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Simulation of loading and wear rate distribution on cutting edges during gears hobbing

Results of complex mathematical and computer simulation of gear hobbing are given. A systematic approach to research allowed for the development of simulation models and sequencing of all aspects of this complex process. Based on the modeling of non-deformable chips, a new analytical method for analyzing hobbing has been proposed. The shear, friction and cutting forces at the level of certain teeth and edges in the active space of the cutter are analyzed depending on the cut thickness, cross-sectional area, intensity of plastic deformation and length of contact with the workpiece has been developed. The results of computer simulations made it possible to evaluate the load distribution along the cutting edge and to predict the wear resistance and durability of the hob cutter, as well as to develop measures and recommendations for both the tool design and the technology of hobbing in general. Changing the shape of cutting surface, or the design of the tooth, can facilitate separation of the cutting process between the head and leading and trailing edges. In this way, more efficient hobbing conditions can be achieved and the life of the hob can be extended.

1. Introduction

Hobs are among the most complex and expensive cutting tools. They are characterized by high costs due to their high accuracy, the quality of the cutting teeth surfaces, including applied wear resistant coatings, as a prerequisite for achieving the high-quality requirements of the machined gears.

The use of precision hobs creates a number of very serious difficulties associated with their design, manufacture and operation. One of the important drawbacks

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of the hobs is the uneven loading of the teeth during work rotation. The tooth farthest from the axial perpendicular “hob-workpiece” cuts off chips of the greatest thickness. When approaching the interaxial perpendicular, the teeth cut off thinner layers. With an increase in feed, more distant teeth enter the cutting zone cutting off the thickest chips. It is believed that the feed rate is limited by the strength of the most loaded tooth. If all the teeth are loaded evenly, that is, when the part of load from the extreme teeth is shifted to the teeth located closer to the central tooth, the cutter is able to work with feeds that are many times higher than those normally accepted. Irregular load of the cutter increases the wear of the cutting edges, at the same time reducing the life of such an expensive tool. Thus, the analysis of the dynamics of loads on the cutting teeth of the hob, along with subsequent consideration of this factor in the design of the tool, is of great scientific and practical importance.

2. Literature review

The complex parameter that most fully allows one to identify the resource of the hob cutter at the level of its individual edges and teeth is the work that is done during the hobbing. This parameter directly depends on the parameters of the cuts and the intensity of shear and friction on the contact surfaces during cutting. It is an indicator of deformation resulting from contact loading, thermal and tribological processes that take place during generative type of gear machining. Obviously, the studies on hobbing have both scientific and practical importance, since they can be used as the basis for both predicting the intensity of wear, ensuring the working capacity and the necessary stability of the hobs, as well as for choosing the optimal cutting parameters, calculating the cutting force and elastic vibrations. These may provide the necessary processing accuracy, ensuring roughness of the machined gear teeth and the specified process productivity, and facilitating the choice of tool material, hardening of the cutting teeth, the type of coating, etc.

The most common parameter that characterizes the working capacity of a cutting tool is wear. Numerous studies have been devoted to investigating the hob tool life. This problem is most fully described in the works [1–9]. A common feature of these studies is an extensive use of modern software packages for simulating non-deformable chips, loading, stress-strain and thermal states, as well as the creation of original programs for simulating such processes based on original mathematical models and algorithms. Another feature of these works is high-quality visualization of geometric shapes, and processes and phenomena that accompany hobbing. At the same time, a critical analysis of the mentioned literature sources shows that the dependences they present are empirical rather than analytical, and that the studies do not sufficiently rely on the basic laws of the cutting theory. For example, K.-D. Bouzakis assumes that the value of the chip thickness ratio is proportional to the hob cutter wear [6], although such a relationship is non-linear. Such an approximation is not scientifically substantiated, since there is a number of non-

linear factors affecting the wear of the hob, such as friction on the rake and flank faces, the increase in intensity of chip deformation and the increased friction resulting from restricted contact cutting effect, the specificity of kinematics of the cutting edges installed at an angle to the plane of the hob rotation, etc.

Another example is the relationship between wear and geometric parameters of the cross sections of the chip. In [5], wear prediction is based on experimentally-derived typical wear laws for five chip group, as a function of the number of successive cuts. These groups are characterized by a specific type of chip formation on the head, and trailing and leading edges. The authors draw conclusions about the influence of geometric parameters of the chips on the wear rate of the tool edges. Experimental data are combined with geometric modeling of non-deformable chips, and one determines the zones of chip layering from side and head edges with the lowest wear resistance. The results of the study are presented as recommendations for the tangential displacement of the hob cutter during gear hobbing for more uniform wear of the cutting teeth. However, there is no quantitative assessment of the parameters of the chips that has been shaped by the side and head edges of the hob's active teeth. Accordingly, there is no analytical wear model based on computer modeling of the geometry of the cut chips, and the influence of thermal and frictional causes of wear is assessed only visually, according to the shape of the cut layers.

In [9], the authors use the thickness and width of the cut layer to simulate the cutting force and its components during finishing gear hobbing, as well as for evaluating the thermal load arising from the action of these forces. However, these parameters are obtained from the planar hobbing diagram, which is very simplified in comparison with the three-dimensional model, so it is not possible to take into account true values of the parameters. In addition, the participation of the side edges of the hob cutter and their influence on the parameters under study aren't taken into account, therefore there appears a discrepancy between the experimental and theoretical data, typically equal to 20%.

Numerous works on this topic [1, 2, 4, 5, 8, 10–13] describe accelerated wear testing methods. The methods are based on hobbing simulation on multi-axis CNC machines, which reproduce the kinematics of the hob cutter using simplified cutting tools – such as a single-tooth linear fly hob, which simulate hobbing cutting on gear hobbing machines and multi-axis CNC machines. However, the main disadvantage of this approach is that the kinematics of cutting with this hob cutter only partially corresponds to the generative type of gear hobbing. The continuity of the process and the regularities of the formation of the gap are violated. In reality, the gap is formed by dozens of teeth on the helical surface of the hob cutter simultaneously. As a result, the chips that are formed by this tool are deformed in comparison with the real ones, and their parameters do not correspond to the actual size and shape, especially at the side edges.

The load calculation according to the Hoffmeister equation [14] based on thickness and length of chip is imperfect. On the one hand, increasing the thickness

of the chips and, accordingly, the cross-sectional area of the cut, as part of the cutting force, leads to an increase in load. However, according to the basic principles of the cutting theory [15], increasing the thickness of the cut causes a decrease in pressure and friction on the rake face. On the other hand, reducing the thickness of the cut leads to an increase in pressure on the rake face of the tool from the running chips; which increases the shear intensity and the cutting force. This factor has a much stronger effect on the cutting force, so it must be taken into account. The intensity of plastic strain increases especially when the thickness of the cut is commensurate with the radius of rounding of the blade, which causes surface crumpling, increased friction on the flank face and, as a result, accelerated tool wear.

According to what has been mentioned, there is a need for a methodology based on the basic principles of cutting theory and using scientific developments in this area for the analytical description of stress-strain, tribological and thermal processes during gear hobbing, taking into account all boundaries and predicting more exactly the place and intensity of wear. This method should take into account the participation of all teeth in the formation of gaps and profiling of gears, as well as the actual shape of the transition surface of the gaps in which the teeth cut the helical surface of the cutter, the thickness and cross-sectional area of the chips, as well as to take into consideration the effect of chip thickness ratio on the generation processes. In addition, it is possible to propose a new scheme for the use of a single-toothed cutter, which will take into account and reproduce all the kinematic movements of the hobbing and the participation of all teeth of the cutter in the chip's formation process.

Thus, the purpose of the present work is to study the operation of the hob cutter at the level of certain elements – teeth and edges, and to identify the possibility of improving tool operability and durability, as well as to investigate the wear development on the hob considering shifting kinematics.

