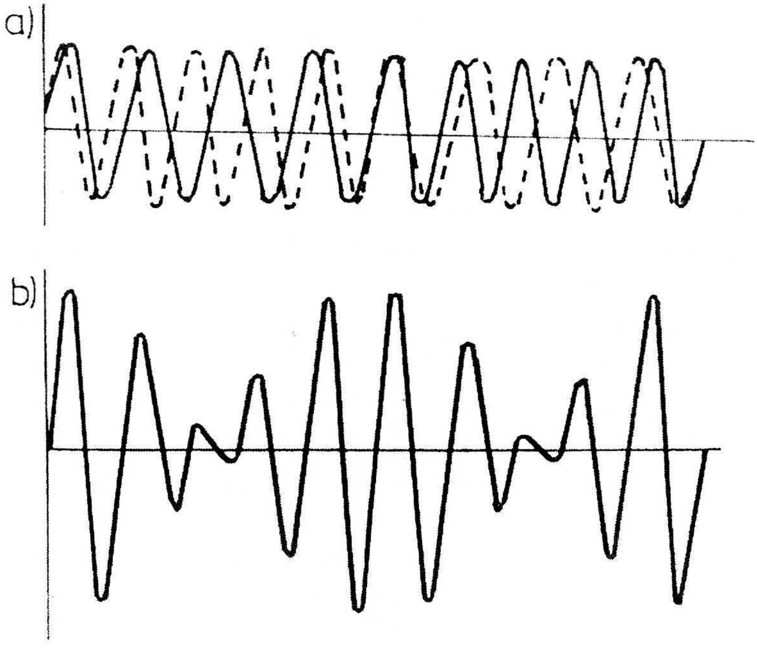

Fig. 2. a) Two pulsation waves of the same amplitude, b) Resultant pressure wave

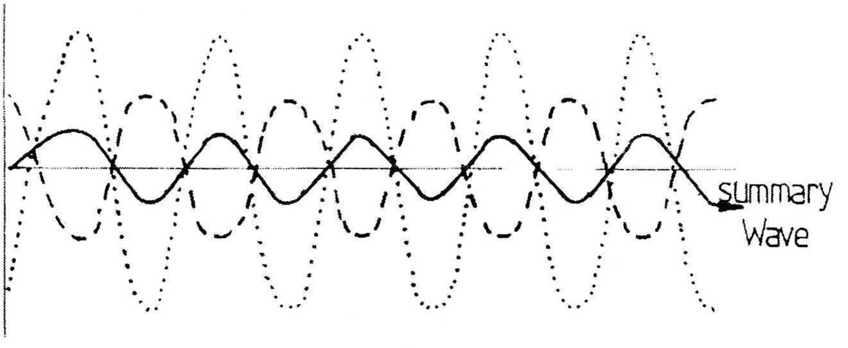

Fig. 3. Two waves showing pulsations of the same frequency and the obtained resultant wave
Amplitude / Frequancy

Pressure pulsations in the form of pressure waves show features of a damped vibration motion. Such a motion is being damped because of a resistance caused by a medium. The differential equation for the damped vibration has the form as follows [7]:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0 \tag{2}$$

where:

β – damping coefficient,

ω_o – specific frequency of the undamped vibrations in the system.

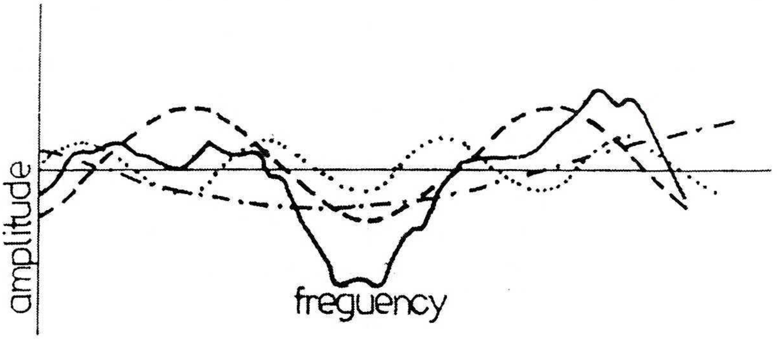

Fig. 4. Three pressure waves and the resultant pulsation

The solution of the equation [2] has different forms depending on either the expression ($\omega_o^2 - \beta^2$) is positive or negative [7];

for $\omega_o^2 - \beta^2 > 0$, the motion is periodical (weak damping) and the solution has the form as follows:

$$x = Ae^{-\beta t}\cos(\omega t + \varphi) \tag{3}$$

where:

ω – angular frequency, $\omega = \omega_o^2 - \beta^2$,

x – coordinate of the point (deflection) in the damped vibration motion,

$Ae^{-\beta t}$ – amplitude of the damped vibration (this amplitude decreases with time),

φ – phase shift (phase displacement),

for $\omega_o^2 - \beta^2 < 0$ the motion is aperiodic (very intensive, strong damping), for $\omega_o^2 - \beta^2 = 0$, the motion is critically damped (the system returns to the rest in the shortest possible time), and the solution has the form as follows:

$$x = Ae^{-\beta t}(1 + \beta t) \tag{4}$$

The Fig. 5 shows dependence of the deflection in the vibrating motion on time [7].

