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Estimation of state variables of the drive system with elastic joint using moving horizon estimation (MHE)

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Abstract. The article presents issues related to the application of a moving horizon estimator for state variables reconstruction in an advanced control structure of a drive system with an elastic joint. Firstly, a short review of the commonly used methods for state estimation in presented. Then, a description of a state controller structure follows. The design methodology based on the poles-placement method is briefly described. Next, the mathematical algorithm of MHE is presented and some crucial features of MHE are analysed. Then, selected simulation and experimental results are shown and described. The investigation shows, among others, the influence of window length on the quality of state variables estimation.

Key words: drive system with elastic joint, torsional vibration, moving horizon estimation (MHE), state variables estimation.

1. Introduction

Mechatronic systems are expected to achieve drive speeds and positions in the shortest time possible while maintaining high precision. This, however, entails the occurrence of high acceleration and speed. These phenomena lead to the appearance of complete rigidity of mechanical connections [1–3]. For the purpose of effective dampening of vibration, it is necessary to use special control structures [3–6].

As it is shown in [3], the system with a PI controller cannot damp the vibrations effectively, so the control structure is usually modified by means of insertion of additional feedback from one selected state variable. This allows to suppress the torsional vibration effectively, yet the settling time cannot be set freely. It results from the fact that there are three control parameters and four closed-loop poles of the system. The control structure with a PID controller has similar properties [3]. In order to place four poles in the desired position, it is necessary to have the structure with a PI controller and two additional feedbacks or a state controller [3, 4].

In the literature more advanced control paradigms can be found. They can be divided into a number of major groups. Robust control techniques, such as sliding-mode control, can be included in the first one. Different frameworks, such as classical, terminate, integral or twisting sliding-mode algorithms are investigated, e.g. in [23]. The second group incorporates methodologies based on the adaptive concept. Two main groups are evident here, namely direct and indirect approaches [24, 25]. Works describing fuzzy or neural adaptive algorithms are particularly popular. The third group encompasses the model predictive control (MPC) approaches. The MPC algorithm adapts to the current operation point of the process, generating an optimum control signal. It allows to take the input and output constraints of the system into consideration directly. The drawback of this method is relatively high computing complexity [6]. All of the above-mentioned control paradigms ensure very effective performance, yet they require precise information about the states (and in some cases parameters) of the plant.

Because the state vector is usually not measurable in practical applications, there is a need to apply a special estimation technique in order to reconstruct it [8–13]. The application of a Luenberger observer or a Kalman filter [8, 9] is quite common in the literature. However, both of these methods have some drawbacks. The Luenberger observer is sensitive to the changes of drive parameters and measurement noises [3]. The Kalman filter requires special characteristics of the noises, a condition which is usually not fulfilled in the industry [10, 11]. Therefore, other approaches are sought.

In the case of drives whose parameters are not known or hard to identify, it is possible to use structures based on artificial neural networks [12]. In this case, very good properties are obtained, however, the design methods of such estimators are complex and require expert knowledge. It is also possible to use simple methods based on the drive model [13], these, however, allow to make the estimation of selected state variable of a drive (shaft torque).

One of the most advanced estimation methods is the moving horizon estimation (MHE) [14, 15, 19]. The technique allows to determine the estimation of not only current information but also of the historical samples taken from a window with a fixed horizon (number of historical samples). The estimator is similar to predictive control in which a current sample is used to determine the future values of state variables. In the world literature there are no works (except for the author's papers) which would present the application of this type of estimator in a drive with an elastic joint with experimental verification of results. The main drawback of the MHE is the high computational complexity of the algorithm. This limits the application of this technique to the system with a low sampling rate. However, due to the con-

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tinuous increase of the computational power of processors, the MHE algorithm can be implemented on-line in industrial drives.

The main goal of the paper is to present a comparative analysis of the properties of the MHE linear observer working in a closed loop control structure with a state controller. Following a short introduction, the systematic methodology of the design of the state controller is presented. The formulas which allow to locate a system's closed-loop poles freely are provided. Then, the mathematical algorithm of the linear MHE and its on-line version are presented and discussed. In the simulation tests, the properties of the MHE are investigated. The possibility of shaping the observer properties by means of suitable selection of its weights is also analysed. Additionally, the influence of the length of the historical sample on the accuracy of state estimation is investigated. The simulation results are confirmed by experimental ones. They show that despite the large computational complexity of the MHE, the estimation algorithm can be implemented on-line for the two-mass system with a high value of sampling frequency.

2. Mathematical model of the drive and the control structure

A dual mass system can be presented in the form of two masses coupled by an elastic joint. The first mass represents the inertia of the drive and the other one stands for the load machine. The shaft is considered inertia-free. The model can be described by the following equations (relative units) [3, 16]:

$$\frac{d}{dt}\omega_1 = \frac{1}{T_1}(m_e - m_s) \tag{1a}$$

$$\frac{d}{dt}\omega_2 = \frac{1}{T_2}(m_s - m_L) \tag{1b}$$

$$\frac{d}{dt}m_s = \frac{1}{T_c}(\omega_1 - \omega_2) \tag{1c}$$

where: ω_1, ω_2 – drive and load speeds, m_e, m_s, m_L – electromagnetic, torsional and load torque, T_1, T_2 – mechanical time constant of the drive and load, T_c – elastic time constant. The main parameters of the system are: $T_1 = T_2 = 0.203$ s and $T_c = 0.0012$ s. The mechanical part is characterized by resonant frequency $f_r = 14.4$ Hz.

The above parameters can be determined using the equations described in [3]. The model can be presented in the form of the following equation of state:

$$\frac{d}{dt}\begin{bmatrix}\omega_1\\ \omega_2\\ m_s\end{bmatrix} = \begin{bmatrix}0 & 0 & -\frac{1}{T_1}\\ 0 & 0 & \frac{1}{T_2}\\ \frac{1}{T_c} & -\frac{1}{T_c} & 0\end{bmatrix}\cdot\begin{bmatrix}\omega_1\\ \omega_2\\ m_s\end{bmatrix} + \begin{bmatrix}\frac{1}{T_1} & 0\\ 0 & -\frac{1}{T_2}\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}m_s\\ m_L\end{bmatrix}. \tag{2}$$

A two-mass drive model seems to be a plant easy to control. It consists of only two masses and the inertia-free shaft. However, in practice when high-performance control is required, high precision is not easy to achieve. A good example of this point is the control problem of a deep space antenna. The observation of the deep universe requires as high precision as possible.

The two-mass drive mechanical model is only a simplification of the real system. Additional factors, such as non-linear friction located on both sides (motor and load machine), the nonlinear characteristic of the mechanical connection (with mechanical hysteresis) and even the imperfections in electromagnetically generated torque (for examples, the torques ripples) all influence the control properties. All these factors decrease the effectiveness of classical methods, which, in turn, provokes the search for other techniques.

