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Research paper

New method for comparing of particle-size distribution curves

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Abstract: The article discusses a new mathematical method for comparing the consistency of two particle size distribution curves. The proposed method was based on the concept of the distance between two graining curves. In order to investigate whether the distances between the particle size distribution curves are statistically significant, it was proposed to use the statistical test modulus-chi. As an example, the compliance of three sieve curves taken from the earth dam in Pieczyska on the Brda River in Poland was examined. In this way, it was established from which point of the dam the soil was washed away. However, it should be remembered that the size of the soil grains built into the dam does not have to be identical to the grain size of the washed out soil, because the fine fractions will be washed away first, while the larger ones may remain in the body of the earth structure.

Keywords: particle-size distribution curve, dam, chi-modulus test, uncertainty of sieves curve

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1. Introduction

The problem associated with comparing particle-size distribution curves emerged as a result of analysing samples collected from soil scoured from the body of the Pieczyska dam at the downstream site in 2020. In 2016, soil was similarly scoured from a downstream part of the dam. Thus, a question emerged of whether the scoured soil originated from the same spot. We have particle-size distribution curves for soil scoured in 2020 and 2016 as well as archive particle-size distribution curves for samples collected from the dam embankment in 1974. In order to investigate where the soil was scoured from, a methodology had to be constructed based on a known particle-size distribution curve. This would be used to confirm whether or not there are statistical differences between the soils, assuming that the particle-size distribution of the scoured material is similar to the original material, which does not necessarily have to be true.

There are many methods for testing grain-size composition [1] and these are continually being developed. The methods include: sieving, decanting, laser diffraction or digital image analysis. Specialist software for investigating particle-size distributions is also being developed [2,3]. The impact of two different sieve mesh description methods on the sieving process was investigated by Alkhaldi et al. [1]. It was demonstrated that the digital image analysis method shows good correlation with sieve analysis [5]. Particle-size distribution curves can be represented by mathematical functions [6]. However, the method for comparing two grading curves has not been considered in terms of there being no statistical difference for soils collected from the same site. Analysis of variance has been used to compare various methods for determining grain-size composition [7]. However, that methods requires the mass accumulated at given sieves to be known, which means that it cannot be applied to archive data where only the grading curve is known.

This article is a presentation of the developed original method for comparing grading curves.

2. A testing method for statistical difference between two grading curves: suggested general principles

In statistics, there exist tests to check whether there is no statistical difference between functions or curves based on the concept of distance. For example, the Kolmogorov test, which is based on the maximum distance between two probability distributions, can be used to test whether there is no statistical difference between a theoretical and an empirical probability distribution. The chi-squared test [8] or the chi-modulus test [9] can be used to compare two curves at a larger number of points. The chi-modulus test is based on the $|\chi|$ statistic.

$$|\chi| = \sum_{i=1}^{n} \frac{|x_i - y_i|}{\sigma(x, y)} \tag{2.1}$$

$$\sigma(x, y) = \sqrt{\sigma^2(x) + \sigma^2(y)} \tag{2.2}$$

where: x_i – value on first curve, y_i – value on second curve, σ – standard deviation for x and y variables, n – number of points at which the curves are compared.

The numerator represents the distance between the curves at the ith point. The denominator normalises the statistic. It represents the minimum distance at which we consider two points to be different. If x and y have been calculated on the basis of measurements, then standard deviation σ is replaced with a standard measurement uncertainty $u(x, y)$ and subsequently Eq. (2.1) and Eq. (2.2) assume the following forms:

$$|\chi| = \sum_{i=1}^{n} \frac{|x_i - y_i|}{u(x, y)} \tag{2.3}$$

$$u(x, y) = \sqrt{u^2(x) + u^2(y)} \tag{2.4}$$

If measurement uncertainties for determining both grading curves can be considered identical, then:

$$u(x) = u(y) = u$$

and Eq. (2.4) can be simplified to

$$u(x, y) = \sqrt{u^2 + u^2} = \sqrt{2} \cdot u. \tag{2.5}$$

Therefore, in order to facilitate a statistical comparison of two particle-size distribution curves, it is necessary to calculate the measurement uncertainty for soil particle percentages.

