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The characteristics of the kinematics of the post-mining dislocation process

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Abstract: The paper concerns the analysis of the kinematics of the dislocation process that affects surface points within the area of underground exploitation. The problem discussed in the paper is the estimation of the changes concerning spatial configuration of a body, forced by underground influence. Observations of the real process show that the trajectories indicating the dislocations of the medium points are irregular. The deterministic description of the examined process, as a rule, generates smooth trajectories of point dislocation. Therefore, as is natural, the analytical representation of the process cannot be approximated to measurement results with arbitrary accuracy. The entropy has been assumed as the measure of the randomness of a given process. It has been shown then that the entropy is not constant. Hence the description of the post-mining dislocation process has been presented as a stochastic model. The quantitative results of the description have been put to a statistical estimation.

Keywords: dislocation, entropy, kinematics, stochastic process, trajectories

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1. Introduction

Formal description of physical processes is usually given by differential equations that contain certain parameters, characterising physical properties of phenomena and the environment (Knothe, 1953; Litwiniszyn, 1956). Identification of a model, i.e. determination of its parameters is based on the results of measurements of a selected process attribute. Unavoidable errors of measurement and the "randomness" of discussed processes are the reason that those elements usually cannot be expressed by means of the function $f(.)$, but they are expressed as a family of functions $f_\omega(.)$. Therefore, description of the majority of real processes leads to replacement of the function $f(.)$ into the random function $f_\omega(.)$ (Bugiel and Piwowarski, 2003; Oksendal, 1998). The parameter ω is sometimes interpreted as an element of the probabilistic space. Similar reformulation frequently includes additional conditions. The procedure of replacement of coefficients in the equations that describe the discussed process with random elements concerns temporal evolution of a large number of material particles – the Hamilton equation. Therefore, for particular solutions usually a set of dynamic variables is introduced to the system of differential equations, which determine the state of the system at the moment t, when the state of the system at the moment $t = t_0$ is determined.

The subject of considerations is an attempt to specify dislocations of material points of a rock mass, as a result of forcing, which leads to deformations of its primary structure. Therefore the description of the non-stationary process of post-mining dislocations is analysed.

2. Characteristics of the deformation process

Bodies – in particular solid bodies – present the ability to resist deformations and damages. On the other hand, they are elastically and plastically deformed under the influence of external forces. Underground exploitation results in changes of the primary conditions of stresses within the surroundings of mining areas, and in induction of changes of the field of dislocations by rheological properties of the rock mass. In such situation post-exploitation voids are tightened, floor layers are often damaged what results in local cavings to the exploited space.

Rheological models of description of the rock mass dislocation field (Kisiel, 1973), although coherent from the formal point of view, have many limitations with respect to their utilisation. Projections based on differential equations (including rheological models) are significantly sensitive to modifications of assumed (idealised) additional conditions concerning the medium. Therefore, dislocations and deformations of the rock mass, caused by underground exploitation, are most frequently determined with the use of the Knothe geometric-and-integral theory (Knothe, 1953). It should be stressed, however, that geometric-and-integral theories do not explicitly consider the medium properties. On the other hand, those models exhibit the important feature (additiveness), what results in their natural use in cases of diversified mining-and-geological conditions.

In the process of consideration of deformations of a body, the surface of deformation is determined. It is a geometric object that, for each point of the deformed body, allows for determination of a relative elongation ε of an elementary linear object surrounding the given point. In general, the deformation surface $\mathbf{P}_0$ is the 2[nd] order surface described by the formula:

$$\mathbf{P}_0 = \sum_{i=1}^{3} \varepsilon_{i,j} x_i^2 + 2 \sum_{i=1}^{3} \varepsilon_{i,j+1} x_i \cdot x_{i+1}$$

where $\varepsilon_{i,j}$ – deformations in principal directions.

In order to describe the level of deformation of an element of a body, dislocations of points of a certain subspace should be determined. In general, the vector of dislocations, which is the continuous function of co-ordinates (Farin, 1993), is described by the relation (1)

$$\bar{u} = \varphi(u_i, \gamma_j)\Big|_{i=1}^{3} \tag{1}$$

where u_i – components of point dislocations,
γ_i – model parameters.