In the paper, the control structure presented in Fig. 1 is used. In order to limit the electromagnetic torque, the incremental version of the state controller is used in the text.

The cascade control concept between the electromagnetic (inner loop) and mechanic (outer loop) systems is evident

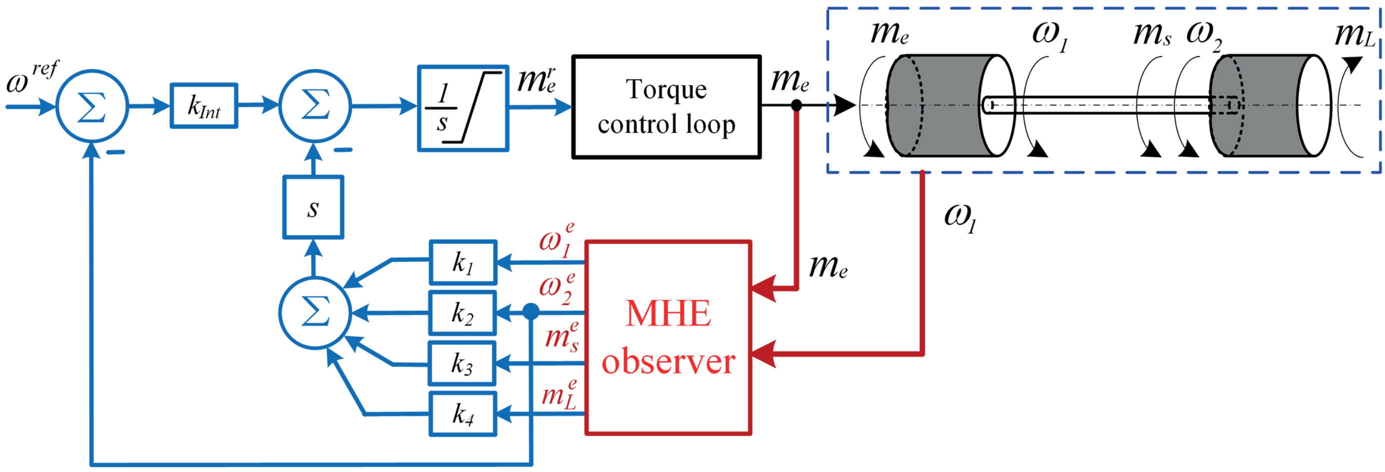

Fig. 1. Block diagram of control structure

in Fig. 1. The inner loop is designed so as to possess much faster dynamics (in the experimental set-up, the torque control loop can be approximated as the first order term with time constant equal to 0.0015 s). The outer loop has much slower dynamics (which can be approximated by first order term with time constant with the value starting from 0.1 s). Therefore, the assumption that the delay caused by torque control can be neglected is justified. This assumption constitutes quite a common approach in the drive with low frequency oscillations [3, 6]. This omission is allowed if the tuning dynamic of the closed system is smaller than the resonant pulsation of the mechanical part. In the case under consideration, this condition is met. Additional information about this problem can be found in [22].

The situation is different in the case of high frequency torsional vibrations. If the period of the oscillations is similar to the delay caused by the torque-control loop, it must be included in the analysis of the drive [16, 18].

The main transfer function of the closed system, shown in Fig. 1, can be presented in the following way (ignoring potential non-linear elements):

$$G^p_{\omega_2} = \frac{\omega_2(s)}{\omega^{ref}(s)} \Rightarrow \\ \Rightarrow \frac{k_{Int}}{s^4 T_1 T_2 T_c + s^3 k_1 T_2 T_c + s^2(k_2 T_2 + T_2 + T_1) + (k_1 + k_3) + k_{Int}} \tag{3}$$

Individual parameters of the state controller can be adjusted using the pole placement method or the Ackermann's formula. In the presented work, the first mentioned algorithm is used because it is convenient and simple in the case of low-order systems. It requires a comparison of the drive system equation with the reference polynomial of the same degree.

The characteristic equation of (3) is compared to the desired polynomial, describing the required dynamics of the closed loop system:

$$\left(s^2 + 2\xi_r \omega_0 s + \omega_0^2\right)\left(s^2 + 2\xi_r \omega_0 s + \omega_0^2\right) = 0 \tag{4}$$

where: ξ_r, ω_0 – required damping coefficient and resonant frequency of the closed-loop system.

After solving four equations, the formulas which allow to set the controller coefficients are obtained (5):

$$\begin{aligned} k_{Int} &= T_1 T_2 T_c \omega_0^4 \\ k_1 &= 4 T_1 \xi_r \omega_0 \\ k_2 &= T_1 T_c\left(2\omega_0^2 + 4\xi_r^2\omega_0^2 - \frac{1}{T_2 T_c} - \frac{1}{T_1 T_c}\right) \\ k_3 &= k_1(\omega_0^2 T_2 T_c - 1). \end{aligned} \tag{5}$$

The additional feedback from the load torque is set with the help of the following formula [3, 4]:

$$k_4 = k_2 + 1. \tag{6}$$

The application of the above control structure allows to freely shape the dynamics of the control system in the linear operation area.

3. Moving horizon estimator

Let us now consider a discrete dynamic system described by the following state equations:

$$\begin{aligned} \mathbf{x}_{t+1} &= \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t + \varsigma_t \\ \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \boldsymbol{\eta}_t \end{aligned} \tag{7}$$

where: $\mathbf{x}_t \in R^{n_x}$, $\mathbf{u}_t \in R^{n_u}$ and $\mathbf{y}_t \in R^{n_y}$ are the state vectors of measurable inputs and outputs, $\varsigma_t \in R^{n_x}$ are unmodellable system interferences, $\boldsymbol{\eta}_t \in R^{n_y}$ are output signal interferences and t is a discrete time index, while n_x, n_u, n_y are the dimension of the space vector, input vector and the output vector, respectively.