3. Gear hobbing computer simulation and research on cutting kinematics data

As is well known [16], the cutting action is performed in three zones: in the shear plane, on the rake face to overcome friction resulting from the chip sliding, and on the flank face to overcome friction between the tool and the newly formed surface. When working with a sharpened tool, friction on the flank face can be conditionally neglected as a non-essential factor. Therefore, we will consider that the total cutting work W is defined by the first two components, which we denote W_{Φ} and W_{γ} , respectively:

$$W = W_{\Phi} + W_{\gamma} . \quad (1)$$

Let's define each of these components.

The plastic deformation work W_{Φ} performed by one cutter tooth of the hob can be described by equation:

$$W_{\Phi} = P p_c, \quad (2)$$

where P is the shear force, N; p_c is the length of the path of arc equal to the length of contact arc between the tool and the workpiece, mm.

At the same time, according to the basic laws of the cutting theory, shear force is a function of the chip cross section, the strength of the workpiece material and the intensity of chip compression ratio, expressed as [15, 17]:

$$P = S \bar{\tau} \xi, \quad (3)$$

where S is the area of the chip cross-section (mm^2), ξ is the chip compression ratio; τ is the ultimate shear strength of the workpiece material, MPa.

Therefore:

$$W_{\Phi} = S \bar{\tau} \xi p_c. \quad (4)$$

Thus, in order to quantify this work, it is necessary to solve the following problems: to find the area, thickness and width of the chip cross sections for the teeth and edges of the hob; to determine the dependence of chip compression ratio intensity on shear thickness; and to set cutting path length at teeth and edges level.

3.1. 3D modeling of non-deformed chips formed during hobbing

For this stage of research, the methodic developed and described in [15, 17] has been used, which takes into account all kinematic movements of generative tooth-cutting and simulates the spatial environment of hobbing. The analysis of chip parameters takes into consideration the peculiarities of the generative cutting process and shaping by hob, which is one of the most complex cutting processes. Simulation results are right for all types of gears, both spur and helical, because cut by the hobs is carried out according to the same geometric and technological principles.

Firstly, hobbing is a multi-tooth cutting process, which depends on the parameters of the hob and machined gear – module, the number of teeth and the tooth inclination angle. At the same time, it is necessary to take into account the fact that one gap is usually cut by dozens of teeth – from 1.5–2 to 60–80. Cutting conditions on each tooth differ from each other and vary periodically in both the hob rotation cycle and the workpiece rotation cycle.

Secondly, it is a multi-edge cutting process. Each tooth of trapezoidal shape has three edges – the head and the sides: the leading edge or the right edge; the initial trailing edge or the left edge and the head edge. Depending on the location of the tooth in the active position of the hob cutter, each of the edges can be use in cutting, and the three edges can be used in different combinations: all the three ones, the head and the leading one, the head and the trailing one, only the side edge (the leading or the trailing one), etc. The chips that these edges create continuously change in

shape and size, depending on the angle of rotation of the gear hob. Accordingly, the stress-strain state, the friction on the surfaces, the contact conditions between the tooth, the chip and the workpiece constantly change. Moreover, the vectors of the constituent cutting forces on each edge and the friction forces on the surfaces of the tooth cutting edge have different directions. This creates a periodic variation of the cutting force and its components, given for both an individual tooth and for the hob cutter and for the machine elastic system as a whole.

Because of complexity of the hobbing process and the features noted above, graphical interpretation of simulation results of chips generated during machining requires some preliminary explanation.

a. All continuous kinematic movements of tooth-cutting can be represented, without loss of simulation accuracy, as a sequence of discrete movements, the value of which is multiple of the hob rotation by one angular pitch. This is the primary elementary movement with the smallest duration: $\tau_h = 360^\circ / Z_h$. In this equation, Z_h is the number of teeth of the hob cutter.

During this time, the workpiece is rotated by the pressure angle $\psi = \frac{360^\circ}{Z_h q}$, also known as the angle of obliquity, and the point on the screw surface of the hob moves along its axis by the value of single movement $\Delta X = \frac{\pi m_n}{q}$ [15]. For the gear hob, this movement corresponds to the axial displacement of the generating rack, to which this tooth belongs, by a similar distance.

b. The zone of the workpiece in which the cutting takes place is three-dimensional. In this space, there are constantly located the following elements:

- one cutter rack characterizing the (potential) actively straight length of this section in the end plane of the workpiece L_{op} ,
- one or more teeth of the hob cutter in the end plane on the circular cutting path, characterizing the circular length of this space (only for a certain tooth). The distance between rack teeth is equal to the pitch, and the number of cutting teeth pertaining to one turn of the hob is equal to q .

c. Chips, which are removed by each tooth of the cutter, change in shape and cross-sectional area according to the profile angle. This area, like the thickness and width of the cut, takes some values. The section which we take into account considering the parameters of the cut layers coincides with the rake face of the rack in the zone with the maximum thickness of the cut. Fig. 1 shows how the volume chips are divided by the section plane into the leading, head and trailing ones, according to the tasks described in this article.

The spatial field of such parameters can be displayed with both spatial graphs (for example, axis “Z” is the section area S , axis “X” is the active length L_{op} , axis “Y” is the hob rotation angle φ_h) and the corresponding planar graphs with two coordinates [15].

d. The pair of gears to be machined with the hob is represented in rectangular projections, as shown in Fig. 2.

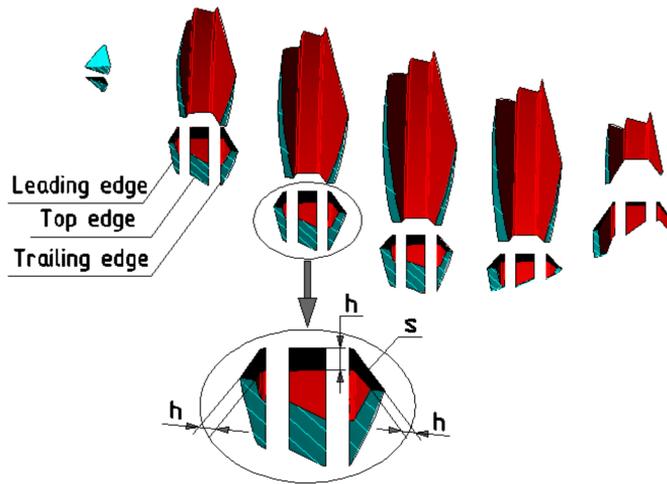


Fig. 1. The scheme of chip separation between the tooth edges of one cutter rack

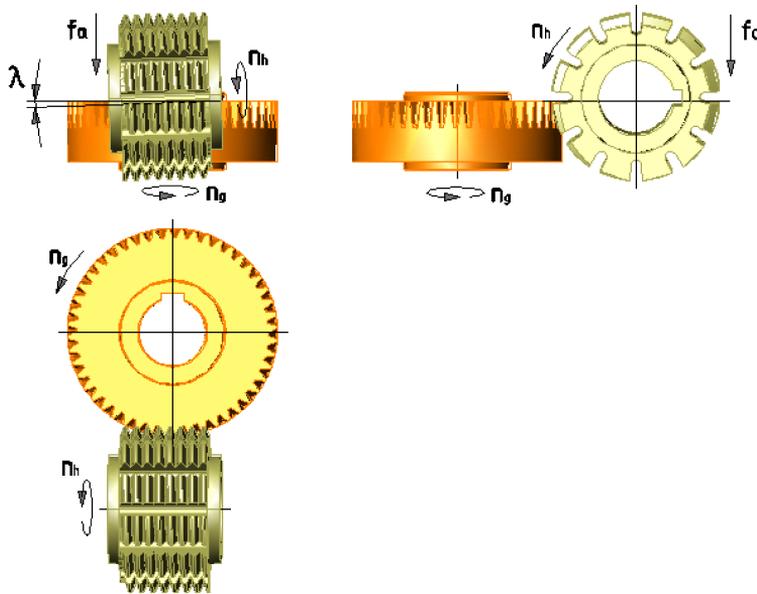


Fig. 2. Simulation scheme of machined gear and hob interposition

The initial data: right handed involute machined gear, normal module $m_n = 3$ mm, number of gear teeth $Z_g = 36$, gear helix angle $\beta = 35^\circ$; hob outside diameter $D_{ah} = 90$ mm and the number of columns (flank teeth) $q = 10$; the hob helix angle is $\lambda = 2.13^\circ$; up-cutting hobbing, axial feed is $f_a = 3$ mm/rev; the depth of cut equals to the full teeth height.

