BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 67, No. 3, 2019
DOI: 10.24425/bpasts.2019.129660

Genetic minimisation of peak-to-peak level of a complex multi-tone signal

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Abstract. This paper presents results of evolutionary minimisation of peak-to-peak value of a multi-tone signal. The signal is the sum of multiple tones (channels) with constant amplitudes and frequencies combined with variable phases. An exemplary application is emergency broadcasting using widely used analogue broadcasting techniques: citizens band (CB) or VHF FM commercial broadcasting. The work presented illustrates a relatively simple problem, which, however, is characterised by large combinatorial complexity, so direct (exhaustive) search becomes completely impractical. The process of minimisation is based on genetic algorithm (GA), which proves its usability for given problem. The final result is a significant reduction of peak-to-peak level of given multi-tone signal, demonstrated by three real-life examples.

Key words: evolutionary computation, genetic algorithms, multi-tone signal, optimisation, peak-to-peak minimisation.

1. Introduction

A possible way of traffic emergency broadcasting is related with so called temporal emergency awareness. The examples can be privileged vehicles approach (police, fire brigade, ambulance) or local traffic hindering (e.g. road works). Specificity and weight of the stated problem lie in its temporariness, both in time and space. In most cases, situations requiring driver's increased attention last from a few to several seconds. This is caused by both self-movement (driving) and/or remote movement of an object requiring attention. Therefore, multiplicity of available user channels can make such emergency broadcasting ineffective, because one cannot be sure which channel (station) a driver's receiver is currently tuned to. Short range, low power broadcasting using all available channels may overcome this limitation. In such case, however, the problem of signal summation from particular channels transmitted simultaneously arises. This problem is especially visible in final stages of radio (RF) transmitters (e.g. linear output amplifier). Signal linearity is required both by citizens' band (CB, AM/FM) and VHF FM (analogue frequency modulation) final signal processing blocks. The main reason is avoidance of intermodulation distortions – thus out-of-band spurious emission. Additionally, the CB AM (amplitude modulation) requires linear signal processing.

2. Problem description

Let us assume analogue signal $s(t)$ being sum of selected number N of tones. Each tone has constant amplitude, particular constant channel frequency and particular variable phase. Amplitudes of all tones are equal:

$$s(t) = \sum_{k=1}^{N} A_k \sin(2\pi f_k t + \varphi_k) \tag{1}$$

where:
A_k – amplitude of k-th tone,
f_k – frequency of k-th tone,
φ_k – phase of k-th tone.

Therefore, there can be expected moments of time, when all tones add in-phase, therefore extremal instantaneous level of signal $s(t)$ can reach $\pm N \cdot A_k$.

It has been assumed, that multi-tone signal $s(t)$ is periodic and, for given period (observation window), peak-to-peak value pp of signal $s(t)$ can be defined as:

$$pp = \max s(t) - \min s(t). \tag{2}$$

Now, a question can arise: for what phase shifts φ_i of particular tones $(i = 1, 2, 3, \ldots, N)$ peak-to-peak value pp of the signal $s(t)$ is minimal?

3. Optimisation process

In order to solve stated optimisation problem, a genetic algorithm (GA) has been used. The GA has proven to be robust and acceptably quick for problems of a multi-parameter optimization [1–19]. However, as all of heuristic methods (except artificial annealing), there is still no formal proof of its global convergence – thus no guarantee that the solution found is definitely an optimal one [1].

There has been used classic elitist GA with schema shown in Fig. 1 [1–3].

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Manuscript submitted 2018-07-11, revised 2018-09-06, 2018-11-26 and 2018-11-29, initially accepted for publication 2018-12-14, published in June 2019.

