

## INFLUENCE OF RECAST LAYER ON THE FATIGUE LIFE OF Ti6Al4V PROCESSED BY ELECTRIC DISCHARGE MACHINING

In this research work, Ti6Al4V alloy material was subjected to electric discharge machining (EDM) and its fatigue life was investigated at low cycle fatigue mode. In order to evaluate the influence of recast layer generated during the machining process on the fatigue life, samples prepared using end milling process were also subjected to similar tests and a comparative analysis is presented. Data were observed in the fully reversed fatigue mode at room temperature using samples fabricated as per ASTM standard E606. The specimen were machined on a spark electric discharge die sink machine which were subjected to fatigue, and the recorded fatigue lives were compared with the fatigue life of end milled specimen. The machined surfaces were examined through optical and scanning electron microscopes, and the roughness was measured with a standard profilometer. It was observed that when the discharge current is augmented, the recast layer formed was in the range of 20 to 70  $\mu\text{m}$  thick. From the results, it is being concluded that fatigue life of the samples fabricated by EDM is less for various load conditions when compared with that of the end milled sample. The milled sample at 160 MPa load exhibited  $2.71 \times 10^5$  cycles, which is 64% more when compared to EDM sample.

*Keywords:* Ti6Al4V, EDM, Recast Layer, Fatigue, Residual Stress

### 1. Introduction

Ti6Al4V has been used for fabricating various aerospace components, such as first-stage compressor fans of gas turbines, small flexures needed to support sensitive instruments and other support structures on space payloads. These components are subjected to large cyclic stresses during their service period and therefore designers need fatigue data while designing the components. Since the components for such applications have intricate profile, electrical discharge machining (EDM) process is increasingly used which enables accurate, quick and repeatable processing of metals. In spite of the advancements in EDM process capabilities, small amount of quantitative data is available in the published literature related to the fatigue life of Ti6Al4V material processed through EDM. Mower [1] has reported about the experimental work in which the decreasing of fatigue strength of Ti6Al4V due to enhanced EDM processing is recorded in axial tension for a load ratio of  $R = 0.1$ . It has been concluded that, compared to finely milled specimens, advanced EDM processing resulted in a reduction of fatigue strength by 30%. In general EDM process induces residual tensile stresses deep into the sub-surface layer. The reason behind this is during cooling the heat affected surface shrinks against the adjacent cooler layer. This effect is predominant in titanium alloy due to its low thermal conductivity. Nanoindentation technique was adopted to measure

the residual stresses induced on the Ti6Al4V specimen machined with micro-EDM, which revealed that tensile stresses of up to 350 MPa was generated at depths up to 12  $\mu\text{m}$  [2].

Klocke et al. [3] have investigated on the fatigue limits of two types of Ti6Al4V specimens. One of the specimens has a surface machined by EDM process and the other specimen with ground surfaces. Even though a low surface roughness values were observed on the surfaces of both types of specimens, the fatigue limits were around 200 MPa for both types of specimen. Strasky et al. [4] have reported about the degradation of fatigue strength for the specimen machined by EDM process. The reason behind the fact was the presence of more number of microcracks and tensile residual stress on the surface of the EDM specimens. Mhatre et al [5] have determined the optimum parameters for electric discharge machining of Ti6Al4V through Grey Relational Optimization technique and found that duty cycle (8%), pulse current (18 Amp), Pulse on time (200  $\mu\text{sec}$ ) and voltage (40 V) resulted in better performance in terms of metal removal rate, electrode wear rate and surface roughness. But no mention was made about the recast layer and the fatigue life of the machined component. Hascalik and Caydas [6] had reported on the EDM of Ti6Al4V with different electrode materials to explore the influence of various parameters on the surface integrity of the machined specimen and it was mentioned that below the recast layer a slightly softening or tempered layer has

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been formed due to low thermal conductivity of the material. Strasky et al. [7] have pointed out that the tensile properties are not substantially varied by EDM but have detrimental effect on fatigue endurance. Poor fatigue performance after EDM irrespective of its original microstructure can be attributed to brittle oxidized surface recast layer, surface microcracks, internal tensile stresses and notch sensitivity of Ti6Al4V. Fatigue properties of Ti6Al4V have been widely investigated [8,9], and it was reported that the fatigue fracture originates at the surface with lower fatigue life region and high stress, whereas in longer fatigue lifetimes and low stress origins are generally sub-surface in nature [10]. Nakamura and Oguma [11] have pointed out that the variation between sub-surface crack and surface crack is the surrounding environment. Sub-surface crack is exposed to the environment without oxidation and gas adsorption. Nakamura et al. [12] investigated on the outcome of the sub-surface fracture around the fatigue crack from environment point of view. Oguma and Nakamura [13,14] have conducted fatigue trails up to a very high cycle regime to find that fracture surfaces had a unique concave-convex agglutinate on the fracture surface of sub-surface fractures.