MHE is the recurrent estimation algorithm of state variables analysed on the basis of a finite data window (N). The problem of state determination is investigated for each moment of time: $N, N+1, \ldots t-1, t$ with account taken of the historical values of the state vector estimate:

$\mathbf{x}_{t-N}\, \mathbf{x}_{t-N+1} \cdots$ on the basis of the initial estimate
$\bar{\mathbf{x}}_{t-N,t}$ and the input information vector
$\mathbf{I}_t = \left[\mathbf{y}_{t-N} \ \cdots \ \mathbf{y}_t, \ \mathbf{u}_{t-N} \ \cdots \ \mathbf{u}_t\right]^T$,

where $N+1$ is the window length. For each moment of time, the linear problem of MHE [20, 21] is formulated in order to minimize the cost function:

$$\begin{aligned} J\left(\hat{\mathbf{x}}_{t-N,t}, \bar{\mathbf{x}}_{t-N,t}, \mathbf{I}_t\right) = \left\|\mathbf{y}_{t-N,t} - \hat{\mathbf{y}}_{t-N,t}\right\| + \\ + \alpha\left\|\hat{\mathbf{x}}_{t-N,t} - \bar{\mathbf{x}}_{t-N,t}\right\| \end{aligned} \tag{8a}$$

$$\begin{aligned} \hat{\mathbf{x}}_{i+1,t} &= \mathbf{A}\hat{\mathbf{x}}_{i,t} + \mathbf{B}\mathbf{u}_i, \qquad i = t-N, \cdots, t-1 \\ \mathbf{y}_{i,t} &= \mathbf{C}\hat{\mathbf{x}}_{i,t}, \qquad i = t-N, \cdots, t \end{aligned} \tag{8b}$$

where: subscript $t-N, t$ defines the historical values of the samples in the N window at the moment t, subscript $i+1, t$ defines the prediction of the state vector at time t for historical samples in window N, $\mathbf{I}_t$ is the input information vector, $\alpha \geq 0$ is a gain factor influencing the effect of the correction from the pre-estimator and $\hat{\mathbf{x}}_{i+1,t}$ in equation (8b) is the prediction of the state vector. For the purpose of limiting the influence of unstable dynamics of the object, or inaccuracies in model determination, it is proposed to introduce the correction of the predicted value of the state vector using the Luenberger observer or the Kalman filter – in equation (8b). After taking account of the above assumption, the problem of the determination of the state estimate using MHE can be presented in the following form:

$$J\left(\hat{\mathbf{x}}_{t-N,t}, \bar{\mathbf{x}}_{t-N,t}, \mathbf{I}_t\right) = \left\|\mathbf{W}\left(\mathbf{y}_{t-N,t} - \hat{\mathbf{y}}_{t-N,t}\right)\right\| + \\ + \alpha\left\|\hat{\mathbf{x}}_{t-N,t} - \bar{\mathbf{x}}_{t-N,t}\right\| \tag{9a}$$

$$\hat{\mathbf{x}}_{i+1,t} = \mathbf{A}\hat{\mathbf{x}}_{i,t} + \mathbf{B}\mathbf{u}_i + \mathbf{L}\left(\mathbf{y}_{i,t} - \hat{\mathbf{y}}_{i,t}\right), \\ i = t-N, \cdots, t-1 \\ \mathbf{y}_{i,t} = \mathbf{C}\hat{\mathbf{x}}_{i,t}, \quad i = t-N, \cdots, t \tag{9b}$$

where: $\mathbf{W} \in R^{n_x \times (N+1)n_y}$ is the matrix of weights differentiating the influence of particular historical samples on the value of the objective function and $\mathbf{L} \in R^{n_x \times n_y}$ is the matrix of observer gains.

The optimum sequence of the estimated state vector which minimizes function (9) can be determined with $\hat{\mathbf{x}}^o_{t-N,t}$. On this basis, the predicted state can be written from equation (7) as:

$$\bar{\mathbf{x}}_{t-N,t} = \mathbf{A}\hat{\mathbf{x}}^o_{t-N,t-1} + \mathbf{B}\mathbf{u}_{t-N-1} + \\ + \mathbf{L}\left(\mathbf{y}_{t-N-1} - \hat{\mathbf{y}}_{t-N-1,t-1}\right) \\ \hat{\mathbf{y}}^o_{t-N-1,t-1} = \mathbf{C}\hat{\mathbf{x}}^o_{t-N-1,t-1} \tag{10}$$

where $\hat{\mathbf{x}}^o_{t-N,t}$ is the start value of the state vector.

For the purpose of estimation of the state variables, the drive model (1) must be transformed to (2) and the vector of state must be extended by load torque so it takes the form of (2).

The above model was discretized with time $T_s = 1$ ms and zero-order extrapolation. The objective function form, which is minimized with each estimation step, can be presented as:

$$\min_X J = (\mathbf{Y} - \mathbf{C}\cdot\mathbf{X})\mathbf{W}(\mathbf{Y} - \mathbf{C}\cdot\mathbf{X})^T + \\ + \alpha\left(\mathbf{X} - \bar{\mathbf{X}}\right)^T \tag{11a}$$

$$\bar{\mathbf{x}}_{t+1,t} = \mathbf{A}\bar{\mathbf{x}}_{t,t} + \mathbf{B}m_{et,t} + \mathbf{L}\left(\omega_{1t,t} - \bar{\omega}_{1t,t}\right) \tag{11b}$$

$$\mathbf{Y} = \left[\omega_{1t-N} \quad \omega_{1t+N} \quad \cdots \quad \omega_{1t}\right]^T \tag{11c}$$

$$\mathbf{X} = \left[\mathbf{x}_{t-N} \quad \mathbf{x}_{t-N+1} \quad \cdots \quad \mathbf{x}_t\right]^T \tag{11d}$$

$$\bar{\mathbf{X}} = \left[\bar{\mathbf{x}}_{t-N-1} \quad \bar{\mathbf{x}}_{t-N+1} \quad \cdots \quad \bar{\mathbf{x}}_{t-1}\right]^T \tag{11e}$$

where: $\mathbf{Y}$ – vector of output signals defined on horizon N, $\mathbf{X}$ – vector of state variables defined on window horizon N, $\bar{\mathbf{X}}$ – vector of state variables predicted in the previous calculation step defined on window horizon N, $m_{e\,t,t}$ – actual sample of electromagnetic torque, $\omega_{1t,t}$ – actual sample of measured motor speed, $\bar{\omega}_{1t,t}$ – actual sample of estimated motor speed.

The estimator considered can be presented in the form of algorithm:

Algorithm 1 (on-line linear MHE observer)

1. At time t measure the actual system input $m_{e\,t,t}$ and output $\omega_{1t,t}$
2. Formulate matrices $\mathbf{Y}$, $\mathbf{X}$, $\bar{\mathbf{X}}$, taking account of the current measurement data,
3. Find the minimum of objective function (11).
4. Download the current value $\mathbf{x}_t$ from $\mathbf{X}$
5. Calculate the prediction of the state vector for the moment $t+1$ (8b)
6. Update $t \leftarrow t+1$ and return to step 1.

To minimize the purpose function, an algorithm based on the Hooke-Jeeves method has been used [21]. To speed up the operation, the previously determined value has been assumed as the starting point.

