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The algorithm of adaptive determination of amplification of the PD filter estimating object state on the basis of signal measurable on-line

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The article presents the algorithm that enables adaptive determination of the amplification coefficient in the filter equation provided by Kalman. The method makes use of an estimation error, which was defined for this purpose, and its derivative to determine the direction of correction changes of the gain vector. This eliminates the necessity to solve Riccati equation, which causes reduction of the method computational complexity. The experimental studies carried out using the proposed approach relate to the estimation of state coordinates describing river pollution using the BOD (biochemical oxygen demand) and DO (dissolved oxygen) indicators).The acquired results indicate that the suggested method does better estimations than the Kalman filter. Two indicators were used to measure the quality of estimates: the Root Mean Squared Error (RMSE) and the Mean Percentage Error (MPE).

Key words: estimation, Kalman filter, adaptive PD filter, filter amplification

1. Introduction

The knowledge of full state vector is a necessary condition for solving a series of problems that occur during exploitation of dynamic systems, including monitoring, diagnosis or control types. In real conditions observation of all the parameters is often not possible in the on-line mode, as measurement of some of them may be time-consuming (e.g. the necessary laboratory service or observation of the biological process), or even impossible (due to technological

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reasons). An example of this type of situation is determination of the biochemical pollution level of a river, which requires BOD measurement (biochemical oxygen demand), as well as DO (dissolved oxygen) measurement. While the DO measurement is instant, the BOD measurement requires 5, 7 or 20 days of laboratory service [15]. Similar situation occurs in the case of temperature measurement and the measurement of solution concentration in a chemical reactor – the temperature measurement is instant, while the concentration measurement requires laboratory work [8]. In such situations, estimation of one coordinate of the state vector (the measurement of which is time-consuming) may be done by using the measurement of the second coordinate and other informations. The issues of state vector estimation are broadly discussed in references; in order to perform it the authors make use of different tools: the Kalman filter and its variations [1, 2, 5, 7, 10, 14, 16, 18], artificial neural networks [6, 7, 17], statistical methods [4, 6, 17, 22], hybrid methods [26]. The Kalman filter and its derivatives seem to be the most popular tool for state estimation. Some of their features, however, seem problematic: the classic Kalman filter requires object linearity and derive of the Kalmans filter require the knowledge of covariations of interferences [13, 25]. Determination of covariance requires numerous studies that encompass statistical measurements and analyses, which in situations dependent for example on weather, are unreliable. The artificial intelligence methods require selection of a representative dataset [21], which is a very difficult task in the case of processes of unobvious dependencies between the features. Very frequently, in order to find an appropriate set that comprises an unambiguous process state signature it is essential to gather extra amount of data and to apply the method of feature selection.

The article makes use of Kalman equations for estimation of the object/process state vector, determining adaptively the amplification coefficient that occurs in it. In the original Kalman method determination of the amplification coefficient requires the Riccati equation. The proposed method frees us from the necessity to know covariance of system and measurement interferences that are required in the Riccati equation. The effectiveness of the proposed approach was studied in the monitoring process of river biochemical pollution- on the basis of the dissolved oxygen (DO) measurement the biochemical oxygen demand (BOD) was estimated. The issue of water region pollution is an extremely important and difficult problem. The difficulty consists in the impossibility of instant measurement of the parameters that are required to determine water pollution; thus, it is necessary to estimate the signals that are difficult to measure.

The Kalman filter (KF) determines optimal state estimation for stationary, linear systems, on the basis of measurements interfered with white Gaussian noise [13]. These requirements pose serious limitations as most systems are nonlinear and measurements/observations are burdened with Levy's noise. The first known and commonly adapted method of the Kalman filter relied on approximation,by means of Taylor series, the non-linearity of system dynamics by the model linearized around the last state estimation. This method was called Extended Kalman Filter (EKF). A key operation performed in the Kalman filter is propagation of the Gaussian random variable by system dynamics. In [12] Julier and Uhlman solved this problem by applying a deterministic approach to sampling, which enables to avoid the errors made by EKF. State distribution is approximated by the Gaussian variable, but in this method, also known as Unscented Kalman Filter (UKF), it is represented by a minimum set of carefully selected samples. These samples fully represent the average and the covariance of the Gaussian random variable to the third order (extended Taylor series) for any non-linearity. The EKF, on the other hand, reaches only the first order accuracy. Another alternative variant of the non-linear Kalman filter is the Divided Difference Kalman Filter (DDKF), presented in [19,20]. An advantage of DDKF over EKF is the fact that it does not require determination of Jacobian non-linear derivatives of dynamic and measurement equations, but it makes use of interpolation with Stirling polynomial, while an advantage over UKF is the fact that it does not have to use the parameters strongly determining estimation accuracy. A solution similar to DDKF is the Central Difference Kalman Filter (CDKF), published by Ito in [11]. In [24] the authors present modification of the Kalman filter, which enables us to apply the method in linear systems with non-Gaussian Levy's filter of non-finite variance. As the non-Gaussian Levy's noise with infinite variance occurs commonly [23], the proposed method seems interesting. Its drawback, however, is the computation cost which exceeds high computation cost of the Kalman filter. An extension of the Kalman filter to the Levy's noise was also proposed by Applebaum and Blackwood in [3]. The main result is that the components of the observation noise that have infinite variance make no contribution to the filtering equations.