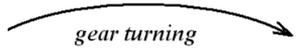
3D structure of non-deformable chips for all hob teeth is simulated in AutoCAD for these initial data based on the technique described above.

3D simulated models of the chip for every second column are shown in Table 1. Hereinafter the column teeth are marked with the numbers -3 , -2 , -1 , 0 , $+1$, $+2$, $+3$, which are the same as the marks for the hob threads, and the same numbers denote the gaps of the gear being machined.

According to our initial data, from among the seven threads that engage with the workpiece during cutting, the first thread works with the teeth No. 4–10, and

Table 1.

3D simulated models of the chip for every second column

Column No.	Thread No.						
	-3	-2	-1	0	1	2	3
1 (-180°)		 11	 21	 31	 41	 51	 61
3 (-108°)		 13	 23	 33	 43	 53	 63
5 (-36°)		 15	 25	 35	 45	 55	 65
7 ($+36^\circ$)	 7	 17	 27	 31	 47	 57	
9 ($+108^\circ$)	 9	 19	 29	 39	 49	 59	
10 ($+180^\circ$)		 11	 21	 31	 41	 51	 61
 <i>gear turning</i>							

the last one works with only a small portion of the trailing edges on the teeth No. 61–64.

Fig. 3 shows graphs describing the change in the maximum cross section area of the edges on the column teeth in the wheel end plane, and, relative to the hob

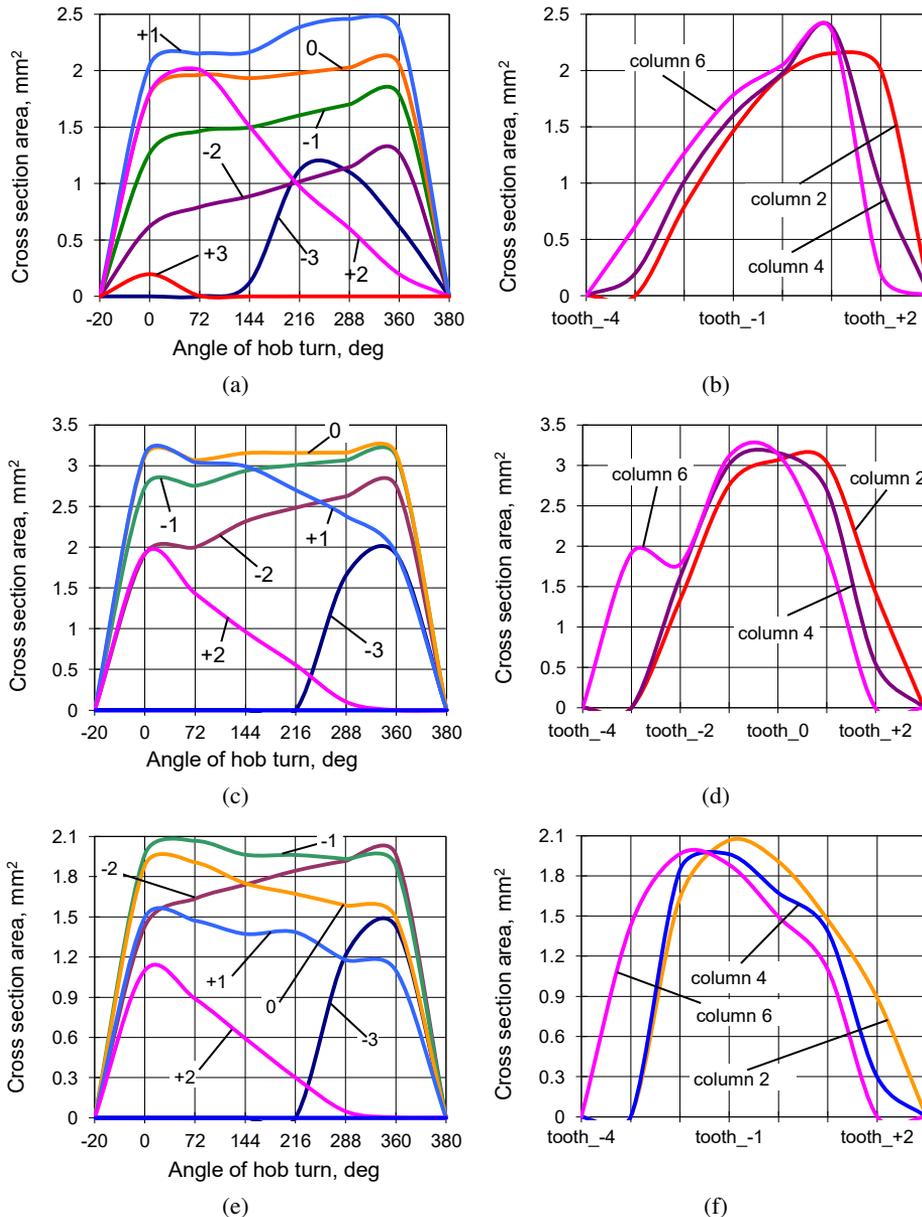


Fig. 3. The maximum transverse cross-sections on the hob columns (a, b – for leading edges; c, d – for head edges; e, f – for trailing edges)

angle rotation, in the tool end plane. In particular, Figs. 3a, 3c, 3e show the change in area relative to the rotation angle of the hob cutter. Figs. 3b, 3d, 3f show graphs from which one can determine the cross-sectional area of teeth numbered -3 to $+3$ for columns 2, 4 and 6.

One can conclude from the graphs (Fig. 3) that the largest total cross-sectional area of the cuts is on the head and trailing edges, and the smallest area is on the leading edges. Peak values of sectional areas on head edges are on central teeth of the column, on the trailing edges are on the teeth located to the right of the central tooth, and on the leading edges are on teeth to the left of central tooth.

3.2. Analysis of the chip compression ratio simulation results

The simulation of the cutting process with variable cutting depth in the Deform 2D that corresponds to the conditions of gear hobbing, and approximation of empirical data allowed us to derive the relationship between changing the parameter chip compression ratio ξ and the thickness of the cut layer [16–19]. The source conditions for the DEFORM pre-processor during simulation of the machining (cutting) operation are: 2D model of machining part (surface); cutting parameters; geometry of the cutting edge, tool material and a covering; mechanical and thermophysical characteristics of the processed material (steel AISI 1045); model of tool's wear; total remeshing criteria (calculated as sum of the reduced errors of modeling on a power vector, vector of speed and admissible geometrical error); type of the strain simulation (Lagrange Incremental); iteration method (Direct Iteration), type of the deformation and temperature solver (Skyline method).

The analytical equation for the chip compression ratio ξ obtained with this software as a result of polynomial interpolation is (Fig. 4b):

$$\xi = -0.0056t^3 + 0.052t^2 - 0.186t + 1.923. \quad (5)$$

This function is consistent with the basic norms of the metal cutting theory, according to which as the shear thickness increases, the specific load and the

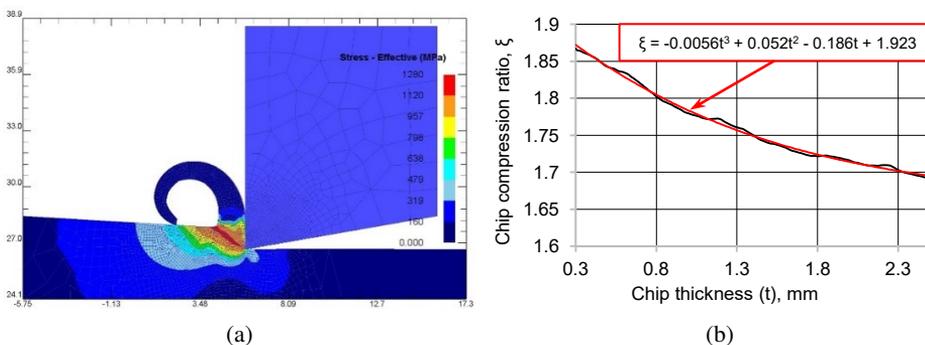


Fig. 4. Simulation the shear zone in Deform 2D (a) and a graph of the dependence of the chip compression ratio on the thickness of the cut (b) during gear hobbing

pressure from the chip side on the rake face decrease and the friction in the secondary chip deformation zone decreases, thereby reducing the value of the chip compression ratio (Fig. 4).