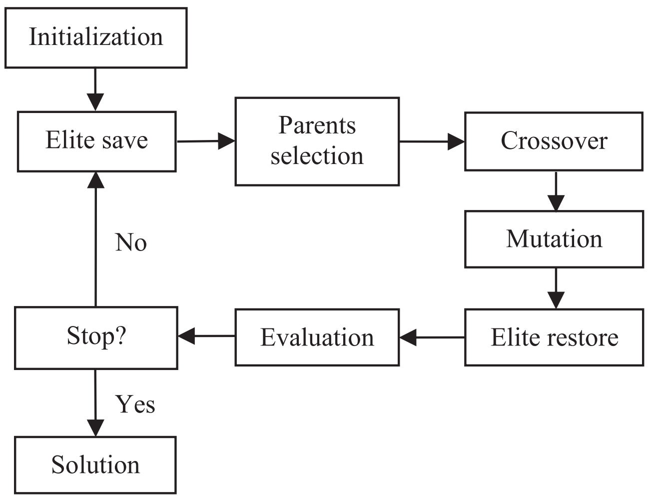

Fig. 1. Schema of the used genetic algorithm (an elitist model) [1–3]

Population size is constant. Each individual contains single linear chromosome, being a vector of genes (bits), and coding particular solution. Length of the chromosome is a trade-off between solution accuracy (bit resolution) and processing time of GA (search space size).

The evolution loop contains following steps (Fig. 1):

- initialization – usually random,
- unique elite individuals are saved. This prevents the best found so far solutions from being destroyed in successive genetic operations (e.g. crossover and mutation),
- parents selection is based on binary tournament. Two individuals are selected randomly (with return) from main population. The one with better (lower) value of a fitness is copied to population of parents,
- reproduction – single-point crossover is applied to randomly selected pairs of parents and offspring population is created,
- succession is trivial – population of offspring fully replaces old main population,
- mutation – each gene is negated with given probability,
- elite is restored into main population (randomly chosen individuals are replaced in order to keep constant population size). Random overwrite does not introduce evolutionary pressure,
- new population is evaluated: fitness of individual is calculated,
- stop criterion – GA stops and returns found solution after specified number of iterations.

4. Demonstration with exhaustive search

The first example is demonstration of the problem and its solution for the simplified case. A four channels from citizens' band (CB), near 27 MHz, are selected and their frequencies are given in the Table 1. Please note, that channel 3*a* is not used in the analysis, because it is reserved for other purposes and is not allowed to be used in the CB band for voice communication.

Table 1
Frequency list of elected signals

Symbol	Channel No.	Frequency [MHz]
f_1	1	26.960
f_2	2	26.970
f_3	3	26.980
	3*a*	26.990
f_4	4	27.000

In this demonstration, there have been assumed:

- phase shift of the first tone (channel 1) is a reference one and always equal 0°,
- phase shifts of the other tones (channels 2, 3 and 4) can take one of four values: –90°, 0°, +90° or 180° (with respect to the channel 1),
- all tones have constant amplitude.

Therefore, there are needed 2 bits to binary encode each phase shift in channels 2–4, so total chromosome length is 6 genes. The coding uses Gray Binary Coding (or Reflected Binary Code). Such coding is preferred over Natural Binary Coding (NBC) in genetic algorithms, because of elimination of Hamming cliffs [1]. The "cliffs" appear when distance (in selected space) between coded (genotype) and decoded (phenotype) values, is non-monotonically related to distance (e.g. number of different genes/bits) between particular chromosomes. One of the effects of NBC coding can be increased non-linear complexity of a genotype search space, which is not desired.

During chromosome decoding, particular phase shifts are calculated according to following equation:

$$\begin{gathered}\varphi_k[^\circ] = Gray\left(b_{1,i}, b_{0,i}\right) \cdot 90 - 90 \\ k = 2, 3, 4 \qquad i = k - 1\end{gathered} \tag{3}$$

where:

φ_k – phase of the *k*-th tone (in degrees) with respect to the tone 1,

$b_{1,i}, b_{0,i}$ – *i*-th pair of bits, coding phase shift for the *k*-th tone. $b_{1,i}$ is the most significant bit (MSb), $b_{0,i}$ is the least significant bit (LSb).

