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The comparison of fault distinguishability approaches – case study

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Abstract. A comprehensive characterization of four selected fault distinguishability methods is presented herein. All considered methods are derived from structural residual approaches referring to model-based diagnostics. In particular, these methods are based on a binary diagnostic matrix, fault isolation system, sequences of symptoms, and their combinations. Fault distinguishability issues are discussed based on an example of four pressure vessel system. Substantial benefits are shown in fault distinguishability figures obtained by utilising extended knowledge regarding fault-symptom relation. Finally, the values of three fault distinguishability metrics are calculated for each method. For the case study, the highest score is achieved using the multivalued fault isolation method combined with a diagnosis utilising information regarding the antecedence of symptoms.

Key words: fault detection and isolation, fault distinguishability, diagnostics of industrial processes, metrics of distinguishability, antecedence of symptoms.

1. Introduction

Online diagnostics of industrial processes are effective for increasing functional safety and reduce economic losses caused by failures or faults. The early recognition of emerging faults has caused process operators to undertake appropriate actions that allow for a relatively rapid and successful recovery of the nominal behavioural state of processes. Hence, blockades and safety measures foreseen and implemented in safety instrumented systems are, in fact, not activated; consequently, process shut-down does not occur. Therefore, the diagnostics guarantee, among others, significant savings in processing.

It is clear that the accurate isolation of faults allows appropriate solutions and repairing actions to be applied. The accuracy of the diagnosis may be determined by adding up the number of faults identified in each elementary block [2]. The smaller the number of faults collected in the elementary block, the more accurate is the diagnosis. The accuracy of the diagnosis depends on the grade of fault distinguishability achievable in a given system. Fault distinguishability is understood as the ability to recognise (distinguish) single faults. Typically, fault distinguishability is based on the analysis of appropriately processed results of diagnostic tests. Diagnostic tests deliver a set of characteristic symptoms associated with the fault or a set of faults.

The fault symptoms will be discussed broadly herein. They can be either defined as a pattern of specific values (symptoms) of diagnostic signals as well as that of specific sequences (orders) of these signals [2]. Clearly, the faults are indistinguishable if all their symptoms are identical. Therefore, fault distinguishability depends on the selection of symptoms.

The required fault distinguishability figures are strongly demanded. The fault distinguishability requirements may be specified, for example, by indicating the set of pairs of faults that must be distinguished. Typically, process operators require the distinguishability of all single faults. However, it is noteworthy that this is not mandatory. In practice, it is sufficient to distinguish those faults, the effects of which are particularly threatening. The other typical requirement is to distinguish faults that might be used to trigger process safety insurance procedures. In this case, faults for which these procedures refer do not need to be distinguished.

The main aim of this paper is to present a case study in which the different methods to the fault isolation are compared in terms of fault distinguishability. The methods will be discussed based on a binary diagnostic matrix, fault isolation system, sequences of symptoms, and their combinations. Fault distinguishability issues will be discussed based on an example of four pressure vessel system. Moreover, the definition of a new metrics for the assessment of fault distinguishability have been proposed for comparative study.

The contribution of the paper comprises in: a) bringing together different fault distinguishability approaches; b) proposing mixed fault distinguishability approaches; c) redefining fault distinguishability metrics; d) showing on example the advantages of proposed approaches in respect to fault distinguishability figures.

The paper is structured as follows: the importance of fault distinguishability in industrial practice is discussed in the introduction. A brief discussion of the methods for increasing fault distinguishability is presented in Section 2. In Section 3, a physical system for further study is described. The analysis of the four fault distinguishability methods applied to the four

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pressure vessel setup is presented in Section 4. Finally, in Section 5 the fault distinguishability metrics obtained for a system are collected and discussed.

2. Methods for increasing distinguishability of faults

Fault isolability has been typically defined in the context of the adopted diagnostic method. It is often analysed based on structural residuals (binary diagnostic matrix, incidence matrix, structure matrix) derived from the linear equations of residuals in the internal form [3–5]. The basic definitions have been formulated by Gertler [4] in terms of the structures of residual sets as well as by Isermann in the context of residual space [6]. The definition of fault isolability was also provided in [7, 8].

Fault distinguishability is an important issue that affects the design flow of diagnostic systems. Clearly, the indication of distinguishable and indistinguishable faults is necessary to verify whether the demanded fault isolability requirements are satisfied.

An appropriate selection of process variables is key in fault detectability and distinguishability issues [9]. Typically, the more process variables are available, the more process models are to be built. It is noteworthy that the development of models that are useful for diagnostics is not trivial. Furthermore, it is a complex issue for complex, large-scale systems. The method of generating the structures of all models by a given set of measurements is discussed in [10]. In practice, heuristic approaches are also used. In this case, both the primary as well as secondary residuals are obtained by combining (linking) neighbouring partial models of the diagnosed system. Partial models are designed for small portions of the diagnosed system. Generally, an increase in the number of models results in an increase of fault distinguishability figures.

Furthermore, fault distinguishability can be increased by utilising multivalued residuals [9, 11–13]. Typically, the tri-valued evaluation of residuals is exploited. In this case, the values of diagnostic signals belong to the set $v = \{0, -1, +1\}$. The diagnostic signal value 0 means that the value of the residual is within some specified and acceptable limit. Such value indicates the nominal behavioural state of the system. Other values are referred to as fault symptoms. The adoption of tri-valued residuals is justified because some faults may cause an increase in residual values above acceptable thresholds $(+1)$, while other faults reduce these values to below acceptable threshold values (-1). Therefore, at the least, the tri-valued evaluation of residuals guarantees better fault distinguishability.