3. Calculating the measurement uncertainty for soil particle percentages

As a result of the sieve analysis, we have n soil samples from particular sieves. By m_i we denoted the soil mass on sieve i. Let us assume that every mass measurement is performed subject to the same measurement uncertainty $u(m)$ stemming from the class of the measuring instrument. The total soil sample mass S is:

$$S = \sum_{i=1}^{n} m_i$$

If total mass is calculated as the sum of masses accumulated on the sieves, then the standard measurement uncertainty $u(S)$ is:

$$u(S) = \sqrt{\sum_{i=1}^{n} [u(m)]^2} = \sqrt{n} \cdot u(m) \tag{3.1}$$

If all the soil is weighed at the beginning and the mass agrees with the sum of the masses accumulated on sieves, then:

$$u(S) = u(m) \tag{3.2}$$

The percentage content Z_i of the $i-$th sieve is:

$$Z_i = \frac{m_i}{S} \cdot 100\% \tag{3.3}$$

The measurement uncertainty for Z_i should be calculated according to the following formula:

$$u_m\left(Z_i\right) = \sqrt{\left(\frac{\partial Z_i}{\partial m_i}\right)^2 [u(m)]^2 + \left(\frac{\partial Z_i}{\partial S}\right)^2 [u(S)]^2} \tag{3.4}$$

Substituting Eq. (3.3) into the Eq. (3.5), we obtain:

$$u_m(Z_i) = \sqrt{\left(\frac{100\%}{S}\right)^2 [u(m)]^2 + \left(\frac{-m_i \cdot 100\%}{S^2}\right)^2 [u(S)]^2}$$

Now we use the Eq. (3.1):

$$u_m(Z_i) = \sqrt{\left(\frac{100\%}{S}\right)^2 [u(m)]^2 + \left(\frac{-m_i \cdot 100\%}{S^2}\right)^2 \cdot n \cdot [u(m)]^2}$$

$$u_m(Z_i) = \frac{100\%}{S} \cdot u(m) \cdot \sqrt{1 + n \cdot \left(\frac{m_i}{S}\right)^2}$$

Next, we substitute the Eq. (3.3) to get:

$$u_m(Z_i) = \frac{100\%}{S} \cdot u(m) \cdot \sqrt{1 + n \cdot (Z_i)^2} \tag{3.5}$$

If we assume that all of the soil tested has been weighed before the sieve analysis and there has been no soil loss during the test procedure, then we substitute Eq. (3.2) into the Eq. (3.4) and we obtain a somewhat simplified version of the Eq. (3.5). It becomes:

$$u_m(Z_i) = \frac{100\%}{S} \cdot u(m) \cdot \sqrt{1 + (Z_i)^2} \tag{3.6}$$

According to the Eq. (3.6) the measurement uncertainty for soil content percentage $u(Z_i)$ of a given grain size depends on the mass measurement uncertainty $u(m)$ or, in other words, on the precision class of the scale used, the percentage of the soil mass accumulated on sieve Z_i and the total mass of the soil tested.

For typical parameters – sample mass of 0.1÷0.5 kg and scale uncertainty measurement for this range of 0.1 g – the uncertainty for calculating the percentage content $u_m(Z_i)$ does not exceed 0.14% (Table 1). For a scale uncertainty measurement of 0.5 g, the uncertainty $u_m(Z_i)$ does not exceed 0.71% (Table 2). If the scale is very precise and its measurement uncertainty does not exceed 0.05 g, then in practice the uncertainty $u_m(Z_i)$ can be omitted. In practice, for small and medium soil samples, laboratories use scales with a measurement accuracy of 0.01 g and an uncertainty of 0.1 g. High accuracy of scale measurement results of residue on given sieves is achievable as long as certain principles are observed pertaining both to prepping the equipment (such as thorough cleaning of sieves after every use and precise taring of containers) and the measurement itself (e.g. repeating the gradation test at least twice, multiple weighing, and weighing the total soil sample after sieving). Applying the principles set forth hereinabove makes it possible to achieve measurement accuracy in the order of a dozen or so hundredth of a % for the percentage content of given fractions for two gradings of the same soil (10).

