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AN IMPROVED ANGLE CALIBRATION METHOD OF A HIGH-PRECISION ANGLE COMPARATOR

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Abstract

Angle calibrations are widely used in various fields of science and technology, while in the high-precision angle calibrations, a complete closure method which is complex and time-consuming is common. Therefore, in order to improve the measurement efficiency and maintain the accuracy of the complete closure method, an improved calibration method was proposed and verified by the calibration of a high-precision angle comparator with sub-arc-second level. Firstly, a basic principle and algorithm of angle calibration based on complete closure and symmetry connection theory was studied. Then, depending on the pre-established calibration system, the comparator was respectively calibrated by two calibration methods. Finally, by comparing E_n values of two calibration results, the effectiveness of the improved method was verified. The calibration results show that the angle comparator has a stable angle position error of $0.17''$ and a measurement uncertainty of $0.05''$ ($k = 2$). Through method comparisons, it was shown that the improved calibration method can greatly reduce calibration time and improve the calibration efficiency while ensuring the calibration accuracy, and with the decrease of measurement interval, the improvement of calibration efficiency was more obvious.

Keywords: angle calibration, position error, complete closure method, method verification.

1. Introduction

As a basic parameter, angle is widely used in various fields such as industry manufacturing, scientific research and national defence [1–3]. Angle calibration is an important part of geometric measurement. The high precision angle calibration technology with sub-arc-second level has been given more and more attention, and plays an important role in the fields of precision engineering, aerospace, basic physics and optical engineering [4–6]. With the development of technologies,

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various angle calibration devices and methods have been proposed, and the measurement accuracy and uncertainty have been greatly improved.

Among the angle calibration devices, the high-precision angle comparator is a kind of continuous angle standard for a full circle based on photoelectrically scanned circular gratings developed in recent years [7–9]. Compared with traditional mechanical angle calibration devices, such as measuring blocks, measuring sticks and dividing disks, etc., it has higher angle measuring accuracy and resolution. Compared with the autocollimator [10–12], the small-angle laser interferometer and other high-precision angle measuring instruments, the angle comparator obtains a wider range of angle measurement. Therefore, many national metrology institutes [13–16] have developed their own angle comparators as national continuous angle standards for a full circle. Among them, Germany is one of the countries with the earliest research and the most advanced achievements in high-precision angle comparators. *Physikalisch-Technische Bundesanstalt* (PTB) [17,18] has cooperated with Heidenhain™ to complete the development of a high-precision angle comparator based on commercial angle encoders and aerostatic bearings. By means of a friction drive, it had a very high motion resolution. Through the precise and stable control of environment and other conditions, the uncertainty of the angle comparator came to $0.01''$ ($k = 2$), and the angle position error was $0.1''$. At present, as angle transmission devices in various countries, most of the angle comparators have had sub-arc-second angle position errors which pose in many problems to angle calibration of comparators. In order to improve the accuracy of angle measurement, it is necessary to adapt methods from environmental control [19, 20], device development [21], method research and other aspects.

There are two kinds of commonly used angle calibration methods of full circle, *i.e.* the autocalibration method and the cross-calibration method. The principle of autocalibration is to distribute several reading heads around the divided circle according to a certain rule, and process the test results of each reading head, so as to eliminate the scaling errors of the divided circle and obtain a higher angle detection accuracy [22–24]. The so-called autocalibration method allows these angular deviations to be quickly and efficiently evaluated. Cross calibration is the fundamental method of angle calibration, which mainly includes two categories: the normal angle method and the comparison method [25]. The complete closure method based on the circle closure principle is the most precise and accurate among the comparison methods. By comparing working angles deviations of two angle devices, the position errors of the reference angle device can be separated from that of the calibrated object with high accuracy [26, 27]. Studies [18] have shown that the well-established complete closure method is well suited for determining the comparator's errors, because both the errors of the divided circle and the measuring errors of the reading heads are combined in this case.