The multivalued evaluation of residuals requires the knowledge of the relation of fault–diagnostic signals in the form of a fault information system (FIS) [9, 11–13]. The fault symptoms are understood as the interpreted values of diagnostic signals. Because, in general, the diagnosed system is a dynamic system, a lag typically occurs between the instant of fault origin and the instant in which the corresponding symptom of this fault appears. This lag depends on the dynamic properties of the diagnosed system. In other words, the same fault affects various diagnostic tests in different time instants. The antecedence of diagnostic signals can deliver additional information that may be useful in the diagnostics [9, 11, 14]. Additionally, it may be beneficial for differentiating faults that are indistinguishable based on the assessment of the residual values. To distinguish any pair of faults, it is sufficient if the antecedence of symptoms for these faults is different.

The computer science and artificial intelligence community has developed numerous fault detection and isolation approaches belonging to the DX family of methods. In these approaches, consistency-based Reiter's theory is utilised [15, 16]. Diagnoses are obtained from conflicts, where a conflict is a set of these system components for which the assumption that all of these components are healthy is inconsistent with observations. The final diagnoses are hitting sets of the conflict sets. The substantial advantage of DX compared with FDI methodologies is that additional measures are not required to isolate multiple faults. Single and multiple faults are isolated in the same manner. In the original approach proposed by Reiter, conflict sets are calculated in the online mode. However, offline extensions have been proposed in [17, 18].

Fault exoneration is frequently assumed in FDI methodologies based on analytic redundancy relations (ARRs) [19]. Under this assumption, if some ARRs are satisfied by observation, then all observation-related components are considered as non-faulty. In DX approaches, such an assumption does not exist. This paper focuses on FDI approaches under the assumption of single faults and fault exoneration.

3. Case study

3.1. Phenomenological process model. Fault distinguishability approaches will be studied based on a plant comprising a series of four interconnected gas buffer vessels. The four-vessels system presented in this paper was inspired by the compressed air and natural gas energy storage installations. Energy storage systems are technologies that are currently used among others for integration of renewable energies for large as well as small scale systems [20, 21]. The energy storage systems make use from installations consisting of batteries of vertical or horizontal interconnected vessels. For the purposes of this paper, the compressed air installation was simplified and idealized. The schematics of the system for diagnosis is depicted in Fig. 1. It is assumed that the phenomenological partial models of the plant are known. Moreover, it is assumed that those models are not affected by faults. The gas flow rate between any two tanks is

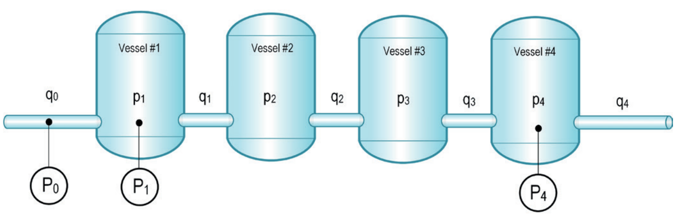

Fig. 1. Schematics of the four pressure vessel setup.

defined by the following formula:

$$q_{(i-1)} = \alpha_i A_{(i-1)} \sqrt{\varsigma_{(i-1)}(p_{(i-1)} - p_i)} \qquad (1)$$

where q_i it the gas inflow rate into the i^{th} vessel, $i \in \{1..4\}$, α_i the flow rate coefficient in the pipe linking $(i-1)^{\text{th}}$ and i^{th} vessels, $A_{(i-1)}$ the cross section of a pipe connecting the $(i-1)^{\text{th}}$ and i^{th} vessels, and ς_i the gas density by pressure p_i in the i^{th} vessel.

3.2. Qualitative model of a process affected by faults. A causal graph of the process illustrating the relationship between process variables is shown in Fig. 2. A set of faults is listed in Table 1; it contains a subset of components and instrument faults. Utilising the set of measurements and faults, we can develop a process graph *GP* [10]. The *GP* graph describes cause-and-effect relations between process variables and explicitly reflects the effects of faults on process variables. The *GP* graph is an extended version of the well-known signed directed graph that is used to represent the cause-and-effect relationship between variables or alarms in technological installations [23]. The *GP* graph of gas flow in a series of four interconnected vessels is shown in Fig. 3.

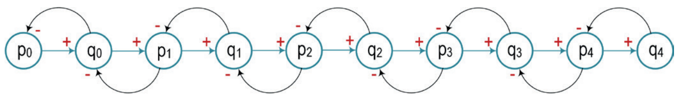

Fig. 2. Causal graph of the process.

Table 1
List of considered faults in the four pressure vessels setup.

Fault	Description
f_1	Leakage of the vessel #1
f_2	Leakage of the vessel #2
f_3	Leakage of the vessel #3
f_4	Leakage of the vessel #4
f_5	Clogging of the q_0 flow
f_6	Clogging of the q_1 flow
f_7	Clogging of the q_2 flow
f_8	Clogging of the q_3 flow
f_9	Pressure sensor P_0 fault
f_{10}	Pressure sensor P_1 fault
f_{11}	Pressure sensor P_4 fault

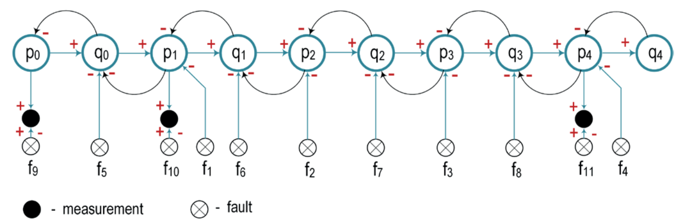

Fig. 3. GP graph of the process reflecting the effects of faults on the values of process variables.