Davim [15] addressed the changes that had occurred during machining with respect to the surface hardness and other mechanical properties. These changes may be attributed to the residual stresses, metallurgical alterations such as microstructural, grain size, precipitation and localized thermo mechanical phase transformation which contributes to the performance of component. Ho et al., [16] had stated that, subsequent the machining of Ti6Al4V, the surface exhibits different property than that of the bulk material property. This alteration is due to thermal effects and phase change because of the rapid cooling in the process. These surface layers consist of the white layer and the recast layer which is higher hardness and different property compared to the bulk material. Sometimes the white layer is beneficial for particular applications such as dental or biomedical medicals but in applications such as aircrafts it is detrimental. This layer is brittle in nature, subsequently it is easy to engender the crack and grow into the bulk material. It also affects the surface and fatigue property of the material [17]. Hascalik and Caydas [18] has studied the effect of peak current on the recast layer thickness (around 11 mm) of Ti6Al4V component. It is perhaps due to the production of deeper and larger craters, consequently melts more amount of material from the work surface and solidifies on the machined surface. The tensile residual stresses originate the crack in the recast layer and penetrate into the bulk material when the stress outstrips the tensile strength of the material. The stresses increase due to the rapid heating and cooling of material by the dielectric fluid. The residual tensile stresses remained in EDM processed specimens results in poor fatigue strength. Pradhan et al., [19] has reported that using EDM process for machining Ti6Al4V is feasible, but the recast layer thickness is increased with an increase in peak current or pulse – on – time. EDM changes the material properties of Ti6Al4V, particularly, the surface roughness of the alloy is increased and the degree of severity of defects is proportional to the applied peak current

[20]. Yang et al. [21] has reported that structural cracks typically start from the stress concentration position which can severely affect the structural strength of engineering components. Therefore, studying the stress concentration factor is crucial for the fracture life assessment of the components processed through EDM [22].

Literature indicates that machinability characteristics study on Ti6Al4V through EDM had been reported by various authors. But, low cycle fatigue characteristics study due to the formation of recast layer hasn't been focused much in the reports. Therefore, the current research is focused on determining the influence of recast layer formed on the work material on the low cycle fatigue life. Experiments are conducted with the combination of different parameters to create various thicknesses of recast layers as mentioned in the subsequent section. Four parameters namely peak discharge current, pulse off time, current pulse duration and gap voltage at three different levels were used for conducting the experimental trials. Further, the fatigue life of EDM samples and milled sample were compared to determine the effect of recast layer (various thicknesses) on the fatigue life.

## 2. Experimentation

The chemical composition and the properties of Ti6Al4V are presented in Table 1 and 2 respectively. The Dimension of the as received material is 20 mm wide, 5 mm thickness and 1000 mm length. The samples are initially cut to 100 mm length each for further processing. Machining experiments were conducted according to Taguchi's L9 orthogonal array and the observed data are presented in Table 3. Figure 1 indicates the experimental setup used for preparing the fatigue specimens. The specimens were trimmed into smaller pieces convenient to observe through Scanning Electron Microscope to study the microstructure and to measure the thickness of the recast layer formed. Rockwell hardness (HRc) was measured on the surfaces of the specimens fabricated from all the experimental trails. In

TABLE 1

Chemical composition of Ti6Al4V

Element	Ti	Fe	C	N	O	H	V	Al
Weight %	Bal.	≤0.30	≤0.10	≤0.05	≤0.02	≤0.015	3.5-4.5	5.5-6.8

TABLE 2

Properties of Ti6Al4V

Parameters	Values
Elastic modulus (GPa)	114
Melting point (°C)	1630
Electrical resistivity (μΩ.cm)	178
Thermal conductivity (W/m.K)	6.70
Density (g/cm <sup>3</sup> )	4.04
Yield strength (kg/mm <sup>2</sup> )	84.2
Tensile strength (kg/mm <sup>2</sup> )	91.3
Elongation (%)	10
Poisson ratio	0.33