Further structure of the paper is as follows: the second part presents the proposed algorithm of adaptive determination of the PD filter, which is proceeded by a brief summary of the equations proposed by Kalman. The third part includes the model of the object used in experimental studies, while the fourth part presents the experiments that compare the estimation quality by means of an adaptive PD filter and the Kalman filter.

2. Determination of the amplification coefficient in the estimation equation

2.1. Kalman filter

The method of state estimation proposed by Kalman requires solving two differential equations [13]:

$$\dot{\hat{x}} = \boldsymbol{A}\,\hat{x} + \boldsymbol{K_F}\left[\boldsymbol{y} - \boldsymbol{C}\,\hat{x}\right], \;\; \hat{x}(0), \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{P} = \boldsymbol{A}\,\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}^T - \boldsymbol{P}\,\boldsymbol{C}^T\boldsymbol{V}^{-1}\boldsymbol{C}\,\boldsymbol{P} + \boldsymbol{D}\,\boldsymbol{W}\boldsymbol{D}^T, \tag{2}$$

where: $\boldsymbol{A}$ – state matrix (the parameters that determine object/process dynamics), $\boldsymbol{K_F}$ – vector of amplification coefficients $\boldsymbol{P}$ – estimation error covariance matrix $\boldsymbol{C}$ – measurement matrix, $\boldsymbol{D}$ – matrix of system interference influence $\boldsymbol{V}$ – covariance matrix of measurement noise $\boldsymbol{W}$ – covariance matrix of system noise.

The measurements $\boldsymbol{y}$ (DO) that appear in equation (1) are described by the following relationship:

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{v}, \tag{3}$$

in which the value $\boldsymbol{v}$ stands for measurement noise.

The equation (1) determines the estimated state vector, while the nonlinear Riccati equation (2) enables to determine the gain in (1) that appears in the equation, according to the dependency:

$$\boldsymbol{K_F} = \boldsymbol{P}\,\boldsymbol{C}^T\boldsymbol{V}^{-1}. \tag{4}$$

Optimality of estimation of the state vector determined by Kalman filter depends on the accuracy of gain calculation ($\boldsymbol{K_F}$), which depends mainly on the precision of statistic estimations of covariance matrices of system interferences ($\boldsymbol{W}$) and measurement interferences ($\boldsymbol{V}$).

2.2. Adaptive calculation of the amplification coefficient

The method of designating the amplification value in equation (1) that was proposed by Kalman (4) is optimal in relation to estimation error covariance. It is determined on the basis of the assumption that interferences have a form of white Gaussian noise, which in reality is not always the case. In the proposed approach determination of the filter amplification vector by means of an additive method that does not require the knowledge of interference nature is assumed:

$$\boldsymbol{K}_{i+1} = \boldsymbol{K}_i + \Delta\boldsymbol{K}_i\,, \tag{5}$$

where: $\boldsymbol{K}_{i+1}$ – new value of the filter amplification vector, $\Delta\boldsymbol{K}_i$ – correction of the filter amplification value in i-th iteration step.

3. Object

Correction of the amplification value $\Delta\boldsymbol{K}_i$ in the proposed adaptive algorithm was made dependant on the value of the adaptive error value ε_i:

$$\boldsymbol{\varepsilon}_i = \boldsymbol{y}_i - \boldsymbol{C}\,\hat{x}_i \tag{6}$$

and its derivative.

The equation that determines the magnitude of the amplification coefficient correction has the form:

$$\Delta \boldsymbol{K}_{i+1} = z_i \left(\boldsymbol{k}_\mathrm{p} \boldsymbol{\varepsilon}_i + \boldsymbol{T}_\mathrm{d} \frac{\mathrm{d}\boldsymbol{\varepsilon}_i}{\mathrm{d}t} \right), \tag{7}$$

where: $\boldsymbol{k}_\mathrm{p}$ – vector of the proportional component of the amplification coefficient, $\boldsymbol{T}_\mathrm{d}$ – vector of the differentiating component of the amplification vector z_i – adaptation coefficient determined on the basis of the current error and its dynamics.