Structure of the chromosome is shown in Fig. 2 and coding details are summarised in the Table 2.

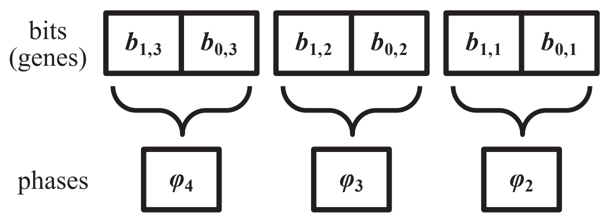

Fig. 2. Chromosome coding 3 phase shifts (for channels 2, 3 and 4) by means of 6 bits (genes)

Table 2
Phase Shift Coding

Code [b_1, b_0]	Phase shift [°]	Code [b_1, b_0]	Phase shift [°]
00	–90	11	90
01	0	10	180

In the following demonstration, the analysed multi-tone signal $s(t)$ contains four components:

$$s(t) = \sum_{k=1}^{4} \sin(2\pi f_k t + \varphi_k)$$
$$\varphi_1 = 0 \wedge \varphi_j \in \{-90, 0, 90, 180\} \tag{4}$$
$$j = 2, 3, 4$$

where:
f_k – frequency of k-th tone (Table 1),
φ_k – phase of k-th tone.
Finally, the signal $s(t)$ is discretized into $s(n)$:

$$s(n) = \sum_{k=1}^{4} \sin\left(2\pi f_k \frac{n}{f_s} + \varphi_k\right) \tag{5}$$

where:
f_k and φ_k – as in (4),
f_s – sampling frequency, equal $27 \cdot 30 = 810$ MHz,
n – samples of $s(n)$.

Such selection of f_s gives approx. 30 samples per period (of a single tone), resulting in acceptably smooth envelope of $s(n)$. The smallest frequency difference between particular components (tones) equals 10 kHz, therefore 1/(10 kHz) = 1/(0.01 MHz) = (100 samples)/(1 MHz) = 100 µs – and it is the repetition period T_r (observation window) of the multi-tone signal $s(n)$.

Initial solution is trivial – all phases are equal: $\varphi_k = 0°$, $k = 1, 2, 3, 4$. Envelope of such signal is presented in Fig. 3 and its peak-to-peak value equals 7.99. This is also the expected worst case, because maximal and minimal instantaneous values of signal $s(n)$ are in-phase and are added, e.g. at time $t = 0$.

In order to compare very close solutions, in this and all following examples, peak-to-peak value of signal $s(n)$ is always rounded to 3 the most significant digits. Resulting precision of 0.1% is sufficient and acceptable for stated problem.

Now, let us examine an "intuitive solution". The tones have very close frequency, so mutual cancellation is expected for pairs with opposite phases, e.g. $\varphi_1 = 0°$, $\varphi_2 = 180°$, $\varphi_3 = -90°$, $\varphi_4 = +90°$. However, peak-to-peak value for such case is still relatively high: 7.03 (Fig. 4). Surprisingly, the greatest reduction ($pp = 5.52$) can be observed for combination: $\varphi_1 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 180°$, $\varphi_4 = 180°$, when tones not in neighbourhood (with closest frequency) have opposite phases (Fig. 5). Other possible combinations and corresponding peak-to-peak values are presented in the Table 3.

