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SEARCHING FOR OPTIMAL PRESTRESSING OF STEEL BAR STRUCTURES BASED ON SENSITIVITY ANALYSIS

V. V. YURCHENKO[1], I. D. PELESHKO[2]

The paper considers parametric optimization problems for the steel bar structures formulated as nonlinear programming ones with variable unknown cross-sectional sizes of the structural members, as well as initial prestressing forces introduced into the specified redundant members of the structure. The system of constraints covers load-bearing capacity constraints for all the design sections of the structural members subjected to all the design load combinations at ultimate limit state, as well as displacement constraints for the specified nodes of the bar system, subjected to all design load combinations at serviceability limit state. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimization problem. A numerical technique to determine the optimal number of the redundant members to introduce the initial prestressing forces has been offered for high-order statically indeterminate bar structures. It reduces the dimension for the design variable vector of unknown initial prestressing forces for considered optimization problems.

Keywords: parametric optimization, redundant member, initial prestressing force, optimal prestressing, sensitivity analysis, gradient projection method

1. INTRODUCTION

The concept of prestressing steel structures is only recently being re-considered, despite a long and successful history of prestressing concrete members. In spite of having many advantages over

[1] Prof., DSc., PhD, Eng., Kyiv National University of Construction and Architecture, Faculty of Civil Engineering, Department of Steel and Timber Structures, Povitroflotskyj av. 31, 03680 Kyiv, Ukraine, e-mail: vitalina@scadsoft.com

[2] Assoc. Prof., PhD, Eng., Lviv Polytechnic National University, Institute of Civil Engineering and Building Systems, Department of Building Production, Karpinskoho st. 6, 79013 Lviv, Ukraine, e-mail: ipeleshko@hotmail.com

prestressed concrete, prestressed steel has not been popular due to the complexity and ambiguity involved in analysis and design calculations, as well as problems arising due to the application of external prestressing technique and fabrication [13].

Early work on the prestressing of steel structures was described by Magnel [12] in 1950, where it is shown experimentally that an improved retrenchment can be achieved by prestressing truss girders. More recent studies have explored the behavior and design of the prestressed steel beams [5], flooring systems [6], columns [20, 23], trusses [1, 9] and space trusses [21, 22]. Studies of the structural response of sub-assemblies and the overall response of prestressed frames with sliding joints have been also carried out [2] with further numerical investigation into the stress-erection process of such systems [11]. Each of the studies described above identifies the potential economies and enhanced performance through the use of prestressing [13].

A number of research works were dedicated to the optimization of prestressed bar structures. Applied optimum design problems for the prestressed bar structures are usually formulated as parametric optimization problems, namely searching problems for unknown structural parameters, which provide an extreme value of the specified purpose function in the feasible region, defined by the specified constraints. For this purpose, research papers [7, 14, 25, 27] used mathematical programming methods, where an optimal design is divided into several stages. At each stage, a search is completed after varying values of a specific group of parameters. Introduction of such stage-by-stage procedures in numerous cases may distort the conditions of optimization tasks.

Prestressing of a statically indeterminate bar system can be created by introducing the initial prestressing forces into the redundant members of a structural system. The number of initial prestressing forces introduced into the bar system can be less or equal to the degree of static indeterminacy of the bar system or the number of the redundant members.

Optimum distribution of the internal forces and material in the bar structure, corresponding to the least structural weight, can be achieved by introducing the initial prestressing forces into all the redundant members of a bar system. However, economical efficiency caused by the regulation of internal forces should be estimated by taking into account additional costs, required to create prestressing in the structural system. The fewer the redundant members in the prestressing process of the structure subjected to the initial deformations, the lower the costs associated with creating prestressing.

Complex high-order statically indeterminate bar systems with a great amount of the redundant members have lots of prestressing variants for them. For such structures, the numerical techniques have been offered to determine an optimal prestressing, which requires a great amount of the calculations related to solving the optimization problems for each prestressing variant or related to the high dimension of the design variable vector for unknown initial prestressing forces. In this paper, a prestressed high-order statically indeterminate bar structure is considered as a *research object*. This object is being investigated to find an optimal number of the redundant members to introduce initial prestressing forces.

Although many papers are published on the parametric optimization of the prestressed bar structures, the development of a numerical technique to determine an optimal number of the redundant members to introduce the initial prestressing forces for high-order statically indeterminate bar structures remains an actual task. Therefore, the main *research goal* is the development of numerical algorithm to solve parametric optimization problems of the prestressed bar structures with the searching for the optimal number of the redundant members to introduce initial prestressing forces. The following *research task* is stated accordingly: to suggest a numerical technique of determination an optimal number of the redundant members to introduce the initial prestressing forces.