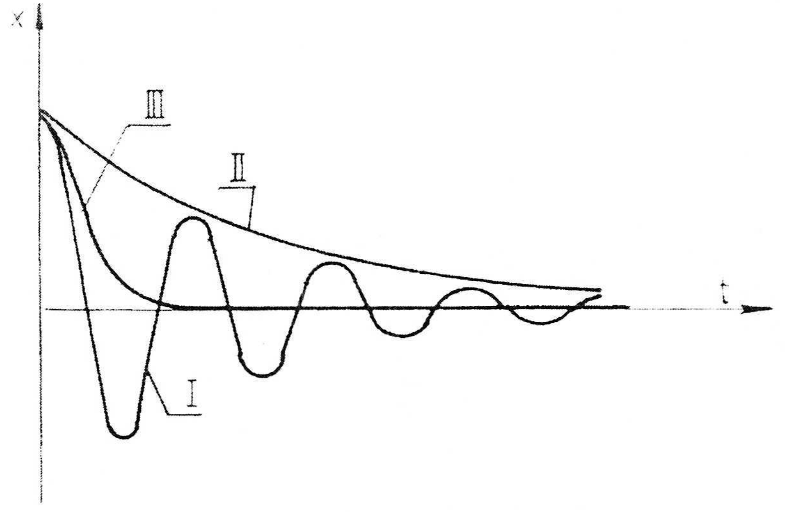

Fig. 5. Damped vibration I – damped vibration ($\omega_0^2 - \beta^2 > 0$), II – aperiodic vibrations ($\omega_0^2 - \beta^2 < 0$), III – damped, critical vibrations ($\omega_0^2 - \beta^2$) = 0

At small frequencies f, in a relation to the specific frequency of the system f_o, the vibration amplitude is also very small. However, as frequencies f increase the amplitude of the vibration also increases, reaching the maximum value at the moment when both frequencies f and f_o converge. In such a case resonance will occur between the vibrating system and a force causing the vibrations. Further increase in frequency of the forcing power (when $f > f_o$) decreases amplitude of the vibration [7].

The Fig.6 shows the resonance curves illustrating dependence of the amplitude A on the frequency ω of the forced vibrations ($\omega = 2\pi f$) for various values of β.

The resonance manifests itself through creation of a standing wave (stationary wave) with considerably increased amplitude of the vibration [9]. The standing waves occur when two series of waves of the same frequencies and amplitudes propagating in the opposite directions meet (Fig.7). Equation for the standing wave has the following form [7]:

$$\zeta = 2A \cos\left(\frac{2\pi}{\lambda} x\right) \cos \omega t \qquad (5)$$

where:

ξ – wave deflection,
A – amplitude,

λ – wavelength,
x – distance of any given point to the wave source,
ωt – phase of the vibrating motion,
ω – angular frequency.

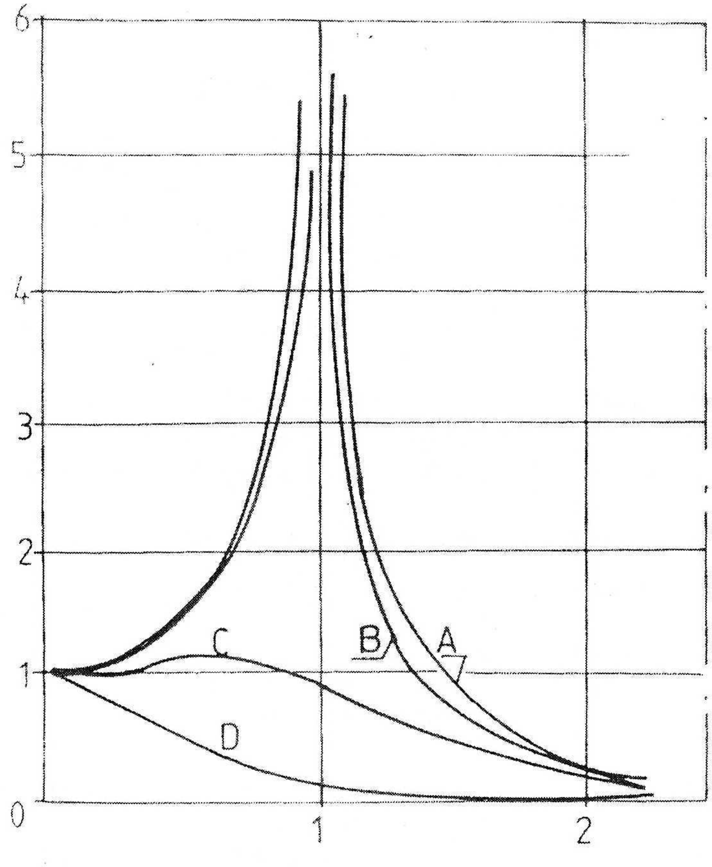

Fig. 6. Dependence of the vibration amplitude on the frequency of vibrations forced for the various values the decay constant (A < B < C < D)

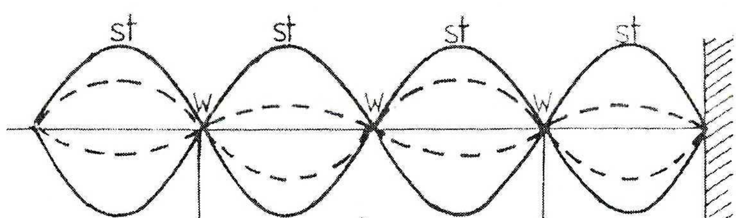

Fig. 7. Stop wave
w – knock of wave, st – maximum amplitude

In the points where $\cos\frac{2\pi}{\lambda}x = 0$, the amplitude of the motion is equal to zero. A distance of the adjoining points with zero amplitude is $\frac{\lambda}{2}$. These points are constantly in rest and we call them "knots". On the other hand, points for which $\cos\frac{2\pi}{\lambda}x = \pm 1$ have a maximum amplitude, i.e. when $x = n\frac{\lambda}{2}$ for n = 0, 1, 2..... Distance of the adjoining points with the largest amplitude is $\frac{\lambda}{2}$. These places are called "arrows" [7].

If in a stock approach system distance between a point where the pressure wave originates in a screen and the slice lip of the headbox is equal to the half of the length of the created pulsation wave then an undesirable standing wave is created.

Superimposition of the preliminary and reflected waves both having sinusoidal character, equal amplitudes, length and frequencies will result in the standing wave with an amplitude twice as high as the amplitude of either advancing wave [9].