Experimental observation

Sample No:	Peak Discharge Current, $I_p$ (A)	Current Pulse Duration, $T_{on}$ ( $\mu$ s)	Gap Voltage, $E_g$ (V)	Pulse-Off Time, $t_{off}$ ( $\mu$ s)	Hardness, HRC	Tensile Strength, $\text{kg/mm}^2$	Surface Roughness, (microns)	Recast layer thickness (microns)	Fatigue Life at 160 MPa		Fatigue Life at 500 MPa	
									Stress (KN)	Life	Stress (KN)	Life
1	6	200	4	150	33.5	84.3	11.2	26.2	8	176897	23	50128
2	6	400	8	100	32	82.7	12.3	18.8	8	169153	23	48981
3	6	810	6	200	32.9	82.6	12.8	34.8	8	142577	23	45933
4	12	200	4	100	33	80.4	7.5	31.2	8	137068	23	52791
5	12	400	6	150	30.6	77.8	8.9	46.2	8	111631	23	47827
6	12	810	8	200	29.4	77.0	8.5	54.2	8	110235	23	50287
7	20	200	6	150	28.8	85.8	6.4	24.8	8	102040	23	64351
8	20	400	4	200	30	78.7	10.6	69.2	8	92040	23	49800
9	20	810	8	100	32	77.4	11.4	57.5	8	89247	23	45233

End milled sample: HRC 36, Tensile Strength 90.8  $\text{kg/mm}^2$

order to avoid the error in measurement, five readings were taken on each sample and the average value was considered for comparison. Mitutoyo 178-923E Sj210 device was used to measure the surface roughness on the machined specimens. Instron Universal Testing Machine (UTM) was used for performing the tests on low cycle fatigue. The UTM is equipped with an adjustable load cell which can be actuated by hydraulic units and static force motor. The test frequency of the UTM was kept at a constant value of 10 Hz i.e. 600 cycles / minute. Further, the cyclic load was adjusted up to a value of 80 kN and a stress ratio of  $R = 0$  was followed which, is a fully reversed condition. The fully reversed load of 160 MPa and 460 MPa were used to evaluate the fatigue behavior of the EDM samples at load below and close to yield point.



Fig. 1. Die sink EDM machine

### 3. Results and discussion

#### 3.1. Surface analysis and fracture study

Figure 2 indicates the SEM images of the specimens fabricated with peak currents of 6 A, 12 A and 20 A respectively. It has been noted that the surface roughness has increased with an increase in the discharge current for all the three  $T_{on}$  conditions namely, 200, 400 and 810  $\mu$ s. This phenomenon is inferred that when  $T_{on}$  time is increased the material removal takes longer time to erode of which leaves a huge defects in the surface which is reflecting on the increase in the surface roughness. The energy level in the machining process will increase for an increase in the peak discharge current. This phenomenon will cause the rapid melting followed by vaporizing of the specimen which results in larger and deeper crater as presented in Figure 2. Therefore, it is indicative that the surface roughness also increases for an increase in the peak discharge current. The SEM images of the fatigue tested specimens fabricated with 6 A, 12 A and 20 A discharge current are presented in Figure 3. The specimen processed with 6 A discharge current contains less number of spots for crack initiation compared to the specimens processed with 12 A and 20 A current. This is attributed to the fact that the specimen processed with 6 A discharge current has a lesser surface roughness value which leads to meager chance for crack propagation. Whereas, many crack initiation spots were recorded on the surface of the specimens that are fabricated with the discharge current of 12 A and 20 A. Figure 4 presents the optical microscopic image from which, different shapes of recast layer with varying depth / craters can be observed. This condition is attributed to higher surface roughness due to higher discharge current. The main theory of failure observed on the specimens fabricated through EDM process is Orowan's theory of fatigue failure where the work piece is observed to have small, weak regions which served as areas favorable for slips to occur and areas for high tensile stress concentration leading to fatigue life reductions. Severe surface pitting and small weak regions are observed on the surfaces of the specimens fabricated

through EDM process which, were not seen on the surfaces of the milled specimen. These defective areas acts as the potential crack initiation regions that shortened the fatigue life of EDM specimens. Under normal service condition, the components are exposed to continuous load during which, the micro level surface defects propagate into a macro level crack. Thus formed cracks results in a stress concentration which gives room for a new localized plastic region where the process gets repeated resulting in further growth in crack and ultimately resulting in

brittle failure. The samples are cut into 3 mm pieces to analyze the grain boundaries at the heat affected zones after the EDM process. It is observed that heat affected zones are seen in all the samples produced during 9 experimentations. Some induced stress is evident from the microstructure of the samples. From Figure 4 it is observed that dense grain boundaries are present near the heat affected zone which confirms the localized melting and rapid solidification took place during the material removal by the EDM process on the samples. From the microstructure

