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# Hydraulic gradient prediction for solid-water mixture flow through horizontal pipelines of different diameters and roughness and solids concentrations

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## Abstract

In wastewater treatment plants, large pumps are often used to accommodate unknown hydraulic properties of solid-water mixture flow. The use of large pumps translates into higher purchasing and operating costs. Wastewater mixture is pumped with solids of different types and concentrations through pipelines. The design of these ducts is mainly based on the hydraulic laws of solid-water mixture which is represented by a corrected friction coefficient corresponding to the concentration of solids in water. This paper experimentally studies hydraulic properties of solid-water mixtures in pipelines by the varying Froude number ( $Fr$ ), which represents the velocity mixture, solid concentration, pipeline diameter and pipeline material type-roughness coefficient. The experiments have been conducted in the wastewater treatment plant where six solid concentrations can be found ranging from 2 to 12% by weight. The pipe diameter ranges between 100 to 300 mm. It has been found that both the friction coefficient and the hydraulic gradient ameliorate with the increase of the pipeline roughness and the solids concentration in the water mixture, whereas the  $Fr$  drop with the diameter of the pipeline. The results are translated into curves and equations to predict the corrected pipeline friction coefficient and the hydraulic gradient of the solid-water mixture flow through horizontal pipelines at various solids concentrations, roughness and diameters.

**Key words:** *deposit limit velocity, hydraulic gradient, hydraulic losses, slurry flow, solid concentrations, suspended solids, wastewater*

## INTRODUCTION

Since water plays a cardinal role in our life, wastewater is directed to treatment plants to remove nutrients, reduce chemical oxygen demand and solids before it is released back into the environment [KAJJUMBA *et al.* 2018; 2019]. The flow characteristics of pure liquids are mathematically described and experimentally verified. However, studies of the flow characteristics of the solid-liquid mixture are very scarce and limited in the literature. The complex nature of solid-liquid mixtures is extremely difficult to predict even with the use of the solid-liquid mixture software [KIM *et al.* 2008; MIRMASOUMI, BEHZADMEHR 2012]. To understand factors that influence the solid-liquid mixture, empirical

examination is necessary. One of the fundamental parameters that influences the flow of water infused with solid particles is the hydraulic gradient ( $i$ ). The hydraulic gradient indicates changes in the inclination of water pipe or a conduit used to convey the mixture. Because of its complexity, different approaches have been applied to measure the  $i$  in different setups.

In subsurface flow of water, conductivity and porosity of solid particles can be used to estimate the  $i$ . Changes in conductivity and porosity can be translated into head loss. KLØVE *et al.* [2005], applied this technique to estimate the  $i$  of the surface flow. The authors have used subsurface flow filters to estimate the residence of a tracer. Using the probability density function, they have been able to provide

a 3-D model of conductivity and porosity. The model has been used to estimate the  $i$ . Such a model is limited to a tracer and is suitable for subsurface measurements. KIM *et al.* [2008] and OSRA [2020] studied the changes in the  $i$  using the Reynolds number. When the critical velocity is lower than the mean velocity, square pipes provide a higher  $i$  than circular pipes [KIM *et al.* 2008]. Thus, during sand transport at low velocity, square ducts offer less resistance compared to circular pipes. Pipe inclinations have a significant effect on the  $i$  of solid-liquid mixture; the lower the inclination, the lower the  $i$  [OSRA 2020].

To understand the relationship between solid-liquid mixture and the  $i$ , prediction equations have been developed. ELTOUKHY *et al.* [2018] developed an equation to determine the  $i$  of sludge-water at the varying Reynolds number (Re) and volumetric loading. The  $i$  is mainly influenced by duct diameter and Re. The  $i$  loss is proportional to the Re, while it is inversely proportional to pipe diameter [ELTOUKHY *et al.* 2018]. MATOUŠEK [2009] studied the effect of deposition within a pipe on the  $i$ . In the study, pressure drop equation in a 100–150 mm pipe diameter has been estimated; the equation predicts the behaviour of the  $i$  with a  $\pm 25\%$  variation. In a two-phase helical flow of slurry, the rotation of slurry improves the drop of the  $i$  [HAN *et al.* 2008]. A higher slurry concentration (5.0%) improves the flow compared to low concentrations of less than 1.0%. This is because the concentration ameliorates specific gravity and viscosity providing an inform shear force in the fluid. Such observations make it hard to forecast the behaviour of the solid-liquid mixture, especially when there is a high variation in slurry like in wastewater treatment plants.