Table 1. The calculated measurement uncertainty for the percentage content subject to total sample weight between 0.1 and 0.5 kg and assuming scale accuracy of 0.1 g

		Soil mass percentage content									
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Sample mass [g]	100	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.13	0.14	0.14
	200	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07
	300	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05
	400	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04
	500	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03

Table 2. The calculated measurement uncertainty for the percentage content subject to total sample weight between 0.1 and 0.5 kg and assuming scale accuracy of 0.5 g

		Soil mass percentage content									
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Sample mass [g]	100	0.52	0.55	0.57	0.59	0.61	0.63	0.65	0.67	0.69	0.71
	200	0.26	0.27	0.29	0.30	0.31	0.32	0.33	0.34	0.34	0.35
	300	0.17	0.18	0.19	0.20	0.20	0.21	0.22	0.22	0.23	0.24
	400	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.17	0.18
	500	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.13	0.14	0.14

Tables 1 and 2 show that for engineering calculations, the adopted value of $u_m(Z_i)$ should be:

- 0.1÷0.14 for a 100 g sample and scale accuracy of 0.1 g,
- 0.5÷0.7 for a 100 g sample and scale accuracy of 0.5 g.

The total uncertainty for determining the percentage content using grading curve $u(Z_i)$ comprises: the uncertainty for calculating the percentage content $u_m(Z_i)$ (Eqs. (3.5) and (3.6)), the uncertainty stemming from an inaccurate, often hand-plotted grading curve $u_p(Z_i)$, and the uncertainty stemming from inaccuracies in determining the soil content percentage using the grading curve $u_r(Z_i)$.

$$u(Z_i) = \sqrt{\left[u_p(Z_i)\right]^2 + \left[u_r(Z_i)\right]^2 + \left[u_m(Z_i)\right]^2} \tag{3.7}$$

The values $u_p(Z_i)$ and $u_r(Z_i)$ have to be assessed each time when comparing curves. It may be assumed that both are equal to k%. For example 2%; however, in some cases they may be higher than that. Thus, the Eq. (3.7) simplifies to:

$$u(Z_i) = \sqrt{k^2 + k^2 + [u_m(Z_i)]^2} = \sqrt{2k^2 + [u_m(Z_i)]^2} \tag{3.8}$$

4. A method for comparing grading curves

Grading curves are compared at m selected points. Essentially any selection of points is feasible, however they should be distributed quite evenly and each fraction should be taken into account: clay, silt, sand, gravel and cobble. The percentage content x_i and y_i on the first and second curve is then determined for each selected grading curve point and for a given soil particle diameter. If Eqs. (2.3), (2.5) and (3.8) are used to test curves for no statistical differences, then:

$$|\chi| = \sum_{i=1}^{m} \frac{1}{\sqrt{2}} \left| \frac{x_i - y_i}{u(Z_i)} \right| = \frac{1}{\sqrt{4k^2 + 2\,[u_m(Z_i)]^2}} \sum_{i=1}^{m} |x_i - y_i| \tag{4.1}$$

5. Calculations

A sample was taken from material washed-out from the downstream site of the earth dam and then a 2020 grading curve was determined (Fig. 1). That grading curve was compared with the 2016 grading curve and the archive grading curve drawn up in 1974. The latter represents typical soil used to build the dam. The grading curves were compared using the method as described above, adopting a comparison of percentage contents for fractions of 14 diameters (Table 3). Percentage soil content values for diameter d were found on appropriate grading curves. The following were adopted for the calculations: $k = 2$, $u_m(Z_i) = 0.5$. The chi-modulus test (Eq. (4.1)) was used to test whether there are no statistical differences between the curves.

Two statistical hypotheses were tested. Hypothesis H_1: there is no statistical difference between the 2020 grading curve and the 2016 grading curve, and hypothesis H_2: there is no statistical difference between the 2020 grading curve and the 1974 grading curve. An $\alpha = 0.05$ significance level was adopted. Number of degrees of freedom (NDF) = 13. This represents the number of points at which the percentage soil content is tested less one. The critical chi-modulus test value for $\alpha = 0.05$ and NDF = 13 is $|\chi|_{\text{crit}} = 14.12$ (Opyrchał, 1999). As the statistic $|\chi|$ calculated for a comparison of the 2020–2016 and 2016–1974 grading curves is more than the test critical value $17.7 > 14.12$ (for curves 2020–2016) and $22.9 > 14.2$ (for curves 2016–1974), the H_1 statistical hypothesis should be rejected. Whereas for hypothesis H_2 the statistic $|\chi|$ calculated for a comparison of the 2020–1974 grading curves is less than the test critical value $5.4 < 14.12$. Therefore there is no basis for rejecting the no statistical difference hypothesis for the 2020 and 1974 grading curves.