4. Simulation results

This section presents a comprehensive analysis of the proposed estimation algorithm. The tests are performed in the following order:

The operation assessment is conducted using an indicator in the form of the sum of state estimation errors averaged by the number of samples:

$$e(\omega_1) = \frac{\sum_{i=1}^{n}\left|\omega_1 - \omega_1^e\right|}{n}; \quad e(\omega_2) = \frac{\sum_{i=1}^{n}\left|\omega_2 - \omega_2^e\right|}{n}; \tag{12a}$$

$$e(m_s) = \frac{\sum_{i=1}^{n}\left|m_s - m_s^e\right|}{n}; \quad e(m_L) = \frac{\sum_{i=1}^{n}\left|m_L - m_L^e\right|}{n} \tag{12b}$$

and the summary estimation index, taking account of all state variable errors, is:

$$e_{sum} = e(\omega_1) + e(\omega_2) + e(m_s) + e(m_L). \tag{13}$$

Because the state variables are expressed in relative terms, there is no need to use scaling factors in (13).

First, open structure tests (controller feedback directly from the object) are carried out.

For the purpose of analysing the influence of the observer on the operation of the closed control structure, an additional quality index is introduced:

$$J_{\omega_2} = \sum_{i=1}^{n}\left(\omega^{ref}(i) - \omega_2(i)\right)^2 t^2. \tag{14}$$

The following test algorithm is selected: in time $t = 0.1$ s the drive is started up to reference speed, next in time $t = 0.4$ s the nominal load torque is applied.

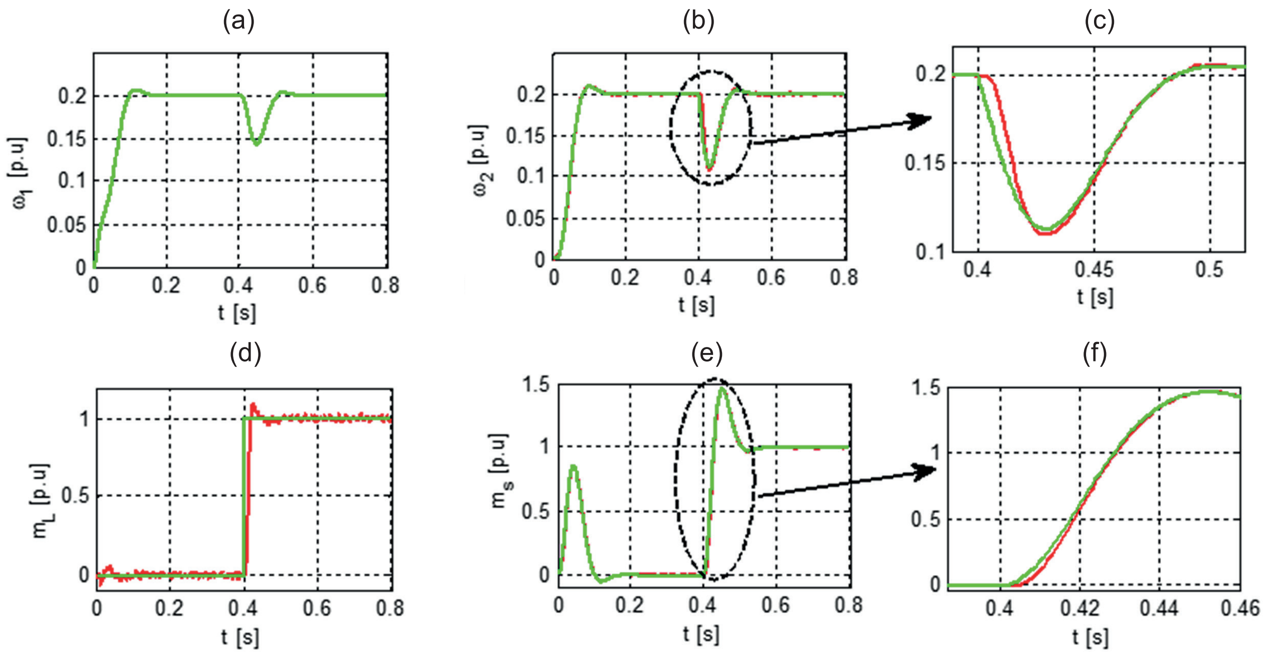

Fig. 2. Waveforms of real (green color) and estimated (red color) values of state variables: a) motor drive speed, b), c) load speed, d) load torque, e),f) torsional moment

The observer gain is set experimentally at the following values: $\mathbf{L} = [1.055; 17.064; -76.89; -318.28]^{\mathrm{T}}$. Additionally, $\alpha = 1000$. The weight value is divided into two components:

$$\mathbf{W} = W_0 \cdot \begin{bmatrix} w_{11} & 0 & 0 & 0 \\ 0 & w_{22} & 0 & 0 \\ 0 & 0 & w_{33} & 0 \\ 0 & 0 & 0 & w_{44} \end{bmatrix} \tag{15}$$

where W_0 is the scaling coefficient and w_{ii} are the weight components of matrix $\mathbf{W}$.

In the first test, the influence of the weight coefficients of matrix $\mathbf{W}$ on estimator properties is tested:

$w_{11} = w_{22} = w_{33} = w_{44} = 1$ (case 1),

$w_{11} = 0.25$, $w_{22} = 0.5$, $w_{33} = 0.75$, $w_{44} = 1$ (case 2),

$w_{11} = 1$ $w_{22} = 0.75$, $w_{33} = 0.5$, $w_{44} = 0.25$ (case 3),

$w_{11} = 1.45$, $w_{22} = 1.75$, $w_{33} = 1.38$ $w_{44} = 0.1$ (case 4).

In this part of the simulation, $W_0 = 1000$ (the values are set experimentally).

Table 1
Estimation quality (10^{-3})

	$e(\omega_1)10^{-2}$	$e(\omega_2)$	$e(m_s)$	$e(m_L)$	e_{sum}
case 1	7.008	2.553	15.7	38.63	56.92
case 2	7.015	2.552	15.6	38.62	56.95
case 3	7.014	2.544	15.5	38.57	56.74
case 4	7.015	2.541	15.4	38.55	56.68

The first three matrices present the even distribution of the influence of historical samples and the current ones (case 1), smaller significance of historical samples (case 2) and larger significance of historical samples (case 3), respectively. The last matrix is selected in an experimental manner.

The obtained estimation quality indices are presented in Table 1 and waveforms of the state variables are presented in Fig. 2.

The results presented (Fig. 2.) show that the estimator works correctly. The estimation error in transient states takes on minimum values and the noise level is very low.

The results presented show that the accentuation of current samples (case 3) results in the improvement of estimator operation in comparison with the weight values accentuating the historical samples (case 2). However, the least favorable results are obtained for the system with the same values of weights. In summary, it can be stated that the use of the weight matrix with unevenly distributed weights leads to obtaining the best results. It should also be said that the differences are relatively small.