In wastewater treatment plants, pumping systems are among the highest energy demand sections. Thus, the optimization of the pumping system is always an ultimate goal during the designing of a treatment plant. To achieve this, sludge characterization is essential to understand the nature of a solid-liquid mixture [ESHTIAGHI *et al.* 2013; SLATTER 2003; 2008]. The main challenge of sludge pumping is related to sludge concentration. SLATTER [2008] developed a transitional equation using a viscos-plastic fluid with a two-layer model to estimate the behaviour of sludge. In a similar study to estimate the behaviour of concentrated slurry, Herschel–Bulkley equation was applied [REMAÎTRE *et al.* 2005]. The authors found that the flow of debris is influenced by the grain size distribution. Although such rheological models can offer a good prediction of slurry behaviour, their practical application is limited. Temporary changes in the slurry can lead to a major error in the model. Changes in slurry source, flow, and composition have a potent effect on linear and nonlinear regression of the Herschel–Bulkley equation [FARNO *et al.* 2018; JIN *et al.* 2003; MAHMOUD *et al.* 2004; RATKOVICH *et al.* 2013].

There are two fundamental challenges for designing of a pumping system in a wastewater treatment plant. Firstly, the changes in pressure result in a change of sludge. During the treatment plant operation, sludge characteristics keep on changing depending on effluent and sludge dewatering systems. Secondly, in most cases, sludge may not obey the Newtonian behaviour making rheological data the only approach to estimate the behaviour. However, such data may

not be available [FARNO *et al.* 2015; SLATTER 1997; 2008]. Therefore, there is a need for determining a reliable pressure drop within the system. Such a task is extremely difficult because the estimating of a pressure drop requires at least two rheological variables besides viscosity alone [MALKIN *et al.* 2004].

The curvilinear behaviour of wastewater sludge, also known as viscoelastic material has been studied using Bingham plastic model (BPM), Herschel–Bulkley model, and modified Herschel–Bulkley. The former model is based on two variables. Thus, the linear curve regression fitting is easy and straight forward. Although the BPM is easy to use, it is subject to major errors due to intermediate and extreme shear forces, like those encountered in sludge management [BAUDEZ *et al.* 2013]. Also, the Herschel–Bulkley model is not suitable at high shear forces, since it underestimates shear forces. Additionally, the exponential parameter in the model makes linear fitting almost impossible and underrates shear forces at extreme ends. The introduction of the third parameter in the modified Herschel–Bulkley model can overcome this challenge. However, the third variable creates ambiguity in the model making the  $i$  calculation difficult [BAUDEZ *et al.* 2013; FARNO *et al.* 2015; WANG *et al.* 2019; ZHANG *et al.* 2018]. Equations (1)–(3) elicit the following model variables. Bingham plastic model:

$$\tau = \tau_B + k\gamma \quad (1)$$

Herschel–Bulkley model:

$$\tau = \tau_H + k\gamma^n \quad (2)$$

Modified Herschel–Bulkley model:

$$\tau = \tau_H + k\gamma^n + \alpha\gamma \quad (3)$$

where:  $\tau$  = shear pressure (Pa);  $\gamma$  = shear rate governed by the flow velocity per a cross section distance;  $\tau_B$ ,  $\tau_H$ ,  $k$ ,  $n$ , and  $\alpha$  = a parameter adjusting to the model.

The presence of large particles in sludge makes it necessary to use BPM and Herschel–Bulkley models. To overcome the BPM, and Herschel–Bulkley challenges, this study has employed the Buckingham- $\pi$ -theorem. The Buckingham model can be used to predict the behaviour of slurry under a non-Newtonian flow regime [GROZDEK *et al.* 2009]. In this paper, experimental runs were conducted in a wastewater treatment plant using wide range of solid concentrations, and pipeline diameters and roughness, to estimate general curves and equations that help to predict the pipeline corrected friction coefficient and hydraulic gradient as a function of the solid concentration.