The values of parameters k_p and $\boldsymbol{T}_\mathrm{d}$ that appear in Eq. (7) are constant and determined heuristically on the basis of object/process knowledge. The proposed concept is an extension of the adaptive approach of determination the filter gain coefficient presented in [9]. Note that this method does not require information about system disturbances (covariance matrix $\boldsymbol{W}$) and measurement disturbances (covariance matrix $\boldsymbol{V}$), but it is based only on the observation of the error and its trend.

Figure 1 presents a flowchart of the algorithm that is used to determine the amplification correction value. The i_last parameter was assumed there as representing the observation period (it is the set value according with the observer's expectations). The algorithm is iterative and it determines, in consecutive steps, the estimation values of signals from the filter equation.

The study assumes a mathematical model that represents quality of water in a river, which is expressed by two indicators of water quality, i.e. the biochemical oxygen demand (BOD) and dissolved oxygen (DO). The kinetics of the oxygen dissolved in water that results from disposal of waste of high BOD level was formulated in many mathematical models for simulations of surface water quality. In research works there dominates the approach suggested by Streeter and Phelps according to whom the distribution of organic substances in water proceeds according to differential equations of the first order physiochemical reaction kinetics. The BOD indicator that represents the pace of organic substance distribution by oxygen microorganisms is described by the following equation:

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = -k_1 x_1(t) + w_1 , \tag{8}$$

while the deficiency of DO is characterized by the following relationship:

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = -k_2 x_1(t) - k_3 x_2(t) + w_2 , \tag{9}$$

where: x_1 – concentration of organic pollution expressed by the value of BOD, x_2 – concentration deficiency of the dissolved oxygen DO, k_1 – reaction rate coefficient of BOD, k_2 – coefficient of the influence of BOD on DO, k_3 – coefficient of the