The change in the chip thickness on the cutting edges depending on rotation angle of the hob cutter (a) and on some threads of columns (b) is shown in Figs. 5–7.

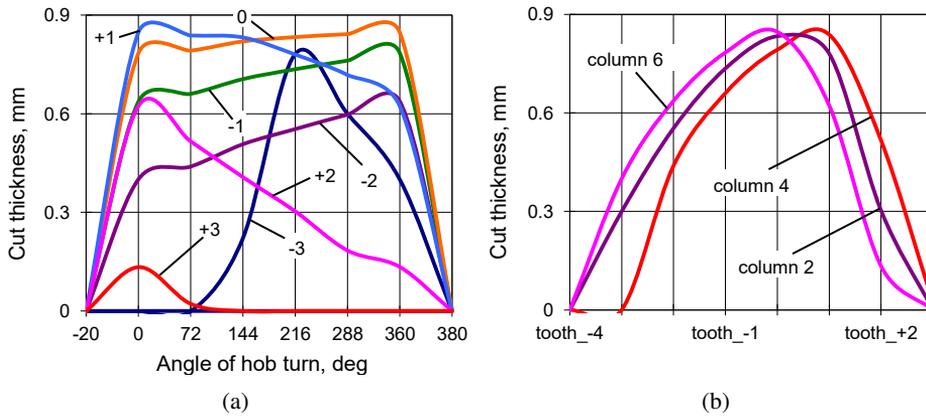


Fig. 5. Thickness of cross-sections on trailing (a, b) edges depending on rotation angle of the hob cutter (a) and on some threads of columns (b)

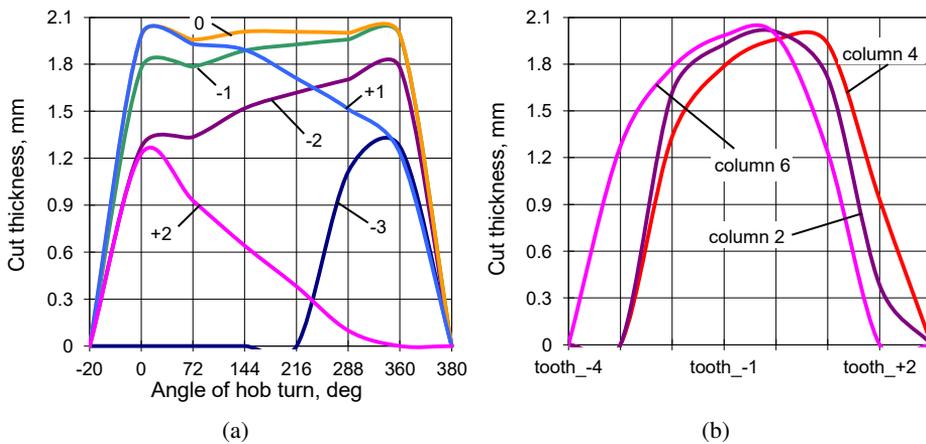


Fig. 6. Thickness of cross-sections on head (a, b) edges depending on rotation angle of the hob cutter (a) and on some threads of columns (b)

The graphs in Fig. 8 contain information about the regularity of changing the section area and thickness of cuttings on the edges of hob teeth during removal of allowance from one gap, that is, along helical line of the cutter (by maximum values). The view of workpiece which contains one gap between adjacent teeth is

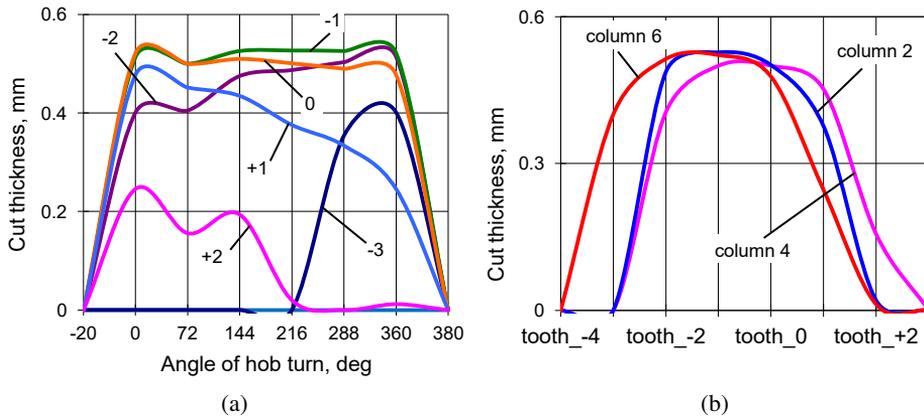


Fig. 7. Thickness of cross-sections on leading (a, b) edges depending on rotation angle of the hob cutter (a) and on some threads of columns (b)

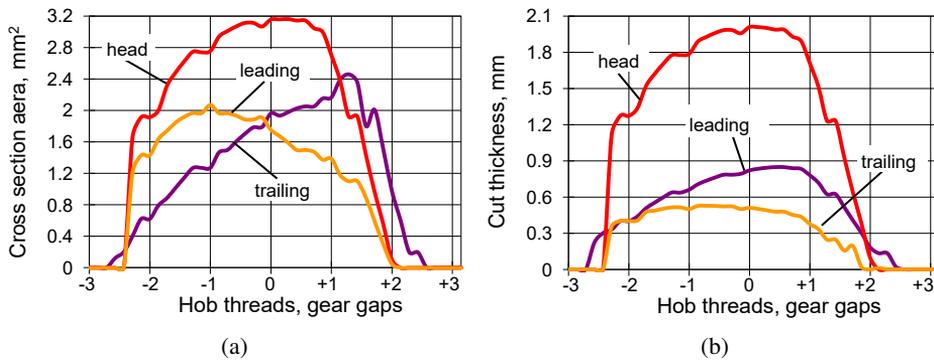


Fig. 8. Maximum cross-sectional area (a) and thickness (b) calculated along the helical line of the hob cutter, in the case of one gap gear cutting

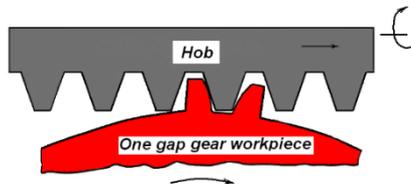


Fig. 9. One gap gear workpiece for the hobbing simulation

shown in Fig. 9. In such a scheme of machining, hobbing occurs only on the edges of one tooth during continuous cutting and with a sequential change of teeth along the thread of the hob.

Fig. 10a shows how the parameter a for the head edges on the columns of hob varies depending on the thickness of the cut layer according to the Eq. (5). As can

be seen, the greatest shear intensity occurs on the outer teeth with the following marks of columns: -3 , -2 , 2 and $+3$, where the shear thickness is smaller than that on the central teeth.