2. FORMULATION OF PARAMETRIC OPTIMIZATION PROBLEMS

Let a parametric optimization problem of a structure consisting of the bar members be considered, which can be formulated as presented below: to find optimum values for bar's cross-section sizes and initial prestressing forces, introduced into the specified redundant members of a bar system, which provide the extreme value of the determined optimality criterion and satisfy all the load-bearing capacities and stiffness requirements. We assume that the structural topology, cross-section types, node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are predefined and constant. The formulated *optimum material and internal forces distribution problem* can be stated as a non-linear programming task in the following mathematical terms: to find the unknown cross-sectional sizes and unknown initial prestressing forces:

$$\vec{X} = \left\{ \vec{X}_{CS}, \vec{X}_{PS} \right\}^T = \left\{ \left\{ X_{CS,\alpha} \right\}, \left\{ X_{PS,\beta} \right\} \right\}^T, \ \alpha = \overline{1, N_{X,CS}}, \ \beta = \overline{1, N_{X,PS}}, \tag{2.1}$$

providing the least value of the determined objective function:

$$f^* = f\left(\vec{X}^*\right) = \min_{\vec{X} \in \Im_\gimel} f\left(\vec{X}\right) \tag{2.2}$$

in the feasible region (search space) $\Im$ defined by the following system of constraints:

$$\boldsymbol{\varphi}\left(\vec{X}\right)=\left\{\phi_{\eta}\left(\vec{X}\right)\leq 0 \mid \eta=\overline{1,N_C}\right\} \tag{2.3}$$

where $\vec{X}$ – vector of the design variables; $\vec{X}_{CS}$ and $\vec{X}_{PS}$ – vectors of unknown cross-sectional sizes and unknown initial prestressing forces, accordingly; f, φ_{η} – continuous functions of the vector argument; $\vec{X}^{*}$ – optimum solution (the vector of optimum values of the structural parameters); f^{*} – optimum value of the objective function; N_C – number of constraints $\varphi_{\eta}\left(\vec{X}\right)$, which define a feasible region in the design space $\Im$; $N_{X,CS}$ – total number of unknown cross-sectional sizes of the structural members; $N_{X,PS}$ – total number of unknown initial prestressing forces, introduced into the specified redundant members of the bar system.

In cases when vector of the design variables $\vec{X}$ consists of unknown cross-sectional sizes $\vec{X}_{CS}$ only, then the *optimum material distribution problem* for the bar system is under consideration.

The specific technical-and-economic index (material weight, material cost, construction cost, etc.) or another determined indicator can be considered as the objective function Eq. (2.2), taking into consideration the ability to formulate it analytical expression as a function of design variables $\vec{X}$.

Load-bearing capacity constraints (strength and stability inequalities) for all design sections of the structural members, subjected to all ultimate load combinations, as well as displacement constraints (stiffness inequalities) for the specified nodes of a bar system, subjected to all serviceability load combinations, should be included into the system of constraints Eq. (2.3). Additional requirements which describe structural, technological, and serviceability particularities of the building structure under consideration can be also included into the system of constraints.

Design internal forces in the bar structural members(components) used in strength and stability inequalities of the system Eq. (2.3) are considered as state variables depending on design variables $\vec{X}$ and can be calculated from the following linear equation system of the finite element method:

$$\mathbf{K}\left(\vec{X}_{CS}\right)\times\vec{z}_{ULS,k}=\vec{p}_{ULS,k}\left(\vec{X}_{PS}\right),\ k=\overline{1,N_{LC}}\ ; \tag{2.4}$$

where $\mathbf{K}\left(\vec{X}_{CS}\right)$ – stiffness matrix of the finite element model of a bar system, which should be formed depending on the unknown (variable) cross-sectional sizes of the structural members $\vec{X}_{CS}$; $\vec{p}_{ULS,k}\left(\vec{X}_{PS}\right)$ – column-vector of the node's loads for k^{th} ultimate design load combination, which should be formed depending on the unknown (variable) initial prestressing forces $\vec{X}_{PS}$; $\vec{z}_{ULS,k}$ – result column-vector of the node displacements for k^{th} ultimate design load combination; N_{LC} – the number of ultimate load combinations.

The design internal forces (axial force, bending moments and shear forces) can be calculated depending on the node displacement column-vector $\vec{z}_{ULS,k}$ for each i^{th}, the design section of j^{th} bar finite element of the structure subjected to k^{th} ultimate design load combination.

Node displacements of a bar system used in stiffness inequalities of the system Eq. (2.3) are also considered as state variables depending on design variables $\vec{X}$ and can be calculated from the following linear equation system of the finite element method:

$$\mathbf{K}\left(\vec{X}_{CS}\right)\times\vec{z}_{SLS,k}=\vec{p}_{SLS,k}\left(\vec{X}_{PS}\right); \tag{2.5}$$

where $\vec{p}_{SLS,k}\left(\vec{X}_{PS}\right)$ – column-vector of the node's loads for k^{th} design load combination of the serviceability limit state, which should be formed depending on unknown (variable) initial prestressing forces $\vec{X}_{PS}$; $\vec{z}_{SLS,k}$ – result column-vector of the node displacements for k^{th} design load combination of the serviceability limit state.

The design vertical and horizontal displacements can be calculated depending on a node displacement column-vector $\vec{z}_{SLS,k}$ for each m^{th} node of the finite element model subjected to k^{th} serviceability design load combination.

The parametric optimization problem stated by Eqs. (2.1) – (2.3) can be successfully solved using gradient projection nonlinear methods [17, 26] in cases when the purpose function and constraints of the mathematical model are continuously differentiable functions, as well as the search space is smooth [10, 19]. The *method of objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations* [8] ensures an effective search for the optimum solution [15]. Additionally, a sensitivity analysis is a useful optional feature [24] that could be used in the scope of numerical algorithms which are developed based on the gradient methods.

