10.24425/acs.2021.137421

Archives of Control Sciences
Volume 31(LXVII), 2021
No. 2*, pages 333–345*

Synchronization of FitzHugh-Nagumo reaction-diffusion systems via one-dimensional linear control law

Adel OUANNAS, Fatiha MESDOUI, Shaher MOMANI, Iqbal BATIHA and Giuseppe GRASSI

The Fitzhugh-Nagumo model (FN model), which is successfully employed in modeling the function of the so-called membrane potential, exhibits various formations in neuronal networks and rich complex dynamics. This work deals with the problem of control and synchronization of the FN reaction-diffusion model. The proposed control law in this study is designed to be uni-dimensional and linear law for the purpose of reducing the cost of implementation. In order to analytically prove this assertion, Lyapunov's second method is utilized and illustrated numerically in one- and/or two-spatial dimensions.

Key words: FitzHugh-Nagumo; synchronization; uni-dimensional control; linear control; reaction-diffusion system; neuronal networks; Lyapunov's second method

1. Introduction

Due to the extreme complexity of the nervous system and its importance in the human body, many biologists, chemists, psychiatrists, computer scientists, physicists, and even mathematicians contributed to the study of this central part

Copyright © 2021. The Author(s). This is an open-access article distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (CC BY-NC-ND 4.0 https://creativecommons.org/licenses/by-nc-nd/4.0/), which permits use, distribution, and reproduction in any medium, provided that the article is properly cited, the use is non-commercial, and no modifications or adaptations are made

A. Ouannas (e-mail: ouannas.adel@univ-oeb.dz) is with Laboratory of Dynamical Systems and Control, University of Larbi Ben M'hidi, Oum El Bouaghi 04000, Algeria.

F. Mesdoui (e-mail: mesdoui@gmail.com) is with Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan.

S. Momani (e-mail: s.momani@ajman.ac.ae) is with Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan and with Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman, UAE.

I. Batiha (corresponding author, e-mail: ibatiha@inu.edu.jo) is with Department of Mathematics, Faculty of Science and Technology, Irbid National University, Irbid 2600, Jordan and Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman, UAE.

G. Grassi (e-mail: giuseppe.grassi@unisalento.it) is with Dipartimento Ingegneria Innovazione, Universita del Salento, 73100 Lecce, Italy.

Received 08.01.2020. Revised 09.03.2021.

of the body. One of the most interesting questions in studying the nervous system is how neurons can synchronize. Synchronization is a process that makes two or more systems oscillate in a harmonious way and have the same behavior over time [13, 25]. In consequence of its efficient implementations in secure communications, laser technology, cryptography, combinatorial optimization and ecological systems [8], different methods were developed as well as various control techniques were implemented to accomplish synchronization in low- [1, 7, 12, 19, 20, 22, 23, 26], or high-dimension domains [10, 14–16, 21, 24, 29, 32, 33].

Like most natural processes, the activities of a neuron can be described by several equations analyzing the evolution of its characteristics over time. This description is usually called a mathematical model. In the literature, a few neuron models have been recently developed to explain neuronal dynamics. One of the simplest modification of the well-known Hodgkin-Huxley model [11] is the Fitzhugh-Nagumo model (FN model) [9, 17] that usefully describes such dynamics. Several significant researches were devoted to analyze the synchronization of the uni-dimensional FN model through describing its dynamics via some appropriate Ordinary Differential Equations (ODEs). For instance, H_∞ universe fuzzy approach [28], single- and two-input control technique [5], feedback control scheme [18], nonlinear feedback and adaptive controls approaches [27], and the internal model technique for spatially homogeneous FN model [31], are some examples of those researches. Although, in real neural modeling, the effect of the spatial component can not be avoided and the FN model must be presented by Partial Differential Equations (PDEs), the synchronization in the spatio-temporal domain of such model remains limited, just some results can be found in [2–4]. However, this work addresses some control techniques and some synchronization's analysis for the FN reaction-diffusion model that was expressed in [30] as follows:

$$\begin{cases} \dfrac{\partial u_1(x,t)}{\partial t} = d_1 \Delta u_1 - u_2 + f(u_1) + I, \\ \dfrac{\partial u_2(x,t)}{\partial t} = d_2 \Delta u_2 + \varepsilon u_1 - \varepsilon \gamma u_2, \end{cases} \qquad x \in \Omega, \quad t > 0. \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ $(n \geqslant 1)$, and $f(u)$ is a nonlinear function given by:

$$f(u) = -u^3 + (1+\alpha)u^2 - \alpha u. \tag{2}$$

In this spatially extended system, the state u_1 corresponds to the membrane potential, while u_2 represents a combination of potassium activation and sodium inactivation at point $(x,t) \in \Omega \times (0, \infty)$. The parameters α, ε and γ are positive constants in which $0 < \alpha < 1$ and $\varepsilon \ll 1$. The parameter I corresponds to the external injected current. At the boundary of Ω, we assume that system (1) satisfies the following homogeneous Neumann conditions:

$$\frac{\partial u_1}{\partial \eta} = \frac{\partial u_2}{\partial \eta} = 0, \quad x \in \partial\Omega, \tag{3}$$

where η is the unit vector normal to $\partial\Omega$.

Due to significant benefit of this model, a suitable control scheme will be next designed in a viable format to achieve synchronization between two neurons. The resultant findings will be analytically proved using some properties of the solution of system (1) together with the Lyapunov's second method. These findings will be then displayed numerically in one- and two-spatial dimensions. However, the structure of this paper is arranged as follow: In section 2, the problem of drive-response is formulated and the error associated with synchronization is defined mathematically. Section 3 presents the main resultant findings of this work. Section 4 illustrates how such findings can be applicable through using several numerical simulations. At the end section, the final conclusion and some concluding remarks are reported.

2. Problem formulation

To assess the possibility of synchronizing couple of FN models, the drive-response method is implemented where these two models can be coupled using certain functions called controllers. The role of these functions is to force the response system's output follows the drive system's output over time. This mechanism is called the complete synchronization and, consequently, the two considered systems are said to be completely synchronized. In particular, the drive system (1) can be coupled with the following response system:

$$\begin{cases} \dfrac{\partial v_1(x,t)}{\partial t} = d_1 \Delta v_1 - v_2 + f(v_1) + I + \boldsymbol{C}, \\ \dfrac{\partial v_2(x,t)}{\partial t} = d_2 \Delta v_2 + \varepsilon v_1 - \varepsilon\gamma v_2, & x \in \Omega, \quad t > 0. \\ \dfrac{\partial v_1}{\partial \eta} = \dfrac{\partial v_2}{\partial \eta} = 0, \quad x \in \partial\Omega, \end{cases} \tag{4}$$

We assume here that the parameters and the nonlinear function are the same as in system (1), but the associate initial conditions are arbitrary. The main aim of this study is to design a suitable controller $\boldsymbol{C}$ in a simplest form, making the control scheme cheaper in the application framework and easier to implement. To express the aforementioned details mathematically, we define the synchronization error between the two systems, system (1) and system (4), as follows:

$$e(x,t) = \begin{pmatrix} e_1 = v_1 - u_1 \\ e_2 = v_2 - u_2 \end{pmatrix}. \tag{5}$$

Afterwards, we intend to prove that this error converges to zero as t goes to infinity. For purpose of clarity, we state the following definition:

Definition 1 *The drive and response systems given, respectively, in (1) and (4) are called completely synchronized if*

$$\lim_{t\to\infty} \|e(x,t)\|_{L^2} = 0. \tag{6}$$

Before we present the main result of this study, it is necessary to note that model (1) is well-posed and its associated solution is uniformly bounded. In particular, the following Lemma confirms and summarised all these affirmations.

Lemma 1 *[6] Model (1) admits a global unique solution (u_1, u_2) and $\exists K \in \mathbb{R}^+$ such that:*

$$u_1(x,t), u_2(x,t) \leqslant K$$

for all (x,t) in $\Omega \times [0,\infty)$.

3. Main results

In a logically equivalent manner to definition 1, we can prove the complete synchronization between system (1) and system (4) by showing that the zero solution of the following synchronization error system:

$$\begin{cases} \dfrac{\partial e_1(x,t)}{\partial t} = d_1\Delta e_1 - e_2 + f(v_1) - f(u_1) + \boldsymbol{C}, \\ \dfrac{\partial e_2(x,t)}{\partial t} = d_2\Delta e_2 + \varepsilon e_1 - \varepsilon\gamma e_2, \end{cases} \qquad x \in \Omega, \quad t > 0 \tag{7}$$

is globally asymptotically stable. At the boundary, it is clear that the above system satisfies the Neumann conditions:

$$\frac{\partial e_1}{\partial \eta} = \frac{\partial e_2}{\partial \eta} = 0, \qquad x \in \partial\Omega. \tag{8}$$

Theorem 1 *The drive and the response systems given, respectively, in (1) and (4) are completely synchronized in accordance with the following one-dimensional linear control law:*

$$\boldsymbol{C} = -\left(3K^2 + 2(1+\alpha)K\right) e_1 + (1-\varepsilon)e_2, \tag{9}$$

where K is positive constant given in Lemma 1.