The condition of continuity of a body imposes limitations on the components of small deformations, what results from the equations of deformations inseparability (Saint-Venant equations). Those equations allow for the determination of the components of dislocations u_i ($i = 1, 2, 3$) when the body occupies the uni-coherent area. In the case of a multi-coherent area additional conditions must be met for each of the cross-sections, which are by convention traced through a given body, in order to obtain the uni-coherent area in vicinity of the conventional cross-section. The rock mass is not a uni-coherent area, what – in formal description – usually leads to singularities. It becomes another issue, which is not the subject of analysis presented in this publication.

3. Trajectories of the dislocation process

Discussion will consider the limit approximation of the results of description of a transient field of dislocations, on the basis of applied theories of rock mass movements, with respect to the results of measurements. It is not assumed that the limit approximation of the representation to the results of measurement is coherent; it is only expected that the appropriate measure of approximation does not exceed specified permissible values. Definition of another formula of projection of the analysed process will be an implication of violation of the permissible measure of approximation.

The projection (1) requires that operators $[\varphi]$ are defined and that parameters $\gamma_i\big|_{i=1}^{3}$ are specified. Identification of γ_i is performed on the basis of measurements of the process trajectories. Let the equation (2) describe dislocation of a point in time. Let us then analyse the solution of the equation (2).

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} = \sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2}; \quad t > 0, \quad x \in R^n \\ &u(0, x) = f(x) \end{aligned}\right\} \tag{2}$$

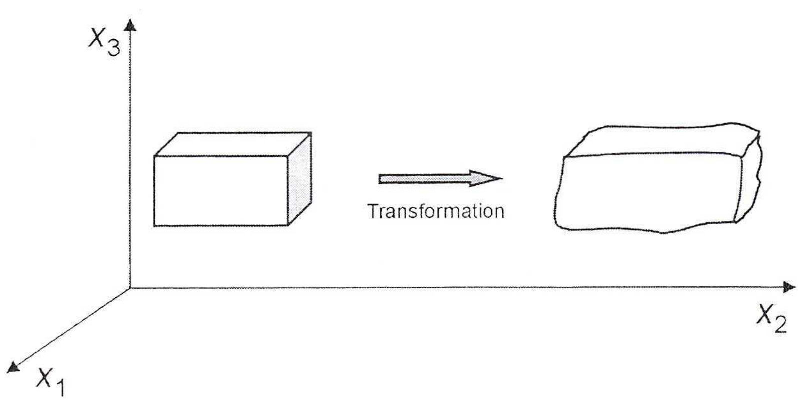

Fig. 1. Evolution of medium configuration within three-dimensional space

The function u: $[0, \infty]\times R^n \to R$ is called the classical solution of the problem (2) (Rudin, 1970), if it fulfils the following conditions:

1) u is continuous in a closed set $[0, \infty]\times R^n$;
2) u has continuous derivatives, i.e. $\exists\ u_t, u_{x_i}, u_{x_i}x_j$; $(i, j = 1, ..., n)$ and fulfils the equation (2) in the set $(0, \infty)\times R^n$;
3) $u < \infty$ and it fulfils the initial condition $u(0, x) = f(x)$; u is limited.

The theory of differential equations proves that $\forall f: R \to R$ continuous and limited, the problem (2) has exactly one solution and for $t > 0$ $x \in R^n$ we obtain

$$u(t,x) = \int_{R^n} \Gamma(t, x - y) f(y)\, dy \quad \text{for} \quad t > 0, \quad x \in R^n \tag{3}$$

where $x_i = R^n$ is the standard, n-dimensional Lebesque measure, and

$$\Gamma(t,x) = \frac{1}{(4\pi t)^{\frac{n}{2}}} \exp\left(-\frac{1}{4t}|x|^2\right) \tag{4}$$

It may be proved that the solution of the equation (2) in the form (3) exists and that it is the unique solution. If $f: R^n \to R$ is continuous and limited, then the classical solution exists, as it was stated earlier.

The differential operator in the equation (2) can be expressed as follows:

$$L = \frac{\partial}{\partial t} - \sum_{i=1}^{n} \frac{\partial^2}{(\partial x_i)^2}$$

After differentiation and $\forall c \in R^n$ the identity occurs:

$$L(\Gamma(t, x - c)) = 0 \quad \text{for} \quad t > 0 \quad x \in R^n \tag{5}$$

Then, continuing the formal differentiation of (3) we get

$$Lu(t,x) = \int_{R^n} L\Gamma(t, x - y)\, f(y) dy \quad \text{for} \quad t > 0, \quad x \in R^n \tag{6}$$

Differentiation is performed with respect to time t and two times with respect to spatial variables x_i. After those operations are completed, all components have the same form:

$$\frac{Q(x,y)}{t^{\alpha}} \exp\left\{-\frac{|x - y|^2}{4t}\right\} \tag{7}$$

where Q – a polynomial of the maximum second order, α – a constant >0.