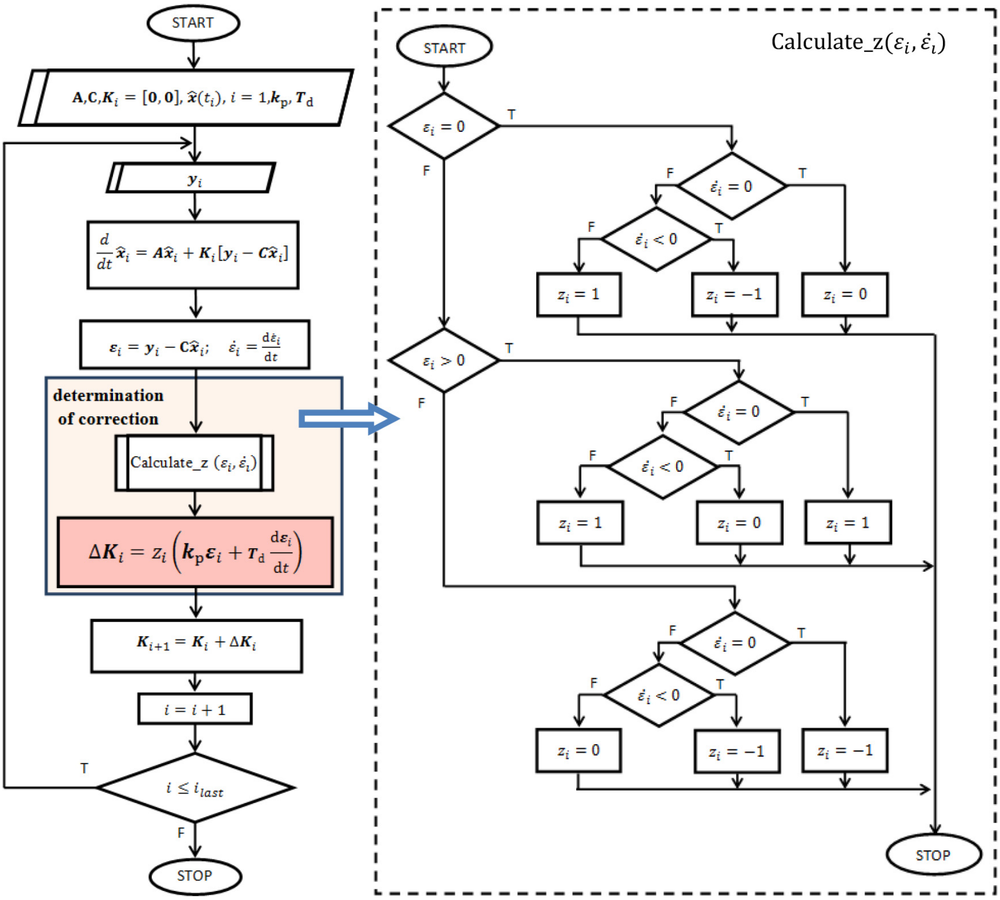


Figure 1: Algorithm flowchart of the proportional-plus-derivative type

rate of changes of DO, w_1 – pollution inflow intensity (BOD interference), w_2 – intensity of sampling/delivery of oxygen from/to water (DO interference).

Coefficients k_1, k_2 and k_3 that appear in Eqs. (8) and (9) represent the dynamics of the natural process of water self-purification and depend on several factors among which the most important is temperature:

$$k_1 = -0.2204 \div -0.3347 \ [\text{mg/l} \cdot \text{day}],$$
$$k_2 = -0.1636 \div -0.2049 \ [\text{mg/l} \cdot \text{day}],$$
$$k_3 = -0.7100 \div -0.8100 \ [\text{mg/l} \cdot \text{day}].$$

In the presented considerations it is assumed that coefficients k_1, k_2 and k_3 have constant values, as the model does not involve the influence of water temperature, which in short time periods is quasi-constant.

When writing down the equations (8) and (9) in the vector form we obtain:

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{w}(t), \tag{10}$$

where: $\boldsymbol{A} = \begin{bmatrix} k_1 & 0 \\ k_2 & k_3 \end{bmatrix}$ – matrix of coefficients $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ – state vector in which x_1 represents BOD, while x_2 the DO deficiency, $\boldsymbol{B}$ – diagonal matrix that subordinates interference signals to the coordinates of state vector, $\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ – the vector of system interference.

Equation (10) describes quality of the studied water by the assumption of environment homogeneity – which is to be understood as a reservoir of particular volume with continuous stirring. In order to model the quality of water in the river it is necessary to consider the changes of state variables, mainly along its length, which leads to partial differential equations:

$$\begin{gathered} \frac{\partial \boldsymbol{x}(t,l)}{\partial t} + \boldsymbol{v}(tl)\frac{\partial \boldsymbol{x}(t,l)}{\partial l} = \boldsymbol{A}(tl)\boldsymbol{x}(t,l) + \boldsymbol{B}\,\boldsymbol{w}(t,l), \\ \text{IC}:\ \boldsymbol{x}(t_0,l), \quad \text{BC}:\ \boldsymbol{x}(t,l_0), \end{gathered} \tag{11}$$

where: l – lenght, $\boldsymbol{v}(tl)$ – diagonal matrix of river stream speed.

Solution of Eq. (11) requires knowledge of the boundary conditions, i.e. the initial condition (IC: $\boldsymbol{x}(t_0,\ l)$) and the boundary condition BC: $\boldsymbol{x}(t, l_0)$.

By performing proper interpretation of the equation (11) it is possible to obtain an ordinary differential equation. By using the method of characteristics it is possible for considerations of the model described with partial differential equations of the hyperbolic type to be solved by means of a set of equivalent and simpler to solve ordinary differential equations. The idea of this method relies on observation of the pollution distribution in a river along the characteristics in the space-time domain. These characteristics are the curves designated by the known speed of flow, which is illustrated in Fig. 2.

When considering the condition of the state of water pollution in a river in the characteristics in the time of $t \in [t_0,\ t_\mathrm{k}]$, $t_\mathrm{k} < \infty$ for a normalised river length $l_\mathrm{z} \in [0, 1]$ the partial differential equations become ordinary differential equations. Thus, the description leads to the necessity of applying a set of characteristics defined in the space-time.

The proposed approach comprised a transformation of partial differential equations to a set of ordinary differential equations along the characteristics. When knowing the river current speed it is possible to obtain solutions of the state equation (10), both in terms of time and length.