3. A METHOD TO DETERMINE OPTIMAL PRESTRESSING VARIANT OF THE BAR STRUCTURE

A certain β^{th} prestressing variant $\mathbf{V}_{\beta}$ of a bar structure can be definitely described by a set of redundant members $\mathbf{r}_{\beta}=\left\{r_{\beta,\mu}\right\},\mu=\overline{1,N_{RM,\beta}}$, and by the value of the initial prestressing force $X_{PS,\beta}$

introduced into these members, $\mathbf{V}_\beta = \{\mathbf{r}_\beta, X_{PS,\beta}\} = \{\{r_{\beta,\mu}\}, X_{PS,\beta}\}$, $\mu = \overline{1, N_{RM,\beta}}$, here $N_{RM,\beta}$ is the number of redundant members for β th prestressing variant $\mathbf{V}_\beta$.

The set of the prestressing variants is $\mathbf{B} = \{\mathbf{V}_\beta\} = \{\mathbf{r}_\beta, X_{PS,\beta}\}$, $\beta = \overline{1, N_{X,PS}}$. The number of initial prestressing forces introduced into the bar system can be less or equal to the degree of static indeterminacy of a bar system N_{DSI}, namely $\sum_{\beta=1}^{N_{X,PS}} N_{RM,\beta} \le N_{DSI}$. The design variables vector $\vec{X}_{PS} = \{X_{PS,\beta}\}$ of the unknown (variable) initial prestressing forces for the considered bar system is formed according to set $\mathbf{B} = \{\mathbf{V}_\beta\}$ of the prestressing variants. An *optimal prestressing variant* for the considered structure can be defined as a combination of some prestressing variants $\mathbf{V}_\beta \subseteq \mathbf{B}$ and presented as subset $\mathbf{\Theta} \subseteq \mathbf{B}$, $\mathbf{\Theta} = \{\mathbf{V}_\delta \mid \mathbf{V}_\delta \in \mathbf{B}\}$, $\delta = \overline{1, \tilde{N}_{X,PS}}$, $\tilde{N}_{X,PS} \le N_{X,PS}$.

In the beginning set $\mathbf{\Theta}$ representing the optimal prestressing variants is $\mathbf{\Theta} = \varnothing$, vector of the initial prestressing forces is $\vec{X}_{PS} = \varnothing$. At each iteration of the proposed algorithm, one of the prestressing variants $\mathbf{V}_\beta \in \mathbf{B}$ is included into the set $\mathbf{\Theta}$, and the optimum material and internal forces distribution problem Eqs. (2.1) – (2.3) in the bar system should be solved.

Let the following function be introduced to the further consideration Eq. (3.1), where it estimates both under-stressing and overstressing in the term of normal stresses for all the structural members:

$$\mathbf{S}_\sigma = \sum_{k=1}^{N_{LC}} \sum_{j=1}^{N_B} \sum_{i=1}^{N_{DS}} \left(\sigma_{x\max,j} - \sigma_{x,ijk}\left(\vec{X}\right)\right)^2; \tag{3.1}$$

where $\sigma_{x,ijk}\left(\vec{X}\right)$ – design value of the local longitudinal stress due to the bending moments and the axial force calculated in i th design section for j th structural member subjected to k th load case combination depending on design variables $\vec{X}$; N_{DS} – number of design sections in structural members; N_B – number of the structural members; $\sigma_{x\max,j}$ – allowable stresses.

An order of the consecutive including the prestressing variants $\mathbf{V}_\beta = \{\mathbf{r}_\beta, X_{PS,\beta}\} = \{\{r_{\beta,\mu}\}, X_{PS,\beta}\}$, $\mu = \overline{1, N_{RM,\beta}}$, from set $\mathbf{B}$ into set $\mathbf{\Theta}$ can be defined by values of the components of the gradient vector for function $\mathbf{S}_\sigma$ Eq. (3.1) with respect to the variable prestressing parameters $\vec{X}_{PS}$. Prestressing variant $\mathbf{V}_m = \{\mathbf{r}_m, X_{PS,m}\} = \{\{r_{m,\mu}\}, X_{PS,m}\} \in \mathbf{B}$, $\mu = \overline{1, N_{RM,m}}$, with maximum value of the gradient of the function $\mathbf{S}_\sigma$ Eq. (3.1) related to the number of redundant members $N_{RM,m}$ should be primarily included into set $\mathbf{\Theta}$. Consecutive including of the prestressing variants from set $\mathbf{B}$ of the predefined prestressing variants into set $\mathbf{\Theta}$ representing the optimal prestressing variants should be performed until the regulation of an internal forces in the structure under consideration, leads to the desired decrement of the objective function.

Let the following algorithm be presented to find the optimal number of the redundant members for introducing initial prestressing forces into the redundant members of the bar structures.

Step 0. $n=0$ is the number of optimization problems solved. The optimal number of the redundant members to introduce the initial prestressing forces for considered bar system is $N_{RM}=0$. The degree of static indeterminacy of the bar system is N_{DSI}.