4. Fault distinguishability analysis

Fault distinguishability will be analysed by assuming the availability of three pressure measurements: P_0, P_1, and P_4, which are marked by black circles in the GP graph in Fig. 3. Let us further assume that for the diagnostics, a set of four partial models (m_i):

$$\widehat{P}_1' = m_1(P_0); \qquad \widehat{P}_4' = m_2(P_0);$$
$$\widehat{P}_4'' = m_3(P_1); \qquad \widehat{P}_1'' = m_4(P_0, P_4)$$

will be built. The partial-models can be implemented be means of different approaches. The only requirement is, that they should reflect the non-linear behavior of the studied process in terms of pressure and flow variation. Therefore, the analytical models based on physical laws, neural, fuzzy, their combinations, as well as additive models are usable. The corresponding residuals are as follows:

$$\begin{cases} r_1 = P_1 - m_1(P_0), \\ r_2 = P_4 - m_2(P_0), \\ r_3 = P_4 - m_3(P_1), \\ r_4 = P_1 - m_4(P_0, P_4). \end{cases} \qquad (2)$$

The sensitivity of residuals to the faults can be determined based on either expert knowledge or reading directly from the GP graph.

$$\begin{cases} r_1 = h_1(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}), \\ r_2 = h_2(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{11}), \\ r_3 = h_3(f_2, f_3, f_4, f_6, f_7, f_8, f_{10}, f_{11}), \\ r_4 = h_4(f_1, f_2, f_3, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}). \end{cases} \qquad (3)$$

The four different inference approaches for faults will be considered further in this study:

- Faults inferred by assuming the binary evaluation of residuals. In this case, either the binary diagnostic matrix *BDM* or rules arising from this matrix will be used for diagnostic reasoning;
- Faults inferred by assuming tri-valued diagnostic signals. In this case, either the *FIS* or rules arising from it will be used for diagnostic reasoning;
- Faults inferred by assuming the binary evaluation of residuals and knowledge regarding the order of symptoms resulting from the *GP* graph. The *BDM* matrix or rules arising from this matrix and elementary sequences of symptoms will be used for diagnostic reasoning;
- Faults inferred by assuming a tri-valued evaluation of residuals and knowledge regarding the order of symptoms resulting from the GP graph. The *FIS* structure or rules arising from this structure and elementary antecedence of symptoms will be used for diagnostic reasoning.

All the above mentioned approaches involve different knowledge depths regarding the relationship between faults and diagnostic signals.

4.1. Fault inference based on BDM. In this case, we assume that the diagnosis is based on the binary evaluation of residuals and knowledge regarding relation fault-symptoms.

BDM determines the relationship between the set of faults $F=\{f_k : k=1,2,\ldots,K\}$ and the set of diagnostic signals $S=\{s_j : j=1,2,\ldots,J\}$. The fault signature is defined as a vector of diagnostic signal values associated with each particular fault. Clearly, each column vector of the *BDM* reflects the signature of the sole fault. This correspondence can be rewritten in the form of a useful rule as follows:

$$\textit{If}\ \ (s_1=v_{1k})\wedge\ldots\wedge(s_j=v_{jk})\wedge\ldots\wedge(s_J=v_{Jk})\ \ \textit{then}\ \ f_k. \quad (4)$$

In [2, 26], definitions of fault distinguishability and indistinguishability were formulated in the *BDM*. The faults $f_k, f_m \in F$ are indistinguishable in the *BDM* (remain in a relationship R_N) iff their signatures are equal:

$$f_k\,R_N\,f_m \Leftrightarrow \forall_{s_j\in S}\left[v_{jk}=v_{jm}\right]. \quad (5)$$

By contrast, the faults are distinguishable (remain in a relationship R_R) if their signatures are different.

$$f_k\,R_R\,f_m \Leftrightarrow \exists_{s_j\in S}\left[v_{jk}\neq v_{jm}\right]. \quad (6)$$

The *BDM* of the studied case is presented in Table 2.For example, according to (4), fault f_1 is concluded from the following rule:

$$\textbf{\textit{if}}\ \{(s_1=1)\wedge(s_2=1)\wedge(s_3=0)\}\wedge(s4=1)\ \textbf{\textit{then}}\ f_1 \quad (7)$$

Table 2
Binary diagnostic matrix of the four pressure vessel systems.

S/F	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
s_1	1	1	1	1	1	1	1	1	1	1	0
s_2	1	1	1	1	1	1	1	1	1	0	1
s_3	0	1	1	1	0	1	1	1	0	1	1
s_4	1	1	1	0	1	1	1	1	1	1	1

Similarly, any row of *BDM* responds to a rule that indicates all possible faults when any diagnostic signal $s_j=1$ occurs. For example, the third row of the matrix shown in Table 2 can be rewritten in the form of the following rule:

$$\textbf{\textit{if}}\ \ (s_3=1)\ \ \textbf{\textit{then}}\ \ (f_2\vee f_3\vee f_4\vee f_6\vee f_7\vee f_8\vee f_{10}\vee f_{11}). \quad (8)$$

The faults $f_k, f_m \in F$ are indistinguishable in a given *BDM* iff their signatures are equal [2, 24, 25]. Meanwhile, any pair of faults $f_k, f_m \in F$ is distinguishable if their signatures are different. For the system being studied, we obtained two elementary blocks (subsets) of indistinguishable faults. In this case, the only three single faults were distinguishable.

$$\{f_1,f_5,f_9\},\ \{f_2,f_3,f_6,f_7,f_8\},\ \{f_4\},\ \{f_{10}\},\ \{f_{11}\}. \quad (9)$$

4.2. Fault inference based on FIS. An FIS is a structure that specifies the set of reference values of diagnostic signals associated with particular faults. In this case, any diagnostic signal is multivalued. The significant extensions of the FIS in relation to the *BDM* are as follows:

- an individual set of values may be associated with each diagnostic signal;
- the set V_j of values of j^{th} diagnostic signal contains a finite number of elements;
- any item $(v_{k,j}\subset V_j)$ in the FIS may be multivalued.

The FIS column constitutes the compound signature, in which each pair of diagnostic signal-fault may be associated with more than one value. A compound signature may be expressed alternatively in the form of the following rule:

$$\textit{If}\ \ (s_1=V_{1k})\wedge\ldots\wedge(s_j=V_{jk})\wedge\ldots\wedge(s_J=V_{Jk})\ \ \textit{then}\ \ f_k. \quad (10)$$

In previous studies [11, 24, 26], unconditional and conditional fault indistinguishability were defined in terms of the FIS. Some of the definitions are provided below.