Proof. Substituting controller (9) in the error system given in (7) yields:

$$\begin{cases} \dfrac{\partial e_1(x,t)}{\partial t} = d_1 \Delta e_1 - \varepsilon e_2 - \left(3K^2 + 2(1+\alpha)K\right) e_1 + f(v_1) - f(u_1), \\ \dfrac{\partial e_2(x,t)}{\partial t} = d_2 \Delta e_2 + \varepsilon e_1 - \varepsilon \gamma e_2, \qquad x \in \Omega, \quad t > 0 \end{cases}$$

To prove the global stability of the zero solution of system (7), we use the Lyapunov's direct (second) method together with the following positive definite Lyapunov functional:

$$V = \frac{1}{2} \int \left(e^T e\right) \mathrm{d}x.$$

This leads us to prove that the derivative of this functional with respect to t is negative definite. Actually, this can be carried out as follows:

$$\begin{aligned} \frac{\partial V}{\partial t} &= \int_\Omega \left(e_1 \frac{\partial e_1}{\partial t} + e_2 \frac{\partial e_2}{\partial t}\right) \mathrm{d}x \\ &= \int_\Omega e_1 \left(d_1 \Delta e_1 - \varepsilon e_2 - \left(3K^2 + 2(1+\alpha)K\right) e_1 + f(v_1) - f(u_1)\right) \mathrm{d}x \\ &\quad + \int_\Omega e_2 \left(d_2 \Delta e_2 + \varepsilon e_1 - \varepsilon \gamma e_2\right) \mathrm{d}x = \int_\Omega \left(d_1 e_1 \Delta e_1 + d_2 e_2 \Delta e_2\right) \mathrm{d}x \\ &\quad - \int_\Omega \left(\left(3K^2 + 2(1+\alpha)K\right) e_1^2 - \varepsilon \gamma e_2^2\right) \mathrm{d}x + \int_\Omega e_1 \left(f(v_1) - f(u_1)\right) \mathrm{d}x. \end{aligned}$$

In view of Eq. (2) and Lemma 1, we can attain the following assertion:

$$\begin{aligned} \frac{\partial V}{\partial t} &= \int_\Omega \left(d_1 e_1 \Delta e_1 + d_2 e_2 \Delta e_2\right) \mathrm{d}x - \int_\Omega \left(\left(3K^2 + 2(1+\alpha)K\right) e_1^2 - \varepsilon \gamma e_2^2\right) \mathrm{d}x \\ &\quad + \int_\Omega \left(|v_1|^2 + |v_1|\,|u_1| + |u_1|^2 + (1+\alpha)\left(|u_1| + |v_1|\right) - \alpha\right) e_1^2 \mathrm{d}x, \\ &\leqslant \int_\Omega \left(d_1 e_1 \Delta e_1 + d_2 e_2 \Delta e_2\right) \mathrm{d}x - \int_\Omega \left((3K^2 + 2(1+\alpha)K) e_1^2 - \varepsilon \gamma e_2^2\right) \mathrm{d}x \\ &\quad + \int_\Omega \left(3K^2 + 2(1+\alpha)K - \alpha\right) e_1^2 \mathrm{d}x, \\ &\leqslant \int_\Omega \left(d_1 e_1 \Delta e_1 + d_2 e_2 \Delta e_2\right) \mathrm{d}x - \int_\Omega \left(\alpha e_1^2 + \varepsilon \gamma e_2^2\right) \mathrm{d}x. \end{aligned}$$

By using Green's formula, one can show that the derivative of the Lyapunov functional V satisfies the following estimate:

$$\frac{\partial V}{\partial t} \leqslant -d_1 \int\limits_{\Omega} |\nabla e_1|^2 \,\mathrm{d}x + d_1 \int\limits_{\partial\Omega} e_1 \frac{\partial e_1}{\partial \eta} \mathrm{d}\sigma - d_2 \int\limits_{\Omega} |\nabla e_2|^2 \,\mathrm{d}x + d_2 \int\limits_{\partial\Omega} e_2 \frac{\partial e_2}{\partial \eta} \mathrm{d}\sigma - \alpha \int\limits_{\Omega} e_1^2 \mathrm{d}x - \varepsilon\gamma \int\limits_{\Omega} e_1^2 \mathrm{d}x.$$