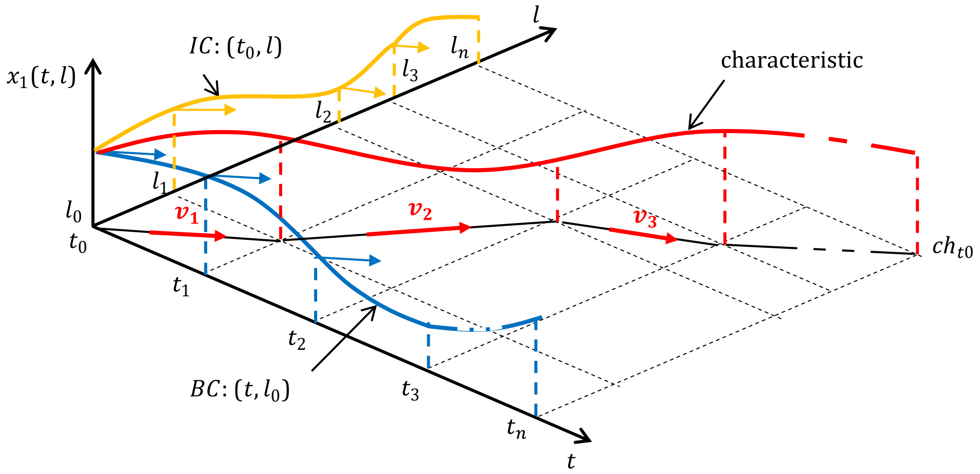


Figure 2: Exemplary characteristics for the state coordinate x_1 in the space-time domain

4. Experimental studies

The aim of the studies was evaluation of estimation accuracy of state variables that represent biochemical oxygen demand (BOD) and dissolved oxygen (DO), performed by means of an adaptive PD filter, the concept which was presented in section 2.2.

In the first stage of studies a comparison of estimates of state variables obtained by the Kalman filter with the estimates obtained with the PD filter was made. In order to do that indicators that enable to asses estimation quality were defined. Two indicators of estiamtion quality were used in the studies:

- Root Mean Squared Error (RMSE):

$$RMSE_j = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_{j,i}^2}, \tag{12}$$

- the second indicator was Mean Percentage Error (MPE):

$$MPE_j = \frac{1}{n}\sum_{i=1}^{n}\left(\left|\frac{x_{j,i} - \hat{x}_{j,i}}{x_{j,i}}\right|\right) \cdot 100\%, \tag{13}$$

where: $e_{j,i} = x_{j,i} - \hat{x}_{j,i}$ – estimation error of the $j \in \{x_1 = \text{BOD},\ x_2 = \text{DO}\}$ signal in the i-th calculations step, n – number of calculations steps.

The RMSE indicator represents the absolute error, while the MPE value is a measure of the relative error percentage. The next stage of the study relied on evaluation of the influence of parameters $\boldsymbol{k}_{\mathrm{p}}$ and $\boldsymbol{T}_{\mathrm{d}}$ and the intensity of system and measurement interference, respectively represented by the covariances $(\boldsymbol{W}, \boldsymbol{V})$.

The simulation experiments were done for a river described with ordinary differential equations, according to the so-called characteristics. A hypothetical river of the length of $l = 1000$ km, flowing with the speed of $v = 25$ km/day was considered with three large side creeks of significant pollution. This tributaries should be understood as forced of the BOD and DO changes (Fig. 3), not as disturbances.

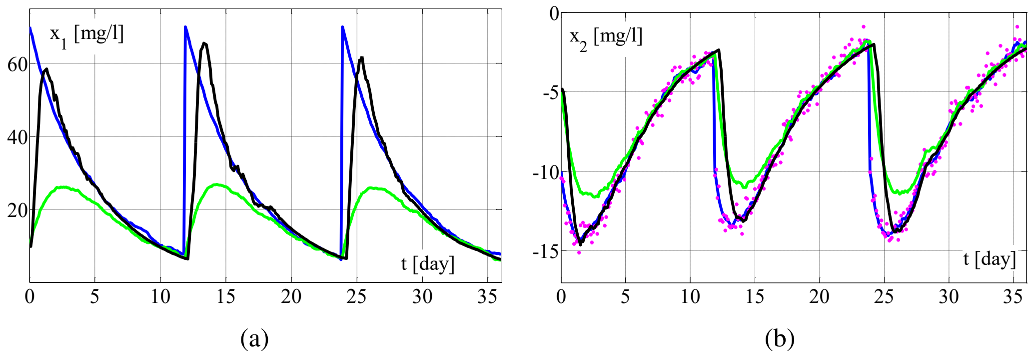


Figure 3: Estimation illustration of state coordinates x_1 that represent BOD (a) and x_2 that represents DO deficiency (b) with the Kalman filter, as well as the filter with PD adaptation. The blue colour – model data, green – estimation with the Kalman filter, black – filter estimation with PD adaptation, pink – observation results

The analysis of the conducted simulations should be started with pointing out that the DO measurement that can be seen in Fig. 3b (pink points) is well reflected by the state coordinate x_2 that represents DO in the numerical model (blue line). The estimations of state variables presented in Fig. 3 and obtained by means of using an adaptive PD filter (black colour) drift faster towards the values of state variables (blue colour). It can be observed in particular in the case of the non-measured state variable x_1 that represents BOD (Fig. 3a). This indicates that the adaptive PD filter performs better in situations burdened with greater number of unknowns.