In the next step, the influence of weight value α on estimation quality when the mechanical time constant of the load machine changes is presented. In these considerations, the following values were tested: $\alpha = 500$, $\alpha = 1000$ and $\alpha = 5000$. The results are presented in Fig. 3. The analysis of the obtained results allows to conclude that the selection of coefficient α below weight W_0 leads to the impairment of the quality of state variable estimation (apart from the drive motor speed). In the case of significant underassessment of plant parameters, the higher value of coefficient α leads to a more correct estimation. However, it should be observed that this value is significantly smaller than the nominal one and that this situation is purely hypothetical.

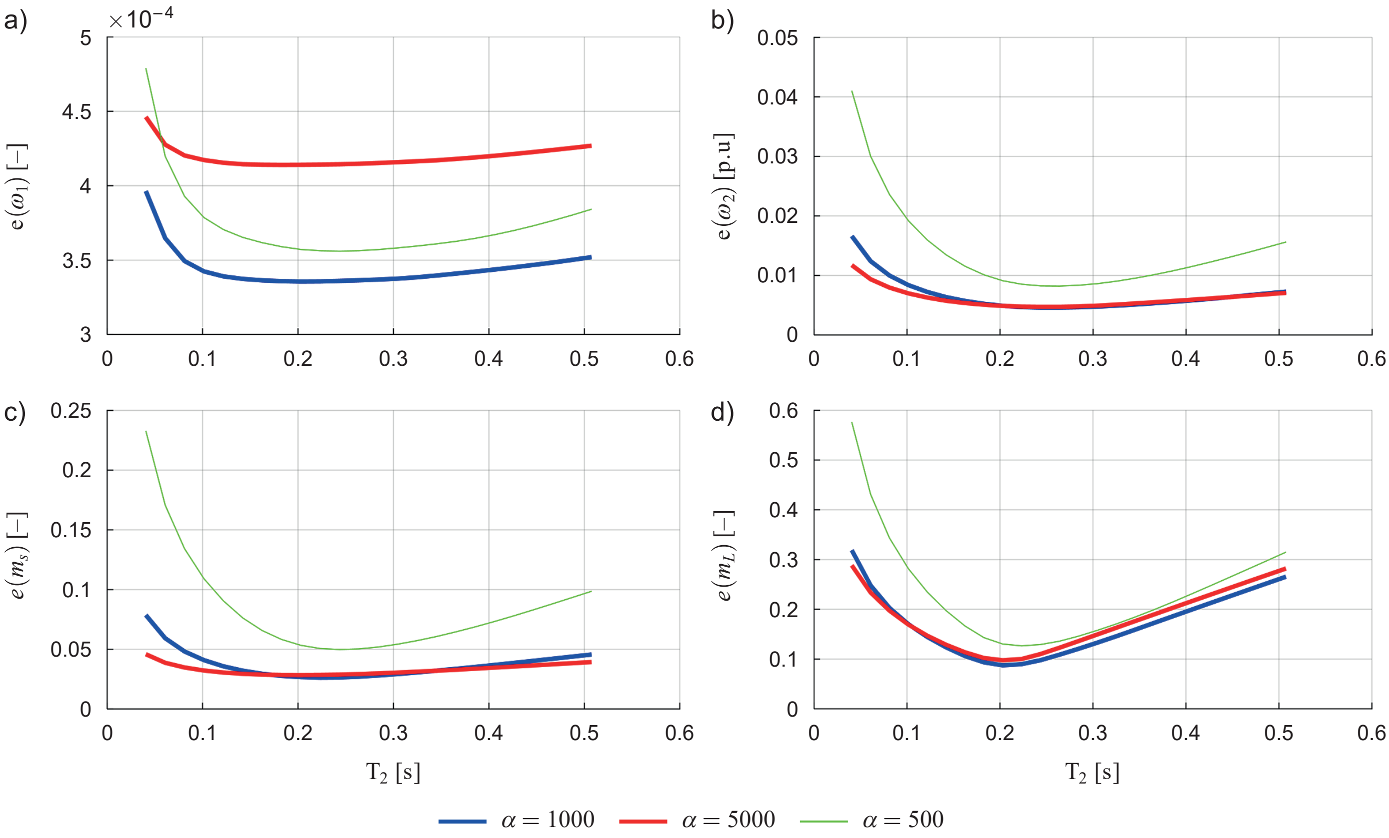

Fig. 3. Influence of coefficient α on estimation quality: motor drive speeds (a), load speeds (b), torsional moment (c), load torque (d), with changes of the mechanical time constant of machinery

Then, the influence of the values of window length on the system operation is checked. In these considerations two values: $N = 0$ and $N = 4$, are selected. The first case corresponds to the situation of a classic observer and the state vector estimation is based only on a current sample. The comparison of the obtained results is shown in Table 2, where there are the data for both the system for which significant measurement noise (labelled "with noise" in Table 2) is taken into account and the system without it (labelled "without noise" in the Table). The noise has been selected at 0.01 level (per unit). Figure 4 presents the waveforms of state variables for the system with and without noise in the open system.

The data presented in Table 2 show that extending the window length causes higher robustness to measurement noise. For the load torque and load speed it is a twofold change. However, in the system without any measurement noise the estimation quality is reduced. Taking into account the fact that measurement noise exists in every real system, it can be stated that the system with a longer window is better.

The above analysis is well illustrated in the waveforms presented in Fig. 4, where one can observe a significantly smaller value of oscillation in the system with the longer window.

Next, the closed control system was tested. In this case, the estimated values were used as control system feedback. The results obtained are presented in Table 3 and Fig. 5. The presented data allow to observe a situation similar to the one in the open structure, the only difference being the more significant influence of the window extension (in the case of load speed, it is nearly threefold).