Table 3
"Intuitive" solutions

φ_1 [°]	φ_2 [°]	φ_3 [°]	φ_4 [°]	Peak-to-peak value
0	180	0	180	6.41
0	180	180	0	6.44
0	**0**	**180**	**180**	**5.52**
0	180	–90	+90	7.03
0	180	+90	–90	
0	–90	180	+90	7.58
0	–90	+90	180	
0	+90	180	–90	
0	+90	–90	180	

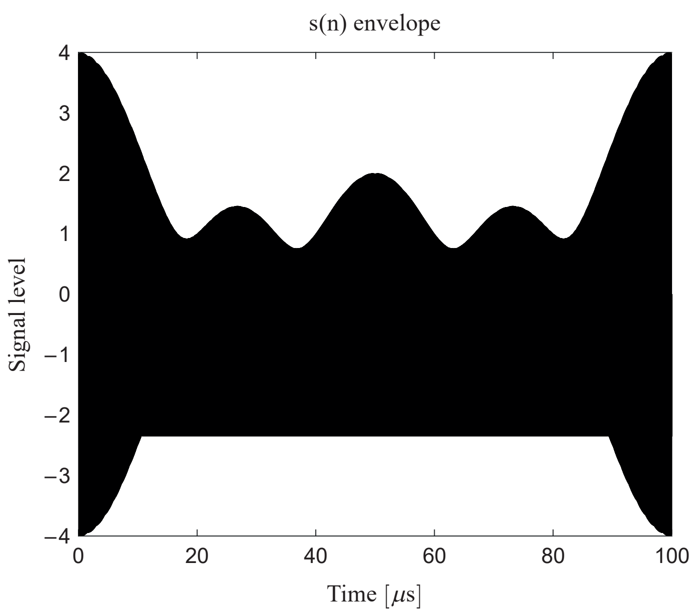

Fig. 3. Signal $s(n)$ envelope for all phases equal 0°

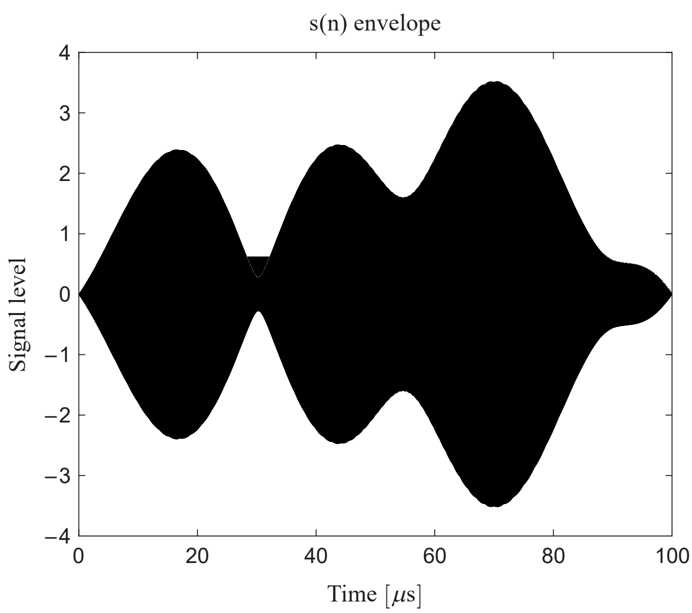

Fig. 4. Signal $s(n)$ envelope for $\varphi_1 = 0°$, $\varphi_2 = 180°$, $\varphi_3 = -90°$, $\varphi_4 = +90°$