Since the kernel $\Gamma(t, x - y)$ has the form (7), the function $u(t,x)$ is continuous for $t > 0$, $x \in R^n$. It is important that the function $u(t,x)$ fulfils the initial condition: $u(0,x) = f(x)$.

Besides, the integral over R^n aims to zero for $t \to 0$. This proves that: $\lim\limits_{t \to 0^+} u(t,x) = f(x)$ with respect to x. Therefore, from the continuity of f results the continuity of the solution in points $(0, x)$, $x \in R^n$. Moreover, from (3) it results that $|u(t,x)| \leq \sup|f|$ what implies the continuity of the solution of the equation (3).

Since trajectories of dislocations obtained from measurements of the process are characterised by deviations around the average value, it is reasonable to analyse entropies of the solutions of the equation (2). Let us consider the family of $\{p^t\}_{t \geq 0}$ operators that are defined as follows (Lasota, 2002):

$$\left.\begin{aligned} p^t f(x) &= \int_{R^n} \Gamma(t, x-y) f(y)\, dy \\ p^0 f(x) &= f(x) \end{aligned}\right\} \tag{8}$$

Operators $p^t : L^1(R^n) \to L^1(R^n)$ are double stochastic operators. For $f \in L^1(R^n)$ the following relation occurs:

$$u(t,.) = p^t f \quad t \geq 0 \tag{9}$$

Let us note, that if $f : R^n \to R$ is continuous and $f < \infty$ then the function $u(t,x) = p^t f(x)$ is a semi-group solution of the initial problem (2). For the equation (2) the strong relation between the classical and the semi-group solutions exists (Lasota, 2002), namely:

$$\lim_{k \to \infty} \sup_{t \geq 0} \|u(t,.) - u_k(t,.)\|_{L^1} = 0 \tag{10}$$

The relation (10) results from the following deduction: continuous functions of compact media create a dense set in $L^1(R^n)$, if $f \in L^1(R^n)$. Let (f_k) be a series of continuous and limited functions $\to f$ in the norm L^1 and $u_k(t,x) = p^t f_n(x)$ and p^t operators are stochastic; therefore:

$$\| u(t,.) - u_k(t,.) \|_{L^1} = \|p^t f - p^t f_k\|_{L^1} \leq \|f - f_k\|_{L^1} \tag{11}$$

The theorem on the increase of entropy will be presented for the needs of the future considerations.

THEOREM (on the increase of entropy): *If the function $\eta : [0, \infty) \to R$ is continuous and concave and the operator $p : L^1(x_1) \to L^1(x_2)$* is double-stochastic, then for each density $f \in D(x_1)$, *such that $\eta^\circ f \in L^1(x_1)$, i.e. fulfilling the condition*:

$$H_\eta(f) > -\infty \tag{12}$$

$\forall t \geq 0$ *entropy $H_\eta(p^t f)$ is determined.*

On the basis of the above equation, if $f \in D(R^n)$ and fulfils the condition (12) then $\forall t \geq 0$ for the entropy $H_\eta(p^t f)$ the relation (13) occurs

$$H(p^{t_2}f) \geq H(p^{t_1}f) \quad \text{for} \quad t_2 \geq t_1 \geq 0 \tag{13}$$

Besides, for an arbitrary initial function $f \in D(R^n)$ the following estimation exists:

$$H(p^t f) \geq \frac{n}{2}(1 + \ln 4\pi t) \quad \text{for} \quad t > 0 \tag{14}$$

Then, estimation of the entropy $H(p^t f)$ from the top equals to (Lasota, 2002):

$$H(p^t f) \leq \frac{n}{2}(1 + \ln 4\pi t) + \frac{M}{4t} \quad \text{for} \quad t > 0 \tag{15}$$

where M – the second finite moment in the form of (16)

$$M = \int_{R^n} |x|^2 f(x)dx \tag{16}$$

If $\exists$ a constant t_0 such that: $\sup\left\{\left|\frac{\beta(t)}{\alpha(t)}\right| : t \geq t_0\right\} < \infty$ then (14) and (15) may be written as:

$$H(p^t f) = \frac{n}{2}(1 + \ln 4\pi t) + O\left(\frac{1}{t}\right); \quad t \to \infty \tag{17}$$

where $\beta(t) = O(\alpha(t))$ for $t \to \infty$ and the density f is specified by (16).