Table 2
Influence of window length and measurement noise level on the estimation quality of state variables in open structure (10^{-3})

$e(\omega_1)10^{-2}$	$e(\omega_2)$	$e(m_s)$	$e(m_L)$		
15.909	6.2	18.6	115.5	with noise	N = 4
3.4037	4.7	15.6	90.3	without noise	
25.094	11.4	28.4	206.0	with noise	N = 0
1.0729	1.7	6.4	49.5	without noise	

Table 3
Influence of window length and measurement noise level on the estimation quality of state variables in closed structure (10^{-3})

$e(\omega_1)10^{-2}$	$e(\omega_2)$	$e(m_s)$	$e(m_L)$	ITAE		
2.4	2.3	7.8	58.5	3.32	N = 0	without noise
1.8	2.5	9.5	56.7	3.31	N = 4	
29.4	15.8	40.0	268.7	4.02	N = 0	with noise
16.3	5.7	19.1	108.7	3.48	N = 4	

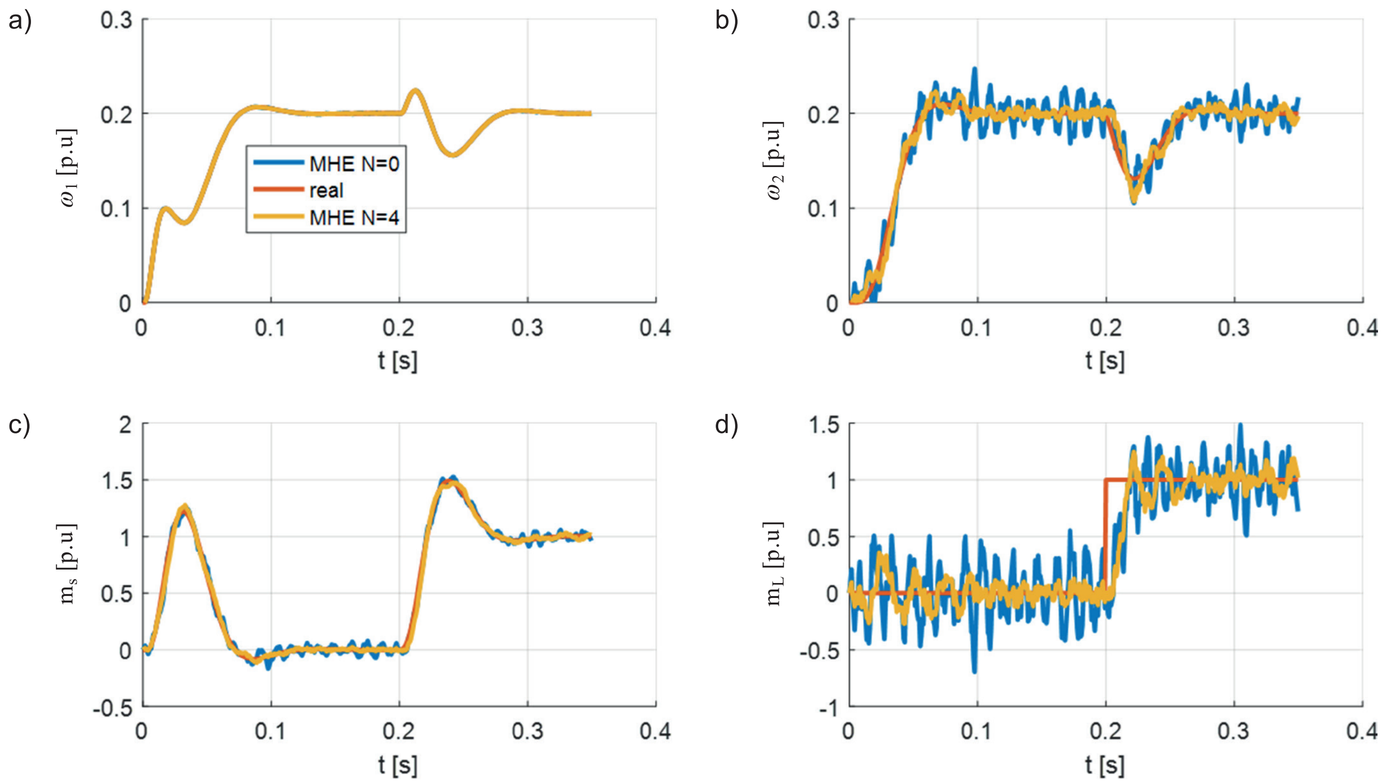

Fig. 4. Waveforms of real and estimated values of state variables for various window length values in open structure: a) motor speed, b) load speed, c) shaft torque, d) load torque

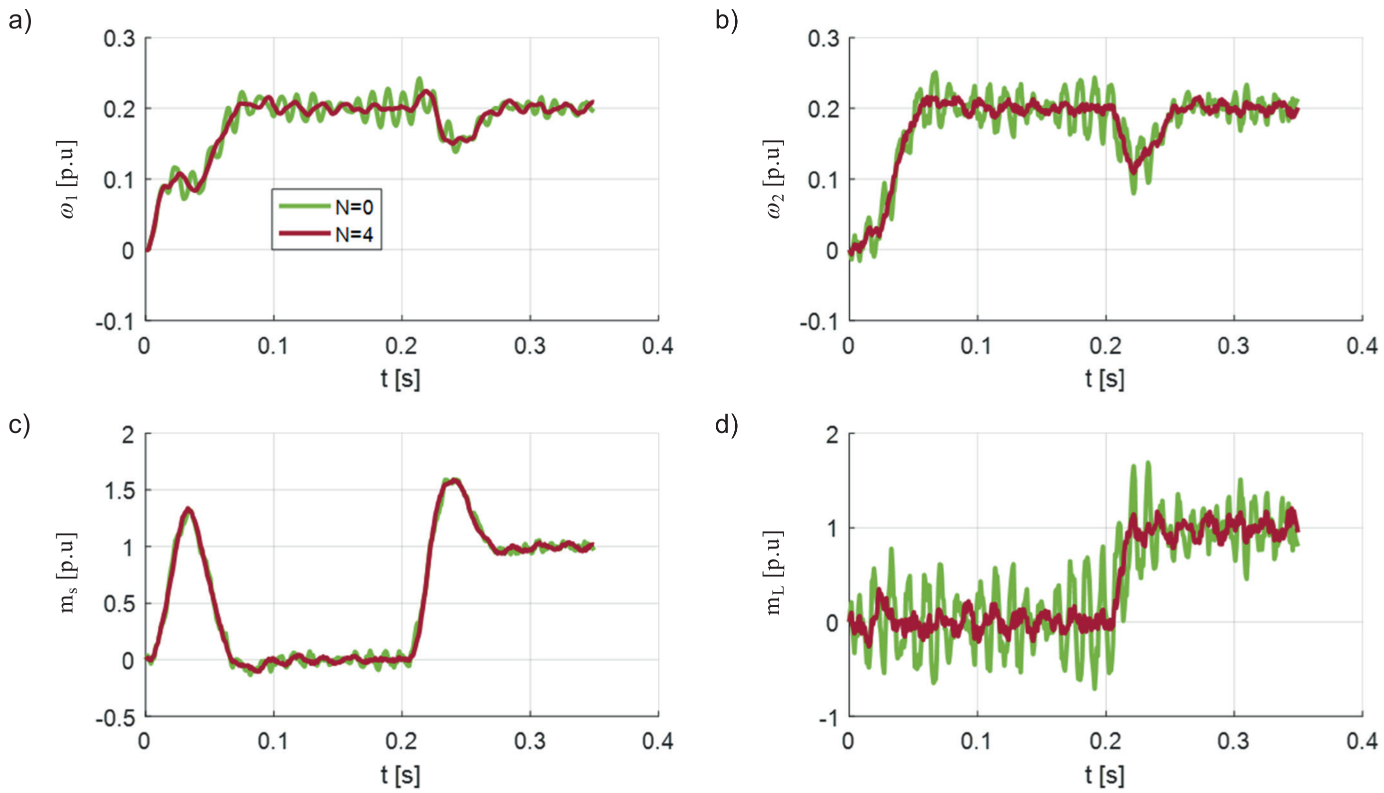

Fig. 5. Waveforms of real and estimated values of state variables for various window length values in closed structure: a) motor speed, b) load speed, c) shaft torque, d) load torque