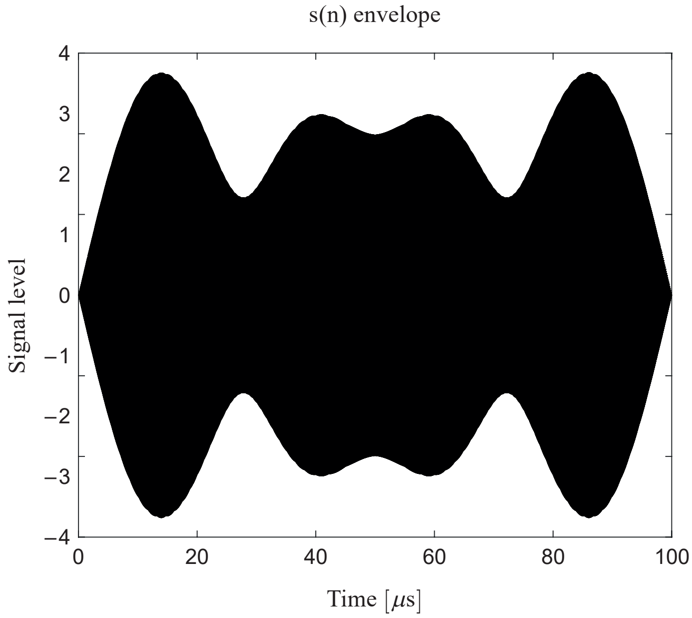

Fig. 5. Signal $s(n)$ envelope for $\varphi_1 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 180°$, $\varphi_4 = 180°$

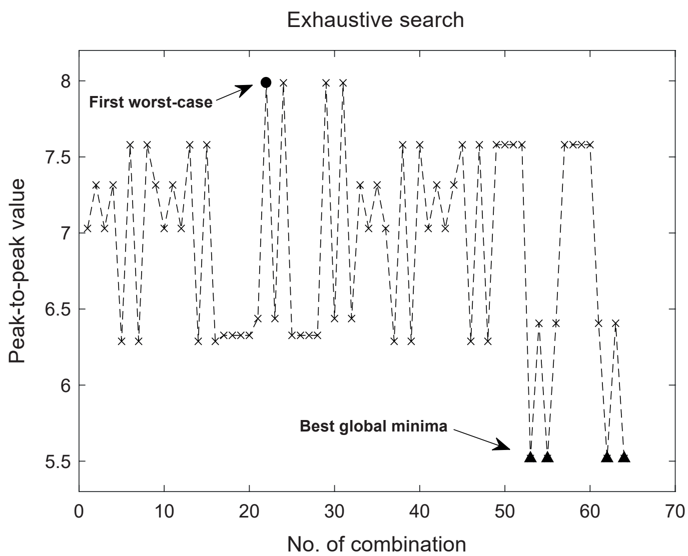

Fig. 6. Result of exhaustive search

Figure 4 shows envelope of the signal $s(n)$ for case of "intuitive solution" when signals should cancel each other (opposite phases): $\varphi_1 = 0°$, $\varphi_2 = 180°$, $\varphi_3 = -90°$, $\varphi_4 = +90°$. Peak-to-peak value for such case is $pp = 7.03$ (Table 3).

In order to definitely find global solutions, an exhaustive search should be performed for all possible combinations of phase shifts. It can be easily calculated that for 3 variable tones and 4 possible phase shifts, total number of combinations is:

$$4^3 = 2^6 = 64. \tag{6}$$

It is obviously the same number of combinations for vector of 6 1-bit (0/1) values.

Figure 6 presents peak-to-peak value of signal $s(n)$ for all 64 combinations of phases 2, 3 and 4. There can be found:
- the (first) worst case for all phases equal 0° (marked with circle),
- another three worst cases (peak-to-peak values approx. equal 8),
- four global minima, marked with triangles.

Table 4 presents phase values for all global minima. It can be noted in the Table 3 (3rd row), that one of the "intuitive" solutions is also one of the global solutions (Table 4, 2nd row).

There can be observed significant reduction of peak-to-peak value ($pp = 5.52$, Table 4) comparing to case of all phases equal ($pp = 7.99$) – nearly by 30%. In radio applications, it allows to increase dynamics of transmitted signal. Either supply voltage can be reduced by 30%, while maintaining unchanged output power or output power can be increased by 70%, for unchanged supply voltage.