The state estimation errors described in Fig. 4 confirm the abovementioned observations – the errors made by an adaptive PD filter drift much faster towards zero than errors of the Kalman filter. This implies that the estimation with PD adaptive filter reflects the reality quicker than estimation with the Kalman filter. This is caused by flexible selection of amplification in the PD adaptive algorithm. Changes in amplification $\boldsymbol{K}$ that correspond to amplification $\boldsymbol{K_F}$ of the Kalman filter for both state variables were shown in Fig. 5. As it can be observed, amplifications vary in a wide range and, what is characteristic, ar the moment of big

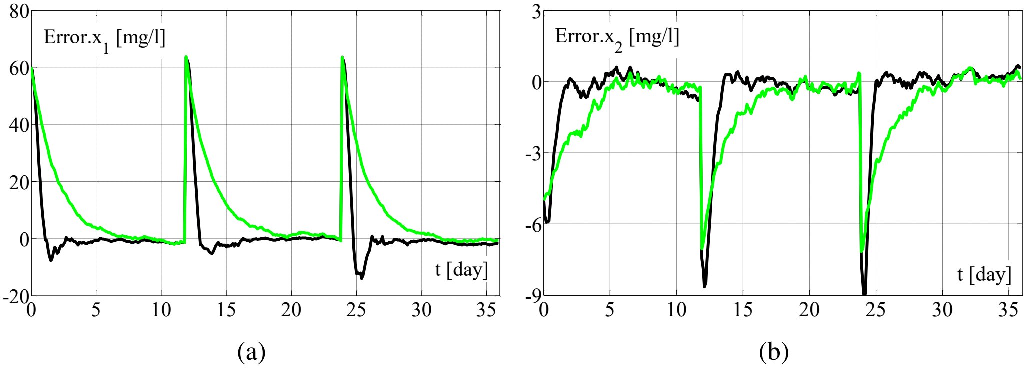


Figure 4: Illustration of estimation errors of state variables x_1 that represent BOD (a) and x_2 that represents DO deficiency (b) with the Kalman filter (green) and the filter with PD adaptation (black)

interference occurrence (side creek) the values rise very significantly and then they decrease gradually. Additionally, changes in the amplification coefficient for the whole state variable that represents BOD vary to a greater extent than for the variable that represents DO deficiency, which is caused by the fact that DO is measured and the algorithm has more complete data on the signal. This observation is proved by all the simulations conducted.

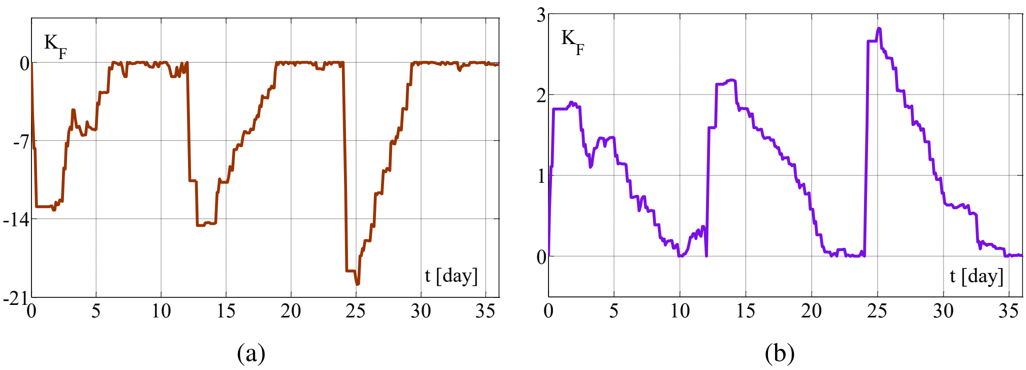
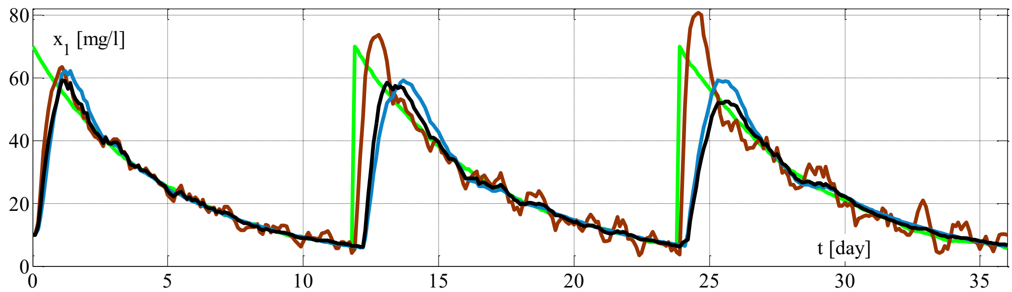
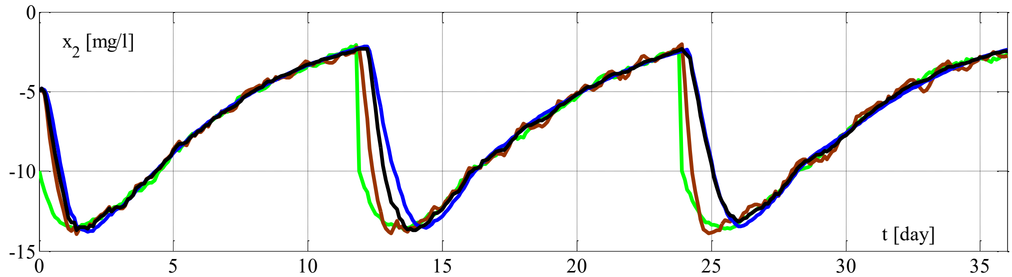