Accordingly, using the homogeneous Neumann conditions given in (8) leads one to obtain:

$$\frac{\partial V}{\partial t} \leqslant -\left(d_1 \int\limits_{\Omega} |\nabla e_1|^2 \,\mathrm{d}x + d_2 \int\limits_{\Omega} |\nabla e_2|^2 \,\mathrm{d}x + \alpha \int\limits_{\Omega} e_1^2 \mathrm{d}x + \varepsilon\gamma \int\limits_{\Omega} e_1^2 \mathrm{d}x \right) < 0,$$

which finishes the proof. □

4. Numerical simulations

This part intends to demonstrate some numerical illustrations in one- and two-spacial dimensions to explain the suitability of the synchronization scheme described in this article. These simulations have been carried out using the finite difference approach and MATLAB software. In such simulations, we let $\Omega = [0,\ 50]$, $t \leqslant 100$ and

$$(d_1, d_2, \alpha, \varepsilon, \gamma, I) = (0.5, 0.8, 0.139, 0.008, 2.54, 2). \tag{10}$$

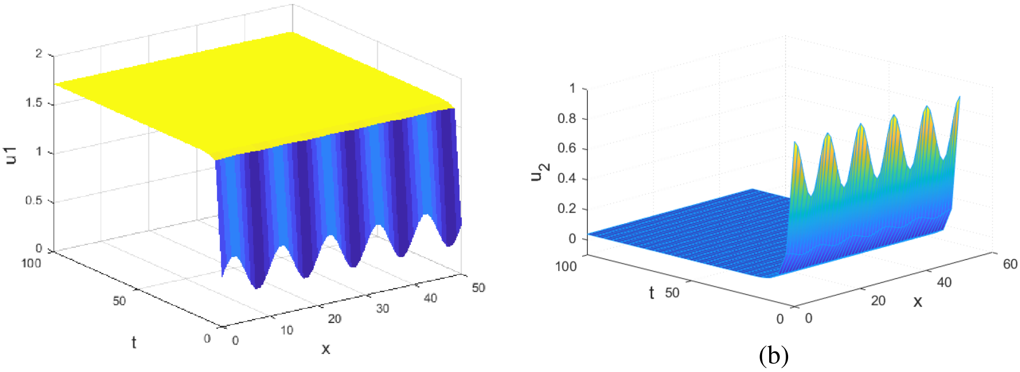
On the other hand, we select the initial condition associated with the drive system to be as follows:

$$(u_1(x,0), u_2(x,0)) = \left(0.5 + 0.1 \sin\left(\frac{\pi x}{5}\right),\ 0.8 + 0.2 \cos\left(\frac{\pi x}{5}\right)\right). \tag{11}$$

The dynamics of the spatio-temporal solutions of system (1) are exhibited in Fig. 1. For more illustration, Fig. 2 depicts these solutions in 2-dimensional space (2D-space). For comparison reasons, we plot the uncontrolled response system (i.e., system (4) with $C = 0$) in one- and two-spatial dimensions (see Fig. 3 and Fig. 4), where the initial condition associated with system (4) is given as follows:

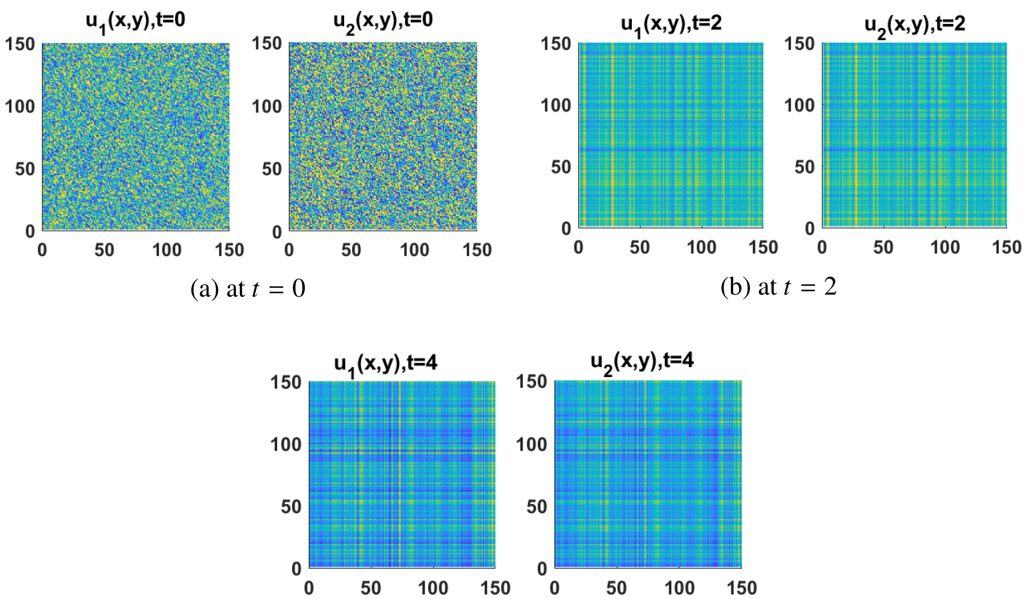
$$(v_1(x,0), v_2(x,0)) = (1.5 + 0.2 \sin(x),\ 0.28 + 0.21 \cos(x)). \tag{12}$$