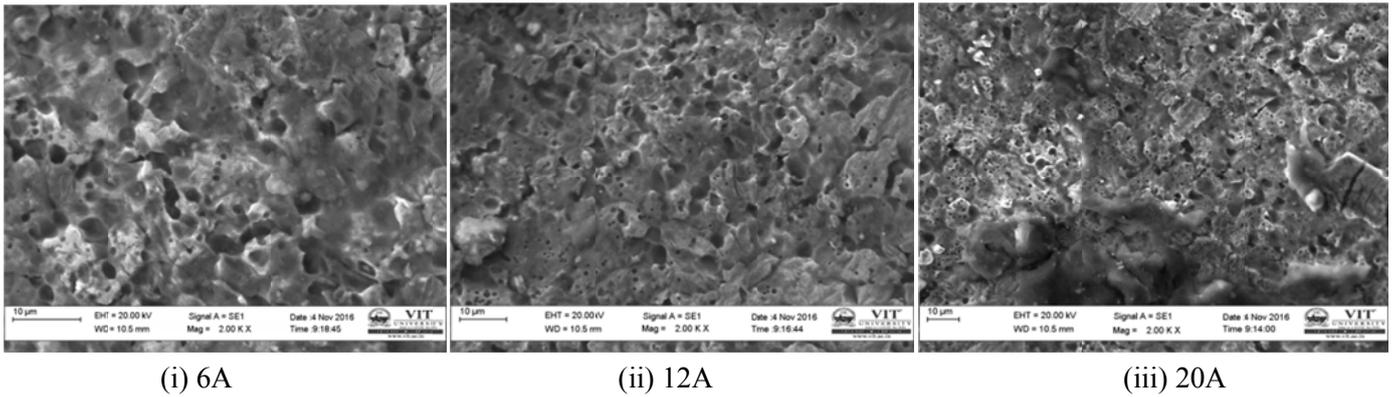


Fig. 2. SEM images of the specimens after EDM process

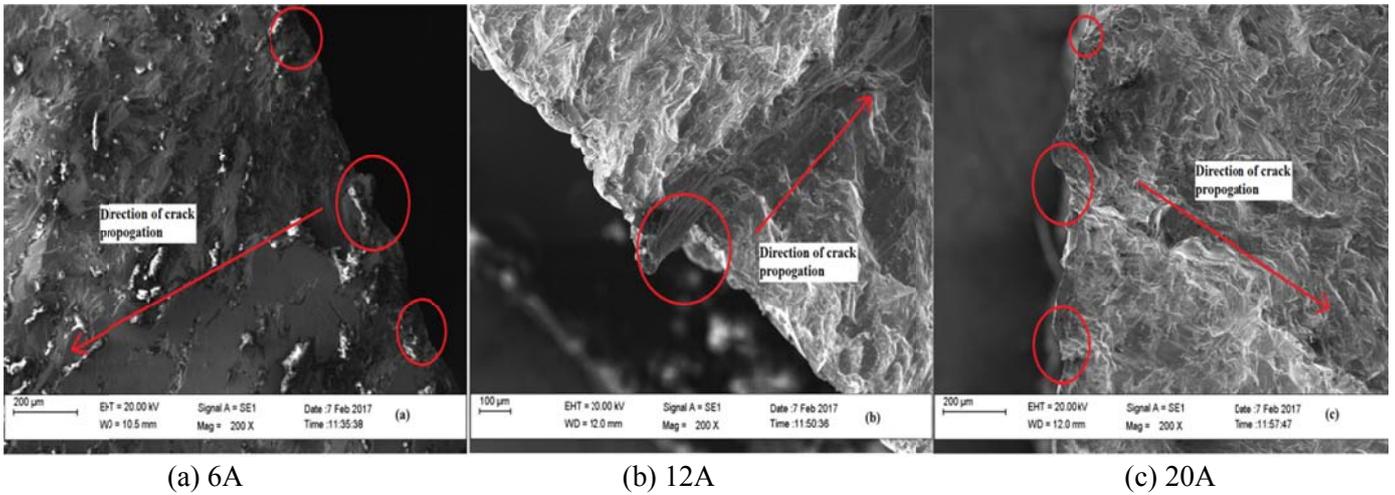


Fig. 3. Fractographs after fatigue test

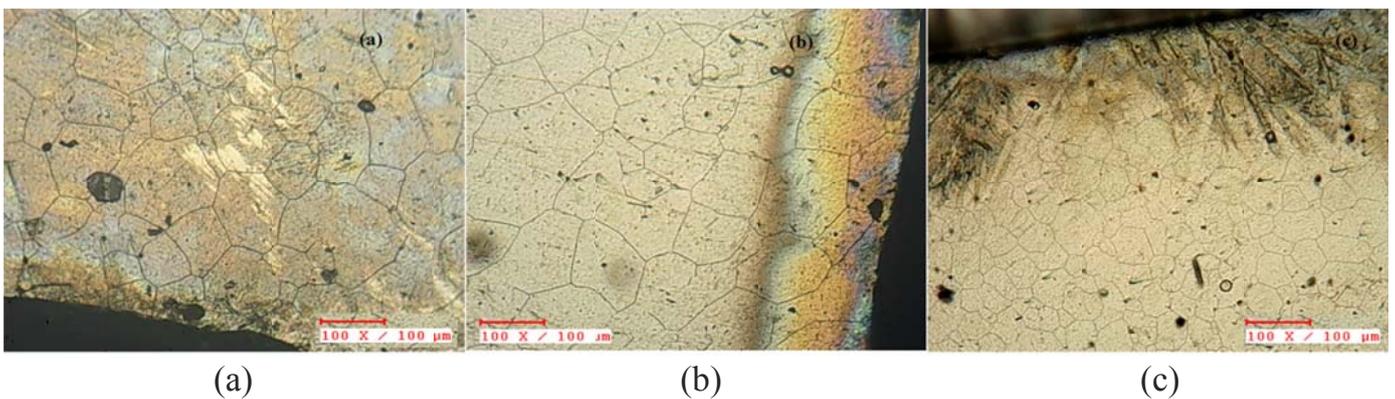


Fig. 4. Microstructure of heat affected zone of the specimens after machining with (a) 6 A (b) 12 A and (c) 20 A

it is understood that the grain boundaries are large in size at the center of the samples and clear as shown in Figure 4. The relationship between microstructure and mechanical properties can be expressed in terms of the various aspects. The resistance to propagation of small surface cracks (micro cracks) is important for the low cycle fatigue properties. The most important microstructural parameter determining the mechanical properties for  $\alpha + \beta$  titanium alloys is the  $\alpha$  colony size. With decreasing the  $\alpha$  colony size, the ductility, yield strength, crack propagation resistance are improved [23].

#### 4. Residual stress analysis

Residual stresses (both tensile and compressive) are induced in all the conventional and unconventional metal cutting processes which have an influence on the fatigue life of the engineering components. In order to correlate the fatigue life with the induced stresses, it is needed to measure the values on the specimens. Stress measurements were performed by adopting X-ray diffraction method. Induced stresses were measured on EDM fabricated specimen and end-milled specimen across the thickness from the surface to the sub-surface depth of 300 microns both in longitudinal and transverse directions.

A maximum compressive stress of 273 MPa was measured on the surface of the milled specimen in the longitudinal direction of the cutting tool axis. A transformation from compressive to tensile stress has occurred as the measurement was recorded deeper into the sub-surface level and finally the stress value reached zero at a depth of 100 microns. It was also