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OPTIMAL DESIGN OF A GROUND-BASED HORIZONTAL CYLINDRICAL TANK WITH ELLIPSOIDAL HEADS

The work is devoted to a horizontal tank composed of cylindrical shell closed with ellipsoidal heads and supported at the ends. The tank is loaded with internal or external pressure. For the first load case, a strength condition was formulated, for the other one – the condition of stability of the structure. An optimization model was formulated, in which the mass of the tank subject to the strength and stability conditions was assumed as an objective function. Optimal proportions of geometric dimensions for a family of the tanks of various capacities provided with heads of various convexities were determined. The results were presented in the form of plots. A function was proposed that approximated the solution and could be useful for purposes of designing of the tanks.

1. Introduction

The designed structures must be safe and reasonable. In the case of the tanks, the first requirement leads, first of all, to observance of the strength and stability conditions. The second one is usually identified with optimal design aiming at achieving a minimal mass. Spence and Tooth [17] discussed fundamentals of designing, first of all the strength and stability problems of pressure vessels. Magnucki [8] laid emphasis on optimization problems of pressure vessels under strength and stability constraints. Życzkowski [19], [20] presented a comprehensive review of the problems related to optimal design of structures. Błachut and Eschenauer [1] specified emerging methods

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for multidisciplinary optimization with special attention paid to designing procedures. Krużelecki and Trzeciak [6], Bochenek and Krużelecki [2] optimized axially symmetric shells, Magnucki and Monczak [12], Magnucki and Lewiński [9], [10], Magnucki, Szyc and Lewiński [15] determined effective thickness and optimal shapes of ellipsoidal heads of pressure vessels. Apart from choosing an appropriate head shape an important problem in the design of ground-based horizontal cylindrical tanks is related to shaping and location of the supports. Magnucki, Stasiewicz and Szyc [14] numerically studied the effect of shaping and location of supports on the strength of a ground-based horizontal cylindrical tank. The strength of such a tank is not a sole problem, another one arises from its stability. Kacperski [5] carried out experimental examination of buckling of thin-walled ground- based horizontal cylindrical tank filled with a liquid. Theoretical research related to buckling of a bending cylindrical shell is much more comprehensive. These problems were discussed by Chan, Tooth and Spence [3] and Huang, Redekop and Xu [4]. Magnucki and Stasiewicz [13] determined critical sizes of ground-based and underground horizontal cylindrical tanks. The problems of optimal shaping of ellipsoidal heads and horizontal cylindrical tanks were presented by Magnucki et al. [7] and Stasiewicz [18].

2. Statement of the optimization problem

Horizontal tanks of circular cylindrical form (Fig. 1) are often used in food and chemical industry for purposes of transportation and storing of liquids. The first of these applications is related to the need of consideration of bigger number of loadcases (longitudinal and transverse loads) and geometric constraints. In the second case, respective tanks are usually of large capacities with the load including hydrostatic pressure of the stored liquid and additional uniformly distributed pressure. Capacity of the tank depends on the relationship between its length, diameter, and head convexities. A question arises on such a choice of geometric parameters of a tank of given capacity as to ensure its minimal mass. In any case, the strength and stability conditions must be met.

The present analysis is related to ground-based horizontal cylindrical tanks supported at both ends, with ellipsoidal heads. Two loadcases are considered:

- total internal pressure $p^{(1)} = p_h^{(\mathrm{int})} + p_0^{(\mathrm{int})}$ – the strength problem,
- total special pressure $p^{(2)} = p_h^{(\mathrm{int})} - p_0^{(ext)}$ – the stability problem,

where: $p_h^{(\mathrm{int})} = \gamma_m a(1 - \cos\varphi)$ – internal hydrostatic pressure,

$p_0^{(\mathrm{int})}$ – internal uniform pressure – the gas pressure of liquid,

$p_0^{(ext)}$ – external additional small pressure – due to a suction pump,

γ_m – specific weight of the medium, a – radius of cylindrical shell,

φ – circumferential coordinate, for $\varphi = 0$ – the upper element of the cylindrical shell, for $\varphi = \pi/2$ – the lower element.