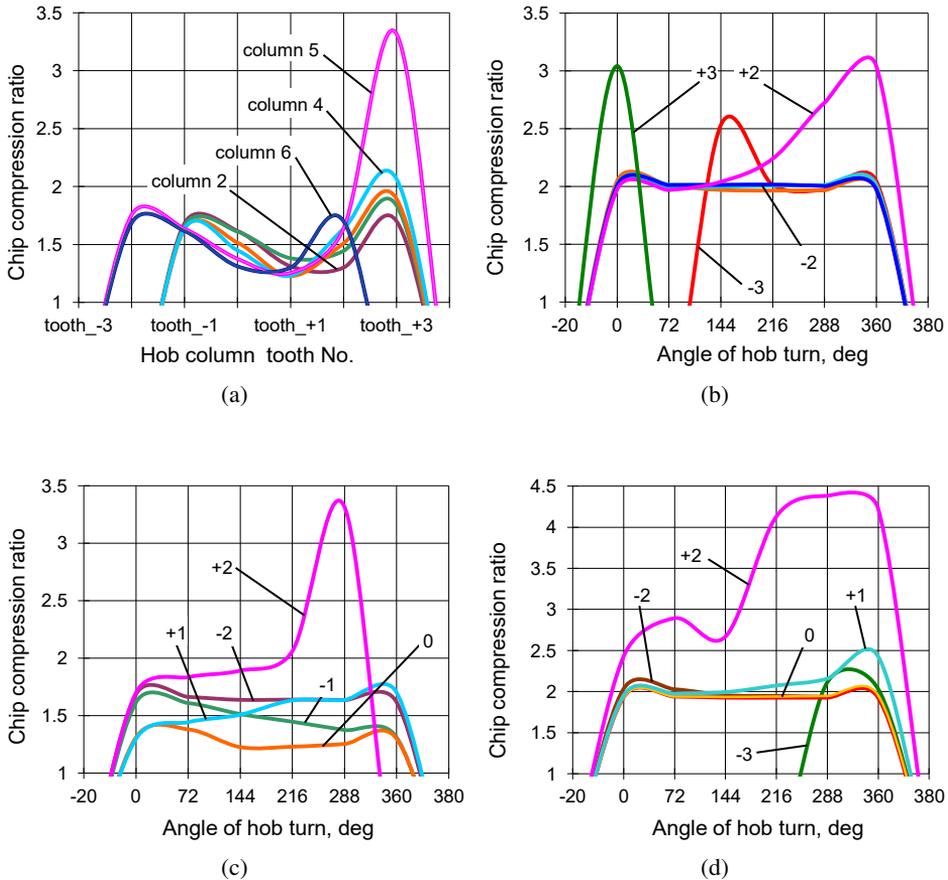


Fig. 10. Chip compression ratio as a function of the cut thickness on hob columns (a) and of the angular motion of the hob cutter (b – trailing edge, c – head edge, d – leading edge)

From the graphs in Fig. 10b, 10c, 10d one can see how the strain intensity of the chips at the trailing (b), head (c) and leading edges (d) varies depending on rotation angle of the hob cutter for threads with the numbers from -3 to $+3$. At the beginning of the cutting by a certain tooth, the intensity is maximal because of small thickness of the cut section. In the central part of the cutting path it is several times smaller, and at the exit of the tooth from the workpiece it grows again. Therefore, the greatest intensity of shear strains is observed on the leading edges of the hob cutter, and the smallest – on the trailing edges.

The analysis of the results obtained at this stage of research allows us to qualitatively evaluate the factors affecting the wear of cutting edges and faces of the teeth.

The first factor is related to the thickness of the cut sections. When the minimum cut thickness is lower than the radius of the blade rounding, there is no cutting, instead, there appears surface crumpling of the contact layer. The graphs in the figure show the teeth and blades on which such conditions can occur: these are teeth No. 1–7 and No. 55–65 on the helical surface of the hob. The leading edges of the teeth No. 1–7 and the trailing edges of the teeth No. 55–65 cut chips less than 0.15 mm thick, which leads to increased sliding at the initial moment of cutting, heating and rapid wear of these edges, despite the fact that the cutting force is very low.

The second factor is the intensity of shear plastic deformation. The result is increased cutting and friction forces in the secondary shear zone at rake face. From the graphs in Fig. 10 it follows that according to this factor the head and both side edges of the teeth No. 54–63 of the second and third threads of the hob have worse cutting conditions. The reason for this is the small thickness of the cut sections, and the result is a significant increase in the local cutting forces on the teeth in this zone of the hobbing.

The third factor is determined by the interaction between chips which simultaneously come off from the side and head edges. The buildup of chips coming off from these edges causes that the resulting chip flows at a different angle, the interaction force between tooth and chip increases and the friction intensity on the rake faces of the teeth at head edges increases. Such conditions can be identified from the graph data shown in Fig. 9. As can be seen in this graph, the leading edges of the teeth of the thread No. (–1) and the trailing edges of the thread No. 1 operate under extreme conditions according to this indicator. At the same time, for the teeth of the central thread, there is a characteristic chips motion both from the head and from the right and left edges. This leads to increased cutting resistance and increased heating of the head edges and the corners of the teeth of this working part of the hob cutter.

3.3. Tool-chip contact length

The duration of contact of the cutting edges with the workpiece under varying conditions of periodic cutting characterizes the ability of the cutting elements to cool and recover during unloaded rotation. The contact between the hob teeth and the workpiece during cutting has the shape of an arc, whose radius is equal to the radius of external diameter of the hob. Fig. 11 shows graphs of the contact length of the head edges of the teeth of individual columns, according to the threads of the hob marked from No. (–3) to No. 3.

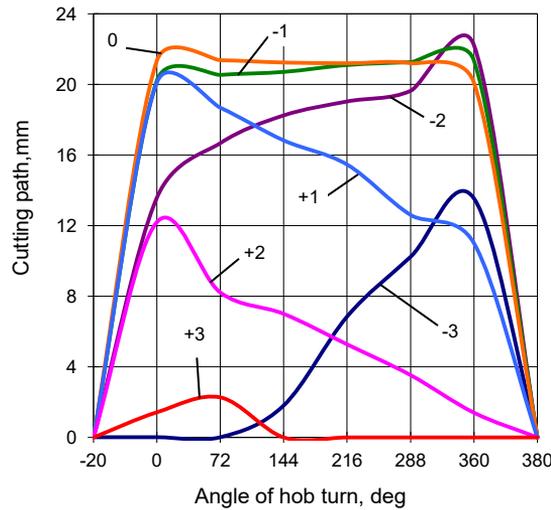


Fig. 11. Tool-chip contact length on the hob threads from No. (-3) to No. 3, depending on the angle of hob rotation

3.4. Work of cutting edges for plastic deformation of shear layers

The work of one edge on the length of its edge-workpiece contact arc during one cutting can be defined as an elementary work of plastic deformation.

The ultimate shear strength $\bar{\tau}$ can be calculated as a half the ultimate tensile strength $\bar{\sigma}$: $\bar{\tau} = 0.5 \bar{\sigma}$. For the gear wheels made from carbon steels $\bar{\sigma} = 600\text{--}650$ MPa, $\bar{\tau} = 300\text{--}330$ MPa; for alloyed steels $\bar{\sigma} = 900\text{--}1000$ MPa, $\bar{\tau} = 450\text{--}500$ MPa. For the used workpiece material – high-carbon steel, we accept $\bar{\tau} = 300$ MPa.

Based on the above data, we can simulate the work of plastic deformation (displacement) performed by the edges of the hob cutter, based on formula (4). For this task, it is advisable to go down from the maximum values of the area and thickness of the cross-section to their average values along the contact arc. Fig. 12 (a, b, c) shows the work of displacement on the edges of the cutter teeth along the rotation angle, respectively, on the trailing, head and leading edges along the threads from No. (-3) to No. 3.

The graphs of changes in the shear of the trailing edges at the end of the gear on the columns 1-3-5-7-9 are shown in Fig. 12d.

From the obtained results it follows that the greatest work of plastic deformation when removing the allowances from the gaps between the teeth of the gear is performed by the head and trailing edges, whereas the leading edges of the same teeth are loaded many times less.