Faults $f_k, f_m \in F$ are unconditionally indistinguishable (remain in a relationship R_N) in the FIS with respect to diagnostic signals $s_j\in\{S\}$ iff their signatures are identical:

$$f_k\,R_N\,f_m \Leftrightarrow \forall_{s_j\in S}\left[V_{jk}=V_{jm}\right]. \quad (11)$$

The faults $f_k, f_m \in F$ are conditionally indistinguishable (remain in a relationship R_{WN}) in the FIS with respect to diagnostic signals $s_j\in\{S\}$ iff for each diagnostic signal, all subsets of its values associated with the faults f_k and f_m are in part common and are not unconditionally indistinguishable:

$$f_k\,R_{WN}\,f_m \Leftrightarrow \forall_{s_j\in S}\left[V_{jk}\cap V_{jm}\right]\neq\emptyset \wedge \exists_{s_j\in S}\left[V_{jk}\neq V_{jm}\right]. \quad (12)$$

Conditional fault indistinguishability implies that v_j may exist, by which both faults are indistinguishable.

$$\forall_{s_j\in S}\ v_j\in\left[V_{jk}\cap V_{jm}\right]. \quad (13)$$

However, other diagnostic signals exist, for which the same faults remain distinguishable. Therefore, the following condition applies:

$$\exists_{s_j\in S}\left[v_j\in V_{jk}\wedge v_j\notin V_{jm}\right]\vee\left[v_j\notin V_{jk}\wedge v_j\in V_{jm}\right]. \quad (14)$$

The faults $f_k, f_m \in F$ in the FIS are unconditionally distinguishable if a diagnostic signal exists, for which the subsets of values associated with these faults are disjoint:

$$f_k\,R_R\,f_m \ \ \Leftrightarrow \exists_{s_j\in S}\left[V_{jk}\cap V_{jm}\right]\neq\emptyset. \quad (15)$$

The FIS developed for the four pressure vessels system under the assumption of tri-valued residual values is presented in Table 3. The content of the Table 3 has been determined based on the analysis of Bernoulli's equations as well as based on expert knowledge. Clearly, the signs of fault symptoms depend on the shape of residual equations. In considered study, they are determined by a set of equations (2) in the form of the differences

Table 3
Fault information system for the four pressure vessel system.

S/F	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
s_1	-1	-1	-1	-1	-1	-1	$+1$	$+1$	$-1,+1$	$-1,+1$	0
s_2	-1	-1	-1	-1	-1	-1	-1	-1	$-1,+1$	0	$-1,+1$
s_3	0	-1	-1	-1	0	-1	-1	-1	0	$-1,+1$	$-1,+1$
s_4	-1	-1	-1	0	-1	-1	$+1$	$+1$	$-1,+1$	$-1,+1$	$-1,+1$

between process and modelled values. If the residuals will be defined as differences between modelled and process values, the sings of symptoms will be opposed. When considering the expert knowledge, the reasoning regarding signs of fault symptoms is quite trivial. In this case, essential is the knowledge of influence of the fault on process values. For example, a clogging of flow q_2 results in increase of pressure P_1 and decrease of pressure P_4. This clarifies the signs of symptoms of fault f_7 in Table 3.

From the FIS, analogously as in the case of the BDM, we can derive rules regarding faults. For example, the rule corresponding to fault f_{10} is as follows:

$$\textbf{\textit{if}}\ \{(s_1 \in \{-1,+1\}) \wedge (s_2 = 0) \wedge (s_3 \in \{-1,+1\})\} \wedge (s_4 \in \{-1,+1\})\}\ \textbf{\textit{then}}\ f_{10}.$$

For the first row of the FIS shown in Table 3, we can get two alternative [26] rules:

$$\textbf{\textit{if}}\ (s_1 = -1)\ \textbf{\textit{then}}\ (f_1 \vee f_2 \vee f_3 \vee f_4 \vee f_5 \vee f_6 \vee f_9 \vee f_{10}),$$
$$\textbf{\textit{if}}\ (s_1 = +1)\ \textbf{\textit{then}}\ (f_7 \vee f_8 \vee f_9 \vee f_{10}).$$

If we transform the FIS into a BDM by substituting tri-valued values by the bi-valued values, then we can obtain the following:

$$\textbf{\textit{if}}\ (s_1 = 1)\ \textbf{\textit{then}}\ (f_1 \vee f_2 \vee f_3 \vee f_4 \vee f_5 \vee f_6 \vee f_7 \vee f_8 \vee f_9 \vee f_{10}).$$

Here, the cardinality of the set of faults indicated is greater than that in the FIS case. Hence, the benefits of applying multi-valued diagnostic signals are evident. When assessing fault indistinguishability in the FIS, the conditional distinguishability and conditional indistinguishability of faults must be considered [2, 9, 12].

In the FIS case shown in Fig. 3, the unconditionally indistinguishable subsets of faults are as follows:

$$\{f_1, f_5\},\ \{f_2, f_3, f_6\},\ \{f_7, f_8\},\ \{f_4\},\ \{f_{10}\},\ \{f_{11}\}.$$

In addition, the fault $\{f_9\}$ is conditionally distinguishable.