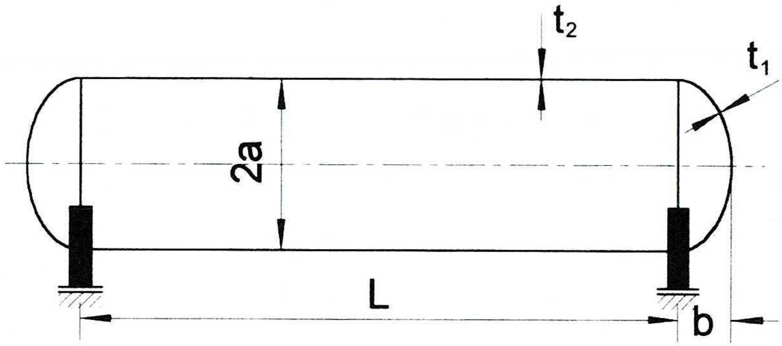

Fig. 1. A horizontal cylindrical tank

Capacity of the tank

$$V_0 = 2V_1 + V_2, \tag{1}$$

where: $V_1 = \frac{2}{3}\pi a^3 \beta$ – capacity of typical ellipsoidal head,

$V_2 = \pi a^3 \lambda$ – capacity of cylindrical shell,

$\beta = \frac{b}{a}$ – relative convexity of the head,

$\lambda = \frac{L}{a}$ – relative length of the cylindrical shell.

From which

$$\lambda = \frac{V_0}{\pi a^3} - \frac{4}{3}\beta. \tag{2}$$

The mass of the tank

$$m_s = 2m_1 + m_2, \tag{3}$$

where: $m_1 = \pi \rho_s a^2 t_1 \cdot c_m$ – mass of the ellipsoidal head,

$c_m = 1 + \beta^2 \frac{\arctan\sqrt{\beta^2 - 1}}{\sqrt{\beta^2 - 1}}$,

$m_2 = 2\pi \rho_s a^2 t_2 \lambda$ – mass of the cylindrical shell, ρ_s – mass density, t_1 – thickness of ellipsoidal head, t_2 – thickness of cylindrical shell.

The optimization criterion was assumed in the form of minimal mass of the tank

$$\min_a \{m_s\}, \tag{5}$$

where the mass (3) – the objective function

$$m_s = 2\pi\rho_s a^2 t_2 (c_m \cdot c_t + \lambda). \tag{6}$$

Coefficients c_m and c_t depend on the head convexity β, the a radius and thickness of the cylindrical part t_2. Taking into account the relationship (2), one can show that for a definite tank capacity V_0 its mass is a function of basic geometric values: i.e. the radius a, relative convexity of the head β and effective thickness t_2. Value of the thickness is determined from the strength and stability conditions.

The tank design belongs to the space of allowable solutions only provided that it meets its basic restrictions. The strength problem of horizontal cylindrical tanks is discussed in the monograph [17]. The strength condition expressed in terms of thickness of cylindrical shell of the tank has a form

$$t_{2\min}^{(1)} \le t_2, \tag{7}$$

where $t_{2\min}^{(1)} = \dfrac{\gamma_m a^2}{4(\sigma_{all} - p_0)} \sqrt{48\left(1 + \dfrac{\alpha_{SC}}{2}\dfrac{p_0}{\gamma_m a}\right)^2 + C_c\left(\dfrac{1}{2}\lambda^2 - 1\right)^2}$ – minimal thickness of the cylindrical shell filled with medium-liquid and supported at both ends [8],

$C_c = \dfrac{6}{1 + \psi^2}$ – correction coefficient – the effect of saddle support on stress state,

$\psi = \dfrac{o_0}{100\,\gamma_m a}$ – dimensionless parameter of uniform internal pressure,

γ_m – specific weight of the medium, σ_{all} – allowable stresses,

α_{SC} – stress concentration factor in the joint of ellipsoidal and cylindrical shells.

Stability problem of a horizontal cylindrical tank filled with liquid is discussed in the [3], [5], [13]. Stability condition expressed in terms of thickness of cylindrical shell of a horizontal tank has a form

$$t_{2\min}^{(2)} \le t_2, \tag{8}$$

where $t^{(2)}_{2\min} = 2.93 \dfrac{a}{1000} \lambda^{0.401}$ – minimal thickness of cylindrical shell for additional uniform external pressure (due to a suction pump, Ref. [30]) $p^{(ext)}_{0} = 0.1$ MPa.

Substituting minimal thickness of the cylindrical shell (7) or (8) into the objective function (6) enables determining of the space (set) of allowable solutions for the horizontal cylindrical tank (Fig.2). Minimal value of the objective function (6) is located at the edge of the set of allowable solutions, in the M_1 point. Location of the point is determined, at the same time, by active strength (7) and stability (8) conditions. The solution is found for two load-cases. On the other hand, for a single load-case, when only total internal pressure $p^{(1)} = p^{(int)}_{h} + p^{(int)}_{0}$ is present, the regard to the sole strength condition (7) is sufficient. Minimal value of the objective function (6) is also located at the M_2 point (Fig.2), but corresponding to an extreme value, i.e. minimum of the objective function. The problem was presented by Magnucki [4] and Stasiewicz [18]. These works did not take into account the stability condition in the form (8) that was considered here based on Ref. [13].