Step 1. A searching problem for optimum cross-section sizes of the considered structure without initial prestressing forces in the redundant members is formulated and solved in the space of the unknown cross-sectional sizes $\vec{X}_{CS}$ only. As a result, those optimum cross-section sizes $\vec{X}^*_{CS,n}$ of the structural members, which provide the least value f^*_n of the objective function Eq. (2.2) and satisfy the system of constraints Eq. (2.3), are defined. $n \leftarrow n+1$.

Step 2. The set of the prestressing variants $\mathbf{B}=\{\mathbf{V}_\beta\}=\{\mathbf{r}_\beta, X_{PS,\beta}\}$, $\beta=\overline{1,N_{X,PS}}$, of the bar system is predefined by a designer. The number of redundant members $N_{RM,\beta}$ for each predefined prestressing variant $\mathbf{V}_\beta=\{\mathbf{r}_\beta, X_{PS,\beta}\}=\{\{r_{\beta,\mu}\}, X_{PS,\beta}\}$, $\mu=\overline{1,N_{RM,\beta}}$ should not exceed the degree of the static indeterminacy of the bar system N_{DSI}, $N_{RM,\beta}\le N_{DSI}$. Auxiliary vector $\vec{Y}_{PS}=\{X_{PS,\beta} \mid X_{PS,\beta}\in\mathbf{V}_\beta\}$ of the unknown initial prestressing forces is formed according to set $\mathbf{B}$. Initial zero value for each component $X_{PS,\beta}$ of vector $\vec{Y}_{PS}$ should be assigned.

Step 3. Set of the optimal prestressing variants is $\mathbf{\Theta}=\varnothing$. Vector of the design variables corresponding to initial prestressing forces is $\vec{X}_{PS,n}=\varnothing$.

Step 4. Vector for the gradient of function $\mathbf{S}_\sigma$ Eq. (3.1) is calculated for all variable prestressing parameters (unknown initial prestressing forces) $\vec{Y}_{PS}$:

$$\nabla\mathbf{S}_\sigma=\left\{\frac{\partial\mathbf{S}_\sigma}{\partial X_{PS,\beta}}\right\}\ \forall X_{PS,\beta}\in\vec{Y}_{PS},\ \beta=\overline{1,N_{X,PS}}. \tag{3.2}$$

Step 5. Whichever prestressing variant $\mathbf{V}_m=\{\mathbf{r}_m, X_{PS,m}\}=\{\{r_{m,\mu}\}, X_{PS,m}\}\in\mathbf{B}$, $\mu=\overline{1,N_{RM,m}}$ that meets the following criteria, should be included into the further consideration:

(3.3) $$\frac{1}{N_{RM,m}} \cdot \left| \frac{\partial \mathbf{S}_{\sigma}}{\partial X_{PS,m}} \right| \rightarrow \max, \; N_{RM,m} \leq N_{DSI};$$

where $N_{RM,m}$ – number of redundant members where initial prestressing force $X_{PS,m}$ is introduced.

If there are no prestressing variants with the number of redundant members $X_{PS,m}$ less than the number N_{DSI} of static indeterminacy degree, then move to step 10 should be executed.

Step 6. Unknown initial prestressing force $X_{PS,m}$, corresponding to prestressing variant $\mathbf{V}_m$, should be added to the design variable vector $\vec{X}_{PS,n}$, $\vec{X}_{PS,n} \leftarrow \vec{X}_{PS,n} + \{X_{PS,m}\}$.

Step 7. The optimum material and internal forces problem Eqs. (2.1) – (2.3) is formulated and solved in the space of the unknown cross-sectional sizes and unknown initial prestressing forces. Those optimum values for cross-sectional sizes $\vec{X}^*_{CS,n}$ and for initial prestressing forces $\vec{X}^*_{PS.n}$ are defined $\vec{X}^*_n = \{\vec{X}^*_{CS,n}, \vec{X}^*_{PS,n}\}^T$, which satisfy the system of constraints Eq. (2.3), and provide the least value of the objective function f^*_n Eq. (2.2). $n \leftarrow n+1$.

Step 8. If $f^*_{n-2} - f^*_{n-1} \geq \varepsilon f^*_{n-1}$, where $\varepsilon \approx 1.05$ is the desired decrement of the objective function value, caused by introducing initial prestressing force $X_{PS,m}$ in the redundant members $\mathbf{r}_m$ of the m^{th} prestressing variant $\mathbf{V}_m = \{\mathbf{r}_m, X_{PS,m}\} = \{\{r_{m,\mu}\}, X_{PS,m}\}$, $\mu = \overline{1, N}_{RM,m}$, then $\mathbf{\Theta} \leftarrow \mathbf{\Theta} + \mathbf{V}_m$, $\vec{Y}_{PS} \leftarrow \vec{Y}_{PS} - \{X_{PS,m}\}$. The optimal number of the redundant members to introduce the initial prestressing forces into the considered bar system is increased, as $N_{RM} \leftarrow N_{RM} + N_{RM,m}$. The degree of static indeterminacy of the bar system is decreased, as $N_{DSI} \leftarrow N_{DSI} - N_{RM,m}$. Move to step 4 should be performed. Otherwise, when $f^*_{n-2} - f^*_{n-1} < \varepsilon f^*_{n-1}$, then move to step 9 should be executed.