The above estimation results (Lasota, 2002) from the following facts:

– From the integral Gibbs inequality:

$$H(p^t f) \leq -\int_{R^n} p^t f(x)\ln\Gamma(t,x)dx = -\int_{R^n} p^t f(x)\left[-\frac{n}{2}\ln 4\pi t - \frac{|x|^2}{4t}\right]dx \tag{18}$$

– $p^t f$ is the density; therefore

$$H(p^t f) \leq \frac{n}{2}\ln 4\pi t + \frac{1}{4t}\int_{R^n} |x|^2 p^t f(x)dx \tag{19}$$

Basing on Fubini theorem (Rudin, 1970) let us calculate the integral for non-negative functions in the equation (19); we get

$$\int_{R^n} |x|^2 p^t f(x)dx = \int_{R^n}\left\{\int_{R^n} \Gamma(t, x-y)f(y)dy\right\}|x|^2 dx = 2tn + M \tag{20}$$

For $t \to \infty$ the relation (17) will have the form:

$$H(p'f) = 0.5n\ln t + O(1) \tag{21}$$

When the components that may be estimated by constants in the equation (17) are neglected, it turns out that the entropy of distribution of the field for asymptotic conditions (large t) considerably depends on the space dimension.

Entropy characterises the randomness of *a priori* results, i.e. before an experiment is performed. As it turns out from considerations concerning the process of dislocations, generated by the equation (2), entropy differs from zero, what means, that the process has the characteristics of the random process.

Physical conditions mean that the process of dislocations depends on time and on mining-and-geological conditions (i.e. on parameters); therefore it may be considered as a stochastic process. The process $u(t, \omega)$ is a measurable function, i.e. $u(t, \omega) := \phi$ with $\phi : \Omega \to U$, where Ω – the space of events, U – the space of implementations of $\omega \in \Omega$; $u \in U$. The process $u(t, \omega) : T \times \Omega \to R$.

If the process $u(t, \omega)$ is a measurable stochastic process, then, following the Fubini theorem, it is measurable as a function of t for almost all $\omega \in \Omega$; therefore implementations (trajectories) are also measurable.

4. An attempt to describe the stochastic process of post-mining dislocations

Let us consider the following random process $\{\zeta_t\}_{t \in T}$

$$\zeta_t = u(\bar{t} - t; \xi_t) \tag{22}$$

Reference to the physical process of rock mass dislocations

Considering the random nature of the process, the equation of the displacement trajectory may be written in the form (23):

$$\begin{cases} \dfrac{d\zeta(t)}{dt} = k[t, \zeta(t)] + \sigma[t, \zeta(t)]\,\xi(t) \\ \zeta(t = 0) = 0 \end{cases} \tag{23}$$

The first component of the equation (23) is the deterministic description, and the second component is the stochastic approach to irregularities of the trajectories of the process.

Following Knothe (Knothe, 1953) let us assume the deterministic part (23) as the solution of the linear differential equation (25)

$$\begin{cases} \dot{\zeta} = f(\zeta_k; \gamma) \\ \zeta(t = 0) = 0 \end{cases} \tag{24}$$

As it is known from the theory of differential equations, the problem (24) has the unique solution:

$$\zeta(x,t) = e^{t\gamma} f(\zeta_k(x)) \tag{25}$$

where $\zeta_k(x)$ – asymptotic value of displacement at the moment t.

It has been assumed in this publication that $\zeta_k(.)$ will be determined according to Knothe theory (Knothe, 1953). For $x \in R^2$ we have:

$$\zeta_k(\xi, t_k; t) = D \sum_{i=1}^{n} \left\{ f(t_i, \Lambda) \frac{\partial}{\partial x^i} \iint_{P[t_i]} \exp\left[-\frac{\pi}{\rho^2}\left((x^1)^2 + (x^2)^2\right) \right] dx^1 \, dx^2 \right\} \tag{26}$$

where $f(t_i, \Lambda)$ – function of time,

D, Λ, ρ – appropriate coefficients and parameters of the theory,

P – the trace of projection of the field of exploitation onto a horizontal plane.