In the next chapter we will determine conditions that have to be fulfilled by the system to avoid the resonance effect.

3. The theoretical model of the pulsating flows in the stock approach system

Let's take into consideration propagation of the stock pressure pulsation in the pipeline with a constant cross-section.

The Euler's equation for the elementary volume of liquid flowing along the x axis, parallel to the pipeline axis, can be written as follows:

$$\rho\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} - \Phi = 0 \tag{6}$$

$$E\frac{\partial u}{\partial x} = \frac{\partial p}{\partial t} \tag{7}$$

where:

ρ – density of the medium,
u – axial speed (longitudinal),
p – pressure,

Φ – friction force related to a unit of volume of the liquid,
E – reduced volumetric modulus of elasticity.

For the pipeline with a constant cross-section the reduced volumetric modulus of elasticity is expressed by equation:

$$E = \frac{1}{\frac{1}{E^{I}_{*}} + \frac{d_0}{\delta} \cdot \frac{1}{E^{II}}} \tag{8}$$

where:

E^{I}_{*} – volumetric modulus of elasticity of the medium,
d_0 – inner diameter of the pipeline,
δ – wall thickness of the pipeline,
E^{II} – volumetric modulus of the pipeline material.

According to Terentjev [9] parameters characterizing a stream of liquid can be expressed as a sum of the components averaged after time and the components of pulsation:

$$\begin{aligned} u &= u^* + u' \\ p &= p^* + p' \\ \Phi &= \Phi^* + \Phi' \end{aligned} \tag{9}$$

Substituting the expressions (9) to the equations (6) and (7) we will obtain:

$$\rho\left(\frac{\partial u^*}{\partial t} + \frac{\partial u'}{\partial t}\right) + \frac{\partial p^*}{\partial x} + \frac{\partial p'}{\partial x} - \Phi^* - \Phi = 0 \tag{10}$$

$$E\left(\frac{\partial u^*}{\partial x} + \frac{\partial u'}{\partial x}\right) = \frac{\partial p^*}{\partial t} + \frac{\partial p'}{\partial t} \tag{11}$$

For the pipeline with a constant cross-section the averaged speed is constant in time and thus:

$$\frac{\partial u^*}{\partial t} = 0 \qquad \frac{\partial u^*}{\partial x} = 0 \qquad \frac{\partial p^*}{\partial t} = 0 \tag{12}$$

In the case of a stationary flow, when all the pulsation components are equated to zero, the equation (5) gives us:

$$\rho \frac{\partial u^*}{\partial t} + \frac{\partial p^*}{\partial x} - \Phi^* = 0 \tag{13}$$

As it was mentioned above, for a fixed cross-section case there is $\frac{\partial u^*}{\partial t} = 0$, thus the following equation has to be satisfied:

$$\frac{\partial p^*}{\partial x} - \Phi^* = 0 \tag{14}$$

Equation (14) indicates that the pressure loss of the flowing stream, averaged over a length of its flow, is primarily due to the friction against the walls of the pipe.

Friction force of the pulsation component Φ_w and the pulsation friction force in the stream Φ_E result in a deformation of the medium along the pipeline.

Assuming that the friction forces on the pipe wall significantly surpass the friction forces required for deformation of the medium we will obtain the following relation:

$$\Phi' = - \rho m u' \tag{15}$$

where:

m – loss factor.

The reduced volumetric modulus of elasticity can be expressed by the acoustic velocity in the medium and density of the medium ρ:

$$E = \rho a^2 \tag{16}$$

Hence the equation system (10) – (11) can be expressed as follows:

$$\rho \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} + \rho m u' = 0 \tag{17}$$

$$\rho a^2 \frac{\partial u'}{\partial x} - \frac{\partial p'}{\partial t} = 0 \tag{18}$$

Differentiating the equation (17) after x, and equation (18) after t we will obtain:

$$\rho \frac{\partial^2 u'}{\partial t \partial x} + \frac{\partial^2 p'}{\partial x^2} - \rho m \frac{\partial u'}{\partial x} = 0 \tag{19}$$

$$\rho a^2 \frac{\partial^2 u'}{\partial x \partial t} + \frac{\partial^2 p'}{\partial t^2} = 0 \tag{20}$$

Accordingly, transforming the equation (20) we will receive:

$$\rho \frac{\partial^2 u'}{\partial x \partial t} = -\frac{1}{a^2} \frac{\partial^2 p'}{\partial t^2} \tag{21}$$

and

$$\rho \frac{\partial u'}{\partial x} = -\frac{1}{a^2} \frac{\partial p'}{\partial t} \tag{22}$$

The equation (19) can be then transformed to a following form:

$$\frac{\partial^2 p'}{\partial x^2} - \frac{1}{a^2} \frac{\partial^2 p'}{\partial t^2} - \frac{m}{a^2} \frac{\partial p'}{\partial t} = 0 \tag{23}$$

This way we have obtained a specific case of the equation for the damped vibration motion [2] that can be described in a general form as:

$$\frac{\partial^2 F}{\partial x^2} - b_0 \frac{\partial^2 F}{\partial t^2} - b_1 \frac{\partial F}{\partial t} - b_2 F = 0 \tag{24}$$

With an assumption of the wave resonance, the equation describing a damped vibration motion permits the special solution in the following form:

$$F(x,t) = e^{\pm ikx} e^{st} \tag{25}$$

where:
$s = \sigma + i\omega_c$ – root of the quadratic equation.

$$b_0 s^2 + b_1 s + (b_2 + k) = 0 \tag{26}$$

In the expressions (25) and (26), for the special case of the equation (23), the following variables are used:

$b_2 = 0$,

k – wave number = $2\pi/\lambda = \omega/a$,

σ – damping constant = $-b_1/2b_0 = -m/2$,

ω_c – specific angular frequency;

$$s = \frac{\sqrt{4b_0k^2 - b_1^2}}{2b_0} = \frac{\sqrt{4\omega^2 - m^2}}{2} \tag{27}$$