Step 9. Introducing the initial prestressing force $X_{PS,m}$ into the $\mathbf{r}_m$ redundant members of the bar system is not effective. Returning to the previous optimum solution should be executed, $\vec{X}^*_n \leftarrow \vec{X}^*_{n-1}$, $f^*_n \leftarrow f^*_{n-1}$. The number of optimization problems solved, should be decremented. $n \leftarrow n-1$.

Step 10. Optimal number of the redundant members to introduce the initial prestressing forces into the considered bar system is N_{RM}. Number of optimization problems solved is n. Optimum material and internal forces distribution corresponds to the design variables vector $\vec{X}^*_n$ and objective function f^*_n.

4. NUMERICAL EXAMPLE

The efficiency of the proposed numerical algorithm is presented to define the optimal number of the redundant members for introducing initial prestressing forces into the bar system, considering parametric optimization of a cross-beam structure (see Fig. 1).

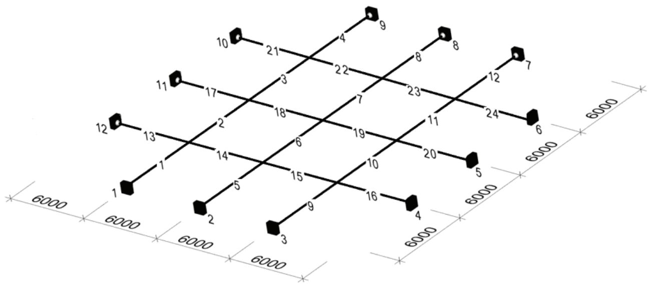

Fig. 1. Design scheme of the cross beam structure with node and bar numbers

The cross-beam structure is subjected to the distributed dead load with characteristic value 5.669 kN/m and distributed live load with characteristic value 243.189 kN/m. Applied loadings on the considered cross-beam structure are transmitted using mezzanine beams arranged with step 1m.

For considered cross-beam structure, steel grade S275 according to EN 10025-2 is used with the following material properties: yield strength $f_y = 275$ N/mm^2, modulus of elasticity $E = 210000$ N/mm^2, Poisson's ratio in elastic stage $\nu = 0.3$ and unit weight $\gamma = 7800$ kg/m^3. For all structural members welded I-beam cross-section type is used. Throat thickness for all beam flange to beam web welded connection is $a_w = 4$ mm.

Sufficient shear buckling resistance for all beam webs has been assumed ensuring by intermediate transverse and longitudinal stiffeners arranged according to EN 1993-1-5 requirements [4]. Class 3 for all cross-sections of the beam structural members is also assumed for considered cross-beam structure.

Cross-section sizes for all beams have been assigned as the same, in order to have load-carrying capacity reserves in the structure, which can be further utilized by prestressing. In practice, such bearing capacity reserves may exist due to requirements of unification, restrictions on the assortment range of rolled steel profiles, etc. It should be noted that there is no need for prestressing, in cases when tapered structural members are used for considered cross-beam structure.

According to item 1 of the algorithm presented above, the optimum material distribution problem is solved for specified initial data. Cross-sectional sizes of the cross-beam structure were considered as design variables $\vec{X}_{CS} = \left(h_w,\ t_w,\ b_f,\ t_f\right)^T$, where h_w is the beam web height, t_w is the beam web thickness, b_f is the beam flange width, t_f is the beam flange thickness. The material weight G was considered as the objective function Eq. (2.2):

$$G\left(\vec{X}_{CS}\right)=\gamma\left(h_w t_w+2b_f t_f\right)L\rightarrow\min; \tag{4.1}$$

where L is the overall length of all beams in the structure, $L=144$ m.

Load-carrying capacity constraints (strength and stability inequalities) for all design sections of the structural members, subjected to the ultimate design load combination, have been included into the system of constraints. The following strength constraints have been considered, formulated for all design sections $\forall i=\overline{1,N_{DS}}$ of all structural members $\forall j=\overline{1,N_B}$ according to requirements [3]:

$$\frac{\sigma_{\max,ijk}\left(\vec{X}\right)\gamma_{M0}}{f_y}-1=\frac{M_{Ed,ijk}\left(\vec{X}\right)\left(h_w+2t_f\right)}{2I_{y,j}\left(\vec{X}_{CS}\right)}\frac{\gamma_{M0}}{f_y}-1\le 0; \tag{4.2}$$

$$\frac{\tau_{\max,ijk}\left(\vec{X}\right)\gamma_{M0}\sqrt{3}}{f_y}-1=\frac{Q_{Ed,ijk}\left(\vec{X}\right)S_{y,j}\left(\vec{X}_{CS}\right)}{t_w I_{y,j}\left(\vec{X}_{CS}\right)}\frac{\gamma_{M0}\sqrt{3}}{f_y}-1\le 0; \tag{4.3}$$

$$\frac{\gamma_{M0}}{f_y}\sqrt{\sigma^2_{Ed,ijk}\left(\vec{X}\right)+3\tau^2_{Ed,ijk}\left(\vec{X}\right)}-1=\frac{\gamma_{M0}}{f_y}\sqrt{\left(\frac{M_{Ed,ijk}\left(\vec{X}\right)h_w}{2I_{y,j}\left(\vec{X}_{CS}\right)}\right)^2+3\left(\frac{Q_{Ed,ijk}\left(\vec{X}\right)b_f t_f\left(h_w+t_f\right)}{2t_w I_{y,j}\left(\vec{X}_{CS}\right)}\right)^2}-1\le 0. \tag{4.4}$$