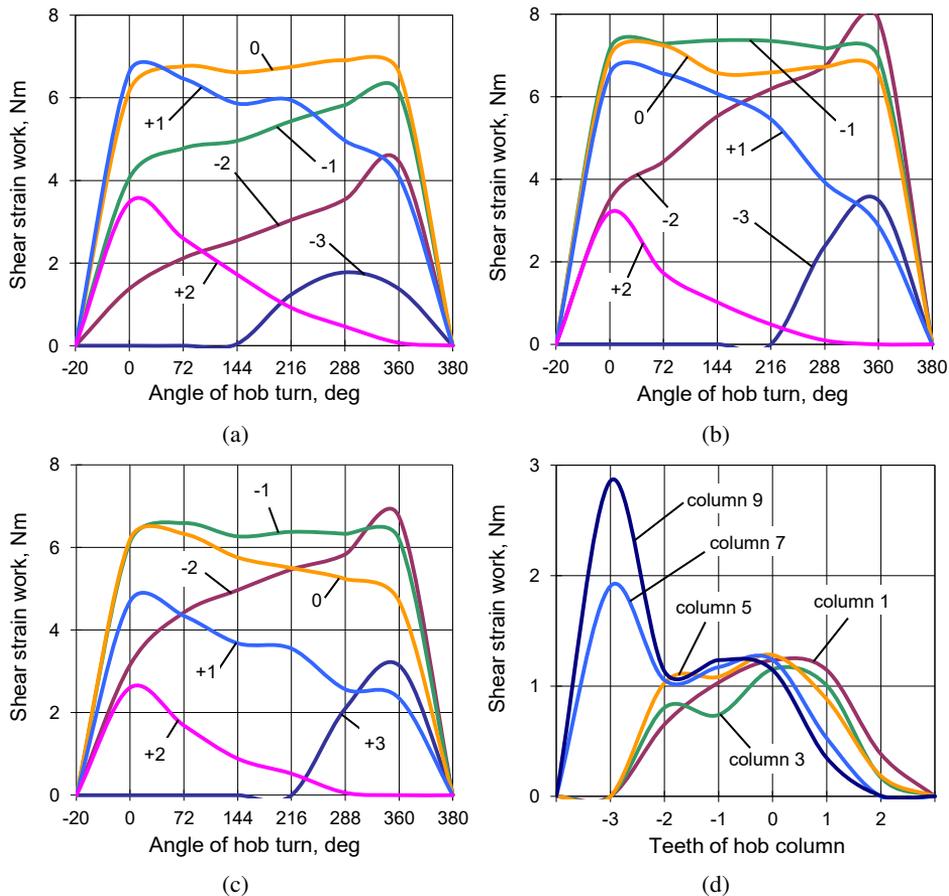


Fig. 12. Shear work on the edges of the cutter teeth relative to the angle of hob rotation (a, b, c) and change the shear work of the trailing edges on the cutter columns (d) (at the end of the gear to be machined)

4. Work done to overcome friction forces on the rake faces of the hob cutter teeth

4.1. Friction force calculation during hobbing

It is generally known [16] that the friction during cutting is determined by the intensity of the contact interaction between the chips and the rake face of the tool. The resulting force of the tool-chip friction depends on the intensity of chip deposition during shear, and the result of it is the deformation of chips in the secondary shear zone.

The friction load on the rake face is determined by the pressure force normal to the edge and the coefficient of friction.

As can be seen from Fig. 13a, the friction angle ρ is equal to:

$$\rho = a \tan(\mu) = \omega + \gamma, \quad (6)$$

where μ is the coefficient of friction; γ is the rake angle of the cutting teeth; ω is the angle of action (the angle between the vectors of the force of chip formation R and the main component of the cutting force (cutting speed)).

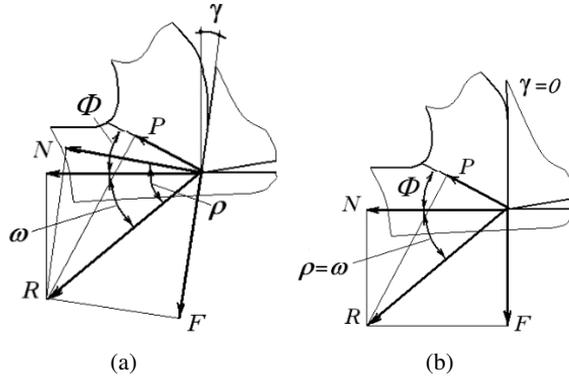


Fig. 13. Forces acting on the cutting edge if $\gamma > 0$ (a) and if $\gamma = 0$ (b)

When $\gamma = 0$, the friction angle ρ is equal to the angle of action ω , and the friction coefficient is: $\mu = \tan \rho = \tan \omega$ (Fig. 13b).

To estimate the value of the action angle ω , we use the relationship between the angle ω and the shear angle Φ , described by the following equation:

$$\omega = I - \Phi, \quad (7)$$

where:

$$I = (i_0 - 6)a^{-0.05}, \quad (8)$$

a is the chip thickness [16].

The value of parameter i_0 has been determined by prof. Silin experimentally for various materials [20]. In our case, it is the high-carbon steel with the carbon content of over 0.25%, for which $i_0 = 50$.

We assume that, when $\gamma = 0$, the angle of shear is $\Phi = \arctan \xi$ and the value of constant is $i_0 = 50$. Then, the graphs of the action angle relative to the angle of hob rotation on the threads and on the columns in the ends of the machined gear are as shown in Fig. 14.

In general, the average value of the angle ω between the cutting speed and the cutting force vector for the trailing edges is 180° ; for the head edges it is 80° and for the leading edges it is 140° .

To determine the friction force on the rake face of the hob, it is necessary to know the value of the normal force, which can be determined as a function of the

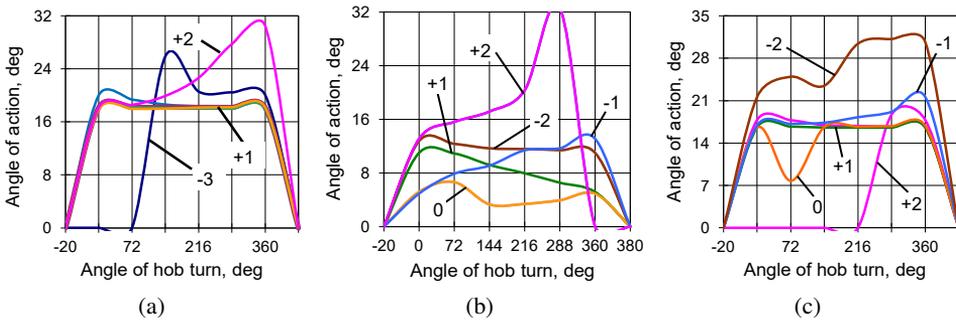


Fig. 14. Graphs of the action angle: a – for the trailing edges; b – for the head edges; c – for the leading edges

shear force, based on the above data. As is known, the resulting chip forming force R is equal to the geometric sum of the friction force F and the normal force N . On the other hand, there is the following relationship between the force R and the shear force P (Fig. 13):

$$R = \frac{P}{\cos(\Phi + \omega)}. \quad (9)$$

Taking into account Eq. (8), we obtain:

$$R = \frac{P}{\cos((i_0 - 6)a^{-0.05})}. \quad (10)$$

The friction force, which is defined as: $F = R\mu = R \sin \omega$, can be described by the formula:

$$F = \frac{P \sin \omega}{\cos I}. \quad (11)$$

The graphs of friction force on the teeth edges for the threads from No. (-3) to (+3) are shown in Fig. 15.

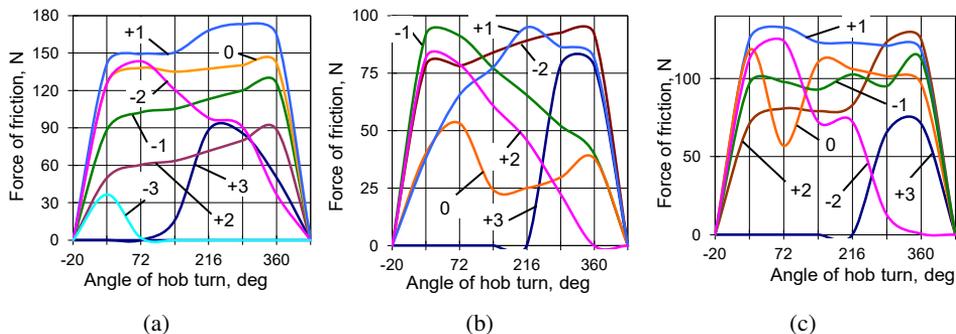


Fig. 15. Graphs of friction force on the hob edges a – for the trailing edges; b – for the head edges; c – for the leading edges

As can be seen from the results of the friction force modeling, their largest values occur on the trailing edges, smaller – on the leading and the smallest – on the head edges. This is due to the different values of the corresponding angles of action.