For the discrete implementation the model (23) may be expressed in the following way:

$$\zeta_{n+1} = \sum_{j=1}^{n} c_j \cdot \zeta_{n-j+1} + \varepsilon_{n+1} \tag{27}$$

where ε_i – the random component,

c_j – parameters resulting from the solution of the initial equation.

The relation (27) represents a model of a forecast of a non-stationary process of post-mining displacements, resulting from underground exploitation.

5. Verification of the process projections

Characteristics of observation lines and underground exploitation

Two observation lines, the line No 1 and the line No 2, located within the "Bogdanka" Coal Mine, have been selected for further analyses. Those lines are located over the caving walls No 0, 1, 2 and 3, which are situated within the central part of the mine (Fig. 2), in the coal bed 382/2. The average thickness of the exploited layer equals to 2.9 m and the average depth of exploitation equals to 690 m. The distance between points of observation lines No 1 and No 2 equals to approximately 10 m.

As a result of identification of parameters of Knothe theory the following values of the optimum parameters tgβ and a have been achieved for the observation lines:

Line 1: tgβ = **1.65**	a = **0.94**	mW = 38.6 [mm]
Line 2: tgβ = **1.89**	a = **0.79**	mW = 32.6 [mm],

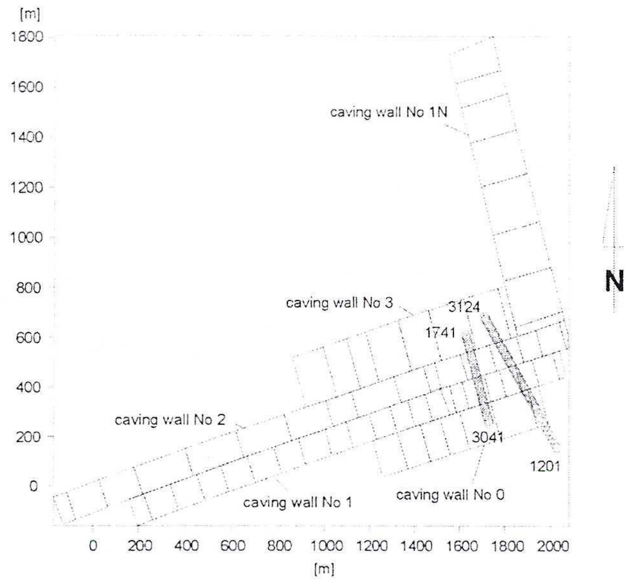

Fig. 2. The scheme showing the development of underground exploitation and the location of measuring points

Verification covers:

- comparison of projections of the process of dislocations according to Knothe theory and description, which considers stochastic disturbances (*model*), with the results of measurements (*meas*),
- implementation of the analysis of correlation between particular variables,
- the statistical T-test for dependent variables,
- diagrams of the range for statistical parameters,

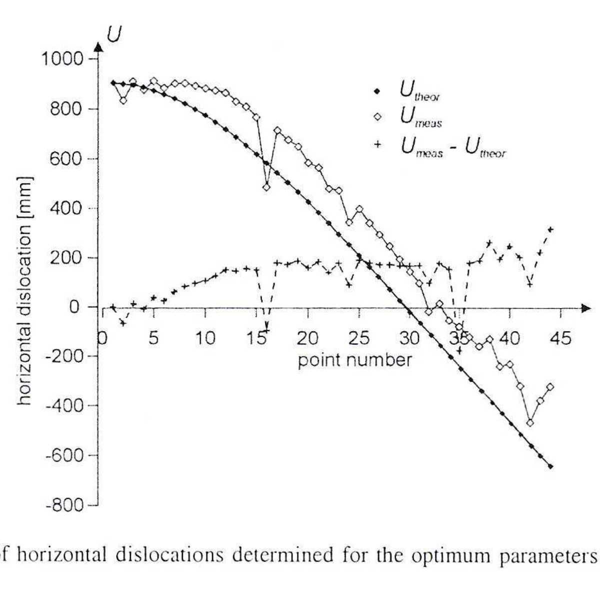

Fig. 3. Distribution of horizontal dislocations determined for the optimum parameters of Knothe theory

– the distribution of deviations between the results of measurements and the theoretical results (*theor*). The STATISTICA TM PL (Słaby and Luszniewicz; 2001) has been used for that purpose.