where $M_{Ed,ijk}\left(\vec{X}\right)$ and $Q_{Ed,ijk}\left(\vec{X}\right)$ – bending moment and shear force acting in i^{th} design section of j^{th} structural member subjected to the ultimate load case combination, $k=1$, calculated from the linear equations system of the finite element method Eq. (2.4), depending on unknown initial prestressing forces $\vec{X}_{PS}$ and unknown cross-sectional sizes $\vec{X}_{CS}$ of the structural members; γ_{M0} – partial factor, $\gamma_{M0}=1$; $S_{y,j}\left(\vec{X}_{CS}\right)$ and $I_{y,j}\left(\vec{X}_{CS}\right)$ – first moment of the half of cross-sectional area and the second moment of inertia, are accordingly calculated depending on unknown cross-sectional sizes $\vec{X}_{CS}$ of the j^{th} structural members.

The following lateral-torsional buckling constraints have been also considered, formulated for all design sections $\forall i=\overline{1,N_{DS}}$ of all structural members $\forall j=\overline{1,N_B}$ according to requirements [3]:

$$\frac{M_{Ed,ijk}\left(\vec{X}\right)\left(h_w+2t_f\right)}{2\chi_{LT,j}\left(\vec{X}_{CS}\right)I_{y,j}\left(\vec{X}_{CS}\right)}\frac{\gamma_{M1}}{f_y}-1\le 0; \tag{4.5}$$

where γ_{M1} – partial factor, $\gamma_{M0} = 1$; $\chi_{LT,j}\left(\vec{X}_{CS}\right)$ – the reduction factor for lateral-torsional buckling calculated depending on the elastic critical moment for lateral-torsional buckling. The latter has been determined based on the cross sectional properties depending on the variable cross section sizes $\vec{X}_{CS}$ and takes into account the distance between lateral restraints equals to 1m.

The following constraints, which reflect maximum width-to-thickness ratios for compression parts for beam webs (internal compression parts) and beam flanges (outstand compression parts), formulated as for the 3rd cross-section class, according to EN 1993-1-1 [3], have been also included into the system of constraints:

$$\frac{h_w - 2a_w}{124 t_w}\sqrt{\frac{f_y}{235}} - 1 \le 0; \tag{4.6}$$

$$\frac{b_f - t_w - 2a_w}{28 t_f}\sqrt{\frac{f_y}{235}} - 1 \le 0. \tag{4.7}$$

The following vertical displacement constraints for specified (all internal) nodes of the cross-beam structure have been also included into the system of constraints:

$$\frac{\delta_{z,lk}\left(\vec{X}\right)}{\delta_{z\max,l}} - 1 \le 0; \tag{4.8}$$

where $\delta_{z,lk}\left(\vec{X}\right)$ is the vertical displacements of l^{th} structural node subjected to k^{th} serviceability load case combination, calculated from the linear equations system of the finite element method Eq. (2.5), $\delta_{z\max,l}$ is the allowable vertical displacement for l^{th} structural node, $\delta_{z\max,l} = 80$ mm.

The dimensions of the considered optimum material distribution problem were 4 design variables and 377 constraints. In order to realize the formulated optimization problem Eqs. (4.1) – (4.8), software OptCAD intended to solve parametric optimization problems for steel structural systems [16, 18] has been used. The following continuous optimum cross-sectional sizes $\vec{X}^*_{CS}$ for all beams has been obtained: $h_w = 2055.60$ mm, $t_w = 17.93$ mm, $b_f = 703.14$ mm, $t_f = 26.47$ mm, corresponded to the material weight $G^* = 83212.0$ kg of the cross-beam structure for the case when the number of redundant members to introduce initial prestressing forces equals to zero, $N_{RM} = 0$. Then the continuous optimum cross-sectional sizes have been discretized and the following discrete optimum cross-sectional sizes $\vec{X}^*_{CS}$ for all beams has been received: $h_w = 2050$ mm, $t_w = 18$ mm, $b_f = 670$ mm, $t_f = 28$ mm, corresponded to the material weight $G^* = 83588.5$ kg of the cross-beam structure.