4.3. Fault inference based on BDM combined with knowledge of elementary sequences of symptoms (ESS). In this case, we will study the fault distinguishability of the system depicted in Fig. 1 using the BDM shown in Table 2. Knowledge regarding the order of fault symptoms can be obtained based on expertise, experimentally or derived from a qualitative process model. However, these models should exhibit a form of causal graph that considers the effects of faults. Let any pair of symptoms $\langle s_j, s_p \rangle$ be referred to as an elementary sequence and further denoted as $es_{j,p}(f_k) = \langle s_j, s_p \rangle$. The faults $f_k, f_m \in F$ are indistinguishable with respect to the order of the symptoms if the corresponding sequences of their symptoms are identical [2, 14].

$$f_k\, R_N\, f_m \Leftrightarrow \forall_{s_j, s_p \in S}\ [es_{j,p}(f_k) = es_{j,p}(f_m)]. \tag{16}$$

Therefore, two faults $f_k, f_m \in F$ are distinguishable if the order of any pair of their symptoms $\langle s_j, s_p \rangle$ is different [14, 35].

$$f_k\, R_R\, f_m \Leftrightarrow \exists\, es_{j,p}(f_k) = \langle s_j, s_p \rangle \wedge \exists\, es_{j,p}(f_m) = \langle s_p, s_j \rangle. \tag{17}$$

Hence, to distinguish between any pair of faults, it is sufficient if any elementary sequence of symptoms $es_{j,p}(f_k)$ and $es_{j,p}(f_m)$ for those faults is different. Conditional distinguishability occurs if the sequence of symptoms for one of the faults is fixed, e.g., $es_{j,p}(f_k) = \langle s_j, s_p \rangle$. Meanwhile, for the second fault, any order of the symptoms may appear: $es_{j,p}(f_m) = \langle s_j, s_p \rangle \cup \langle s_p, s_j \rangle$. In the case of sequence $\langle s_p, s_j \rangle$, fault f_m is indicated; meanwhile, in the case of sequence $\langle s_j, s_p \rangle$, faults f_k and f_m are not distinguishable. Knowledge regarding elementary sequences may increase the distinguishability of faults that are indistinguishable based on inference performed solely through the appropriate evaluation of diagnostic signals. For example, from the BDM shown in Table 2, we can obtain the following subsets of indistinguishable faults: $\{f_1, f_5, f_9\}, \{f_2, f_3, f_6, f_7, f_8\}, \{f_4\}, \{f_{10}\}, \{f_{11}\}$. Then, we can analyse the order of symptoms of undistinguishable faults in a subset $\{f_2, f_3, f_6, f_7, f_8\}$ based on the graph GP. The order of symptoms depends on the length of the path starting from the process variable that is used for residual computation and is affected by a fault. Clearly, this order depends on the dynamic properties of the system, which are hidden behind the arcs of the graph. To simplify our considerations, we will assume that the volumes of all vessels as well as the lengths and diameters of connecting pipelines are approximately equal. Next, we analyse the paths of the faults that affected the modelled process variables (see Fig. 3). The path of fault f_6 that affects process variable P_1 (used for calculation of residual r_1) is short: $f_6 \to q_1 \to p_1 \to P_1$. The same fault affects the variable P_4 (related to residual r_2) by spreading through a much longer path $f_6 \to q_1 \to p_2 \to q_2 \to p_3 \to p_4 \to P_4$. Considering similar dynamics of pressure-flow and vice versa processes, in the case of fault f_6, the symptom s_1 should precede the symptom s_2. Similar dependency occurs in the case of fault f_2, for which the paths are as follows: $f_2 \to p_2 \to q_1 \to p_1 \to P_1$, $f_2 \to p_2 \to q_2 \to p_3 \to q_3 \to p_4 \to P_4$. However, the cases of faults f_3 and f_8 are different. The path from the fault f_8 to the process variable P_1 (associated with the residual r_1) is relatively long: $f_8 \to q_3 \to p_3 \to q_2 \to p_2 \to q_1 \to p_1 \to P_1$. This implies a long formation time of the symptom of this fault. Meanwhile, the path from the fault f_8 to the variable P_4 (related to residual r_2) is short: $f_8 \to q_3 \to p_4 \to P_4$. The same applies in the case of fault f_3: $f_3 \to p_3 \to q_2 \to p_2 \to q_1 \to p_1 \to P_1$, $f_3 \to p_3 \to q_3 \to p_4 \to P_4$. Therefore, in the case of faults f_8 and f_3, the symptom s_2 precedes the symptom s_1. As such, the following elementary sequences are legitimate:

$$\begin{cases} es_{1,2}(f_2) = \langle s_1, s_2 \rangle, \\ es_{1,2}(f_6) = \langle s_1, s_2 \rangle, \\ es_{1,2}(f_3) = \langle s_2, s_1 \rangle, \\ es_{1,2}(f_8) = \langle s_2, s_1 \rangle. \end{cases} \tag{18}$$

This provides the distinguishability of pairs of faults $\{f_2, f_6\}$ and $\{f_3, f_8\}$. The following two sequences are expected for the fault f_7.

$$\begin{cases} es_{1,2}(f_7) = \langle s_1, s_2 \rangle, \\ es_{1,2}(f_7) = \langle s_2, s_1 \rangle. \end{cases} \tag{19}$$

Therefore, a conditional indistinguishability of the fault f_7 occurs with respect to pairs of faults $\{f_2, f_6\}$ and $\{f_3, f_8\}$. For the sequence $\langle s_1, s_2 \rangle$, the fault can be inferred in the subset $\{f_2, f_6, f_7\}$; meanwhile, for the sequence $\langle s_2, s_1 \rangle$, we can infer that one of the faults from subset $\{f_3, f_7, f_8\}$ has occurred.

The sequences of symptoms s_1 and s_3 are identical to those of s_1 and s_2, respectively. Therefore, these sequences do not contribute to fault distinguishability. Similarly, a detailed analysis of other sequences shows that other sequences that enables faults to be distinguished in the subset $\{f_1, f_5, f_9\}$ do not exist.

4.4. Fault inference based on FIS combined with knowledge of elementary sequences of symptoms. In this section, we will analyse the fault distinguishability in the diagnostic system designed for the four pressure vessels setup as shown in Fig. 1, using the *FIS* structure shown in Table 3 and knowledge regarding elementary sequences of symptoms. The fault distinguishability obtained solely based on the *FIS* was described in Section 4.2. In the current subsection, the elementary sequences presented in the previous section will be considered. In this case, the increase in fault distinguishability is caused by both the multivalued evaluation of residuals and utilising knowledge regarding elementary sequences.