observed that a tensile residual stress of 125 MPa was recorded at a depth of 35 microns while measuring in transverse direction which indicates that the milling process generates higher stresses in the direction of the cutting tool movement. Contrary to the quantum of stresses induced in different directions in milled specimen, EDM processed specimens have induced similar stresses in both transverse and longitudinal directions. This phenomenon attribute to the fact that EDM processes specimens exhibit non-directional behavior. A maximum tensile residual stress of 724 MPa was induced in both directions of the specimen fabricated through EDM process. It was also observed that there is a smooth gradient in the tensile stress values which was concentrated

within a narrow layer of 150 microns deep from the top surface and further reduced to zero stress at a deeper sub-surface. The tensile residual stress induced by the EDM process on the surface of the specimen results in surface defects which propagates and promotes the fatigue crack growth rate. The tensile residual stress on the material surface tries to further open the surface defects such as voids and surface cracks formed after EDM during tension cycle in fatigue loading. Fatigue crack mostly initiate at the defects on the surface. Having tensile residual stress fastens the crack propagation and reduces the fatigue life. EDM process results in a plastically deformed layer of certain thickness because of rapid cooling and localized melting of the material. Cracks don't initiate in a plastic environment. So crack initiation point is driven further below the surface where stresses are lower compared to tensile residual stress at the surface induced by EDM. Tensile residual stress further amplifies the quantum of stress on the surface of the component due to which cracks instigate and propagate during fatigue loading, resulting in the reduction in the fatigue life of the EDM processed Ti6Al4V.