The last step of demonstration is the presentation of the genetic algorithm, successfully finding the global minima returned by the exhaustive search. Parameters of the GA have been the following:

Table 4
The global minima

Peak-to-peak value	φ_1 [°]	φ_2 [°]	φ_3 [°]	φ_4 [°]
5.52	0	–90	0	180
	0	**0**	**180**	**180**
	0	90	0	180
	0	180	180	180

- initial population: uniformly random,
- selection operator: rank (binary tournament with return),
- crossover: single-point,
- succession: complete,
- mutation: uniform (mutation probability for single gene: 0.01),
- elite saving: yes (one or more individuals with best fitness and unique genotype),
- population size: 100 individuals (constant),
- size of parents and offspring population: equal to size of the main population,
- stop criterion: maximal number of iterations (generations) equal 50.

Fitness of each individual (coding particular solution) has been calculated in following steps:
- decode particular phase shifts (φ_2, φ_3 and φ_4) from their binary representation in Gray code (3).
- simulate discrete signal $s(n)$ in time window 0 ÷ 100 μs (5).
- find maximal peak-to-peak value in the abovementioned time window.
- round this value to 3 most significant digits.

Figure 7 presents progress of exemplary GA run. Average fitness of population decreases, which is desirable behaviour. Individuals coding bad solutions (high peak-to-peak ≡ low fitness) "die out" during evolution progress. Individuals coding good solutions (low peak-to-peak ≡ high fitness) have greater survival probability, thus reproduce and propagate in population. The best (minimal) fitness is monotonic non-increasing function, because of used elitism. Discovery of global solution ($pp = 5.52$) can be observed in 4-th generation. The maximal possible fitness value equals the worst case: all tones are in-phase and add.

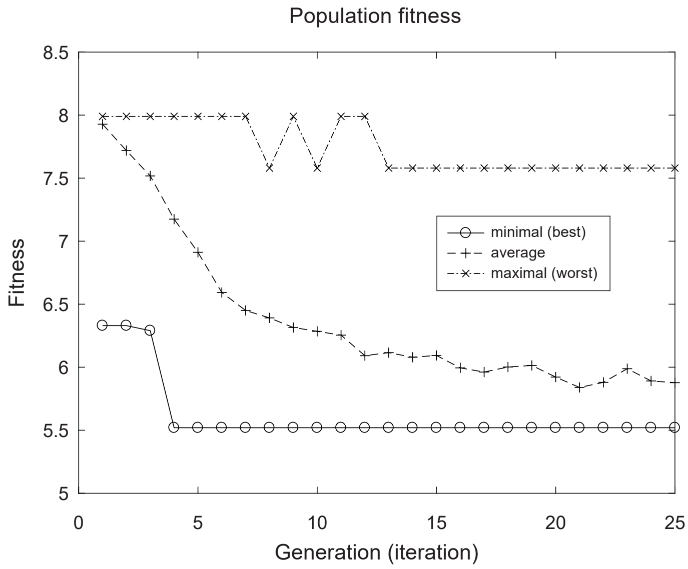

Fig. 7. Fitness progress of exemplary GA evolution

Total 100 GA runs have been used for demonstration. Each run resulted in discovery of at least one global solution: minimal peak-to-peak value equal 5.52. However, not for every run all global solutions have been found. Table 5 summarises 100 demonstration runs of the GA.

Table 5
Summary of 100 GA runs

No. of found global solutions	1	2	3	4 (all)
No. of cases	7	59	16	18
Min. iterations	4	3	4	4
Avg. iterations	7.3	8.2	9.6	8.0
Max. iterations	13	19	22	16

It can be observed that average just a few GA iterations have been necessary to find at least one global solution. In most cases up-to several iterations are needed and this relation is weakly dependent on number of found global solutions. Of course, the demonstration is example with very low complexity. Search space contains only 64 points, thus it is faster to perform exhaustive search for this case. However, complexity of next, real-life examples, grows exponentially and exhaustive search becomes impossible.

5. Example 1 – 40 CB Channels

The first real-life example covers simultaneous emergency transmission on all 40 channels of Citizens' Band (CB). The band is available in most European countries and has moderate popularity in many parts of the world, especially among truck drivers.