λ – wavelength $= \dfrac{2\pi}{\omega} a.$

The equation (26) has the following roots:

$$s_{1,2} = \frac{-b \pm \sqrt{b_1^2 - 4b_0(b_2 + k)}}{2b_0} \tag{28}$$

Taking into consideration the expression (27) we will receive

$$s_{1,2} = -\frac{m}{2} \pm \frac{\sqrt{m^2 - 4\omega^2}}{2} \tag{29}$$

If b_0, b_1, b_2 are real numbers, than the roots s_1 and s_2 for $m^2 - 4\omega^2 < 0$ will be complex numbers, thus it is more convenient to present the solution (25) in the form (2):

$$p(x,t) = e^{\sigma t}(A\cos\omega t + B\sin\omega t) = p_0 e^{\sigma t}\sin(\omega t \pm kx) \tag{30}$$

where:

p_o – pulsation amplitude of the interference source.

Assuming that $p(x,t) = 0$ for $t = 0$, equation (30) can be converted to the form:

$$p(t) = p_0 e^{\sigma t}\sin\omega t = p_0 e^{\frac{m}{2}t}\sin\omega t \tag{31}$$

In our example distance x and time t are interdependent, $t = x/a$, and for the practical calculations it is favorable to express relation (31) in the form (9):

$$p(x) = p_0 e^{-\frac{m}{2a}x} \sin \omega t \tag{32}$$

In the general case, besides the solution (30), we should also take into consideration the variants for which $m^2 > 4\omega^2$ and $m^2 = 4\omega^2$. This is possible for the very high values of the loss factor (e.g. in the viscous liquids). Respectively, the solutions for those variants are given in the form:

$$p(t) = p_1 e^{s_1 t} + p_2 e^{s_2 t} \tag{33}$$

$$p(t) = (p_1 + p_2) e^{-\frac{b_1}{2b_0}t} \tag{34}$$

Earlier performed evaluation of the stock approach system operating at frequencies > 10Hz has indicated that relation $m^2 < 4\omega^2$ is always fulfilled, accordingly either solution (31) or (32) should be used.

Acoustic velocity of the wave in the medium in the equation (32) can be described by the reduced volumetric modulus of elasticity E from the equation (16). After taking into consideration relation (18) we will obtain

$$a = \frac{1}{\sqrt{\rho\left(\frac{1}{E^{l}} + \frac{d_0}{\delta}\frac{1}{E^{n}}\right)}} \tag{35}$$

Let described relation $m/2\alpha$ as β_x, called a damping coefficient (9), where β_x characterizes effect of the dissipation factor on the weakening of the wave in the unite of the pipeline length.

In the calculation it is more convenient to use relationship (9):

$$\beta\lambda = \int_0^{\lambda} \beta_x \, dx \tag{36}$$

It is the logarithmic damping decrement,
Since

$$\lambda = \frac{2\pi a}{\omega}$$

then:

$$\beta\lambda \cong 2\pi \frac{a}{\omega} \beta_x \tag{37}$$

For the small values of a damping coefficient:

$$\beta\lambda \cong 2\pi \left[\frac{1}{2} \left(\frac{m}{\omega} + \frac{1}{\mathrm{Re}_\alpha} \right) \right] \tag{38}$$

where:

m/ω – nondimensional parameter characterizing losses over a length of the wave, described by the friction of the flowing stream against the pipeline wall,

I/Re_α – parameter characterizing losses from friction in the wave, described by the longitudinal deformation of the stream velocity.

Value Re_α is called the Reynold's acoustic criterion and can be calculated from the relationship:

$$\mathrm{Re}_\alpha = \frac{\rho}{2\pi} \frac{a\lambda}{\mu_{ef}} \tag{39}$$

where:

μ_{ef} – effective dynamic viscosity composed from viscosities for the molecular and turbulent components.

Loss factor m is a reference to the pulsating force of the friction against the pipeline wall to the magnitude of the vibrating component of the mass velocity:

$$m = \frac{\Phi_{w'}}{\rho u'} = \frac{\frac{P}{F} \tau_{w'}}{\rho u'} \tag{40}$$

where:

P – circuit of the pipeline cross-section,

F – cross-section area,

τ_w – pulsation component of the tangential stress on the wall (shearing stress).

For approximate evaluations a method of the harmonic linearization may be used

$$\tau_w^{'} = A \sin \omega t \tag{41}$$

However, in order to use the above approximation an impulse of the friction force on the pipeline wall in the direction of the semi-period (half-period) of the variations $^1/_2$ To = π/ω has to become persistently weaker and weaker:

$$\int_0^{\pi/\omega} \tau_w \, dt = const \tag{42}$$

In this case the following relation can be used to approximately determine the loss factor:

$$m = \xi \left(\frac{u_0}{d_0} + \frac{\pi u'}{8d_0} \right) \tag{43}$$

Over an industrially relevant range of the operational conditions of the stock approach system to the paper machines the following inequality [9] comes true:

$$\pi u' / 8d_0 << u_0 / d_0 \tag{44}$$

just like the reference $u^1/u < 10^{-3}$.
Thus the equation (43) can be simplified to the following form:

$$m \cong \xi \frac{u_0}{d_0} \tag{45}$$

where:

ξ – drag coefficient.

Substituting λ, from equations (39) and (45) into equation (25), we can obtain a relation for the damping coefficient:

$$\beta = \frac{\omega}{2a} \left(\xi \frac{u_0}{d_0 \omega} + \frac{\omega \mu_{ef}}{\rho a^2} \right) \tag{46}$$

In relation (46) term $\zeta u_0/(d_0\omega)$ characterizes decrease in the amplitude of the longitudinal sound wave as a result of the friction against the pipeline walls, and term $\omega\mu_{ef}/\rho a^2$ describes decline in the amplitude as a result of the energy lost for the viscous friction inside the stream.
Let us discuss the characteristic cases of the pulsating flows in the pipeline.