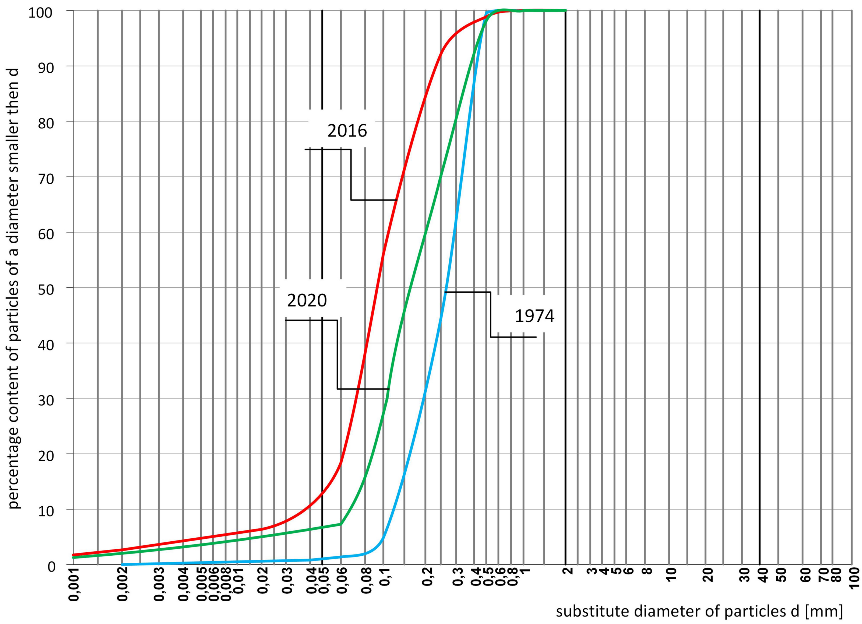

Fig. 1. Particle-size distribution curves

Table 3. Diameters and fraction percentage content values for given diameters d. f_{1974} – percentage content of fractions in the 1974 sample. f_{2016} – percentage content of fractions in the 2016 sample. f_{2020} – percentage content of fractions in the 2020 sample. $|\chi|$ – the calculated chi-modulus statistic

fractions	d [mm]	f_{2020}	f_{2016}	f_{1974}	$\|f_{2020}-f_{2016}\|$	$\|f_{2020}-f_{1974}\|$	$\|f_{2016}-f_{1974}\|$
clay	0.001	0	2	0	2	0	2
	0.002	0	3	0	3	0	3
silt	0.005	0	4	0	4	0	4
	0.01	0	5	0	5	0	5
	0.05	3	13	2	10	1	11
sand	0.1	22	56	3	34	19	53
	0.3	79	92	78	13	1	14
	0.5	98	99	98	1	1	1
	1.0	100	100	100	0	0	0
				$\|\chi\|$	17.7	5.4	22,9

6. Discussion

The presented new method for comparing grading curves is simple to apply. It is also suitable for use with archive data where the masses collected on given sieves are not known and variance analysis would not be feasible. All that one has to do is find the percentage content for selected fractions on the graph and adopt an uncertainty for that value.

A chi-squared test may be used in place of the chi-modulus test used for the distance between two grading curves, however it is less robust to outliers [8] than the suggested chi-modulus test.

Even though the suggested method may be used to compare soil samples collected in order to determine if they are the same, one has to remember that the particle-size distribution of the soil constituting the dam does not have to be identical to the washed-out soil, as the smaller fractions will be washed out in the first place, with larger fractions remaining in the dam body. On the other hand, the largest diameters for the original soil are similar to the largest diameters in the washed-out soil. This suggests that the method is valid.

However, the proposed method requires further verification using both, laboratory examinations and practical examinations performed on real hydraulic buildings. The confirmation of its effectiveness would provide a tool that would significantly facilitate dam renovations because archive particle-size distribution curves developed during dam construction are available for the majority of dams. In case of soil scouring, by comparing particle-size distribution curves of the scoured soil with archive particle-size distribution curves, it is possible to approximately define the region of origin of the soil. This is very important information because it limits the area of necessary geotechnical analyses which have to be performed in order to define the deconsolidation of the dam body, which reduces the renovation costs.

A certain drawback of the proposed method is the lack of the uncertainty of determining particle-size distribution curves. This remark refers not only to archive studies but also to those performed nowadays. An element which seems very important is ensuring that engineers pay more attention to uncertainty assessment because a measurement is not only a numerical value, but also a range in which, with assumed probability, the measured value is present.