Figure 5: Illustration of PD adaptive filter amplification changes during estimation of the state variable x_1 that represents BOD (a) and x_2 that represents DO deficiency (b)

The value of the parameter k_p influences the performance of the adaptive PD filter (Fig. 6). Assumption of too high or too low values causes a change in waveforms of the estimated signals. In order to do that indicators that enable to asses estimation quality were defined. This influence is, practically speaking, small in the case of DO deficiency estimation, which, as previously mentioned, results from the fact that DO is a measured signal and the estimation has more information on its character.



(a)



(b)

Figure 6: Illustration of the estimate of state variables x_1 (BOD) (a) and x_2 (deficiency of DO) (b) with the adaptive PD for different amplification values k_p. The green colour shows data from the model. We are showed assumed values: Black colour for the variable $x_1 \rightarrow k_p = -0.9$, $x_2 \rightarrow k_p = 0.12$. Brown colour $x_1 \rightarrow k_p = -1.3$, $x_2 \rightarrow k_p = 0.16$. Blue colour $x_1 \rightarrow k_p = -0.6$, for $x_2 \rightarrow k_p = 0.07$

The studies also encompassed the influence of the intensity changes of measurement and system interference, expressed by means of their covariance, on the estimation quality measured with two indicators: $RMSE$ (12) and MPE (13). The selected research results are presented in Fig. 7.

In the case of estimation of the non-measured state coordinate x_1 that represents BOD one can notice apparent advantage of the adaptive PD filter over the Kalman filter (Figs. 7a, 7c). The value of both error indicators is significantly lower for the analysed cases, irrespectively from the values of covariances of system errors $\boldsymbol{W}$ and measurement errors $\boldsymbol{V}$. In the case of the estimation measured for the state variable x_2 that represents DO for both filters the indicators acquire comparable values.