In our previous work [4,28], first, some key technologies were studied, such as aerostatic bearings preloaded with vacuum, friction driving, multi-mode control strategy and so on. Secondly, based on the key technologies, a high-precision angle comparator was developed. Finally, through the complete closure method, the comparator was calibrated in order to obtain high-precision compensation data. The calibration process of the complete closure method includes a complex algorithm, time-consuming detection and complicated calculation work. For example, the frequency of cross comparisons in the measurement process increases with a quadric function of the measurement intervals. The compact measurement interval will lead to a significant increase of measurement time, which poses a great challenge to the stability of measurement environment and affects the measurement efficiency and accuracy. Therefore, it is necessary to improve the method to fast accomplish high-accuracy calibration of the angle comparator.

In this paper, an improved complete closure method based on the circle closure principle and symmetry connection theory was introduced. In order to verify the correctness and effectiveness

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of this method, a verification test was carried out on a stable angle comparator. And an angle calibration system based on an autocollimator and an indexing table, the same as instruments in the complete closure method, was built to reduce the comparison error caused by instruments. Through the comparison of calibration results, it was proved that the improved calibration method had considerable accuracy and faster measurement.

2. Calibration method

With the premise of ensuring measurement accuracy, in order to improve measurement efficiency, it is necessary to reduce the frequency of measurements on the complete closure principle. Therefore, a constant angle symmetrical connection method based on the selection of two large constant angles (mutually prime) under certain conditions is proposed. When the required angle is divided into two different types, forming two series, each series is measured separately, and through establishing the connection between the two constant angle measurement series of mutual quality, the angle deviation is finally obtained. In this method, the small interval error can be obtained by observing and connecting several large constant angles so as to improve the measurement accuracy and efficiency.

Therefore, assuming not changing the basic principle of complete closure method, the symmetrical connection theory is applied to form an improved complete closure method, which combines the complete closure method with the symmetrical connection method. This method, based on symmetry connection theory, adopts the closeness and accuracy of the circle closure principle, at the same time, the large angle of the symmetrical connection method is used to find the small angle so as to improve measurement efficiency.

The basic principle of the improved complete closure method is to decompose the original n measurement sequences into p-type sequence and q-type sequence. The number p and q should be coprime, and

$$p \cdot q = n. \tag{1}$$

In order to establish the connection between the p-type sequence and q-type sequence, the q groups of complete closure measurements are carried out in the p-type sequence, and the p groups of complete closure measurements are carried out in the q-type sequence. In this way, there are n single point evaluation values in each series, and then, the two series of measurement data are connected by coordinate translation and zero-point correction. There is

$$\begin{cases} \varphi^q = \omega_\varphi^q + \omega_\varphi^{p,q} \\ \varphi^p = \omega_\varphi^p + \omega_\varphi^{q,p} \end{cases}, \tag{2}$$

where, ω_φ^q is the graduation error measured in the full combination series of q-type series at φ position, $\omega_\varphi^{p,q} = \frac{1}{q}\sum \omega_\varphi^p$ is the zero correction number of the series of measurement point errors in the p-type series, ω_φ^p is the graduation error measured in the full combination series of p-type series at φ position, $\omega_\varphi^{q,p} = \frac{1}{p}\sum \omega_\varphi^q$ is the zero correction number of the series of measurement point errors in the q-type series. The division error of each measuring point is

$$\Delta\varphi = \frac{1}{2}\left(\varphi^q + \varphi^p\right). \tag{3}$$

It is easy to prove that the number of measurements $(p+q)n$ of the improved method is smaller than that of the complete closure method n^2, improving measurement efficiency with the same measurement accuracy. Table 1 shows that under the common number of optical polygon faces (12, 24 and 36), compared with the complete closure method that has the same calibration accuracy, the application of the improved complete closure method can effectively reduce the number of measurements and the amount of measurement data, thus reducing the measurement time and greatly increasing the calibration efficiency. In addition, with the decrease of measurement interval, the improvement of calibration efficiency is more obvious.

Table 1. Efficiency improvement of different faces.