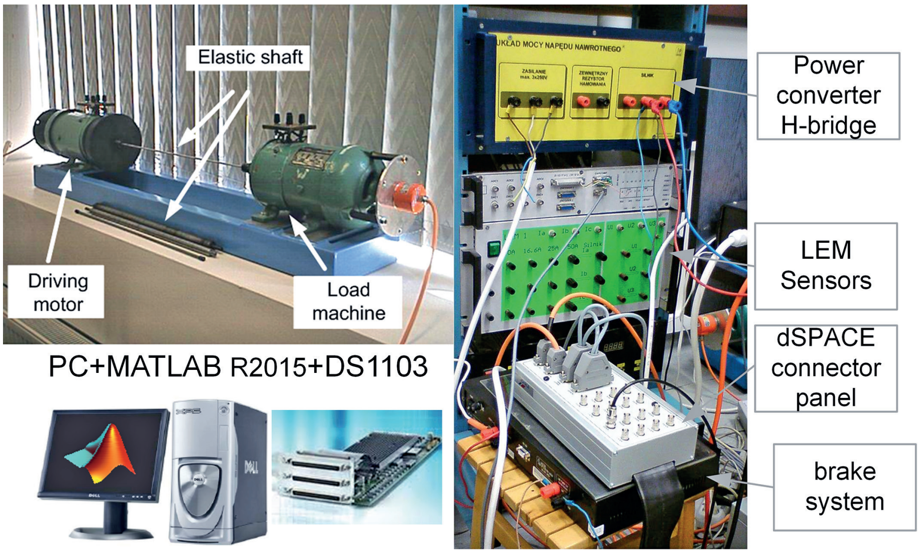

Fig. 6. Pictorial view of experimental setup

5. Experimental research

The experimental test is conducted on an experimental setup with DC motors, 0.5 kW, which are connected by means of a long shaft ($l = 0.6$ m, $\phi = 0.005$ m). The drive motor is fed by an *H* bridge converter. The converter is controlled by a pulse width modulator, carrier frequency 13 kHz, and cooperated with a PI-type current controller. Drive motor speed is measured by a Kuebler incremental encoder with a resolution of 36 000 pulses per revolution. Currents are measured by an LEM sensor (a Hall effect sensor). Experimental setup is presented in Fig. 6. The control algorithm and the analysed estimator are implemented on the *DS*1103 rapid prototyping card.

In the first tests the control structure is calculated with a 1 ms step. Next, the discretization step is adapted to estimator calculation time. First of all, it was checked how much time was necessary for the MHE observer to calculate the estimated value.

Additionally, the time is compared with the value necessary to calculate the Kalman filter. The results obtained are presented in Table 4.

Table 4
Time necessary to determine estimates by selected estimators

	Kalman	MHE
T_{calc}^{aver} [ms]	0.055	1.6

As can be seen, the level of the algorithm complexity is quite high and exceeds the calculation step of the controller itself that had been adopted earlier. This is why the sampling time is changed to 0.002 s.

Next, the operation of the system is tested in the closed-loop structure. The test comprises cyclical speed reverses with the reference value which is one-half of the nominal speed. In addition to this, load torque is applied in the steady state. The results are presented in Fig. 7. As can be concluded from the results presented, the control structure (controller and observer) operates in an accurate and stable manner. Speed fluctuations are insignificant.

Additionally, Fig. 8a presents the waveform of the objective function *J* and Fig. 8b – the waveform of the algorithm realization time.

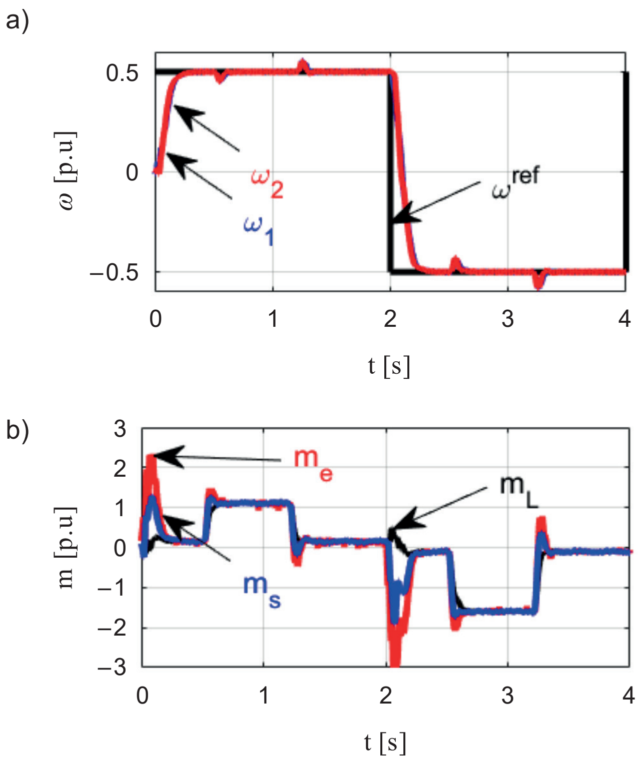

Fig. 7. Experimental results a) reference, motor and load speeds, b) electromagnetic, shaft and load torques in a closed structure with an *MHE* observer