The faults $f_k, f_m \in F$ are indistinguishable [2] in the *FIS* and by the sequence of symptoms if the signatures and sequences of these faults are equal:

$$f_k R_N f_m \Leftrightarrow \left[\forall_{s_j \in S}\, V_{j,k} = V_{j,m}\right] \wedge \left[\forall_{s_j, s_p \in S}\, es_{j,p}(f_k) = es_{j,p}(f_m)\right]. \tag{20}$$

The faults $f_k, f_m \in F$ are unconditionally distinguishable in the *FIS* and by the sequence of symptoms, if one diagnostic signal exists at the least, for which the subsets of values corresponding to these faults are disjoint or distinguishable elementary sequences of symptoms exist.

$$f_k\, R_R\, f_m \Leftrightarrow \left[\exists_{s_j \in S}\, V_{j,k} \cap V_{j,m} = \emptyset\right] \vee \left\{\left[\exists\, es_{j,p}(f_k) = \langle s_j, s_p \rangle\right] \wedge \left[\exists\, es_{j,p}(f_m) = \langle s_p, s_j \rangle\right]\right\}. \tag{21}$$

In the remaining cases, the faults are unconditionally indistinguishable [2].

Without providing details, we obtain the following unconditionally distinguishable subsets of faults: $\{f_1, f_5\}$, $\{f_2, f_6\}$, $\{f_3\}$, $\{f_4\}$, $\{f_{10}\}$, $\{f_{11}\}$. The conditional distinguishable are faults $\{f_7\}$ and $\{f_8\}$ as well as $\{f_9\}$ and $\{f_1, f_5\}$.

4.5. Qualitative comparison of fault distinguishability approaches. Definitely, the lowest fault distinguishability provides *BDM*. Inference with *BDM* leads to identification of a class of subsets of indistinguishable faults which are mutually unconditionally distinguishable. In considered study we obtain 5 subsets of unconditionally distinguishable faults: $\{f_1, f_5, f_9\}, \{f_2, f_3, f_6, f_7, f_8\}, \{f_4\}, \{f_{10}\}, \{f_{11}\}$. The fault distinguishability obtainable from the *BDM* will be confronted further with other approaches based on the *FIS*, and fusion of *BDM* and *ESS* as well as *FIS* and *ESS*. The distinguishability of faults obtained for the same process by means of combined *BMD* and *FIS* approaches yields better results compared to these achieved for *BDM* itself. In this case, the subsets of faults $\{f_2, f_3, f_6, f_7, f_8\}$ and $\{f_7, f_8\}$ are distinguishable. In addition, fault $\{f_9\}$ is conditionally distinguishable from the faults $\{f_1, f_5\}$. This valuable result may be explained by exploiting of additional diagnostically useful knowledge injected by application of tri-valued assessment of residuals.

The use of combined *BDM* and *ESS* approaches allows for getting yet better results. This results in unconditional distinguishability of subsets $\{f_2, f_6\}$ and $\{f_3, f_8\}$ and in addition in conditional distinguishability of fault $\{f_{10}\}$ from pairs of indistinguishable faults $\{f_2, f_6\}$ and $\{f_3, f_8\}$.

The best fault distinguishability was observed in case of fusion of *FIS* and *ESS*. In the *FIS* solely, the faults $\{f_2, f_3, f_6\}$, $\{f_7, f_8\}$ are indistinguishable. By fusion of *FIS* and *ESS* we got unconditional distinguishability of subsets $\{f_2, f_6\}$, $\{f_3\}$ and conditional distinguishability of faults $\{f_7\}$ and $\{f_8\}$.

Given above remarks concerned a qualitative comparison of fault distinguishability obtainable for the studied process by means of four fault isolation approaches. However, it is advisable to introduce some measures which allow to quantify fault distinguishability. This allows for carrying out the ranking of the fault isolation methods what should be appreciated by implementation of diagnostic systems.

5. Metrics of fault distinguishability

Problems in the quantitative assessment of fault distinguishability in a given diagnostic system has been discussed widely [2, 9, 13, 25, 27–31], in which numerous definitions of fault distinguishability metrics have been defined. Three of them were adopted in this study to illustrate the trends of evolution of the metric values depending on the selection of fault isolation method. In order to copy with multivalued diagnostic signals and sequences of symptoms, the redefinition of metrics (31) is proposed together with a proposition of transformation of the structure of residual sets into a multivalued fault isolation system. This makes allowance for generalization of all incidence matrix based approaches to fault distinguishability. The calculated values of the selected fault distinguishability metrics are presented in Table 5. Below, we briefly introduce the definitions of these metrics.

5.1. Theoretical mean diagnosis accuracy. The theoretical mean diagnosis accuracy d_{acc} was introduced in [31]. It originates from the statement that a diagnosis can be interpreted as a superset of elementary diagnoses. An elementary diagnosis is understood as a set of faults that are indistinguishable. In addition, the set of all undetectable faults together with a fault free behavioural state of the diagnosed system constitutes a single elementary diagnosis d_1. According to [31]:

$$d_{acc} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{L_i} \tag{22}$$

where N is the number of elementary diagnoses, L_i the number of faults in the i^{th} elementary diagnosis.

In this study, all faults are detectable; therefore, $L_1 = 1$. Additionally, (22) is extremely easy to calculate. However, it is noteworthy that d_{acc} is not the best selection when searching for the optimal sensor placement in diagnostics [27, 32–34] because it does not reflect the sensitivity to the strength of fault isolation.

5.2. Normalised mean distinguishability index. The independent diversification of fault signatures regardless of whether they are isolated unidirectionally or bidirectionally is necessary in fault isolability based on structural residuals. The measure (22) does not imply whether the structure of the residual sets is weakly or strongly isolated. Therefore, in general, it is overvalued. To obtain metrics that will better reflect the isolability features of the structures of residual sets, the normalised mean distinguishability index $\mathfrak{d}^N$ was introduced in [25]. We will briefly describe it below.