Number of faces (n)	p	q	$(p+q)n$	n^2	Efficiency Improvement
12	3	4	84	144	41.67%
24	3	8	264	576	54.17%
36	4	9	468	1296	63.89%

3. Experiment

3.1. Experiment process

A complete calibration system consistent with the complete closure method was placed on a granite base. All arrangements were the same as those in reference [4], and the instrument installation is shown in Fig. 1. The central axis of the optical polygon, the indexing table and the angle comparator were installed coaxially with a deviation of no more than 0.02 mm.

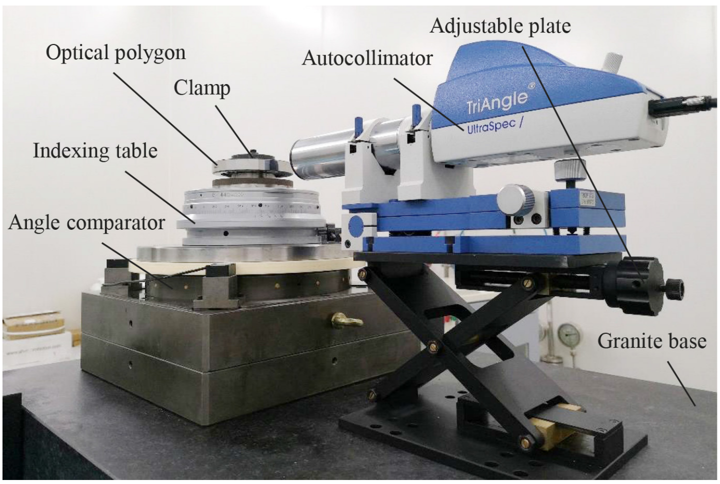

Fig. 1. Arrangement of main devices of the calibration system.

The test flow of the improved complete closure method is shown in Fig. 2. It includes q groups of p-type complete closure measurements and p groups of q-type complete closure measurements.

The calibration experiment of angle comparator based on the improved complete closure method was carried out through 12-faces of the optical polygon. Setting $p = 3$ and $q = 4$, then the experiment was equivalent to carrying out the complete closure experiment of $n = 12$. The experiment parameters are shown in Table 2.

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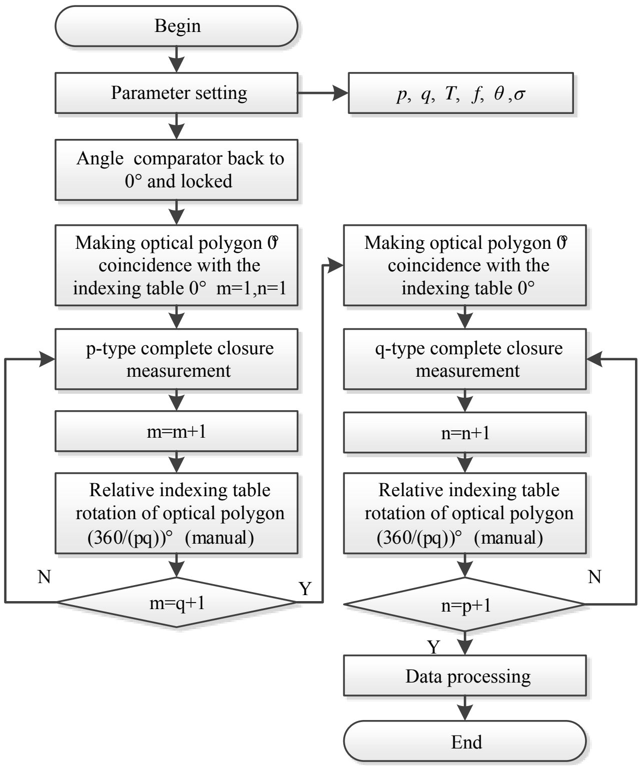

Fig. 2. Experiment Process.

Table 2. Experiment parameters.