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Nowa metoda porównywania krzywych uziarniena gruntu

Słowa kluczowe: krzywa uziarnienia gruntu, zapora, rozkład moduł-chi, niepewność krzywej uziarnienia

Streszczenie:

Problem porównywania krzywych uziarnienia powstałpo wypłukaniu na dolne stanowisko niewielkiej ilości gruntu z korpusu Zapory w Pieczyskach w 2020 r. Ponieważ w 2016 r. również został wypłukany z dolnej części zapory grunt, zadano sobie pytanie, czy na podstawie znajomości krzywej uziarnienia można stwierdzić, że wypłukany materiał pochodzi z tego samego miejsca. W celu zbadania zgodności krzywych uziarnienia zaproponowano metodę statystyczną opartą o koncepcję odległości pomiędzy tymi krzywymi. Za odległość pomiędzy krzywymi uziarnienia rozumiana jest suma wartości bezwzględnych procentowych zawartości masy przechodzącej przez dane sito podzielona przez odchylenie standardowe wyznaczenia zawartości procentowej (wzory (2.1)–(2.4)). Wzór (2.4) upraszcza się gdy procentowa zawartość masy jest obliczana z takim samym odchyleniem standardowym dla każdej krzywej (2.5). Odchylenie standardowe identyfikowane jest z niepewnością standardową wynikającą z pomiaru masy gruntu zebranego na sitach. Kolejne kroki obliczenia tej wartości podane są we wzorach (3.1)–(3.5). Wzór (3.6) to końcowy rezultat obliczenia niepewności standardowej. Zależy ona od pomiarowej niepewności masy oraz od samej masy zebranej na sicie. Tabela 1 oraz tabela 2 prezentują wartość niepewności obliczoną dla typowych mas pobieranych próbek gruntu (100–500 g) i procentowych zawartości gruntu na sicie, dla dokładności pomiarowej wagi 0,1 i 0,5 g. Następnie porównywano krzywe przesiewu próbek pobranych z wypłukanego gruntu w 2016 i 2020 r. oraz krzywej archiwalnej z 1974 r. reprezentującej typowy grunt korpusu zapory (rys. 1).

W celu porównani krzywych przesiewu przyjęto, że na całkowitą niepewność odczytania procentowej zawartości gruntu z krzywej przesiewu $u(Z_i)$ składa się: niepewność obliczenia zawartości procentowej $u_m(Z_i)$, niepewność wynikającą z niedokładności, wykreślenia krzywej przesiewu $u_p(Z_i)$, oraz niepewność wynikającą z niedokładności odczytu z krzywej przesiewu $u_r(Z_i)$. Uwzględniając te niepewności otrzymujemy końcowy wzór (4.1) służący do obliczenia miary odległości pomiędzy krzywymi przesiewu. Następnie testowana jest hipoteza, czy odległość pomiędzy krzywymi wyrażona parametrem $|\chi|$ może być wartością losową. Duża wartość parametru $|\chi|$ świadczyć będzie przeciwko hipotezie tożsamości krzywych, mała wartość parametru $|\chi|$ nie da podstaw do odrzucenia hipotezy równości. Obliczoną wartość parametru $|\chi|$ dla poszczególnych krzywych przesiewu

podano w tabeli 3. Następnie porównano ją z wielkością teoretyczną rozkładu moduł-chi dla poziomu istotności $\alpha = 0,05$, która wynosi $|\chi|_{\text{crit}} = 14.12$. Warunek $|\chi| < |\chi|_{\text{crit}}$ spełniają tylko krzywe z lat 2020 i 1974.

W pozostałych przypadkach hipotezę równości krzywych przesiewu należy odrzucić. Proponowana metoda wymaga dalszego potwierdzenia zarówno laboratoryjnego jak i badan polowych. Jeżeli jej skuteczność zostanie potwierdzona będzie stanowić użyteczne narzędzie do planowania zakresu remontu. Archiwalne krzywe przesiewu, pobrane w trakcie budowy zapory są znane. Stosując te metodę będzie można w przypadku wypłukania gruntu określić rejon zapory, skąd wypłukany grunt pochodzi, co znacznie ograniczy obszar niezbędnych badan geotechnicznych, które należy przeprowadzić w celu określenia stref rozgęszczenia.

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