## METHODS

### EXPERIMENTAL SETUP

In the experimental runs, six solid concentrations of 2, 4, 6, 8, 10, and 12% (weight by weight) from a wastewater treatment plant were investigated. Slurry was collected from the Abou Tig wastewater treatment plant, Assiut Governorate, Egypt. Pure water was used as a control. The diameter of the pipe tested ranged from 100 to 300 mm, as

**Table 1.** Experimental matrix used in this study

Pipe diameter $d$ (mm)	Pipe roughness $\varepsilon$ (mm)	Solid concentration $C$
100	estimating pipeline roughness (5 runs)	0
100	0.01, 0.012, 0.015, 0.02, 0.03	0, 2, 4, 6, 8, 10, and 12% by weight
150	0.01	0, 2, 4, 6, 8, 10, and 12% by weight
200	0.01	0, 2, 4, 6, 8, 10, and 12% by weight
250	0.01	0, 2, 4, 6, 8, 10, and 12% by weight
300	0.01	0, 2, 4, 6, 8, 10, and 12% by weight

Source: own elaboration.

shown in Table 1. Pipes on which head loss was determined were of 2.0 m.

Head loss estimation offers a faster and reliable approach to comparing the effect of different variables. Upon establishing the critical velocity of sludge, the solid-liquid mixture is then operated at a point higher than the critical velocity to avoid the settling of particles. Table 1 shows the experimental setup.

The following procedures were used in the experiments:

- for each pipe diameter, two pressure gauges were installed 2.0 m apart from each other,
- sludge mixture was pumped into the pipe at a speed greater than the settling velocity,
- at a given sludge mixture, the reading of the gauge was recorded,
- Buckingham theorem was used to establish the curve equations that can be used to predict the  $i$  at different concentrations, duct diameters and roughness.

#### HYDRAULIC GRADIENT $i$

The hydraulic gradient  $i$  in a duct depends on roughness  $\varepsilon$ , velocity  $v$ , diameter of the pipe  $d$ , and gravity  $g$ , as shown in Equation (4).

$$i = F(\varepsilon, v, d, g) \quad (4)$$

Applying dimension analysis, the  $i$  can be elicited using Equation (5).

$$\frac{(P_1 - P_2) d}{\frac{1}{2} \rho v^2 L} = F \left[ \frac{v}{\sqrt{g d}}, \frac{\varepsilon}{d} \right] \quad (5)$$

where:  $P_1$  and  $P_2$  = the pressure at point 1 and 2, respectively;  $L$  = the length of the pipe;  $\rho$  = density.

From Equation (5), the first part of the function represents Froude number (Fr) while the second portion shows surface roughness. For wastewater sludge slurry, sludge concentration, which is another parameter influencing the  $i$ , can be added (Eq. 6).

$$\frac{(P_1 - P_2) d}{\frac{1}{2} \rho v^2 L} = F \left[ \frac{v}{\sqrt{g d}}, \frac{\varepsilon}{d}, C \right] \quad (6)$$

To determine the effect of non-dimensional variables on the  $i$  friction factor  $f$  is plotted against Fr at a given duct diameter and sludge concentration.

$$f = \frac{(P_1 - P_2) \pi^2 g d^5}{8 Q^2 L} \quad (7)$$

where:  $Q$  = sludge flowrate.

$$Re = \frac{\rho v d}{\mu} \quad (8)$$

where: Re = Reynolds number;  $\rho$  = density,  $v$  = velocity;  $d$  = diameter of the pipe;  $\mu$  = dynamic viscosity.

## RESULTS AND DISCUSSION

### ESTIMATING THE PIPELINE ROUGHNESS $\varepsilon$

Five experimental runs were carried out to estimate  $\varepsilon$  at different pipeline diameters. The discharge  $Q$  was pumped through the pipeline and pressure levels at the two points (1.0 m apart) were recorded. The pipeline friction coefficient  $f$  was calculated with Equation (7) and the Moody chart that relates Re and pipeline roughness  $\varepsilon$ . Re was estimated using Equation (8). The estimated values of pipeline roughness for different pipeline types are listed in Table 1.

### PIPELINE CORRECTED FRICTION COEFFICIENT $f$

The Abou Tig wastewater treatment plant is a high-density sludge processing facility. Based on preliminary studies, the sludge does not obey the Newtonian flow. This shows the complexity of designing a pumping system for the expansion of the plant. Rheological variables have been used to estimate  $f$  and  $i$  estimation curves. Figure 1 shows changes in the friction coefficient and Froude number (Fr) at different pipe diameters; as Fr plummeted,  $f$  soared. For example, at duct diameter of 100 mm and 8% sludge concentration, an increase of Fr from 1.0 to 2.0 (100%) has been recorded together with a drop of the friction coefficient from 0.10 to 0.03 (97%). However,  $f$  has increased with sludge concentration. The duct diameter of 100 mm and Fr = 1.5, increases the sludge intensity from 2 to 4%, ameliorates  $f$  from 0.02 to almost 0.03.