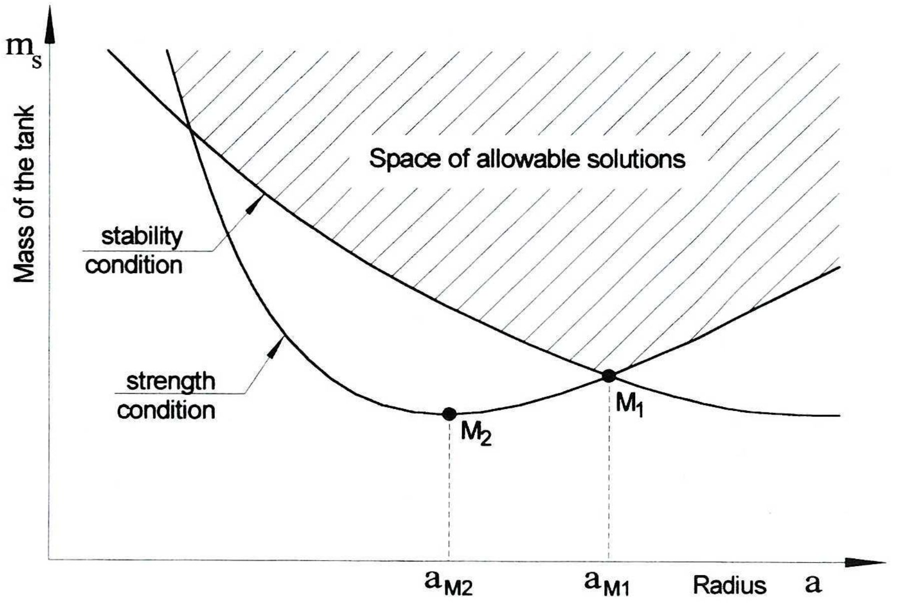

Fig. 2. The space of allowable solutions

3. Numerical analysis

Numerical analysis was carried out for the family of steel tanks filled with water of the capacities in the range $50\ m^3 \leq V_0 \leq 300\ m^3$, for allowable stresses

$\sigma_{all} = 330$ MPa, specific weight of water $\gamma_m = 9.81$ kN/m^3, uniform internal pressure – $p_0^{(int)} = p_0 = 2.5$ MPa, and additional uniform external pressure $p_0^{(ext)} = 0.1$ MPa. The value of stress concentration factor in the joint of ellipsoidal and cylindrical shells, based on Ref. [10], [11], was assumed as a linear function of the head convexity β

$$\alpha_{SC} = -0.17656\beta + 1.1994.$$

For the cases subject to the analysis, the c_t (equation 4) may be written in the form:

$$c_t = \begin{cases} 1.427(1 + 0.109\alpha - 0.024\alpha^2), & \text{for } \beta = 0.5 \\ 1.0036, & \text{for } \beta = 0.6 \\ 0.7744, & \text{for } \beta = 0.7 \\ 0.674996, & \text{for } \beta = 0.8,\ 0.9,\ 1.0 \end{cases},$$

where $\alpha = \dfrac{a}{100t_2}$.