Figure 5 shows the residual stress values for samples (Fig. 5(a)) EDM sample at 6 A, (Fig. 5(b)) EDM samples at 12 A and (Fig. 5(c)) EDM samples at 20 A. As mentioned earlier, the recast layer induced is the major source for crack initiation, which reduces the fatigue life of the component. Even though the recast layer is at micron scale, the tensile residual stresses induced on the surface and sub-surface regions will influence the fatigue life of the samples. The induced residual stress is also evident from the figure 4 where the closer and dense grain boundaries are observed at the heat affected zone of the EDM samples.

#### 5. Fatigue analysis

S-N curve generated using the sample machined through end – milling process and the samples fabricated by EDM process using the nine experimental conditions is presented in Figure 6. Five data points for each of the stress levels were used to plot the S-N curve. A range of 550 to 900 MPa stress level was selected to evaluate the EDM specimens. The S-N curve indicates that the fatigue life of the specimens fabricated through EDM decreases when compared to the milled specimen. The specimens fabricated through EDM process resulted in a highest fatigue life of

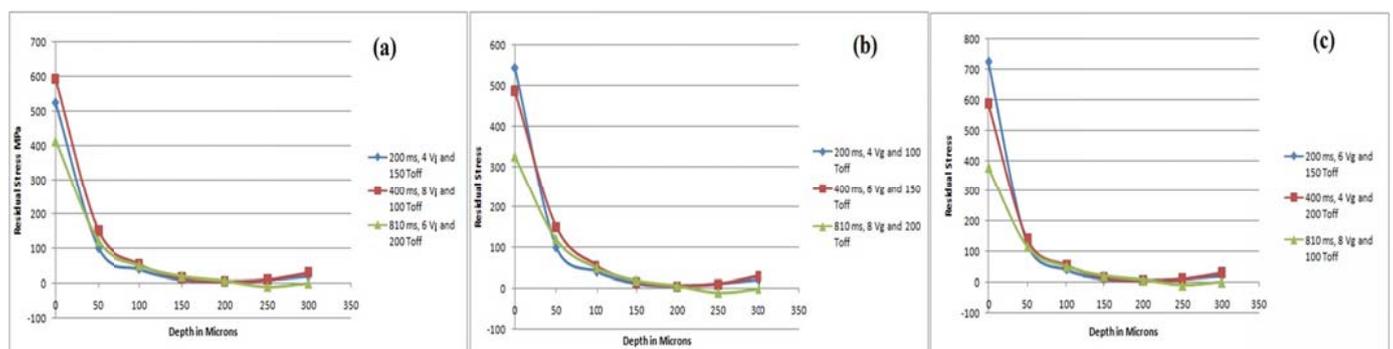


Fig. 5. Residual Stress measurement after EDM in samples (a) 6 A (b) 12 A and (c) 20 A

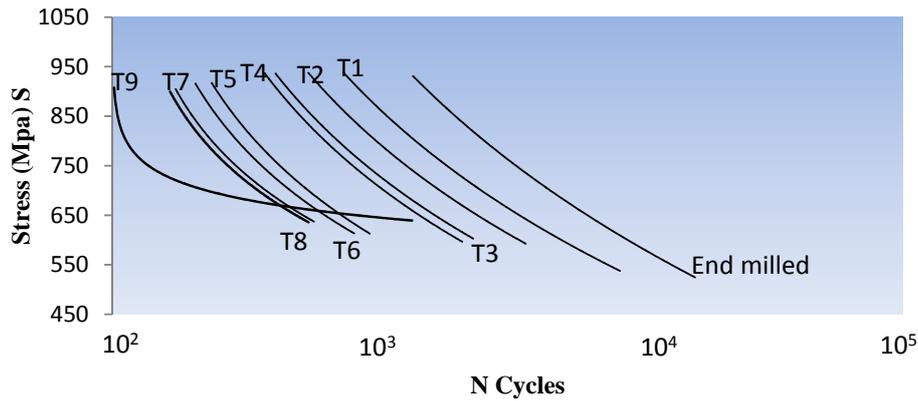


Fig. 6. S-N curve of EDM samples compared with end milled

$1.76 \times 10^5$  fatigue cycles for a minimum load of 160 MPa (8 KN) and  $5.02 \times 10^4$  fatigue cycles for a load of 500 MPa (23 KN) which is closer to yield point. Likewise, the milled specimen exhibited  $2.71 \times 10^5$  fatigue cycles for a load of 160 MPa (8KN) which is 64% higher when compared to EDM specimens. At load of 500 MPa (23 KN) the milled sample exhibited  $9.07 \times 10^4$  fatigue cycles which is 54% more than EDM specimens. Fatigue life of similar trend was recorded for all values of peak discharge current under various combinations of  $T_{on}$ , gap voltage and  $T_{off}$ .