Comparing with the dynamics of the drive system (Fig. 1 and 2), one might notice that both figures, Fig. 3 and Fig. 4, have confirmed that the uncontrolled response



(a) (b)

Figure 1: Dynamic behavior of the drive system (1) with $d_1 = 0.5$, $d_2 = 0.8$, $\alpha = 0.139$, $\varepsilon = 0.008$, $\gamma = 2.54$, $I = 2$ according to the initial conditions given in (11)



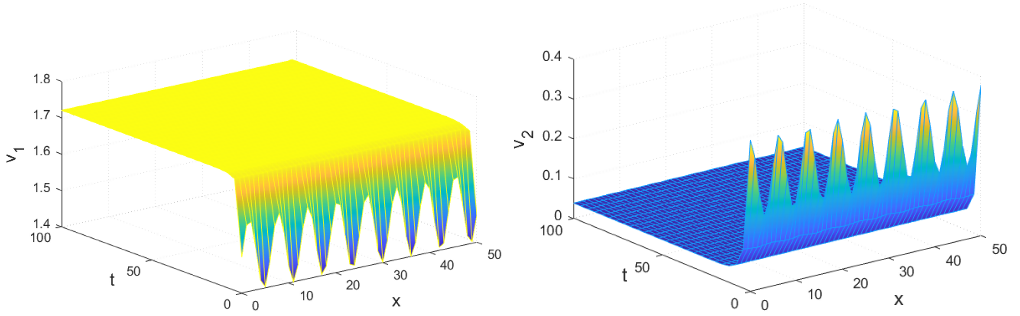
(a) at $t = 0$ (b) at $t = 2$

(c) at $t = 4$

Figure 2: Solution of the drive system (1) in 2D-space

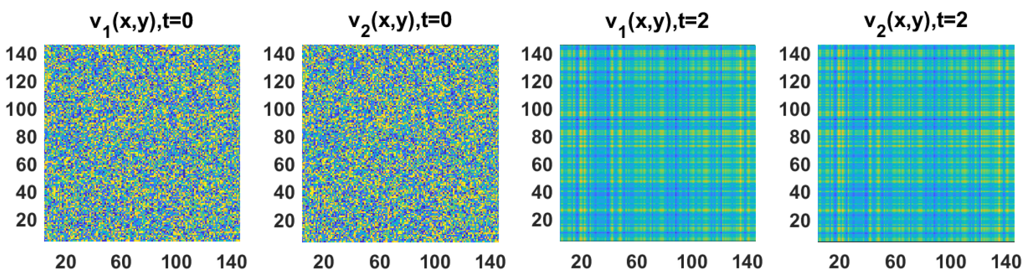
system (4) does not synchronize with the drive system (1). In the same perspective and based on Theorem 1, if one selects $K = 0.2$, then the uni-dimensional linear controller will be designed as follows:

$$\boldsymbol{C} = 3.259e_1 + 0.992e_2$$

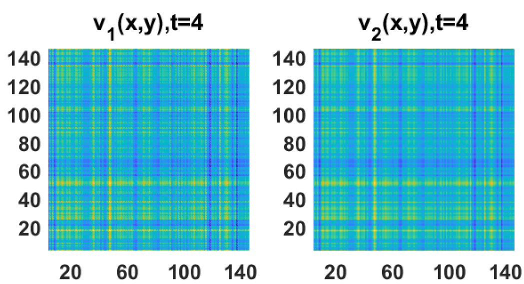


(a) (b)