Parameters	Symbol	Specification
p-type	p	3
q-type	q	4
Number of observations	n	12
Acquisition time	T	10 s
Sampling frequency	f	1 per second
Permissible value of closure error	θ	0.1″
Permissible value of standard deviation	σ	0.05″

3.2. Calibration results

A high precision angle comparator (designed precision: 0.2") was calibrated by the established calibration system. Four groups of 3-type experiments with 120° measurement interval were completed and 36 test data were obtained. Three groups of 4-type experiments with 90° measurement interval were completed and 48 test data were obtained. Compared with the complete closure experiment of $n = 12$, the number of measurement points decreased by 41.67%, and the measuring time reduced synchronously, which was conducive to the control of the influencing factors.

In order to reduce the measurement errors, the possible influencing factors, such as temperature fluctuation (< 0.5°C), humidity fluctuation (±1‰), flatness error of the optical polygon (better than 50 nm), tilt error of the optical polygon reflecting face around the rotation axis (< 35″) were strictly controlled during the test. The control parameters were consistent with the complete closure method. After data processing of coordinate translation and zero-point correction, the calibration results of angle comparator were obtained (Fig. 3), and it can be seen that the angle position error reached 0.17″

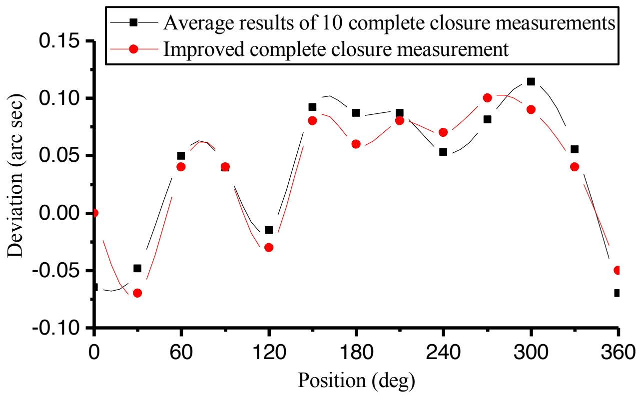

Fig. 3. The position error of the angle comparator.

Apart from adopting the improved method to calibrate the angle comparator, ten repeated calibration experiments were conducted consecutively with the complete closure method of $n = 12$ in five days. The average results are shown in Fig. 3, It can be seen from the figure that the average angle position error of the angle comparator is 0.18″, this is very close to the calibration result obtained with the improved method.

Ten instances of calibration results are shown in Fig. 4, and the measurement results obtained with the improved complete closure method are included in the measurement results distribution domain formed by ten measurement results. In addition, the maximum value of standard deviation

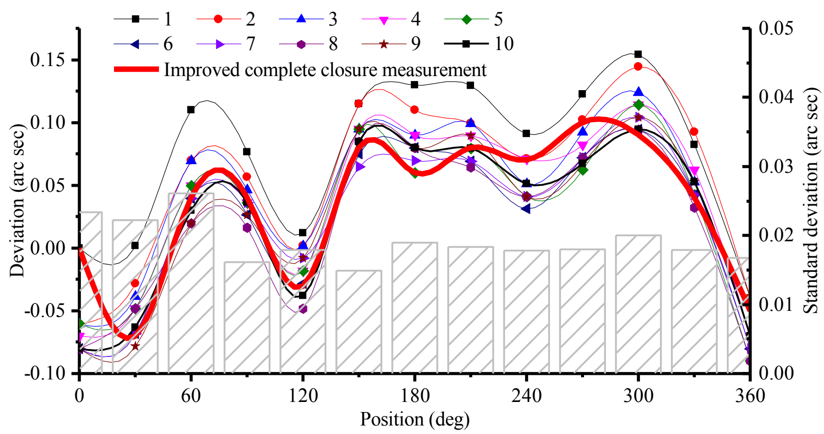

Fig. 4. Comparison of calibration results of two methods.