Assuming that losses of the pulsating energy resulting from diffusion in a viscous liquid are significantly lower than losses from friction against the walls of the pipeline, we will get a following relation:

$$\xi \frac{u_0}{d_0\omega} >> \frac{\omega\mu_{ef}}{\rho a^2} \tag{47}$$

and the equation (41) can be written as follows:

$$\beta = \xi \frac{u_0}{2ad_0} \tag{48}$$

Coefficient ξ has to be determined experimentally as a loss of pressure in each specific case.

It should be mentioned that, when the condition (47) is satisfied, the damping coefficient β will no longer depend on the pulsation frequency. It is fundamentally influenced by hydraulic and geometric parameters of the pipeline and the reduced volumetric modulus of elasticity of the system that determine the velocity of propagation of the longitudinal acoustic wave in the pipeline.

Another example of the energy loss for the pulsation flow can be diffusion of the energy at a cost of the viscosity. Then:

$$\xi \frac{u_0}{d_0\omega} << \frac{\omega\mu_{ef}}{\rho a^2} \tag{49}$$

and the relation (41) can be approximated as:

$$\beta = \frac{\omega^2\mu_{ef}}{2\rho a^3} \tag{50}$$

Formula (50) indicates that, with this type of a flow, the damping coefficient is essentially affected by frequency of the pulsation, an effective viscosity and velocity of the propagation of the acoustic pulsation in the medium.

In the considered case, the losses of the pulsation energy will in reality spread inside the stream at the cost of the viscous friction. The values of the damping coefficient will increase with an increase in the pulsation frequency and with an increase in viscosity of the liquid.

The presence of air in the stream of the fibrous suspension may result in a twofold influence.

Firstly – it can decrease the effective viscosity of the suspension leading to a lower value of the damping coefficient.

Secondly – it can decrease density (concentration) of the suspenssion and, what is most essential, it can reduce the volumetric modulus of elasticity of the liquid. This will result in a decrease in the velocity of propagation of the acoustic wave and thus to an increase in the damping coefficient.

The third case of the flow of the pulsation stream is a case when magnitudes of $\xi \frac{u_0}{d_0 \omega}$ and $\frac{\omega \mu_{ef}}{\rho a^2}$ are commensurable.

In such a case the energy of pulsation will be lost to friction both inside of the stream and at the walls of the pipeline.

Since β is used to describe m/2a, the final equation for calculating amplitude of the pressure pulsation at the distance x from the source of the interference has a following form:

$$p(x) = p_0 e^{-\beta x} \tag{51}$$

From analysis of the equation (51), together with the equation (45), it can be stated that declining of the pulsation amplitude is proportional to the frequency, average stream velocity, hydraulic coefficient of the pipeline resistance, and effective viscosity of the liquid. On the other hand, decrease in the pulsation amplitude is also inversely proportional to the pipeline diameter, medium density and velocity of the acoustic wave propagation in the pipeline.

To calculate attenuation of the pulsation amplitude the following input data are required:

p_o – amplitude of the interference source,
u_o – average velocity of the stream,
d_0 – diameter of the pipeline,
ω – frequency of the pulsation,
ρ – density of the liquid (suspension),

μ_{eff} – effective viscosity of the liquid (in the turbulent stream it is composed of the molecular viscosity and the turbulent viscosity),
δ – wall thickness of the pipeline,
E^{I} – volumetric modulus of elasticity of the liquid,
E^{II} – modulus of elasticity of the pipeline material,
Δp – pressure losses at the computational pipeline selection.

Some of those parameters are provided by the reference book and some have to be obtained experimentally.

So far we have discussed a case for propagation of the pressure pulsation of an acoustic character in the infinitely long pipeline. In the real-world systems the length of the pipeline is limited.

Sudden changes of the boundary conditions, e.g. at the pipeline entrance to a tank, create a reflected wave. If length of the pipeline l is equal to the multiple value of the half-length of the vibration wave, than an inevitable resonance effect will occur in the closed pipeline. This condition may be expressed in the following form:

$$l = n\frac{\lambda}{2} \qquad (n = 1,2,3,...) \tag{52}$$

The resonance effect manifests itself through creation of the standing wave with significantly increased amplitude. Magnitude of this amplitude varies across the pipeline.

The superimposition of the primary and reflected waves both having a sinusoidal character, an equal amplitude, length and frequency results in formation of the standing flat wave with an amplitudes twice as large as the amplitude of any of each moving wave.

$$p_n + p_{od} = 2p_0 \cos kx \cos \omega t \tag{53}$$

where:

p_n – amplitude of the primary wave,
p_{od} – amplitude of the reflected wave,
p_0 – amplitude of the wave in the point of change of the boundary conditions.

The length of the vibration wave depends on velocity of the sound propagation in the medium and on the frequency. Half-length of the wavelength is given by:

$$\frac{\lambda}{2} = \frac{\pi}{\omega} \frac{1}{\sqrt{\rho\left(\frac{1}{E^{1}} + \frac{d_0}{\delta}\frac{1}{E^{11}}\right)}} \tag{54}$$

In order to prevent the wave resonance effect in the pipeline system the following condition should be satisfied:

$$l \neq \frac{n\pi}{\omega} \frac{1}{\sqrt{\rho\left(\frac{1}{E^{1}} + \frac{d_0}{\delta}\frac{1}{E^{11}}\right)}} \tag{55}$$

In practice this problem becomes even more complicated because the oscillations are usually poly-harmonic, i.e. there are several harmonic components present at the same time.

Thus the pipeline diameter varies over its length.