4.2. The work of friction on the rake faces of the hob teeth

The value of the elementary work on the rake face of the hob tooth W_f , which must be performed to overcome the friction force F , is determined by the equation:

$$W_f = 10^{-3} F p_c, \text{ Nm}, \tag{12}$$

where p_c is the path of the tooth in contact with the workpiece during cutting, which is equal to the length of the contact arc.

According to the previously given data and on the basis of formula (12), the graphs that characterize the work to overcome the friction along the edges of the cutting teeth relative to the angle of action take the form presented in Fig. 16.

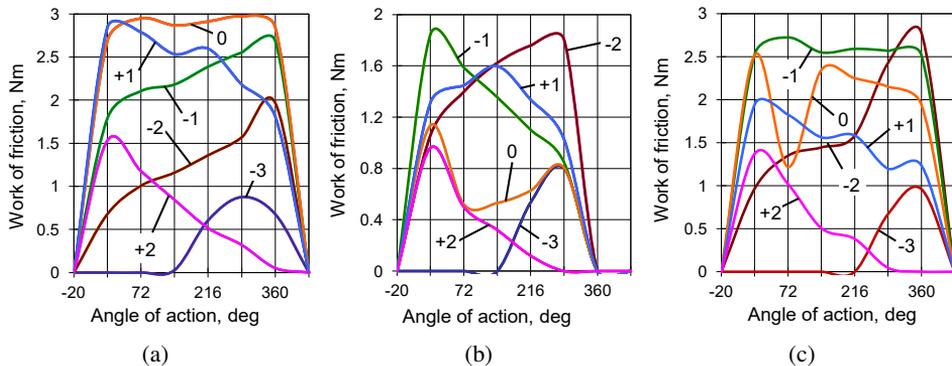


Fig. 16. The work required to overcome the friction load on the rake face of the cutting teeth, which is performed by the edges: trailing (a), head (b) and leading (c)

As we can see from these graphs, friction forces are distributed over the edges in the following way, according to the intensity: the greatest friction occurs on the trailing edges, the average one – on the leading edges and the smallest one – on the head edges.

5. Total work performed by cutting edges and hob teeth

Referring to Eq. (1) and using the simulation results given above, we obtained graphics that characterize the total work distributed between the edges, which is performed by all hob teeth involved in cutting (Fig. 17).

Fig. 18 shows the total work of cutting teeth on threads from (-3) to (+3) relative to the angle of hob rotation on the trailing, head and leading edges. An

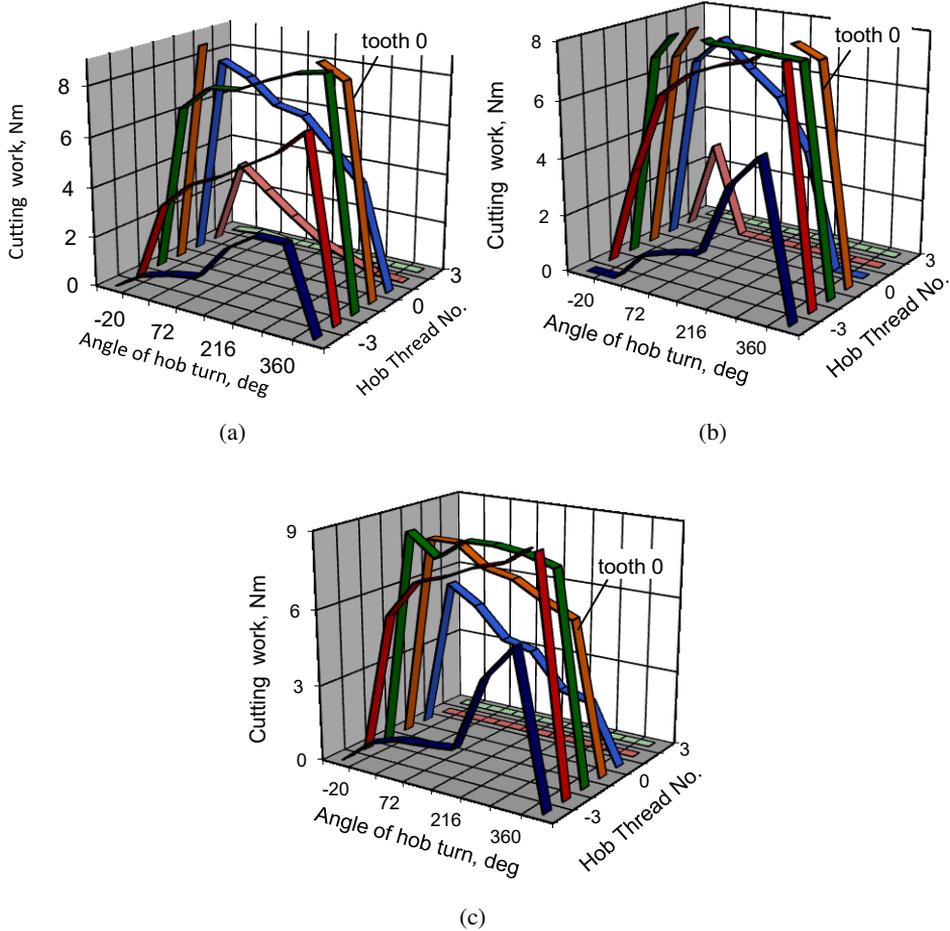


Fig. 17. Total cutting work of trailing (a), head (b) and leading (c) edges

additional information is provided by the graphs shown in Fig. 19 and Fig. 20. The change in the total work of the cutting teeth on the hob threads, i.e., during the continuous rotation of the hob cutter when machining in one gap between two adjacent teeth of the machined gear, is shown in Fig. 19. As can be seen from these graphs, the largest cutting work is performed by the trailing (left) teeth (average value is 4 Nm). The greatest value of this work falls on the teeth located after the zero tooth of the cutter rack, i.e., on its trailing part. Slightly less cutting work is performed by the head edges (average value is 3.7 Nm), and even less work is performed on the leading edges of the hob teeth (average value is 3.4 Nm). The maximum values of the work appear when cutting-in of the hob into the workpiece, i.e., before the tooth No. 0.

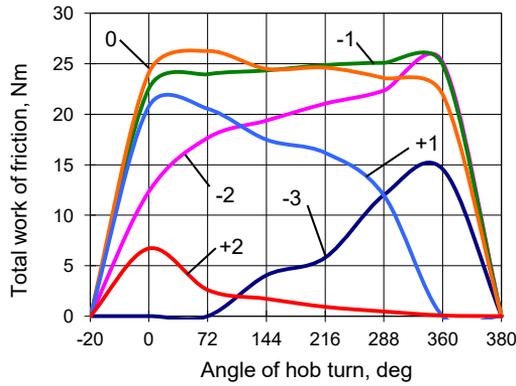


Fig. 18. Total work performed by the cutting teeth on threads during hobbing

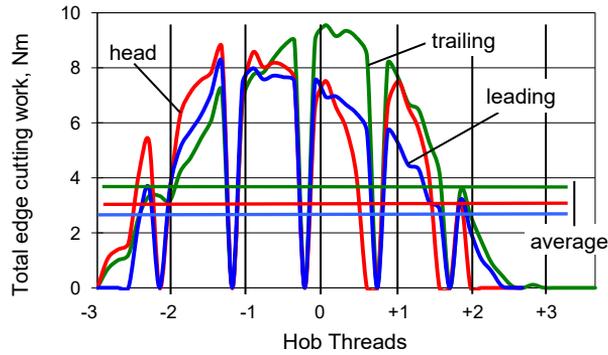


Fig. 19. Work of cutting edges on the hob helical surface during machining of the one gap

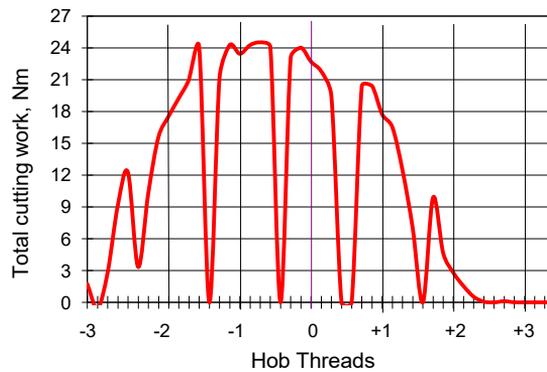


Fig. 20. The total work of the cutting teeth on the hob helical surface when machining one of the gap

6. Analysis of the work of active part of cutting edges during hobbing

The analysis of the work of the cutting edges and the teeth in the hobbing process would not be complete unless we consider the work of the side edges of the teeth of the hob cutter.