Figure 3: Dynamic behavior of the response system (4) with $d_1 = 0.5$, $d_2 = 0.8$, $\alpha = 0.139$, $\varepsilon = 0.008$, $\gamma = 2.54$, $I = 2$ according to the initial conditions given in (12)



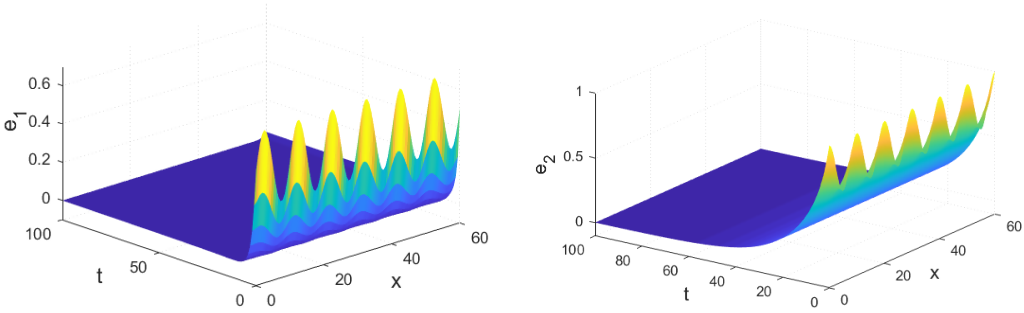
(a) at $t = 0$ (b) at $t = 2$



(c) at $t = 4$

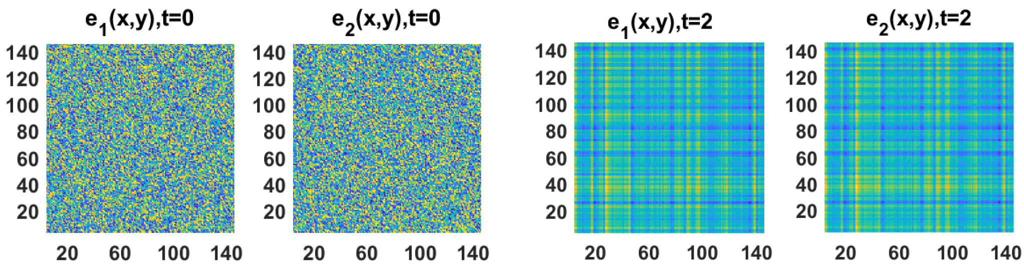
Figure 4: Solution of the response system (4) in 2D-space

with noting that the drive system (1) and the response system (4) will be also globally synchronized. To illustrate this numerically, the spatiotemporal solutions of the error synchronization system (7) are provided in Fig. 5 and Fig. 6 in one- and two-dimensional space. This evolution indicates, consequently, that all errors go to 0 as t goes to $+\infty$.

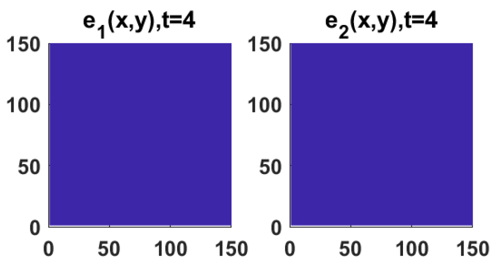


(a) (b)

Figure 5: Dynamic behavior of the solutions of the spatiotemporal synchronization error system (5) with $d_1 = 0.5$, $d_2 = 0.8$ and $K = 0.2$



(a) at $t = 0$ (b) at $t = 2$



(c) at $t = 4$

Figure 6: Solution of the spatiotemporal synchronization error system (5) in 2D-space

5. Conclusion

Over the past few years, several researchers were focused on studying the nervous system, especially in how neurons synchronize with each others. In this paper, we have developed a novel control scheme to achieve synchronization between two neurons in spatially extended domain. This contribution has been proved rigorously with the use of the uniform boundedness of the unique solution of the Fitzhugh-Nagumo model and the Lyapunov's second method. In order to reduce the implementation cost, this controller has been designed in unidimensional and linear form, and then it has been illustrated numerically in one- and two-spatial dimensions. There is no question that the findings of this study will motivate us to discuss this subject further. For this reason, we are planning to analyze synchronization in many types of spatially extended systems, including lattice maps, stochastic and fractional-order models.