However, knowing frequencies of different parts of the system leading to the maximum pulsations (a frequency of the pumps, screens etc.) and omitting changes in the pipe diameters, it is possible to approximate the pipeline length which is far from the one that would cause the resonance effect using a method of result calculation for each frequency.

5. Effect of the stock pressure pulsation on its distribution on the paper web surface

In this chapter an analytical method will be presented for evaluation of the effect of pressure pulsation of the stock flowing out from the slice of the pressure headbox on distribution of this stock on the surface of $1\mathrm{m}^2$ of the paper web (paper grammage).

In construction of the modern high-speed paper machines (with good paper properties), aspects of the stock outflow onto the machine wire constitute a separate and important problem.

The main requirements for the modern headbox of the paper machines are:

a) an uniform distribution of the paper stock over the width of the paper machine wire. Fulfillment of this requirement ensures uniformity of the stock distribution in $1\mathrm{m}^2$ of the paper web over width of the machine;
b) a stable discharge of the paper stock stream flowing out on the wire over time (stream stability), this will minimize variation of the stock distribution in $1\mathrm{m}^2$ of paper web over the length of the machine;
c) a high degree of dispersion of the stock stream being fed on the machine wire.

Figure 8 illustrates movement of the stock suspension stream in the slice lip of the pressure headbox. This movement can be considered as a movement between two parallel surfaces. Let's use the general equation describing motion of the paper stock. According to Tieretjev [9], the general equation describing motion of the fibrous suspensions, in vector form, is expressed as follows:

$$\rho\frac{d\overline{v}}{dt} = p\overline{F} - gradP + div\left(\sum_{k=1}^{n+1} A_k e^{-\alpha_k\gamma_{ij}} + \mu\gamma_{ij}\right) \tag{56}$$

where:

$\overline{v}$ – vector of the absolute velocity,
t – time,
$\overline{F}$ – vector of the mass forces,
P – average hydraulic pressure in the stream,
ρ – density of the suspension.

Equation (56) differs from Navier and Stokes equation by term $\sum_{k=1}^{n+1} A_k e^{-\alpha_k\gamma_{ij}}$, which takes into consideration the anomaly (abnormality) of the stock suspensions and characterizes the internal stresses of the cross-linked stream.

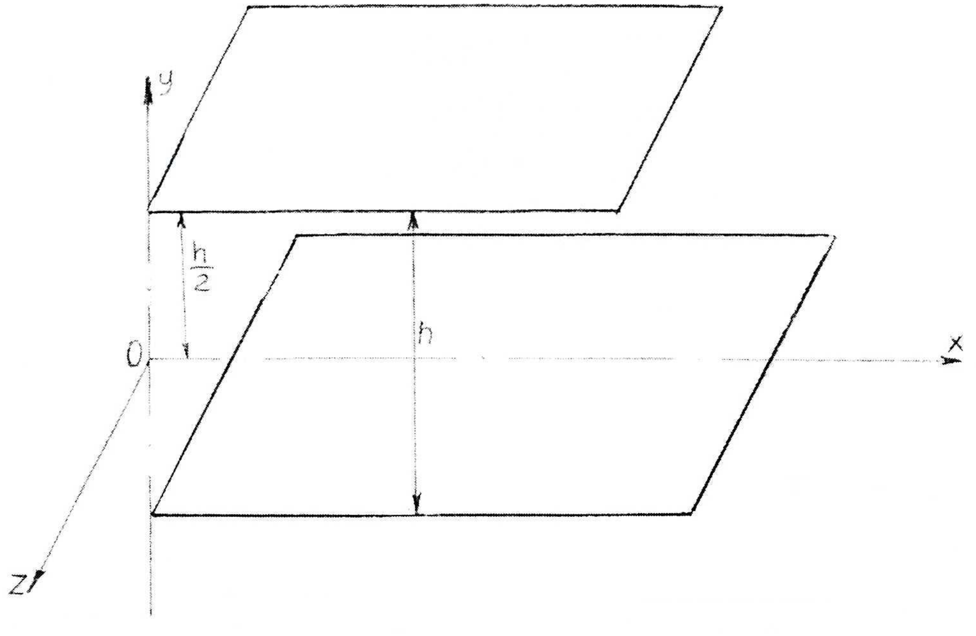

Fig. 8. Diagram of flow between two parallel, stationary planes

The bracketed term of the equation (56) is obtained through substitution in the stress tensor of the rheological equation (57) which is true for the three – dimensional motion of the fibrous suspension [9]:

$$\tau_{ij} = \sum_{k=1}^{n+1} A_k e^{-\alpha_k \gamma_{ij}} + \mu \gamma_{ij} \tag{57}$$

where:

τ – shear stress,

A_k, α_k – coefficients of the rheological equation,

γ_{ij} – speed of displacement (shearing rate),

μ – dynamic viscosity coefficient for the dispersed suspension.

The dynamic state of the fibrous structure can also be described using the following equation [9]:

$$\tau = A_1 e^{-\alpha_1 \gamma} + A_2 e^{-\alpha_2 \gamma} + A_3 e^{-\alpha_3 \gamma} + \mu \gamma$$

where:

A_1 – stress from the initial friction of the fibrous core against the wall (at start),

A_2 – stress characterizing an internal durability (strength) of the fibrous core (at start),

A_3 – stress characterizing durability of the flocks,

α_1, α_2, α_3 – time constants characterizing a period of existence of the adequate stresses A in the stream.

Let us consider a flat movement in the direction of the axis OX, i.e. on the surface OXY (Fig. 8).

It can be stated that $\omega = v = 0$.

The mass forces are neglected, i.e. $F_x = F_y = F_z = 0$.