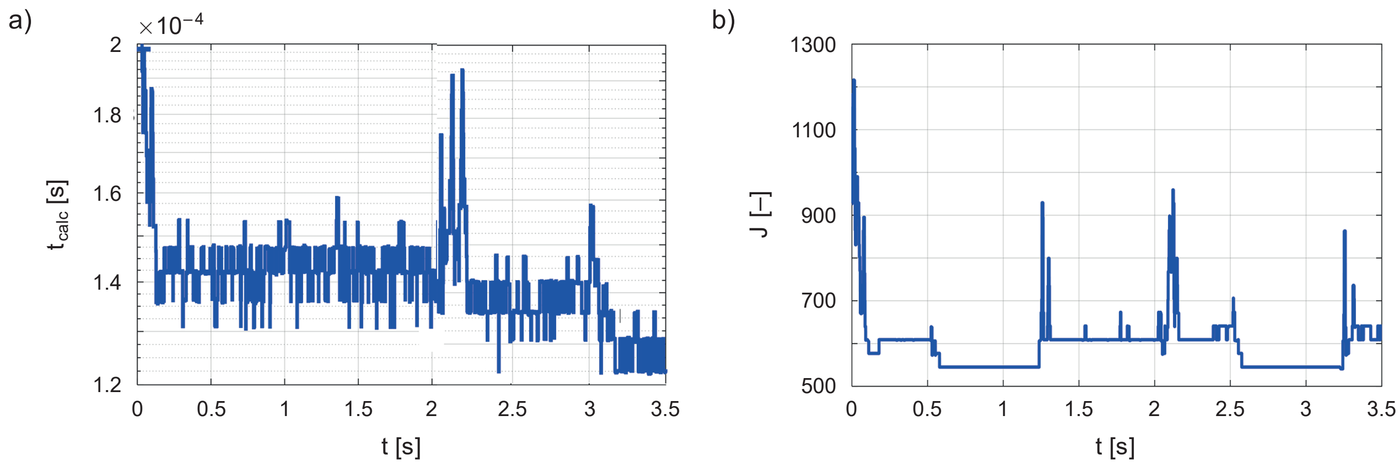

Fig. 8. Waveforms of calculation time (a) and values of indicator *J* (b) in closed structure with *MHE* observer

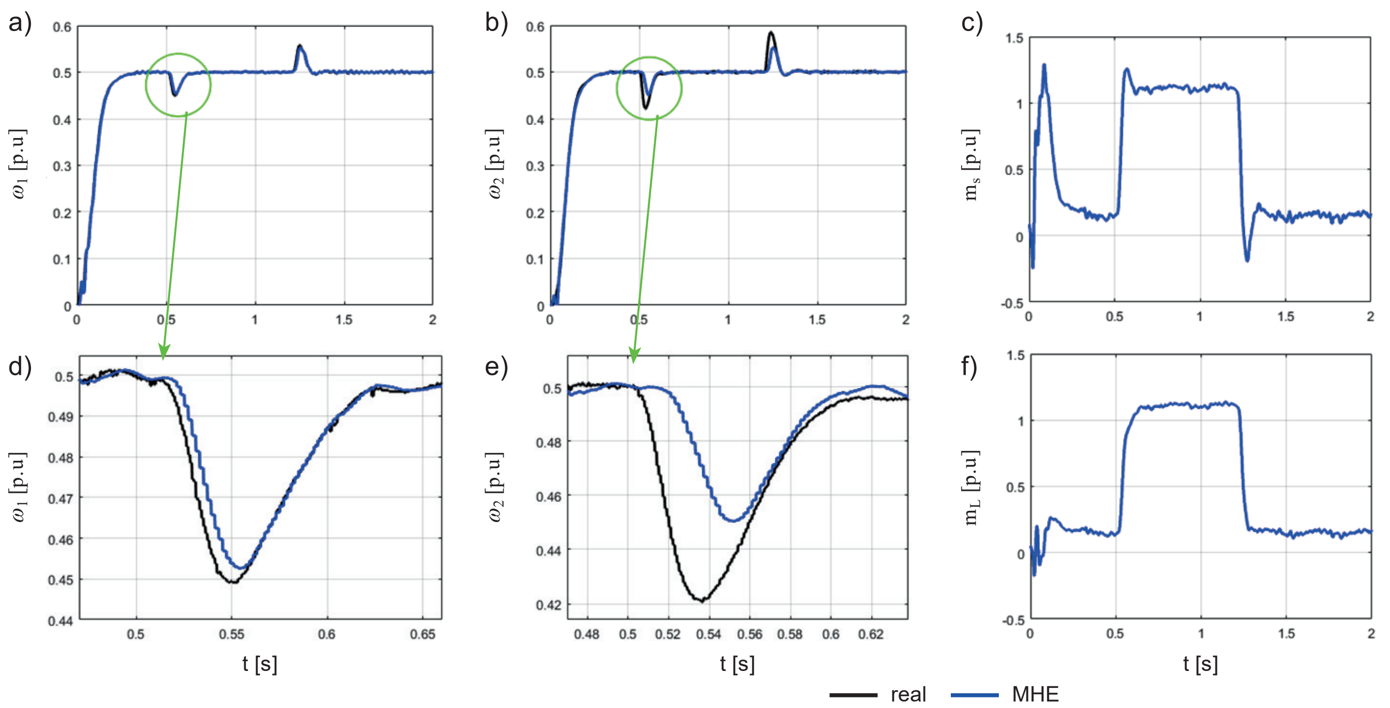

Fig. 9. Experimental comparison of real and estimated waveforms of: motor speed (a, d) and load speed (b, e), torsional moment (c), load torque (f)

It can be clearly seen that in the initial phase of the structure operation, the time necessary to determine control (including the estimation of the state vector) is significantly longer than at subsequent stages. The next such time increment is observed at the reversal time. This results from a quick dynamic state in which the differences between real and estimated signals are large.

Figure 9 presents the increase in the estimated state variables and the comparison of real values and their estimates (both speeds were measured). As can be seen, the differences are small and they quickly disappear. The maximum momentary error of the second speed determination was at the level of 7% of the real value.

Finally, it is experimentally checked how a change of drive parameters influences the estimation quality. In this research, the nominal mechanical time constant along with half and double values of this parameter were used. The results obtained are presented in Table 5.

Table 5
Influence of the changes in mechanical time constant on estimation of motor and load speed

$e(\omega_1)$	$e(\omega_2)$	Comments
0.02009	0.01890	$T_2 = T_{2N}$
0.0211	0.02168	$T_2 = 2T_{2N}$
0.0210	0.02231	$T_2 = 0.5T_{2N}$

6. Summary

The paper presents issues related to the development and testing of the MHE controller for a drive with an elastic coupling. The synthesis of the estimator and comprehensive simulation and experimental tests in the open and closed structure have been presented. As a result the following conclusions can be drawn:

- Despite the high complexity of the algorithm, it is possible to practically implement it in a drive system in which the time constants are small enough, and it can also work in a closed structure.
- The basic parameters which can influence the dynamic parameters are the gain matrix **L**, the weight matrix **W** and coefficient α. The selection of gains is key for estimator stability, however, it can be conducted using algebraic methods (like in the case of the classical observer).
- The introduction of the time window has a significant influence on the reduction of measurement noise impact and leads to the improvement in drive operation.

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