References

[1] S.K. Agrawal and S. Das: A modified adaptive control method for synchronization of some fractional chaotic systems with unknown parameters. *Nonlinear Dynamics*, **73**(1), (2013), 907–919, DOI: 10.1007/s11071-013-0842-7.

[2] B. Ambrosio and M.A. Aziz-Alaoui: Synchronization and control of coupled reaction–diffusion systems of the FitzHugh–Nagumo type. *Computers & Mathematics with Applications*, **64**(5), (2012), 934–943, DOI: 10.1016/j.camwa.2012.01.056.

[3] B. Ambrosio, M.A. Aziz-Alaoui, and V.L.E. Phan: Global attractor of complex networks of reaction-diffusion systems of Fitzhugh-Nagumo type. *Discrete & Continuous Dynamical Systems*, **23**(9), (2018), 3787–3797, DOI: 10.3934/dcdsb.2018077.

[4] B. Ambrosio, M. A. Aziz-Alaoui, and V.L.E. Phan: Large time behaviour and synchronization of complex networks of reaction–diffusion systems of FitzHugh–Nagumo type. *IMA Journal of Applied Mathematics*, **84**(2), (2019), 416–443, DOI: 10.1093/imamat/hxy064.

[5] M. Aqil, K.-S. Hong, and M.-Y. Jeong: Synchronization of coupled chaotic FitzHugh–Nagumo systems. *Communications in Nonlinear Science and Numerical Simulation*, **17**(4), (2012), 1615–1627, DOI: 10.1016/j.cnsns.2011.09.028.

[6] S. Bendoukha, S. Abdelmalek, and M. Kirane: The global existence and asymptotic stability of solutions for a reaction–diffusion system. Nonlinear Analysis: Real World Applications. 53, (2020), 103052, DOI: 10.1016/j.nonrwa.2019.103052.

[7] X.R. Chen and C.X. Liu: Chaos synchronization of fractional order unified chaotic system via nonlinear control. *International Journal of Modern Physics B*, **25**(03), (2011), 407–415, DOI: 10.1142/S0217979211058018.

[8] D. Eroglu, J.S.W. Lamb, and Y. Pereira: Synchronisation of chaos and its applications. *Contemporary Physics*, **58**(3), (2017), 207–243, DOI: 10.1080/00107514.2017.1345844.

[9] R. Fitzhugh: Thresholds and Plateaus in the Hodgkin-Huxley Nerve Equations. *The Journal of General Physiology*, **43**(5), (1960), 867–896, DOI: 10.1085/jgp.43.5.867.

[10] P. García, A. Acosta, and H. Leiva: Synchronization conditions for master-slave reaction diffusion systems. *EPL*, **88**(6), (2009), 60006.

[11] A.L. Hodgkin and A.F. Huxley: A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol*, 117, (1952), 500–544, DOI: 10.1113/jphysiol.1952.sp004764.

[12] T. Kapitaniak: Continuous control and synchronization in chaotic systems. *Chaos, Solitons & Fractals*, **6** (1995), 237–244, DOI: 10.1016/0960-0779(95)80030-K.

[13] A.C.J. Luo: *Dynamical System Synchronization*. Springer-Verlag, New York. 2013.

[14] D. Mansouri, S. Bendoukha, S. Abdelmalek, and A. Youkana: On the complete synchronization of a time-fractional reaction–diffusion system with the Newton–Leipnik nonlinearity. *Applicable Analysis*, **100**(3), (2021), 675–694, DOI: 10.1080/00036811.2019.1616694.

[15] F. Mesdoui, A. Ouannas, N. Shawagfeh, G. Grassi, and V.-T. Pham: Synchronization Methods for the Degn-Harrison Reaction-Diffusion Systems. *IEEE Access.*, **8** (2020), 91829–91836, DOI: 10.1109/ACCESS.2020.2993784.

[16] F. Mesdoui, N. Shawagfeh, and A. Ouannas: Global synchronization of fractional-order and integer-order N component reaction diffusion systems: Application to biochemical models. *Mathematical Methods in the Applied Sciences*, **44**(1), (2021), 1003–1012, DOI: 10.1002/mma.6807.

[17] J. Nagumo, S. Arimoto, and S. Yoshizawa: An active pulse transmission line simulating nerve axon. *Proceedings of the IRE*, **50**(10), (1962), 2061–2070, DOI: 10.1109/JRPROC.1962.288235.