The results of a survey and of the modelling of vertical dislocations within the area of underground exploitation affect are presented in Table 1 (line 1) and in Table 2 (line 2).

Table 1. The results of modelling and of dislocation surveying of Line 1

Line 1									
Point No	U_{theor} [mm]	U_{meas} [mm]	U_{max} [mm]	$U_{max} - U_{theor}$ [mm]	Point No	U_{theor} [mm]	U_{meas} [mm]	U_{max} [mm]	$U_{max} - U_{theor}$ [mm]
1210	904	904.0	974	0	1221	295	473.6	428	178.6
3058	900	832.7	973	– 67.3	3069	253	342.0	392	89.0
1211	896	908.3	971	12.3	1222	208	396.9	356	188.9
3059	887	878.4	964	– 8.6	3070	163	340.9	323	177.9
1212	874	911.8	954	37.8	1223	117	288.1	293	171.1
3060	859	883.1	942	24.1	3071	71	242.3	267	171.3
1213	842	903.2	927	61.2	1224	24	189.8	247	165.8
3061	822	905.3	909	83.3	3072	– 22	144.0	236	166.0
1214	799	893.8	889	94.8	1225	– 69	99.2	233	168.2
3062	775	881.2	867	106.2	3118	– 117	– 20.8	241	96.2
1215	747	873.1	842	126.1	1226	– 163	13.2	257	176.2
3063	718	868.1	814	150.1	3119	– 209	– 55.5	281	153.5
1216	687	831.8	785	144.8	1227	– 255	– 77.5	310	177.5
3064	654	810.6	755	156.6	3120	– 301	– 124.2	343	176.8
1217	619	769.4	722	150.4	1228	– 348	– 161.0	379	187.0
3065	583	487.2	688	– 95.8	3121	– 391	– 128.7	416	262.3
1218	544	714.8	651	170.8	1229	– 435	– 242.0	454	193.0
3066	505	677.4	615	172.4	3122	– 477	– 230.0	492	247.0
1219	467	652.8	581	185.8	1230	– 520	– 318.8	530	201.2
3067	426	585.0	544	159.0	3123	– 561	– 468.1	568	92.9
1220	383	566.0	504	183.0	1231	– 600	– 378.6	605	221.4
3068	340	480.5	467	140.5	3124	– 641	– 324.6	645	316.4

Table 2. The results of modelling and of dislocation surveying of Line 2

Line 1									
Point No	U_{theor} [mm]	U_{meas} [mm]	U_{max} [mm]	$U_{max}-U_{theor}$ [mm]	Point No	U_{theor} [mm]	U_{meas} [mm]	U_{max} [mm]	$U_{max}-U_{theor}$ [mm]
3010	888	888.0	889	0	3029	142	249.8	146	107.8
3011	855	894.6	856	39.6	3030	108	226.6	112	118.6
3012	819	1205.2	820	386.2	3031	76	150.1	82	74.1
3013	780	993.0	781	213.0	3032	43	87.4	51	44.4
3014	739	837.8	740	98.8	3033	10	44.8	29	34.8
3015	699	809.6	701	110.6	3034	−23	−163.6	34	−140.6
3016	657	779.0	658	122.0	3035	−54	−76.9	59	−22.9
3017	614	774.7	615	160.7	3036	−88	52.0	91	140.0
3018	570	774.2	572	204.2	3037	−122	−96.8	124	25.2
3019	528	710.1	530	182.1	3038	−156	−234.9	157	−78.9
3020	484	705.0	486	221.0	3039	−191	−304.1	191	−113.1
3021	441	631.8	443	190.8	3040	−226	−350.8	226	−124.8
3022	399	591.6	401	192.6	3103	−263	−367.1	264	−104.1
3023	361	544.0	363	183.0	3104	−303	−316.3	303	−13.3
3024	321	467.6	323	146.6	3105	−344	−443.3	344	−99.3
3025	282	395.9	285	113.9	3106	−387	−720.6	387	−333.6
3026	245	393.4	247	148.4	3107	−430	−1148.3	430	−718.3
3027	209	370.3	212	161.3	3108	−475	−1726.3	475	−1251.3
3028	177	277.1	180	100.1	3109	−520	−2033.9	520	−1513.9

The distribution of theoretical horizontal dislocations (Fig. 3) $u_i(t = \cos t, x)$ is the U_{theor} representation based on appropriate equations of Knothe theory (Knothe 1953). The results of measurements of vertical dislocations have been used for the determination of the optimum parameters of equations. Parameters for horizontal movements have not been identified since the equations describing the process are the closed entity.