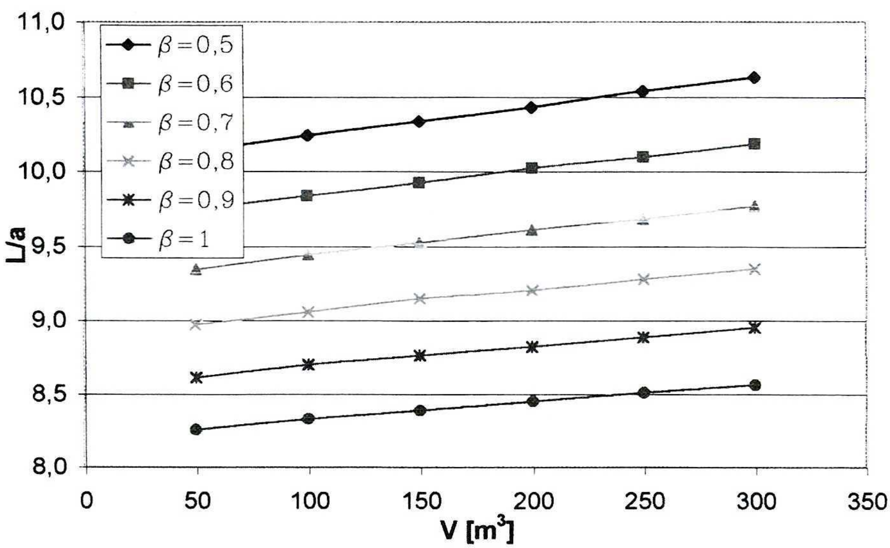

Fig. 3. Relative length as a function of the tank capacity

Optimal proportions i.e. (L/a) – the M_1 – point – have been determined for the vessels of different capacities. Results of the analysis are shown in Fig. 3. It should be assumed that the proportions of optimal dimensions linearly depend on the V_0 capacity. Figure 4 shows a plot of the objective function (6) corresponding to optimal (L/a) proportions, according to head convexity β in the vessels of various capacities. In the case of the assumed model, the vessels of hemispherical heads have minimal masses, irrespective of their capacities.

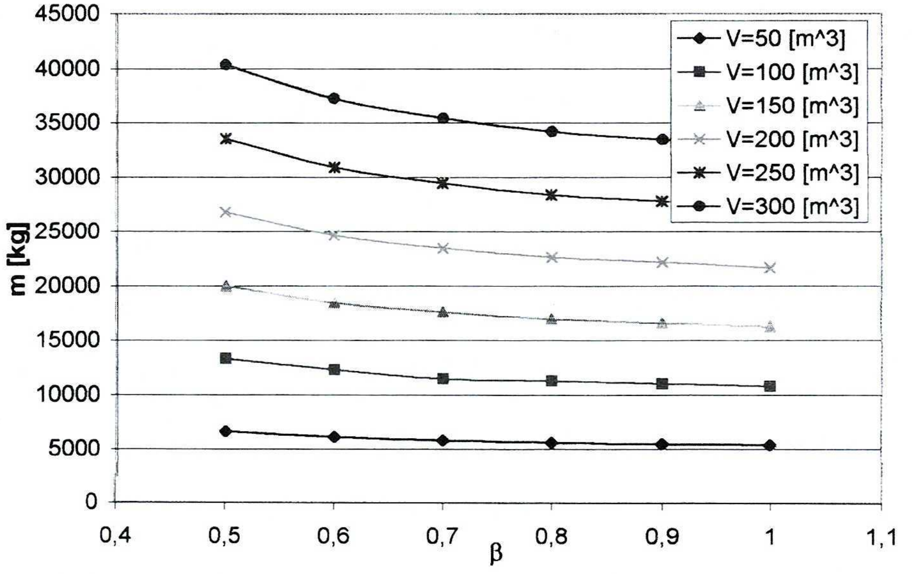

Fig. 4. Mass of the tank as a function of the relative convexity of the head

4. Conclusion

Optimal proportions (L/a) depend on the capacity V_0 and relative convexity of the head β. From the practical point of view, the use of a single function of two variables is more convenient than application of a set of diagrams (Fig. 3). Optimal proportions may be approximately defined as follows

$$\left(\frac{L}{a}\right)_{opt} = (-0.00153 \cdot \beta + 0.00273)\,\bar{V}_0 - 3.6747 \cdot \beta + 11.862,$$

where: $\bar{V}_0 = \dfrac{V_0}{1\,\mathrm{m}^3}$, $V_0\,[\mathrm{m}^3]$ – vessel capacity.

The above relationship may be used for design purposes of horizontal cylindrical vessels.

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Optymalna konstrukcja poziomego zbiornika walcowego z dnami elipsoidalnymi

Streszczenie

Poziomy zbiornik zbudowany jest z walcowej powłoki zamkniętej elipsoidalnymi dnami i podparty na obu końcach. Uwzględnione są dwa przypadki obciążenia zbiornika: ciśnieniem wewnętrznym lub zewnętrznym. Dla pierwszego przypadku sformułowano warunek wytrzymałości, dla drugiego natomiast – warunek stateczności. Sformułowano również model matematyczny optymalizacji, w którym jako funkcję celu przyjęto masę zbiornika. Obszar rozwiązań dopuszczalnych ograniczają warunki wytrzymałości i stateczności. W badaniach numerycznych wyznaczono optymalne proporcje geometrycznych wymiarów rodziny zbiorników o różnych pojemnościach, wyposażonych w dna o różnych głębokościach. Wyniki przedstawiono w postaci wykresów. Zaproponowano funkcję przybliżającą rozwiązanie, która może być przydatna w praktycznym projektowaniu takich zbiorników.