### HYDRAULIC GRADIENT

The hydraulic gradient  $i$  has been estimated based on experimental data collected at different solid intensity and duct diameters. Critical head loss and reduced velocity have been assessed at different sludge intensity ranging from 2 to 12%. Elevated temperatures affect the hydraulic gradient; however, this effect has not been considered in this study. The hydraulic gradient has a direct relation with the Fr and the quantity of solids in the liquid mixture as shown in Figure 2. For example, for the solid concentration of 8% and duct diameter of 150 mm, a change of Fr from 0.746 to 1.212 increases the  $i$  from 0.0297 to 0.0359; 62.47% increase in Fr leads to increases in the hydraulic gradient by 21.02%. At 200 mm pipe diameter, a 100% increase in solid concentration (from 6 to 12%) has been recorded together with an increased hydraulic gradient by 66.67% (0.016665–0.027775). Table 2 summarizes the corrected pipeline friction coefficient and hydraulic gradient equations with varying Froude number and solid concentrations.

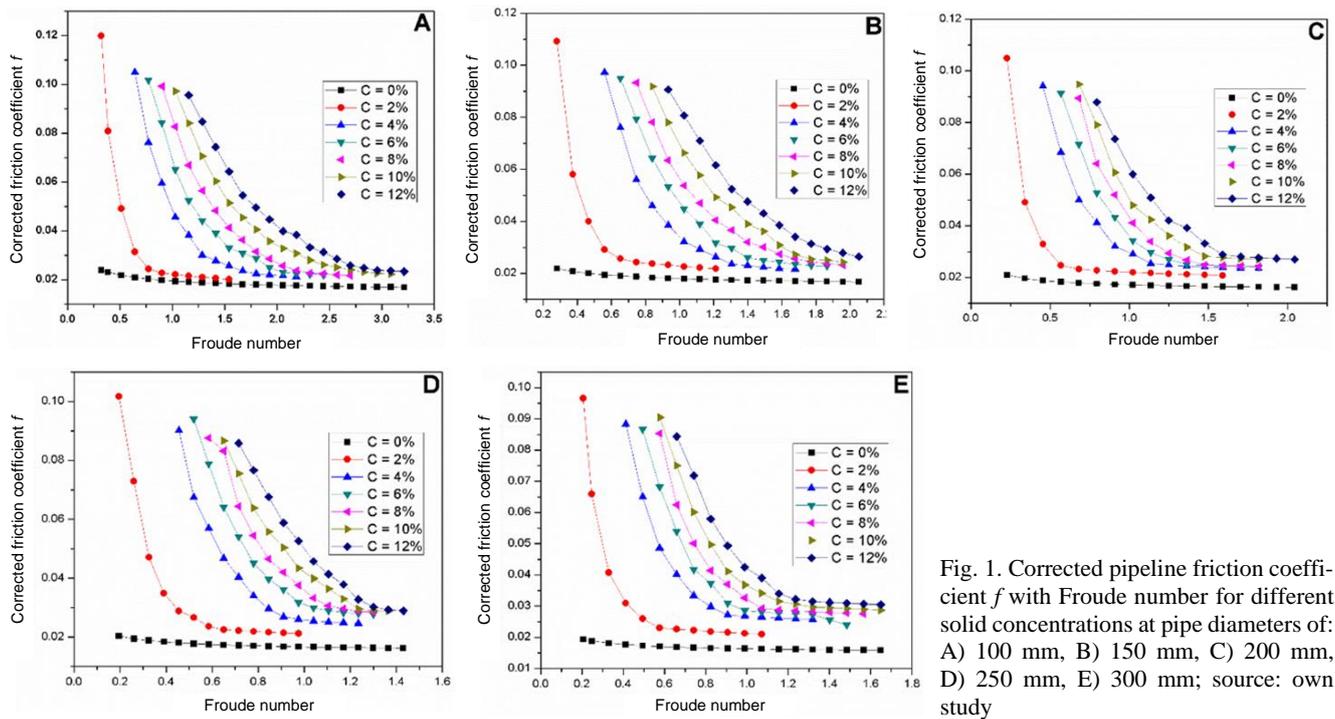


Fig. 1. Corrected pipeline friction coefficient  $f$  with Froude number for different solid concentrations at pipe diameters of: A) 100 mm, B) 150 mm, C) 200 mm, D) 250 mm, E) 300 mm; source: own study