Taking into consideration the above assumptions with the equation (56), the dynamics of the fibrous suspension for the narrow pipeline can be written as follows:

$$\frac{\partial u}{\partial t} - \upsilon \frac{\partial^2 u}{\partial y^2} + \alpha_k A_k \frac{1}{2} e^{-\frac{\alpha_k}{2} \frac{\partial u}{\partial y}} \frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho} \frac{\partial p}{\partial x} \tag{58}$$

Obviously, at the high speed in the turbulent conditions of the motion the stock suspension (with concentration up to 1%) moves in the dispersed state.

This leads to a large speed gradient ∂, in the result of which the term $\alpha_k A_k \frac{1}{2} e^{-\frac{\alpha_k}{2}\frac{\partial u}{\partial y}}$ in the equation (58) becomes sufficiently small and can be neglected.

Thus equation (58) can be transformed into the following form:

$$\frac{\partial u}{\partial t} - \upsilon \frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{59}$$

On the surface of the plane the velocity u equals zero, thus the following boundary conditions can be introduced:

$$u = 0 \text{ at } y = h \tag{60}$$

As it was shown above (26) when a pulsation motion stabilizes, in an accordance with the harmonic law, the change in the pressure drop can be described as:

$$f(t) = \rho B \sin \omega t$$

In our case function f(t) corresponds to the pressure drop ∂p over a length ∂x. Thus the last equation will take the form:

$$\frac{\partial p}{\partial x} = -\rho B \sin \omega t \tag{61}$$

where:

B – amplitude of the pressure drop,

ω – angular frequency of the vibrations (pulsations).

In order to transform equation (59) to the more practical form for the final solution, we need to introduce a function u(y,t) in the form of:

$$u(y,t) = \varphi(y,t) = -B/\omega \cos \omega t \tag{62}$$

where:

$\varphi(y,t)$ – certain auxiliary function.

Substituting relation (62) in equation (59) and taking into account that

$$\frac{\partial u}{\partial t} = \frac{\partial \varphi}{\partial t} + B \sin \omega t \tag{63}$$

e will obtain:

$$\frac{\partial \varphi}{\partial t} + B \sin \omega t - \upsilon \frac{\partial^2 \varphi}{\partial y^2} = - \frac{1}{\rho} \frac{\partial p}{\partial x} = B \sin \omega t \tag{64}$$

Finally, after taking into account the boundary conditions (60), equations (64) and (62) will get the following form:

$$\begin{gathered} \frac{\partial \varphi}{\partial t} = \upsilon \frac{\partial^2 \varphi}{\partial y^2} \\ \varphi(h, t) = \frac{B}{\omega} \cos \omega t \\ \varphi(-h, t) = \frac{B}{\omega} \cos \omega t \end{gathered} \tag{65}$$

After substitution of the variables

$$\begin{aligned} y &= h - y_1 \quad &\text{for } y \geq 0 \\ y &= y_1 - h \quad &\text{for } y \leq 0 \end{aligned} \tag{66}$$

and introduction of the following terms

$$\begin{aligned} \psi_1(x) &= \varphi(h - y_1) = \varphi(y) \quad &\text{for } y \geq 0 \\ \psi_2(x) &= \varphi(h_1 - y) = \varphi(y) \quad &\text{for } y \leq 0 \end{aligned} \tag{67}$$

we will receive

$$\begin{gathered} \frac{\partial \varphi(y)}{\partial t} = \frac{\partial \varphi(h - y_1)}{\partial t} = \frac{\partial \psi_1}{\partial t} \\ \frac{\partial \varphi(y)}{\partial t} = \frac{\partial \varphi(y_1 - h)}{\partial t} = \frac{\partial \psi_2}{\partial t} \\ \frac{\partial \varphi(y)}{\partial y} = \frac{\partial \varphi(h - y_1)}{\partial (h - y_1)} = \frac{\partial \psi_1}{\partial y} (h - y_1) = - \frac{\partial \psi_1}{\partial y} \end{gathered} \tag{68}$$

$$\frac{\partial^2 \varphi}{\partial^2 y} = \frac{\partial}{\partial (h - y_1)}\left(-\frac{\partial \psi_1}{\partial y}\right) = \frac{\partial^2 \psi_1}{\partial y^2} \tag{69}$$

Equations (60) can be also written as:

$$\frac{\partial \psi_1}{\partial t} = \upsilon \frac{\partial^2 \psi_1}{\partial y^2}$$

$$\psi_1(0,t) = \frac{\beta}{\omega} \cos \omega t \qquad \text{for y} \geq 0 \tag{70}$$

$$\frac{\partial \psi_2}{\partial t} = \upsilon \frac{\partial^2 \psi_2}{\partial y^2}$$

$$\psi_2(0,t) = \frac{\beta}{\omega} \cos \omega t \qquad \text{for y} \leq 0 \tag{71}$$

The obtained dependencies (70) and (71) represent the standard equations for thermal (heat) conductivity. The general solution of this equation system has the following form:

$$\psi(y,t) = \frac{B}{\omega} e^{-\sqrt{\frac{\omega}{2\upsilon}} y} \cos\left(\sqrt{\frac{\omega}{2\upsilon}} y - \omega t\right) \tag{72}$$

By using the previous variable, we can also write equation (72) as:

$$\varphi(y,t) = \frac{B}{\omega} \exp\left[-\sqrt{\frac{\omega}{2\upsilon}}(h - y)\right] \cos\left[\sqrt{\frac{\omega}{2\upsilon}}(h - y) - \omega t\right] \quad \text{for y} \geq 0 \tag{73}$$

$$\varphi(y,t) = \frac{B}{\omega} \exp\left[-\sqrt{\frac{\omega}{2\upsilon}}(h + y)\right] \cos\left[\sqrt{\frac{\omega}{2\upsilon}}(h + y) - \omega t\right] \quad \text{for y} \leq 0 \tag{74}$$