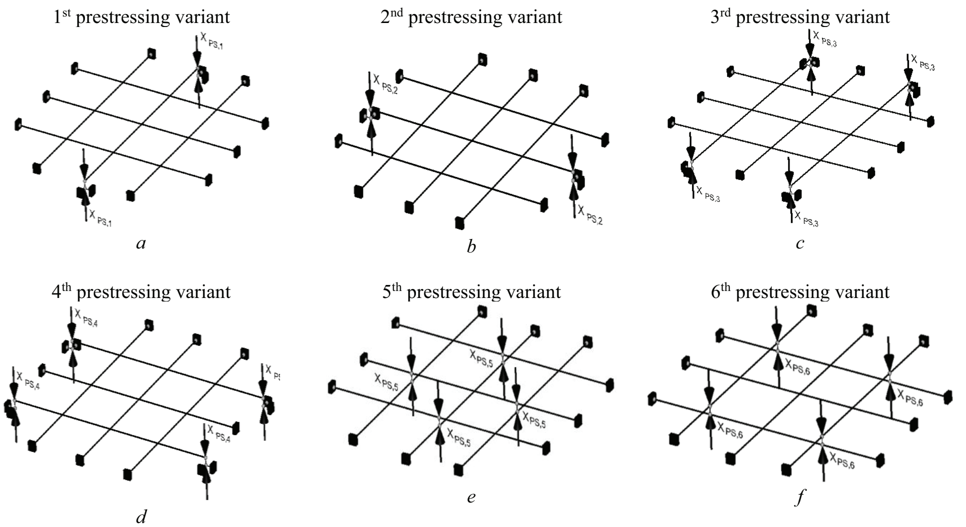

Fig. 2. Prestressing variants for the cross-beam structure by: *a* – *d* – lowering external supports; *e* – *f* – vertical shifting of the beams relative to each other at their mutual intersections

The set of prestressing variants for the considered cross-beam structure $\mathbf{B} = \{\mathbf{V}_\beta\} = \{\mathbf{r}_\beta, X_{PS,\beta}\}$, $\beta = \overline{1, N_{X,PS}}$, $N_{X,PS} = 6$ has been predefined (see Fig. 2) , according to item 2 of the algorithm presented above. The corresponding auxiliary vector $\vec{Y}_{PS} = \{X_{PS,\beta} \mid X_{PS,\beta} \in \mathbf{V}_\beta\}$, $\beta = \overline{1, N_{X,PS}}$ of the unknown initial prestressing forces with initial zero values for all components $X_{PS,\beta}$ was formed. According to item 3, vector for the gradient of function $\mathbf{S}_\sigma$ Eq. (3.1) has been calculated for all variable prestressing parameters (unknown initial prestressing forces) $\vec{Y}_{PS}$ when variable cross-section parameters $\vec{X}_{CS}$ of the cross-beam structure were fixed at the level of $\vec{X}^*_{CS}$ (see Table 1). An order of the consecutive inclusion of the prestressing variants $\mathbf{V}_m = \{\mathbf{r}_m, X_{PS,m}\} = \{\{r_{m,\mu}\}, X_{PS,m}\}$, $\mu = \overline{1, N_{RM,m}}$ from set $\mathbf{B}$ of the predefined prestressing variants into set $\mathbf{\Theta}$, representing the optimal prestressing variants has been determined based on the values of the criteria Eq. (3.3) (see Table 1).

Table 1. Determination of the order of the consecutive including of the predefined prestressing variants into the set of optimal prestressing variants

Prestressing variant, β	Number of redundant members $N_{RM,\beta}$	Initial prestressing force $X_{PS,\beta}$	$\nabla \mathbf{S}_\sigma$ components,×10^-6^, Eq. (3.2)	Criteria Eq. (3.3)	Order
1	2	$X_{PS,1}$	8.1133	4.0566	2
2	2	$X_{PS,2}$	8.1170	4.0585	1
3	4	$X_{PS,3}$	–7.7961	1.9490	3
4	4	$X_{PS,4}$	–7.7877	1.9469	4
5	4	$X_{PS,5}$	3.0895×10^{-4}	0.7723×10^{-4}	5
6	4	$X_{PS,6}$	2.6861×10^{-6}	0.6715×10^{-4}	6

According to item 6 of the algorithm presented above, the unknown initial prestressing force $X_{PS,2}$ that corresponded to the 2nd prestressing variant (see Fig. 2, *b*), has been added to the design variable vector in the first instance. The optimum material and internal forces problem Eqs. (4.1) – (4.8) has been formulated and solved in the continuum space of the unknown cross-sectional sizes and unknown initial prestressing forces. The following continuous optimum cross-sectional sizes for all beams and optimal initial prestressing force has been obtained: $h_w = 2008.63$ mm, $t_w = 17.52$ mm, $b_f = 687.08$ mm, $t_f = 25.86$ mm, $X_{PS,2} = -580.66$ kN, corresponded to the material weight $G^* = 79453.2$ kg of the cross-beam structure for the case when the number of redundant members to introduce the initial prestressing force is $N_{RM} = 2$. Then the continuous optimum cross-sectional sizes have been discretized and the following discrete optimum cross-sectional sizes for all beams and optimal initial prestressing force has been received: $h_w = 2060$ mm, $t_w = 18$ mm, $b_f = 650$ mm, $t_f = 26$ mm, $X_{PS,2} = -580.66$ kN, corresponded to the material weight $G^* = 79612.4$ kg of the cross-beam structure. Introducing the initial prestressing force into the redundant members of the cross-beam structure according to the 2nd prestressing variant has ensured the material economy 4.76% compared to the weight of the structure without prestressing.