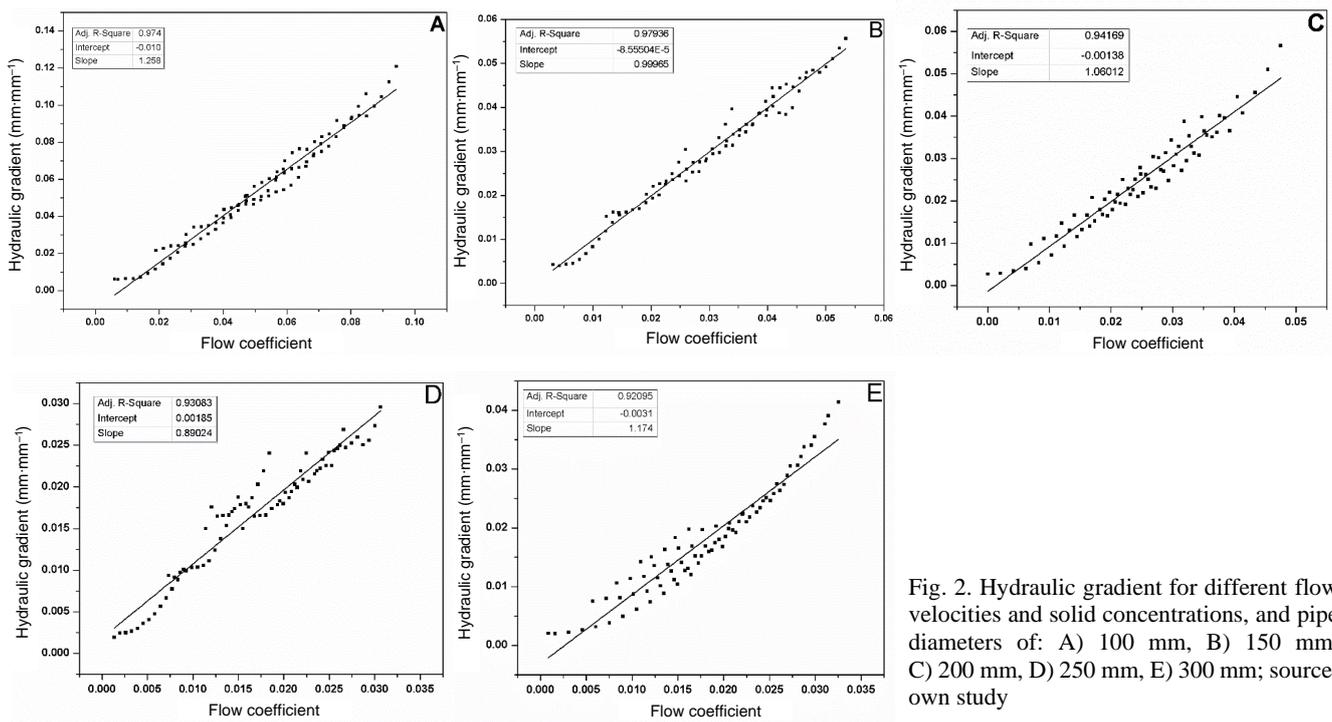


Fig. 2. Hydraulic gradient for different flow velocities and solid concentrations, and pipe diameters of: A) 100 mm, B) 150 mm, C) 200 mm, D) 250 mm, E) 300 mm; source: own study

**Table 2.** Curve fitting of solid-water coefficient and hydraulic gradient

Diameter (mm)	Corrected friction coefficient	Hydraulic gradient $i$
100	$1.851Fr + 0.358C - 0.007$	$2.3282Fr + 0.45C - 0.017$
150	$0.3666Fr + 0.288C - 0.006$	$0.232Fr + 0.29C - 0.016$
200	$0.232Fr + 0.147C - 0.007$	$0.246Fr + 0.156C - 0.0084$
250	$0.0511Fr + 0.121C - 0.0035$	$0.0455Fr + 0.1072C - 0.001$
300	$0.068Fr + 0.057C - 0.004$	$0.798Fr + 0.0667C - 0.0077$

Source: own study.