Testing the conditions for residuals between the standard (results of measurements), U_{meas} and the representation, which considers the random component of the process, U_{theor}.

The vector of residuals ε

$$\varepsilon \overset{df}{=} U_{meas} - U_{theor}$$

(i) $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ independent,

(ii) $E(\varepsilon_i) \cong 0$,

(iii) ε_i – quasi-normal distribution – what results from histograms.

Quasi-normality of the distribution of residuals states, that the model, which considers the randomness, is the good approximation of a standard (results of surveying measurements).

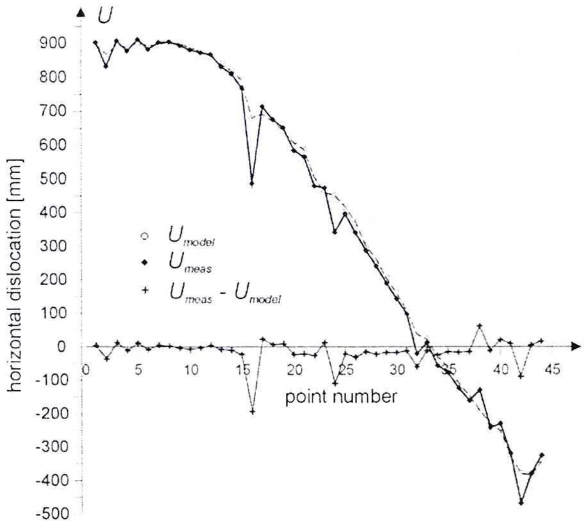

Fig. 4. Distribution of horizontal dislocations $U(x; t = const)$

Measures of deviations with respect to the central trend Q_i have been determined. The following values have been obtained for the results of measurements and modelling of the horizontal dislocation:

Quantile Q_i	U_{meas}	U_{model}
$Q_{1/4}$	140	145
$Q_{1/2}$	200	198
$Q_{3/4}$	260	262
$Q_{4/4}$	320	315

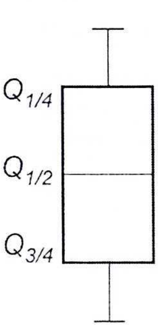

Fig. 5. The diagram showing quantiles Q_i

Deviations between $U_{meas} - U_{model}$ and $U_{model} - U_{theor}$ have been respectively cumulated, thus creating the separable series: $x_i \in (-100, -90, ..., 180)$, with $n_i \in (1, 2, ..., 12)$. Therefore it is possible to generate the histogram of the structure: $hist(x) \overset{df}{=} n_i$, if $x \in \{u_{i-1}, u_i\}$. This definition has the following meaning:

$$\int_{u_0}^{u_n} hist(x)\,dx = 1 \tag{28}$$

The distribution of the x variable in the form: $hist(x) = n_i$, referred to the normal distribution, presents evident discrepancies. Therefore the interrelation (U_{meas}, U_{model}) has been investigated. Appropriate calculations have been performed using the STATISTICA ™ PL software package; results have been listed in Table 3 for discrete variables. The results obtained indicate the existence of the strong interrelation between the variables U_{meas} and U_{model}. In the formal sense, the following inequality is satisfied: $|\langle U_{meas}, U_{model}\rangle| \le ||U_{meas}|| \cdot ||U_{model}|| \Rightarrow U_{meas}, U_{model}$ are linearly dependent. The values U_{meas}, and U_{model} are respective vectors, whose co-ordinates are listed in Table 1. Parameters of the deterministic model U_{model} have been iteratively determined with reference to the results of measurements of vertical dislocations.

The distribution of horizontal dislocations model (27) $U(x; t = const.)$ (Fig. 4), has not been investigated with respect to statistical measures since distortion with respect to the measurement results is not important in this case. Adequate statistical measures have been determined for the field of vertical dislocations for the line 1 (Table 3, Fig. 5). The line 2 can be analysed in a similar way.