In the second iteration of the searching process for the optimal prestressing variant of the cross-beam structure, the unknown initial prestressing force $X_{PS,1}$, that corresponded to the 1st prestressing variant (see Fig. 2, *a*), has been added to the design variable vector. The optimum material and internal forces problem Eqs. (4.1) – (4.8) has been formulated and solved in the space of the unknown cross-sectional sizes and unknown initial prestressing forces. The following continuous optimum cross-sectional sizes for all beams and optimal initial prestressing forces has been obtained: $h_w = 2150.72$ mm, $t_w = 18.76$ mm, $b_f = 475.18$ mm, $t_f = 27.99$ mm, $X_{PS,2} = -653.83$ kN, $X_{PS,1} = -662.0$ kN, corresponded to the material weight $G^* = 75211.6$ kg of the cross-beam structure for the case when the number of redundant members to introduce initial prestressing forces is $N_{RM} = 4$. Then the continuous optimum cross-sectional sizes have been discretized and the following discrete optimum cross-sectional sizes for all beams and optimal initial prestressing force has been received: $h_w = 2055$ mm, $t_w = 18$ mm, $b_f = 550$ mm, $t_f = 28$ mm, $X_{PS,2} = -655.95$ kN, $X_{PS,1} = -414.6$ kN, corresponded to the material weight $G^* = 76141.7$ kg of the cross-beam structure. Introducing the initial prestressing force into the redundant members of the cross-beam structure, according to the 2nd (see Fig. 2, *b*) and the 1st prestressing variants (see Fig. 2, *a*), ensured the material economy 9.8%, compared to the weight of the cross-beam structure without prestressing, and material economy 4.6%, compared to the weight of the cross-beam structure with the 2nd prestressing variant only.

In the third iteration of the searching process for optimal prestressing variant of the cross-beam structure, the unknown initial prestressing forces $X_{PS,3}$ and $X_{PS,4}$ corresponded to the 3rd (see Fig. 2, *c*) and 4th (see Fig. 2, *d*) prestressing variants accordingly, have been added to the design variable vector. The optimum material and internal forces problem Eqs. (4.1) – (4.8) has been formulated and solved in the space of the unknown cross-sectional sizes and unknown initial prestressing forces. The following continuous optimum cross-sectional sizes for all beams and optimal initial prestressing forces has been obtained: $h_w = 1941.52$ mm, $t_w = 16.94$ mm, $b_f = 664.08$ mm, $t_f = 25.0$ mm, $X_{PS,2} = -401.61$ kN, $X_{PS,1} = -410.9$ kN, $X_{PS,3} = 245.54$ kN, $X_{PS,4} = 260.04$ kN, corresponded to the material weight $G^* = 74228.1$ kg of the cross-beam structure for the case when the number of redundant members to introduce initial prestressing forces is $N_{RM} = 12$. Then the continuous optimum cross-sectional sizes have been discretized and the following discrete optimum cross-sectional sizes for all beams and optimal initial prestressing force has been received: $h_w = 2065$ mm, $t_w = 18$ mm, $b_f = 610$ mm, $t_f = 24$ mm, $X_{PS,2} = -323.71$ kN, $X_{PS,1} = -332.99$ kN, $X_{PS,3} = 326.67$ kN, $X_{PS,4} = 341.17$ kN, corresponded to the material weight $G^* = 74636.6$ kg of the cross-beam structure. Introducing the initial prestressing force into the redundant members of the cross-beam structure, according to the 2nd (see Fig. 2, *b*), 1st (see Fig. 2, *a*), 3rd (see Fig. 2, *c*) and 4th (see Fig. 2, *d*) prestressing variants, has been ensured the material economy 12.0%, compared to the weight of the cross-beam structure without prestressing, as well as material economy 2.0%, compared to the weight of the cross-beam structure with previous 1st and 2nd prestressing variants only.

Since, decrement of the objective function value is not significant (less than 1.5%) compared to one for considered structure with previous prestressing variants, so introduction of the initial prestressing forces into the redundant members according to 3rd and 4th prestressing variants (see Fig. 2, *c*, *d*) is not effective. Search for the optimal prestressing variant of the considered cross-beam structure can be finished. Thus, the optimal prestressing variant of the considered cross-beam structure consists of the 1st and 2nd prestressing variants (see Fig. 2, *a*, *b*) and can be created by lowering external 2nd, 5th, 8th and 11th supports (see Fig. 1). The optimal number of the redundant members for introducing the initial prestressing forces is 4 accordingly.

In order to define the optimal prestressing variant for considered cross beam structure, three optimum material and internal forces distribution problems only have been solved with the number of variable initial prestressing forces 1, 2 and 3 accordingly.

As it has been shown in the presented numerical example, the suggested numerical technique to determine the optimal number of the redundant members to introduce initial prestressing forces ensures the reduction of the dimension for the design variable vector of unknown initial prestressing forces for considered optimization problems.

4. CONCLUSION

A numerical technique to determine the optimal number of the redundant members to introduce initial prestressing forces has been offered for high-order statically indeterminate bar structures. An idea to form an optimal prestressing variant for the considered bar structure by consecutive introduction of the initial prestressing forces into the redundant members and subsequent solving of the optimum material and internal forces distribution problems has been suggested. An order of the consecutive including of the initial prestressing forces into the redundant members can be defined by values of the components of the gradient vector for the function that estimates both under-stressing and overstressing in term of longitudinal stresses for all structural members of the bar system with respect to the variable prestressing parameters.

The suggested numerical technique to determine the optimal number of the redundant members to introduce initial prestressing forces provides the reduction of the dimension for the design variable vector of unknown initial prestressing forces for considered optimization problems.

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LIST OF FIGURES AND TABLES:

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