**THE EFFECT OF THE PIPELINE DIAMETER**

The effect of the pipe diameter on the  $f$  and  $i$  is shown in Figure 3A. It has been found that the increase in  $f$  and pipe roughness, and  $\epsilon$  has been proportional to the duct diameter. Figure 3A reveals similar trends for all the curves;  $f$  abated as  $Fr$  ameliorated. While the  $i$  increased as  $Fr$  increased (Fig. 3B), the  $i$  has been inversely proportional to the increase in the pipe diameter. Changing duct diameter from 150 to 250 mm at Froude number of 0.7461 and solid concentration of 8% leads to a decrease in a hydraulic

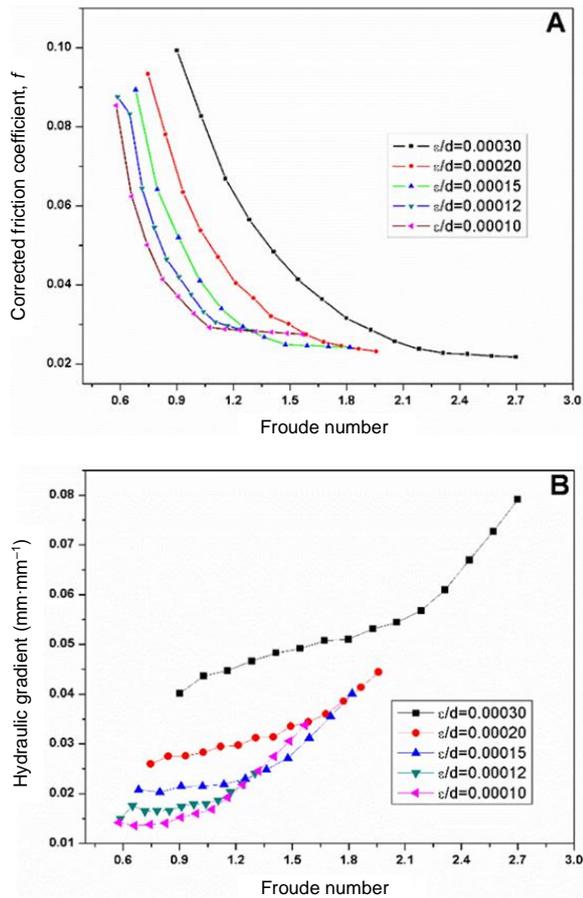


Fig. 3. (A) Variation of corrected friction coefficient  $f$  with Froude number for different pipe diameters (B) hydraulic gradient variation with Froude number for the pipeline diameter of 100 mm and the solid concentration of 8%; source: own study

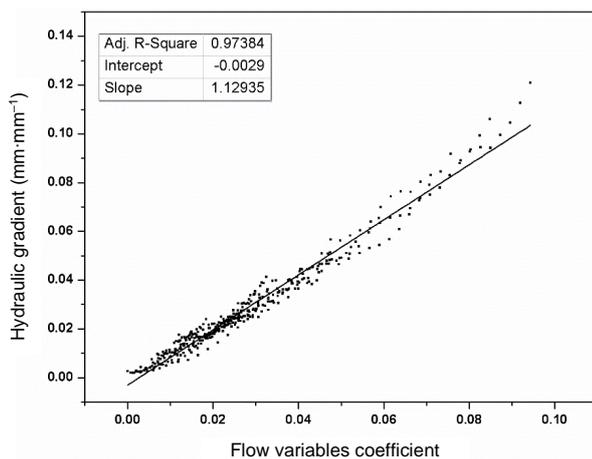


Fig. 4. Hydraulic gradient for different pipeline diameter, roughness, solid concentrations, and solid water mixture discharge; source: own study

gradient from 0.025974 to 0.016586, and the corrected friction coefficient decreases from 0.093321 to 0.054495. Thus, a 66.67% increase in the pipe diameter resulted in a 36.14% and 41.61% decrease in the hydraulic gradient and corrected friction coefficient, respectively.

## GENERAL HYDRAULIC GRADIENT (HG) EQUATION

Combining all the findings and Buckingham's theorem (Tab. 2), general equations to predict hydraulic gradient for a given sludge concentration and pipe diameter have been developed (Eqs. 9 and 10). The equations are governed by sludge concentration  $Fr$  and duct diameter. Figure 4 shows the testing of the equations; the equations yielded a perfect correlation with  $R^2 > 0.95$ .

$$f = 0.8724Fr + 0.2302C - 0.0077 \quad (9)$$

$$HG = 0.89162Fr + 0.2353C - 0.033 \frac{\epsilon}{d} + 0.001 \quad (10)$$

## CONCLUSIONS

The purpose of this paper is to analyse the combined effect of solid concentration, pipe diameter and roughness on the hydraulic gradient variation. Based on the results, the following conclusions have been drawn.