It is known that the width of the blade does not affect the intensity of shear deformation and the cutting force, but this parameter is one of the important factors of the blade's participation in the total work of cutting, both for an individual tooth and for the hob in general. Fig. 21a, 21b shows graphs of changes in the active width of the side edges for the seven threads of the cutter relative to angle of hob rotation, and the total width of the head and side edges are shown Fig. 21c.

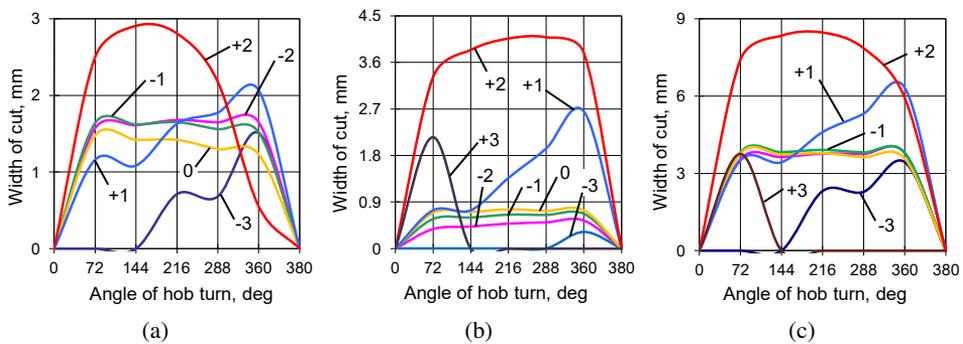


Fig. 21. Width of cut sections on trailing (a), leading (b) edges and the total width of the active part of the cutting edges on to the threads from (-3) to (+3) (c)

The potential cut width of one tooth is equal to its perimeter (from the circuit of initial rack). According to the initial data ($m = 3$ mm, $q = 10$, the number of threads of the hob cutter engaged with the workpiece is 7), the perimeter of the edges of one tooth is 16 mm. The total perimeter of the edges of the teeth for one thread is 160 mm, and for seven threads is 1120 mm. At the same time, according to the results of this study, the total width of all cuts is 81 mm, i.e., only 7.2% of the potential length of all edges on the active part of the hob cutter takes part in cutting. The use of the perimeter of the teeth in certain threads is shown in Fig. 19. The portions of edges in their total length are as follows: the trailing edges (total width is 44.5 mm) are used in 9%; leading edges (total width is 38.1 mm) are used in 7.5%. Only the head edges are maximally involved in cutting (approximately in 98.5%). As can be seen, the maximum use of the perimeter of the edges takes place in the second thread after the zero tooth, but here the percentage of use does not exceed 25% (Fig. 22).

Therefore, only a small part of the costs spent on manufacturing of hobs is expedient, i.e., these costs are excessive and irrational.

The obtained results provide a great deal of information that can be used in further studies on the processes and phenomena that occur during hobbing.

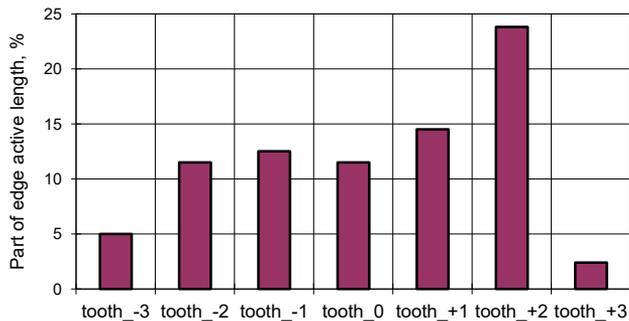


Fig. 22. Portions of potential width of edges on hob threads

Based on the analysis of operation of adjacent edges, the intensity of the constrained cutting can be determined on each tooth. As is known, such cutting conditions lead to the layering of chips on the rake face in the area and cause an increase in friction, heating and accelerated wear in these zones, in particular, in the upper part of the cutting teeth. Therefore, these data make it possible to more accurately predict the wear and durability of hob cutters.

Information on the working conditions of the cutting edges can serve as the basis for changing the design of the hob teeth. For example, the design of teeth that perform the main cutting work can be changed to separate the flow of chips that come from the side and head edges. These are the tooth numbered from “-25” to “-35” on the helical surface of the hob cutter. In order to do so, an additional groove can be milled for the chips on each of these teeth, and cutting teeth sizes can be changed: by reducing the first part of the tooth and excluding the head edge from the cutting part; narrowing the second part of the tooth, and excluding the side edges from cutting. Such hobs are made, in particular, by Fette Compacting GmbH [21], and our research can provide data for introducing such changes with reasonable cutting correction parameters.

7. Conclusions

Based on the results of the research presented in this article, the following conclusions can be drawn.

1. According to the results of the cutting work simulation, the greatest wear appears on the trailing edges of the teeth located in the center of the hob cutter and slightly to the right of the “zero” tooth. They are followed by the leading (right) edges of the teeth on the threads No. (-1) and (-2), that is, on its leading part. In addition, there appears an additional wear of these edges due to zero relief angles, which leads to increased friction and accelerated wear rate of the cutting teeth. Such results coincide very well with numerous experimental data. However, using the modeling method described in the article allows one to accurately identify the

most loaded edges of cutting teeth, facilitates design and making technological decisions for optimal redistribution of these loads.

2. A very small portion (for described above example, it is 7.2%) of the total length of the edges (perimeter) of the cutter is used during hobbing. This means that significant costs spent on manufacturing of hobs are excessive and unreasonable. These costs are spent on machining, cutting of the screw and chip grooves, relieving, hardening, finish grinding and application of protective coatings on the operational surfaces of hob. Only a small portion of cutting surfaces, which really used during hobbing, is active and affects the accuracy, quality of toothed surfaces and productivity of the gear's machining process.

3. The next conclusions that follow from the preliminary ones are: the cost of manufacturing of the hob can be reduced if the highest required accuracy, surface hardness and roughness is ensured not for the entire cutter, and not even for all surfaces of the cutting teeth, but only for that part of the active perimeter of each tooth that directly participates in cutting, i.e., in generation of the gear's teeth geometry.

4. Due to the fact that obtaining the highest hardness and the most effective coating of cutting tooth surfaces is generally very expensive, it should be applied only for the certain surfaces of a few threads that do the greatest work of machining. The remaining part of surfaces of the cutting teeth can be uncoated, if the required hardness and strength can be ensured by applying standard methods, such as thermal and chemical-thermal treatment (hardening, nitriding, cyanidation, etc.). In this way, the wear rate can be equalized for the cutting teeth along the entire active length of the hob.

5. One of the main causes of wear of the most loaded cutting teeth is the simultaneous operation of the head, right and left edges, which leads to increased friction on the rake faces and accelerated wear in endangered tip-to-flank regions (corner areas) of the tooth. Therefore, it is proposed to change the design of the rake face of these cutting teeth. Changing the shape of this surface or the design of the tooth one may achieve separation between the cutting processes of the head and side edges. This can be done by changing the design of the hob cutter, such as proposed by company "Fette Compacting GmbH", in the following way: in one tooth – make an additional chip groove, for the first part of the tooth – leave only the head edge, and for the second tooth – leave only two side edges. Cutting a groove along the head edge can be an acceptable solution to this task, as well. As a result, the tooth begins to cut by the side edges, and the head edge works with a delay. The same effect can be achieved by cutting grooves along the side edges of the cutting tooth. In this case, the cutting will start at the head edge, and the side edges will lag behind. This will partially separate the chips from the three edges in space and in time and reduce friction loading.

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