T a b l e 3. Statistical measures

Variable	INTERRELATION MATRIX				
	U_{meas}	U_{model}	U_{theor}	$U_{meas} - U_{model}$	$U_{model} - U_{theor}$
U_{meas}	**1**	**1**	**0.99**	0.19	−0.23
U_{model}		**1**	**0.99**	0.12	−0.24
U_{theor}			**1**	0.11	−0.36
$U_{meas} - U_{model}$				**1**	0.09
$U_{model} - U_{theor}$					**1**

where U_{meas} – results of measurements of the horizontal component of dislocations,
U_{model} – results of modelling of the horizontal component of dislocations, with consideration of randomness,
U_{theor} – results of modelling of the horizontal component of dislocations, in accordance to Knothe theory.

The T-test for dependent samples

Table 4. Results of Student's T-test

Variable	Mean	Standard deviation	Number of measurements N	Differences	T	Probability p
U_{meas}	203.07	396.36	44			
U_{model}	202.59	394.50	–	0.48	0.105	0.917
$U_{meas} - U_{theor}$	18.70	410.19	–	15.93	1.658	0.105
$U_{meas} - U_{model}$	1.29	24.36	–			
$U_{model} - U_{theor}$	15.45	52.86	–	– 14.16	– 1.670	0.102

6. Results of the analysis

The paper presents analysis of the kinematics of a non-stationary process of horizontal dislocations of points of a measuring line, within the areas of the influence of mining exploitation. The analysis concerns particular mining-and-geological conditions. The discussed modelling procedure of a non-stationary process is not limited with respect to the geometry of the exploitation area that generates the field of dislocations. The standard of the horizontal component of mining dislocations is created by the results of measurements of dislocations of points within the entire time interval of observations $\{t_0, ..., t_k\}$.

Usually the accuracy of description of non-stationary vertical dislocations, based on the deterministic projection – Knothe theory considering the development of underground exploitation – is not a fully acceptable approximation in a sense of the mean error measure, i.e. $m_u > m_{u\ accept}$. The relation $m_u > m_{u\ accept}$ (non-requested relation) is confirmed by the results of estimation of the entropy of the process of dislocations. Variability of the entropy of the process, resulting from the formal analysis, creates arguments concerning the stochastic nature of the process of dislocations. It turns out from analysis that strong limitations exist within the obtained accuracy of deterministic description with respect to results of measurements.

Therefore an attempt was made to build a formula, which would consider the random irregularity of the process. It has been assumed that the investigated process is a combination of a deterministic component and a random value. Deviations around the average value have been estimated on the basis of the chronologically ordered set of the results of measurements of dislocations, assigned to a corresponding moment:

$$\{u_i(t_0), u_i(t_1), ..., u_i(t_n)\} \rightarrow E\{u_i(t_{n+1}) \,|\, u_i(t_0), u_i(t_1), ..., u_i(t_n)\}.$$

Numerical results of modelling of the kinetics of horizontal dislocations with use of the modified formula within an area of the developing underground exploitation have been performed in the form of diagrams. Qualitative characteristics of the description are satisfactory in this case. Statistical evaluation of projection as well as appropriate measures and the T-test are the basis for drawing the conclusions, that the modified formula of description of the investigated process is a better approximation to the results of measurements than deterministic models.

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Charakterystyka kinematyki procesu przemieszczeń pogórniczych

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Streszczenie

Praca dotyczy analizy kinematyki procesu przemieszczeń punktów powierzchni terenu w obszarze oddziaływania eksploatacji podziemnej. Rozważono problem określenia zmian konfiguracji przestrzennej ciała wymuszonych wpływem eksploatacji podziemnej. Obserwacje realnego procesu wskazują, że trajektorie przemieszczania się punktów ośrodka cechuje tu nieregularność. Z kolei opis deterministyczny badanego procesu generuje z reguły gładkie trajektorie przemieszczania się punktu. Z natury więc nie można przybliżyć odwzorowania procesu, z dowolną dokładnością do wyników pomiaru. Analizowano więc entropię procesu charakteryzującą losowość danego zjawiska. Wykazano, że entropia nie jest stała. Przestawiono więc próbę opisu danego procesu jako złożenie odwzorowania deterministycznego i procesu losowego.