Material Removal Rate (MRR) and the surface roughness showed an increasing trend when the discharge current was increased. It has also been noted that MRR showed a decreasing trend in proportion to the discharge current and voltage, whereas the roughness increases for an increase in pulse duration ( $T_{on}$ ). Quality of surface roughness is also driven by pulse off duration that is attributed to the unstable nature of the spark due to shorter pulse off-time which is concurrent with the findings reported by Klocke et al., [3].

The analysis on the experimental data indicates that surface roughness increases for an increase in pulse duration for any constant peak current value. From Table 3, it can be seen that by keeping the same machining conditions, there is a decrease in tensile strength for an increases in peak discharge current. It is to be noted that for an increase in peak discharge current, there is an increase in surface roughness which indirectly re-

duces the tensile strength. It is also inferred that the hardness of the specimens shows a decreasing trend as the peak discharge current increases which is attributed to the formation of recast layer over the surface of the components. The de-ionized water which is used as the dielectric fluid for the EDM process undergo an electrolysis resulting in oxidation and thereby reducing the surface hardness.

Figs 7-9 indicate the recast layer formed on the specimens machined at 6 A, 12 A and 20 A respectively. As it was mentioned in Table 3, the surface finish is reasonably good with shallow and flattened crater on the surface for a lesser peak discharge current. When compared to the milled specimen, the EDM specimens have lesser hardness and tensile strength values which is attributed to the generation of recast layer. The thickness of recast layer is increased by 35% from sample to sample for an increase in peak discharge current as shown in table 3 and Figs. 7-9.

## 6. Conclusion

In general, the electric discharge machining process results in the formation of recast layer on the surface of the machined specimen. This layer has a detrimental effect on the surface integrity and also on the fatigue life of the component. Current research was focused on determining the influence of recast

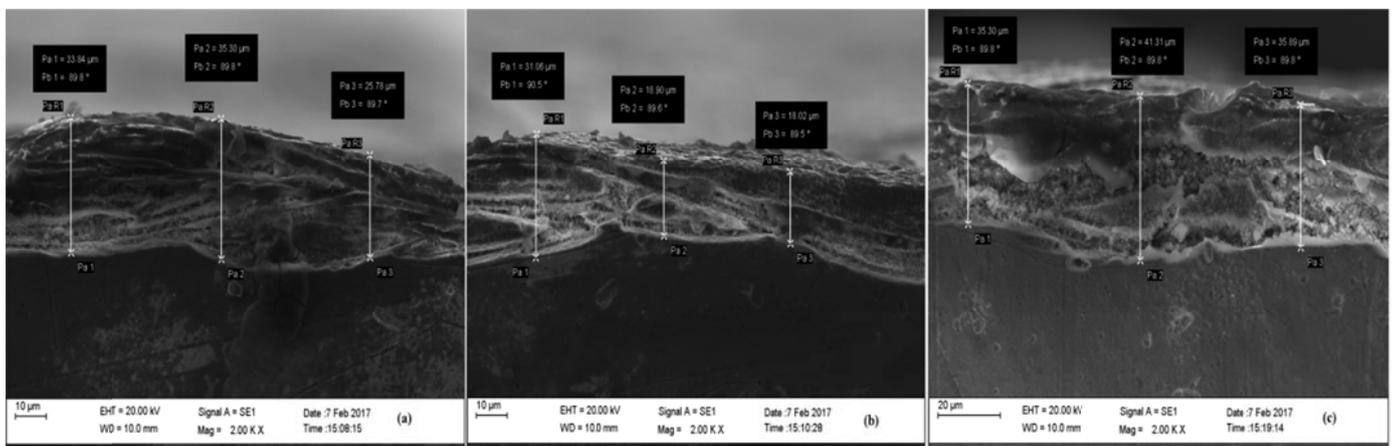


Fig. 7. Recast layer format ion on 6A sample of (a) 200  $\mu$ s (b) 400  $\mu$ s (c) 810  $\mu$ s variat ion