The $\mathfrak{d}^N$ index is based on the definition of a single fault diversity matrix $\mathbf{M}[m:n]$. Each column of the matrix $\mathbf{M}$ contains binary valued numbers expressing the distinguishability of each pair of single faults regardless of their unidirectional or bidirectional characterisation. It has been proposed that the measure of diversity of signatures will be an algebraic sum of all nonzero entries in each column of matrix $\mathbf{M}$. Let us denote this number as $\mathfrak{d}_{i,k}$.

$$\mathfrak{d}_{i,k} = \sum_{j=1}^{m} m_{j,i,k} \tag{23}$$

where $m_{j,i,k} = v_{j,i} \otimes v_{j,k}$ is the entry of matrix M; $v_{j,i}$ is the entry of the alternative fault signature of fault f_i.

Next, we create diversity vector $\mathfrak{D}$:

$$\mathfrak{D}[1:c] = \left[\mathfrak{d}_{1,2}, \mathfrak{d}_{1,3}, .., \mathfrak{d}_{1,n}, \mathfrak{d}_{2,3}, \mathfrak{d}_{2,4}, .., \mathfrak{d}_{(n-1),n}\right] \tag{24}$$

and isolability vector $\mathfrak{I}$:

$$\mathfrak{I}\left[\mathfrak{i}_1, \mathfrak{i}_2, .. , \mathfrak{i}_{n-1}\right], \tag{25}$$

where

$$\mathfrak{i}_i = \sum_{k=i+1}^{n-1} \mathfrak{d}_{i,k}\,. \tag{26}$$

To determine the minimal number of differences between signatures of each pair of faults $\langle f_i, f_k\rangle$ for which $k > i$, we create $(n-1)$ sets $\mathfrak{d}_i$ referred to as sets of distinctiveness of the i-th fault.

$$\mathfrak{d}_i = \{\mathfrak{d}_{i,(i+1)}, .. , \mathfrak{d}_{i,n}\}; \quad i = [1..(n-1)] \tag{27}$$

The mean value of the isolability metric of single faults $\mathfrak{d}$ is as follows:

$$\mathfrak{d} = \frac{1}{(n-1)}\sum_{i=1}^{n-1}\mathfrak{i}_i \tag{28}$$

where $(n-1)$ is the number of elements of the isolability vector $\mathfrak{I}$.

By substituting $\mathfrak{i}_i$ in (26) and $\mathfrak{d}_{i,k}$ in (23), we obtain

$$\mathfrak{d} = \frac{1}{(n-1)}\sum_{i=1}^{n-1}\sum_{k=i+1}^{n-1}\sum_{j=1}^{m} m_{j,i,k} = \frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{m} m_{j,i} \tag{29}$$

where $I = \binom{n}{2}$ Finally, the normalised mean isolability index of single fault metric $\mathfrak{d}^N$ can be defined as

$$\mathfrak{d}^N = \frac{1}{m}\mathfrak{d}\,. \tag{30}$$

Therefore, the metric $\mathfrak{d}^N$ is calculated based on adding up the minimal number of differences between signatures for any pair of faults. This metric should be regarded as one that reflects the worst case isolation result in a given isolating structure. This is a highly important feature, particularly for applications that place importance on functional safety issues.

5.3. Normalised isolability metrics. The normalised isolability metrics proposed in [27] aspires to be a universal measure for the assessment of isolability of the FDI based on structural residual approaches regardless diagnostic signal type. Therefore, it is applicable for assessment of isolability of binary, multivalued, fuzzy, and continuous diagnostic signals. In addition, it is useful for the assessment of fault isolability when information regarding the sequences of symptoms is known. Owing to its inherent flexibility, it is particularly applicable for obtaining the optimal sensor placement in diagnostic systems [27, 32], in which different fault isolation approaches are used.

Normalised isolability metrics [27] is based on a trivial statement, i.e. isolability can be characterised by the normalised mean value of all events in which fault f_i excludes fault f_k, for all ordered pairs of faults $\langle f_i, f_k\rangle$ and for all alternative [26] signatures of fault f_i related to the maximal theoretical number of all exclusions. In fact, this definition corresponds to the normalised mean isolability index $\mathfrak{d}^N$ with the exception that metrics (30) is calculated for $k > i$, and the normalised isolability metrics are calculated for all $k \neq i$. According to [27], the normalised isolability metrics for binary diagnostic matrices is calculated according to the following formula:

$$\psi = \frac{1}{K\cdot(K-1)} \sum_{(i=1)}^{K} \sum_{(k-1,\, i\neq k)}^{K} D(f_i, f_k) \tag{31}$$

where K is the number of faults, $D(f_i, f_k) = 1$ if the appearance of all symptoms of fault f_i excludes the fault f_k; if this is not true, then $D(f_i, f_k) = 0$. It is noteworthy that formula (31) allows one to distinguish between unidirectional strong and weak fault isolabilities. If $\{(D(f_i, f_k) = 1) \vee (D(f_k, f_i) = 1)\} = 1$, then faults f_i and f_k are weakly isolable. If $\{(D(f_i, f_k) = 1) \wedge (D(f_k, f_i) = 1\} = 1$, then faults f_i and f_k are unidirectionally strongly isolable. Therefore, the measure (31) has a property that distinguishes weak and strong isolabilities.

In the case of multivalued diagnostic signals, the conditional isolability should be considered. To solve this problem, $D(f_i, f_k)$ assumes any value from the range $[0..1]$ and we propose that $D(f_i, f_k)$ will be calculated as follows:

$$D(f_i, f_k) = \frac{|\{\phi(f_i) : \phi(f_i) \in \Phi(f_i) \wedge \phi(f_i) \neq \phi(f_k)\}|}{|\Phi(f_i)|} \tag{32}$$

where $\phi(f_i)$ is any alternative signature of the fault f_i, and $\Phi(f_i)$ is the set of all alternative signatures of fault f_i.