Returning to the velocity u(y,t) and substituting value of the function φ(y,t) into equation (62) we will receive:

$$u(y,t) = \frac{B}{\omega} \exp\left[-\sqrt{\frac{\omega}{2\upsilon}}(h - y)\right] \cos\left[\sqrt{\frac{\omega}{2\upsilon}}(h - y) - \omega t\right] - \frac{B}{\omega} \cos \omega t \quad \text{for y} \geq 0 \tag{75}$$

$$u(y,t) = \frac{B}{\omega} \exp\left[-\sqrt{\frac{\omega}{2\upsilon}}(h+y)\right] \cos\left[\sqrt{\frac{\omega}{2\upsilon}}(h+y) - \omega t\right] - \frac{B}{\omega}\cos\omega t \quad \text{for } y \le 0 \tag{76}$$

After integration of the equation (75) we will determine deviations of the rate of flow, counted in seconds, from the nominal flow at the stock outflow from the pressure headbox:

$$Q = \int_{-h}^{h} u(y,t)\,dy = \int_{-h}^{h} \left\{ \frac{B}{\omega} \exp\left[-\sqrt{\frac{\omega}{2v}}(h-|y|)\right] \times \cos\left[\sqrt{\frac{\omega}{2v}}(h-|y|) - \omega t\right] - \frac{B}{\omega}\cos\omega t \right\} dy \tag{77}$$

Function u(y,t) is described as a parity function i.e. u(y,t) = u(-y, t)
Thus equation (72) can be written in the form:

$$\int_{-h}^{h} u(y,t)\,dy = 2\int_{0}^{h} u(y,t)\,dy \tag{78}$$

Substituting cosines of the sum in the equation (78) and taking into account parity of the function u(y,t) we will receive:

$$Q = 2\frac{B}{\omega}\left\{\sqrt{\frac{v}{2\omega}}\cos\omega t\left[1 - \cos\sqrt{\frac{\omega}{2v}}h + \sin\sqrt{\frac{\omega}{2v}}h\right] \times e^{-\sqrt{\frac{\omega}{2\upsilon}}h} + \right.$$
$$\left. + \sqrt{\frac{\upsilon}{2\omega}}\sin\omega t\left[1 - \cos\sqrt{\frac{\omega}{2v}}h - \sin\sqrt{\frac{\omega}{2v}}h\right] \times e^{-\sqrt{\frac{\omega}{2\upsilon}}h} - h\cos\omega t\right\} \tag{79}$$

Returning to equation (61) and describing:

$$\frac{\partial p}{\partial x} = \frac{\Delta p}{l} \tag{80}$$

where:

Δp – pressure drop over a unit length l.

we can substitute equation (75) in relationship (56):

$$\frac{\Delta p}{l} = -\ \rho\beta \sin \omega t \tag{81}$$

Let us proceed from variability in the flow rate over time to variability in the flow rate of the fibrous suspension, just at the outflow from the slice of the pressure headbox on the wire, over a length of the wire and assume that the wire moves with a constant speed v, then

$$t = \frac{x}{v} \tag{82}$$

where:

v – wire speed,

x – coordinate along direction of the wire movement.

The equation (79) may be written in the form

$$Q(t) = Q\left(\frac{x}{v}\right) = Q_m(x) = 2\frac{B}{\omega}\left\{\sqrt{\frac{v}{2\omega}}\cos\omega\left(\frac{x}{v}\right)\left[1 - \cos\sqrt{\frac{\omega}{2v}}h + \right.\right.$$

$$\left. + \sin\sqrt{\frac{\omega}{2v}}h\right] \times e^{-\sqrt{\frac{\omega}{2\upsilon}}h} + \sqrt{\frac{\upsilon}{2\omega}}\sin\omega\left(\frac{x}{v}\right) \times \left[1 - \cos\sqrt{\frac{\omega}{2v}}h - \sin\sqrt{\frac{\omega}{2v}}h\right] \times$$

$$\left. \times\ e^{-\sqrt{\frac{\omega}{2\upsilon}}h} - h\cos\omega\left(\frac{x}{\mathrm{v}}\right)\right\} \tag{83}$$

In case of the variability on a low, molecular scale, the value $\sqrt{\frac{\omega}{2v}}h$ is small and, for practical considerations, can be neglected.

Then the relationship (83) will take a form:

$$Q(t) = 2\frac{B}{\omega}h\cos\omega\left(\frac{x}{v}\right) \tag{84}$$

To transit from the intensity of flow over time, provided by equation (84), to variability in a mass distribution in 1 m^2 of the paper web we will use:

$$\Delta q = Q(t) bc\rho \tag{85}$$

where:

Δq – deviation of the weight in 1 m^2 of the paper web,

b – slice width in the pressure headbox,

c – concentration of the fiber suspension in the pressure headbox,

ρ – density.

The obtained equation (85) may be used for the practical considerations about dependence of the amplitude of pressure pulsation on an equivalent variability of the basic weight in 1 m^2 of the paper web, because intensity of the stock flow rate over time is proportional to the amplitude of pressure pulsation.

This equation makes it possible to determine an acceptable degree of the pressure pulsation in the paper stock stream, and to develop the most effective means and methods for damping those pulsations.

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Analiza pulsacji ciśnienia masy doprowadzanej do wlewu maszyny papierniczej

Streszczenie

W pracy przedstawiono charakterystykę pulsacji ciśnienia masy papierniczej przepływającej w układzie jej doprowadzania do wlewu maszyny papierniczej i samym wlewie. Opisano teoretyczny model przepływów pulsacyjnych masy w rurociągach i podano warunki jakie musi spełniać układ rurociągów, aby uniknąć powstania falowego rezonansu. Opracowano zależności określające wpływ pulsacji ciśnienia masy na jej rozkład na powierzchni wstęgi papieru formowanej na sicie maszyny papierniczej. Analiza tych zależności umożliwia właściwy wybór środków tłumienia pulsacji ciśnienia masy przed jej dostarczeniem na sito maszyny papierniczej.