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of angle position is only 0.026″, indicating that the angle comparator has excellent positioning repeatability. Therefore, the reliability of the measurement system has been verified. The repeatability can also be used to verify the reliability of subsequent measurement uncertainty evaluation, because the repeatability is due to a variety of measurement errors in the measurement process. The experiment results provide conditions for subsequent verification.

4. Discussion

In order to verify the correctness of the improved method, the comparison method of E_n value between results obtained with two calibration methods was used to compare the position errors of the angle comparator. The evaluation rule was that when $E_n \leq 1$, the comparison results were satisfactory, otherwise, the comparison results were unsatisfactory, and the E_n value could be calculated with

$$E_n = \frac{\left|y_{imp} - y_{ave}\right|}{\sqrt{U_{imp}^2 + U_{ave}^2}}, \tag{4}$$

where y_{imp} was the improved complete closure measurement result, y_{ave} was the average result of 10 complete closure measurements, U_{imp} was the extended uncertainty of y_{imp}, U_{ave} was the extended uncertainty of y_{ave}.

In the previous study [4], we obtained the measurement uncertainty of no more than 0.05″ ($k = 2$). In this test, we used the same instrument and installation. At the same time, the environment, closure error and tower difference (the verticality errors of the reflecting surface and base plane) were controlled within the allowable range. According to the calculation method of measurement uncertainty in reference [4], using the mathematical model of (3), the contributions of indication error and quantification error of the autocollimator, repeatability of angle comparator, flatness error of the reflecting surface, tower difference, and closure error on measurement uncertainty were analysed, and the measurement uncertainty of the improved method was easily obtained. As a result, the measurement uncertainty obtained through evaluation was also 0.05″ ($k = 2$). Figure 5 shows the calculated E_n value curve, in which E_n did not exceed the limit value and the comparison results were all satisfactory.

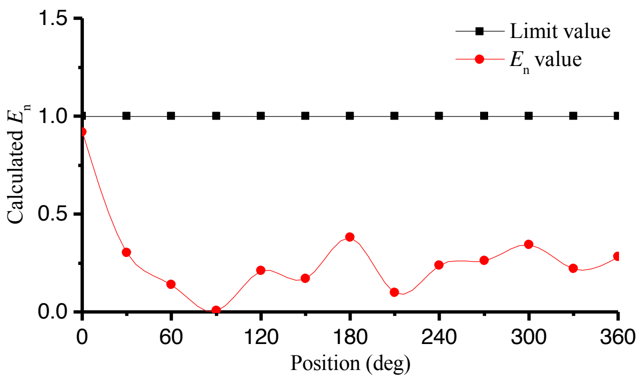

Fig. 5. The curve E_n value.

It can be seen from Fig. 5 that the E_n value reaches 0.9 when position is 0°. The reasons may include the following aspects:

a) Repeatability: when the improved method is used to calibrate the precision angle comparator, the fixed-point static measurement is used, and the fixed-point repeatability of the tested object has a great influence on the measurement uncertainty.
b) Fluctuation: considering fluctuation of measurement environment and instruments, there are some points with large measurement errors, which lead to the deviation of E_n value at some points.
c) Process control: the relative errors between the two calibration results are amplified by the calculation method of normalized deviation E_n value, which leads to large deviation of the value without the whole process control of the two comparison tests.

5. Conclusions

This paper introduced an improved complete closure method based on a symmetry connection theory. This method ensured the calibration accuracy and improved the calibration efficiency by reducing the calibration time, and with the decrease of the measurement interval, the improvement of calibration efficiency was more obvious. In order to verify the correctness and effectiveness of this method, relying on the same angle calibration system as the complete closure method, a stable high-precision angle comparator was calibrated through the complete closure method and the improved method. Compared with that of the complete closure experiment of $n = 12$, the number of measurement points of the improved method decreased by 41.67%, and the measuring time reduced synchronously. The experiment showed that the angle position error of the comparator reached $0.17''$, and the measurement uncertainty was only $0.05''$ ($k = 2$). Through comparison of calibration results obtained with the two calibration methods, it was proved that the improved calibration method has considerable accuracy and high efficiency.

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