As shown, (32) takes exclusively values 0 or 1 for the binary evaluated diagnostic values. This is because the cardinalities of all sets of alternative signatures in this case are equal to 1.

When utilising knowledge regarding the antecedence of symptoms, we assume that each sequence provides potentially useful information regarding fault isolation. Therefore, this information could deliver valuable and supplementary diagnostic signals to existing diagnostic systems. While the sequences are associated with faults, it is clear that they can be easily incorporated in the two-dimensional structure of residual sets. Subsequently, the fault isolability metric ψ can be calculated immediately from (31) and (32). In general, it is convenient to transform the structure of residual sets into a multivalued *FIS* by additional diagnostic signals and according to the following rules:

- add a new row (virtual diagnostic signal) in the *FIS* structure for each unique elementary sequence $es_{j,p}$;
- assign values of this diagnostic signal to each fault as follows:
  - $\{0\}$ if both symptoms in the elementary sequence are not relevant to this particular fault,
  - $\{-1,+1\}$ if both symptoms are sensitive to this particular fault but their order is any or not known,
  - $\{-1\}$ or $\{+1\}$ if both symptoms are sensitive to a given fault and the order of symptoms is fixed. In this case, values of $\{-1\}$ or $\{+1\}$ must be applied consequently to each fault in the entire *FIS*,
  - $\{-1\}$ or $\{+1\}$ if only one symptom is sensitive to a given fault and this symptom precedes in the elementary sequence. This distinction enables the interpretation of fault isolability properties with and without the exoneration assumption.

An example of the procedure above is illustrated in Table 4. It is noteworthy that three indistinguishable faults $\{f_1, f_2, f_3\}$ could be weakly $\{f_1\},\{f_2\}$ distinguishable if additional knowledge regarding the sequence of symptoms are used in the diagnostic system.

Table 4
Illustration of transformation of knowledge regarding sequences of symptoms.

S/F	f_1	f_2	f_3	f_4
s_1	+1	+1	+1	+1
s_2	+1	+1	+1	
s_3	$\langle s_1, s_2\rangle$	$\langle s_2, s_1\rangle$	1	0

$\Rightarrow$

S/F	f_1	f_2	f_3	f_4
s_1	+1	+1	+1	+1
s_2	+1	+1	+1	
s_3	+1	−1	−1, +1	+1

6. Summary

The main aim of this study was to emphasise the role and effect of the selected fault isolation approaches in the context of obtaining fault distinguishability metrics. This study focused on fault isolation methods that were suitable for implementation in automatised diagnostic systems intended for industrial applications. Therefore, the availability of the partial models of the system was assumed instead of that of the global one. In addition, it was assumed that knowledge regarding the fault–symptom relation was mainly based on expertise. Fault distinguishability was analysed for four different cases where the degree of knowledge regarding the relationship between the faults and values of diagnostic signals were different:

- inference based on the *BDM*;
- inference based on the *FIS*;
- inference based on the *BDM* and additional knowledge regarding the sequences of symptoms resulting from the *GP* graph;
- inference based on the *FIS* and additional knowledge regarding the sequence of symptoms resulting from the GP graph.

Table 5 summarises the calculated values of different fault distinguishability metrics presented in Section 5:

- the normalised mean theoretical diagnosis accuracy d_{acc};
- the normalised mean distinguishability index $\eth^N$;
- the normalised isolability metrics ψ.

Table 5
List of calculated fault distinguishability metrics.

No	Approach	d_{acc}	$\eth^N$	ψ
1	*BDM*	0.71	0.24	0.49
2	*BDMS*	$0.71^{1)}..0.76^{2)}$	$0.33^{1)}..0.43^{2)}$	$0.55^{1)}..0.58^{2)}$
3	*FIS*	$0.76^{1)}..0.79^{2)}$	$0.34^{1)}..0.64^{2)}$	$0.65^{1)}..0.79^{2)}$
4	*FISS*	$0.86^{1)}..0.90^{2)}$	$0.37^{1)}..0.71^{2)}$	$0.69^{1)}..0.93^{2)}$

1) unconditional fault distinguishability,
2) maximal value of conditional fault distinguishability.

As can be seen from Table 5, the values of fault distinguishability metrics differ significantly even for the same incidence matrix. This might be confusing. It is rather expected that fault distinguishability should not depend on distinguishability metrics applied. However, this results from the fact that all metrics characterized in Section 4 are differently sensitive to properties of diagnostic system such as e.g. unconditional and conditional distinguishability or weak and strong distinguishability of faults.

From the other hand, it is also easy to see from Table 5 that all metrics display the same trends and this is not contradictory with the rule: the better fault distinguishability, the higher is value of distinguishability metrics. This allows for recommendation the usage consequently of the one chosen distinguishability metrics for evaluating and tracking fault distinguishability property of diagnostic system particularly in its development phase.

The values of all distinguishability metrics discussed in this paper are bounded in the range [0..1]. It follows from the fact, that all of them are constructed in such a manner that converge to value of one in case of unconditional distinguishability of all faults. This make allowance for applying them for example for solving optimal sensor placement in order to achieve maximal fault distinguishability in a system with budgetary constraints [27, 32].

As shown in Table 5, information regarding the sequences of symptoms may be highly useful for the enhancement of fault distinguishability. Multivalued residual evaluation enabled faults that were indistinguishable in bivalued evaluations of residuals to be distinguished. An additional increase in fault distinguishability could be obtained using information regarding the antecedence of symptoms.

The presented methods for increasing fault distinguishability indicated the directions for the development of diagnostic systems. This study demonstrate that it is profitable to apply direct measurements of faults. This is because, currently, only a few faults could be measured directly as indirect fault isolation are more common. The dominant effect on fault distinguishability has a set of available measurements. In fact, it determines the set of possible diagnostic tests. Hence, the more process variables are measured, the more models can be built that will be advantageous for the generation of residuals, thus increasing of fault distinguishability.

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