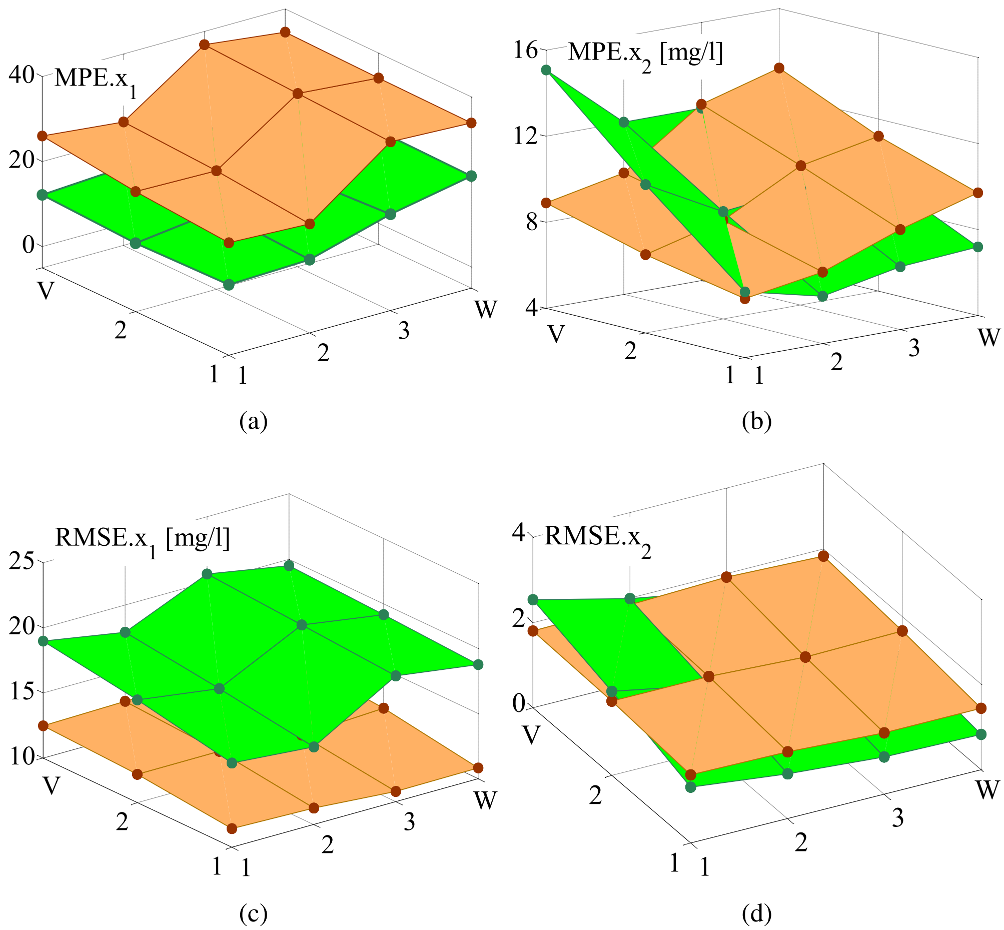


Figure 7: Illustration of the changes of estimation quality indicators (RMSE) and (MPE), depending on the covariance of system and measurement interferences

The values on the axis W are represented by system interference covariance matrices of the values of: $1 \rightarrow \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$; $2 \rightarrow \begin{bmatrix} 6 & -4 \\ -4 & 21 \end{bmatrix}$; $3 \rightarrow \begin{bmatrix} 9 & -4 \\ -4 & 6 \end{bmatrix}$; $4 \rightarrow \begin{bmatrix} 18 & -6 \\ -6 & 9 \end{bmatrix}$; the values on the axis V are represented by covariances of measurement interference of the values: $1 \rightarrow 0.1$; $2 \rightarrow 0.3$; $3 \rightarrow 0.6$. Green colour – the Kalman filter, light brown – adaptive PD filter

5. Summary

The paper discusses the algorithm of adaptive determination of the amplification filter that estimates object state on the basis of the state coordinate measured online. A model of a river described by ordinary differential equations, according to the so-called characteristics, with three big side creeks of significant pollution was used as the study object.

The proposed iterative algorithm of adaptive determination of the amplification filter does not require the knowledge of system ($\boldsymbol{W}$) and measurement ($\boldsymbol{V}$) covariance matrices, which make up for significant non-conformity in the case of using the Kalman filter. Determining values of the $\boldsymbol{W}$ and $\boldsymbol{V}$ matrices requires long-term statistical research and their values are often burdened with a large error. These disadvantages do not appear in the approach proposed in this paper. The algorithm presented in the work require any information about system and measurement disturbances, but only the expert knowledge needed to select $\boldsymbol{k}_{\mathrm{p}}$ and $\boldsymbol{T}_{\mathrm{d}}$. The algorithm has a form of additive filter amplification correction and for the purpose of its selection it makes use of the predefined adaptive error and its derivative; thus it has a proportional-differential character (PD). In all the investigated cases the adaptive filter provided better estimation results, measured with quality indicators RMSE and MPE, than the Kalman filter. It was particularly evident for the non-measured estimation of the state coordinate. Better convergence of estimates in the adaptive filter for the state variable values, in the case of high variations, is the result calculated by the algorithm of temporarily high values of amplifications.

Performance of the adaptive PD filter was stable in a wide range of changes of the proportional amplification parameters $\boldsymbol{k}_{\mathrm{p}}$ and differentiation time $\boldsymbol{T}_{\mathrm{d}}$

It is noteworthy that the proposed method does not require a significant amount of calculations and the computation complexity is lower by one order than the Kalman filter.

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