1. Duct friction coefficient ameliorates as the solid intensity  $C$  increases.
2. The hydraulic gradient  $i$  is directly related to sludge amount in the liquid.
3. An increase in the pipeline diameter increases the corrected pipeline friction coefficient.
4. The hydraulic gradient is inversely proportional to the pipeline diameter.
5. Equation (4) can be used to predict the solid water mixture hydraulic gradient for a given solid concentration  $t$  mixture discharge, pipeline diameter, and roughness.

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## REFERENCES

- BAUDEZ J.C., SLATTER P., ESHTIAGHI N. 2013. The impact of temperature on the rheological behaviour of anaerobic digested sludge. *Chemical Engineering Journal*. Vol. 215–216 p. 182–187. DOI [10.1016/j.cej.2012.10.099](https://doi.org/10.1016/j.cej.2012.10.099).
- ELTOUKHY M.A.R., ALKHUZAI K.A., ELKASHEF A. 2018. Head losses ratio prediction for hydraulic sludge transport in horizontal pipes for different concentrations and pipeline sizes. *Australian Journal of Basic and Applied Sciences*. Vol. 12(11) p. 92–101. DOI [10.22587/ajbas.2018.12.11.19](https://doi.org/10.22587/ajbas.2018.12.11.19).
- ESHTIAGHI N., MARKIS F., YAP S.D., BAUDEZ J.C., SLATTER P. 2013. Rheological characterisation of municipal sludge: A review. *Water Research*. Vol. 47. Iss. 15 p. 5493–5510. DOI [10.1016/j.watres.2013.07.001](https://doi.org/10.1016/j.watres.2013.07.001).
- FARNO E., BAUDEZ J.C., PARTHASARATHY R., ESHTIAGHI N. 2015. Impact of temperature and duration of thermal treatment on different concentrations of anaerobic digested sludge: Kinetic similarity of organic matter solubilisation and sludge rheology. *Chemical Engineering Journal*. Vol. 273 p. 534–542. DOI [10.1016/j.cej.2015.03.097](https://doi.org/10.1016/j.cej.2015.03.097).
- FARNO E., COVENTRY K., SLATTER P., ESHTIAGHI N. 2018. Role of regression analysis and variation of rheological data in calculation of pressure drop for sludge pipelines. *Water Research*. Vol. 137 p. 1–8. DOI [10.1016/j.watres.2018.02.059](https://doi.org/10.1016/j.watres.2018.02.059).
- GROZDEK M., KHODABANDEH R., LUNDQVIST P. 2009. Experimental investigation of ice slurry flow pressure drop in