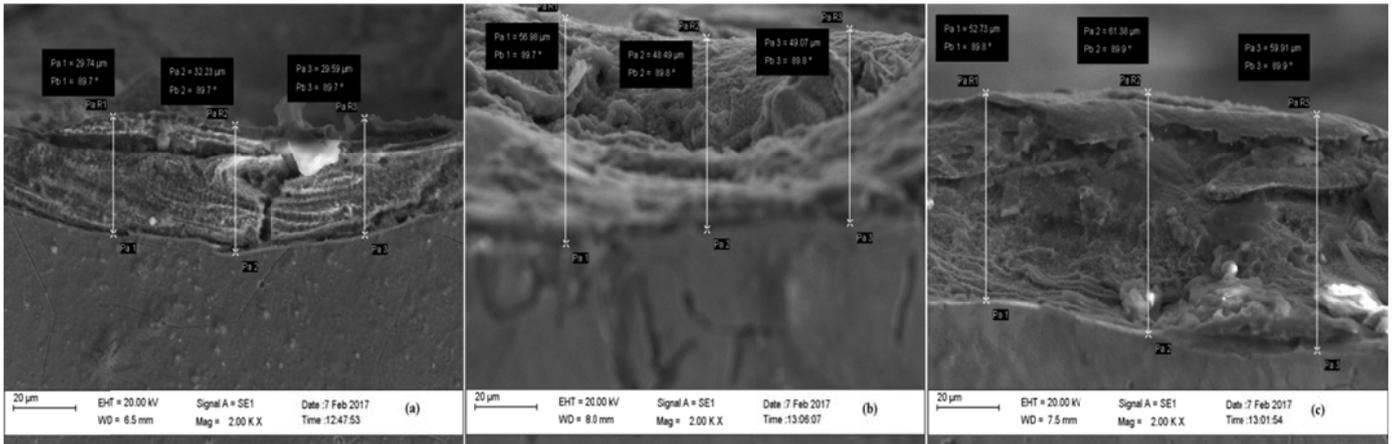


Fig. 8. Recast layer format ion on 12A sample of (a) 200  $\mu\text{m}$  (b) 400  $\mu\text{m}$  (c) 810  $\mu\text{m}$  variat ion

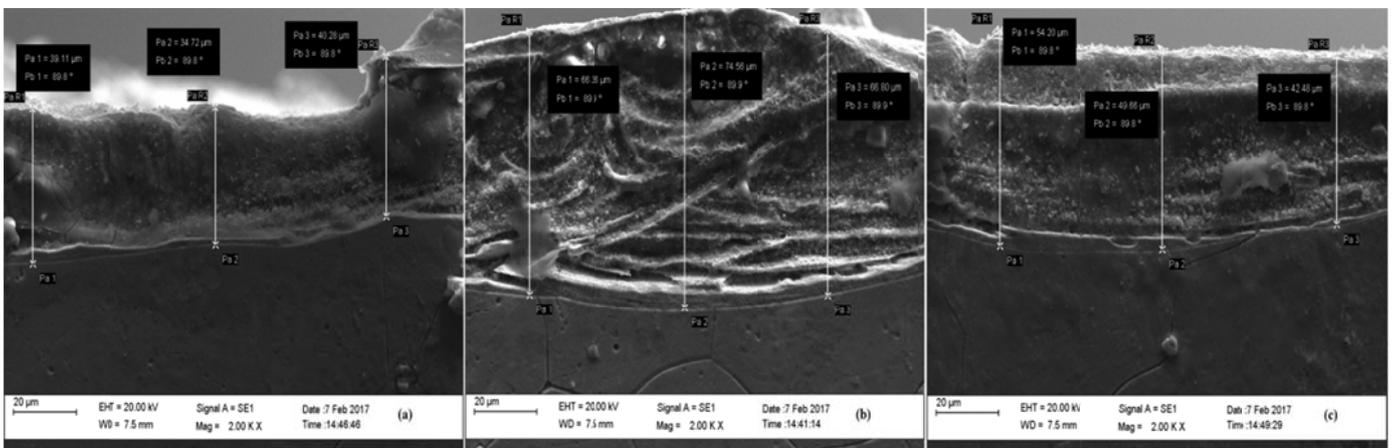


Fig. 9. Recast layer format ion on 20A sample of (a) 200  $\mu\text{m}$  (b) 400  $\mu\text{m}$  (c) 810  $\mu\text{m}$  variat ion

layer on surface integrity and fatigue life of Ti6Al4V specimens machined by EDM process. Experimental data reveals that the average thickness of recast layer generated was in the range of 20 to 70  $\mu\text{m}$ . The analysis performed on the observed data indicates that a significant variation in fatigue and mechanical properties of Ti6Al4V is inevitable as a result of the recast layer formed due to the EDM process. The recast layer formed on the surfaces acts as a potential crack initiation zones which results in the reduction of tensile strength. In order to understand the impact of the recast layer on the fatigue life of EDM specimen, similar tests were conducted on a specimen fabricated by end milling process. It is being concluded that the milled specimen has 64% better fatigue life when compared to the specimen fabricated by EDM process

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