[18] L.H. Nguyen and K.-S. Hong: Synchronization of coupled chaotic FitzHugh–Nagumo neurons via Lyapunov functions. *Mathematics and Computers in Simulation*, **82**(4), (2011), 590–603, DOI: 10.1016/j.matcom.2011.10.005.

[19] Z.M. Odibat: Adaptive feedback control and synchronization of non-identical chaotic fractional order systems. *Nonlinear Dynamics*, **60**(4), (2010), 479–487, DOI: 10.1007/s11071-009-9609-6.

[20] Z.M. Odibat, N. Corson, M.A. Aziz-Alaoui, and C. Bertelle: Synchronization of chaotic fractional-order systems via linear control. *International Journal of Bifurcation and Chaos*, **20**(1), (2010), 81–97, DOI: 10.1142/S0218127410025429.

[21] A. Ouannas, M. Abdelli, Z. Odibat, X. Wang, V.-T. Pham, G. Grassi, and A. Alsaedi: Synchronization Control in Reaction-Diffusion Systems: Application to Lengyel-Epstein System. *Complexity*, (2019), Article ID 2832781, DOI: 10.1155/2019/2832781.

[22] A. Ouannas, Z. Odibat, N. Shawagfeh, A. Alsaedi, and B. Ahmad: Universal chaos synchronization control laws for general quadratic discrete systems. *Applied Mathematical Modelling*, **45** (2017), 636–641, DOI: 10.1016/j.apm.2017.01.012.

[23] A. Ouannas, Z. Odibat, and N. Shawagfeh: A new Q–S synchronization results for discrete chaotic systems. *Differential Equations and Dynamical Systems*, **27**(4), (2019), 413–422, DOI: 10.1007/s12591-016-0278-x.

[24] N. Parekh, V.R. Kumar, and B.D. Kulkarni: Control of spatiotemporal chaos: A study with an autocatalytic reaction-diffusion system. *Pramana – J. Phys.*, **48**(1), (1997), 303–323, DOI: 10.1007/BF02845637.

[25] L.M. Pecora and T.L. Carroll: Synchronization in chaotic systems. *Physical Review Letter*, bf 64(8), (1990), 821–824, DOI: 10.1103/PhysRevLett.64.821.

[26] M. Srivastava, S.P. Ansari, S.K. Agrawal, S. Das, and A.Y.T. Leung: Anti-synchronization between identical and non-identical fractional-order chaotic systems using active control method. *Nonlinear Dynamics*, **76** (2014), 905–914, DOI: 10.1007/s11071-013-1177-0.

[27] J. Wang, T. Zhang, and B. Deng: Synchronization of FitzHugh–Nagumo neurons in external electrical stimulation via nonlinear control. *Chaos, Solitons & Fractals*, **31**(1), (2007), 30–38, DOI: 10.1016/j.chaos.2005.09.006.

[28] J. Wang, Z. Zhang, and H. Li: Synchronization of FitzHugh–Nagumo systems in EES via H^{∞} variable universe adaptive fuzzy control. *Chaos, Solitons & Fractals*, **36**(5), (2008), 1332–1339, DOI: 10.1016/j.chaos.2006.08.012.

[29] L. Wang and H. Zhao: Synchronized stability in a reaction–diffusion neural network model. *Physics Letters A*, **378**(48), (2014), 3586–3599, DOI: 10.1016/j.physleta.2014.10.019.

[30] J. Wei and M. Winter: Standing waves in the FitzHugh-Nagumo system and a problem in combinatorial geometry. *Mathematische Zeitschrift*, **254**(2), (2006), 359–383, DOI: 10.1007/s00209-006-0952-8.

[31] X. Wei, J. Wang, and B. Deng: Introducing internal model to robust output synchronization of FitzHugh–Nagumo neurons in external electrical stimulation. *Communications in Nonlinear Science and Numerical Simulation*, **14**(7), (2009), 3108–3119, DOI: 10.1016/j.cnsns.2008.10.016.

[32] F. Wu, Y. Wang, J. Ma, W. Jin, and A. Hobiny: Multi-channels coupling-induced pattern transition in a tri-layer neuronal network. *Physica A: Statistical Mechanics and its Applications*, **493** (2018), 54–68, DOI: 10.1016/j.physa.2017.10.041.

[33] K.-N. Wu, T. Tian, and L. Wang: Synchronization for a class of coupled linear partial differential systems via boundary control. *Journal of the Franklin Institute*, **353**(16), (2016), 4062–4073, DOI: 10.1016/j.jfranklin.2016.07.019.