- horizontal tubes. *Experimental Thermal and Fluid Science*. Vol. 33. Iss. 2 p. 357–370. DOI [10.1016/j.expthermflusci.2008.10.003](https://doi.org/10.1016/j.expthermflusci.2008.10.003).
- HAN S.M., KIM Y.J., WOO N.S., HWANG Y.K. 2008. A study on the solid-liquid 2 phase helical flow in an inclined annulus. *Journal of Mechanical Science and Technology*. Vol. 22 p. 1914–1920. DOI [10.1007/s12206-008-0732-y](https://doi.org/10.1007/s12206-008-0732-y).
- JIN B., WILÉN B.M., LANT P. 2003. A comprehensive insight into floc characteristics and their impact on compressibility and settleability of activated sludge. *Chemical Engineering Journal*. Vol. 95. Iss. 1–3 p. 221–234. DOI [10.1016/S1385-8947\(03\)00108-6](https://doi.org/10.1016/S1385-8947(03)00108-6).
- KAJJUMBA G.W., AYDIN S., GÜNEYSU S. 2018. Adsorption isotherms and kinetics of vanadium by shale and coal waste. *Adsorption Science and Technology*. Vol. 36 p. 936–952. DOI [10.1177/0263617417733586](https://doi.org/10.1177/0263617417733586).
- KAJJUMBA G.W., YILDIRIM E., AYDIN S., EMİK S., AĞUN T., OSRA F., WASSWA J. 2019. A facile polymerisation of magnetic coal to enhanced phosphate removal from solution. *Journal of Environmental Management*. Vol. 247 p. 356–362. DOI [10.1016/j.jenvman.2019.06.088](https://doi.org/10.1016/j.jenvman.2019.06.088).
- KIM C., LEE M., HAN C. 2008. Hydraulic transport of sand-water mixtures in pipelines. Part I. Experiment. *Journal of Mechanical Science and Technology*. Vol. 22 p. 2534–2541. DOI [10.1007/s12206-008-0811-0](https://doi.org/10.1007/s12206-008-0811-0).
- KLØVE B., XU S., LINDAHL A., WÖRMAN A., SØVIK A.K. 2005. A study of K variability and its effect on solute transport in subsurface-flow sand filters by measurement and modelling. *Journal of Environmental Science and Health, Part A. Toxic/Hazardous Substances and Environmental Engineering*. Vol. 40. Iss. 6–7 p. 1123–1132. DOI [10.1081/ESE-200055599](https://doi.org/10.1081/ESE-200055599).
- MAHMOUD N., ZEEMAN G., GIJZEN H., LETTINGA G. 2004. Anaerobic stabilisation and conversion of biopolymers in primary sludge – Effect of temperature and sludge retention time. *Water Research*. Vol. 38. Iss. 4 p. 983–991. DOI [10.1016/j.watres.2003.10.016](https://doi.org/10.1016/j.watres.2003.10.016).
- MALKIN A.Y., MASALOVA I., PAVLOVSKI D., SLATTER P. 2004. Is the choice of flow curve fitting equation crucial for the estimation of pumping characteristics? *Applied Rheology*. Vol. 14 p. 89–95. DOI [10.1515/arh-2004-0005](https://doi.org/10.1515/arh-2004-0005).
- MATOUŠEK V. 2009. Predictive model for frictional pressure drop in settling-slurry pipe with stationary deposit. *Powder Technology*. Vol. 192. Iss. 3 p. 367–374. DOI [10.1016/j.powtec.2009.01.017](https://doi.org/10.1016/j.powtec.2009.01.017).
- MIRMASOUMI S., BEHZADMEHR A. 2012. Effects of nanoparticles mean diameter on the particle migration and thermo-hydraulic behavior of laminar mixed convection of a nanofluid in an inclined tube. *Heat and Mass Transfer. Und Stoffuebertragung*. Vol. 48 p. 1297–1308. DOI [10.1007/s00231-012-0978-x](https://doi.org/10.1007/s00231-012-0978-x).
- OSRA F.A. 2020. A laboratory study of solid-water mixture flow head losses through pipelines at different slopes and solid concentrations. *South African Journal of Chemical Engineering*. Vol. 33 p. 29–34. DOI [10.1016/j.sajce.2020.04.001](https://doi.org/10.1016/j.sajce.2020.04.001).
- RATKOVICH N., HORN W., HELMUS F.P., ROSENBERGER S., NAESSENS W., NOPENS I., BENTZEN T.R. 2013. Activated sludge rheology: A critical review on data collection and modelling. *Water Research*. Vol. 47. Iss. 2 p. 463–482. DOI [10.1016/j.watres.2012.11.021](https://doi.org/10.1016/j.watres.2012.11.021).
- REMAÎTRE A., MALET J.P., MAQUAIRE O., ANCEY C., LOCAT J. 2005. Flow behaviour and runout modelling of a complex debris flow in a clay-shale basin. *Earth Surface Processes and Landforms*. Vol. 30. Iss. 4 p. 479–488. DOI [10.1002/esp.1162](https://doi.org/10.1002/esp.1162).
- SLATTER P. 2003. Pipeline transport of thickened sludges. *Water*. Vol. 21. p. 56–57.
- SLATTER P. 2008. Pipe flow of highly concentrated sludge. *Journal of Environmental Science and Health, Part A. Toxic/Hazardous Substances and Environmental Engineering*. Vol. 43. Iss. 13 p. 1516–1520. DOI [10.1080/10934520802293636](https://doi.org/10.1080/10934520802293636).
- SLATTER P.T. 1997. The rheological characterisation of sludges. *Water Science and Technology*. Vol. 36. Iss. 11 p. 9–18. DOI [10.1016/S0273-1223\(97\)00663-X](https://doi.org/10.1016/S0273-1223(97)00663-X).
- WANG H., ZHAO C., CHEN S. 2019. Method for determining the critical velocity of paste-like slurry filling into goaf using computational fluid dynamics. *Arabian Journal of Geosciences*. Vol. 12, 528. DOI [10.1007/s12517-019-4664-x](https://doi.org/10.1007/s12517-019-4664-x).
- ZHANG X., LI Z., ZHANG Z., LI Y. 2018. Discrete element analysis of the rheological characteristics of self-compacting concrete with irregularly shaped aggregate. *Arabian Journal of Geosciences*. Vol. 11, 597. DOI [10.1007/s12517-018-3960-1](https://doi.org/10.1007/s12517-018-3960-1).