The signal $s(n)$ is calculated according to (5). This example uses East European frequency allocation: 26.960–27.400 MHz with 10 kHz step. Signal $s(n)$ contains 40 tones (channel frequencies), except five so called "α-channels" with frequencies [MHz]: 26.990 (channel 3*a*), 27.040 (7*a*), 27.090 (11*a*), 27.140 (15*a*) and 27.190 (19*a*). The α-channels are not permitted for voice operation, therefore have not been used in optimisation.

Sampling frequency $f_s = 810$ MHz (30 samples per period) and repetition period (observation window) equals 100 μs.

Parameters of the GA have been following:

- population size: 100 individuals,
- mutation probability for single gene: 0.01,
- stop criterion: maximal number of iterations (generations) equal 1000.

Other GA parameters and fitness calculation have been the same as in the demonstration above. Phase shift of the first tone (channel 1) equals 0° (reference). There have been analysed 3 cases for phase shifts of the other 39 tones. The GA has been run 3 times for each case.

5.1. Case 1: $\Delta\varphi = \pm 90°$. In Case 1, phase shifts of remaining 39 tones are assumed to take four possible values: –90°, 0°, +90° or 180° (with respect to the channel 1). Therefore, chromosome (problem) coding is the same as in the demonstration (Fig. 2, Table 2). However now, 39 phase shifts × 2 bits = 78 bits, thus total number of combinations is far greater:

$$4^{39} = 2^{78} \cong 3 \cdot 10^{23}. \quad (7)$$

5.2. Case 2: $\Delta\varphi = \pm 45°$. In the second case, phase shifts of the remaining 39 tones can take eight possible values: –135°, –90°, –45°, 0°, +45°, +90°, +135° or 180° (with respect to channel 1). Problem coding requires 3 bits/phase, so chromosome length is 39 phase shifts × 3 bits = 117 bits. The coding is shown in Table 6.

Table 6
Phase shift Gray-coding (case 2)

Code [b_2, b_1, b_0]	Phase shift [°]	Code [b_2, b_1, b_0]	Phase shift [°]
000	–135	110	45
001	–90	111	90
011	–45	101	135
010	0	100	180

Size of search space (total number of combinations) equals then:

$$8^{39} = 2^{117} \cong 1{,}7 \cdot 10^{35}. \quad (8)$$

The minimal found value of peak-to-peak and summary for this case are shown in the Table 7.

Table 7
Results for 40 CB channels

Case	1	2	3
Peak-to-peak upper boundary	80		
Peak-to-peak for all phases equal	79.86		
Minimal found peak-to-peak	18.6	17.6	17.9
Relative peak-to-peak decrease	4.3	4.5	4.4
Average single evaluation time [ms]	5		
Time of exhaustive search [years]	$4.8 \cdot 10^{13}$	$2.7 \cdot 10^{25}$	$4.4 \cdot 10^{60}$

5.3. Case 3: $\Delta\varphi = \pm 5.625°$. In the last case, phase shifts of remaining 39 tones can take 64 possible values with 5.625° of phase shift step (with respect to channel 1). Problem coding requires 6 bits/phase, so chromosome length is 39 phase shifts × 6 bits = 234 bits. Size of search space (total number of combinations) equals then:

$$64^{39} = 2^{117} \cong 2{,}8 \cdot 10^{70}. \tag{9}$$

The minimal found value of peak-to-peak and summary for the last case are shown in Table 7.

It can be observed significant (at least 4.3 times) reduction of peak-to-peak values of the multi-tone signal, comparing to the case, where all phases are equal (Fig. 8, $pp = 79.86$). In radio and signal processing applications, it allows compression of signal dynamics by factor of 4.3 ÷ 4.5 or power increase by factor of 18.5 ÷ 20.3 (equivalent to 12.7 ÷ 13.1 dB), while maintaining supply voltage unchanged.

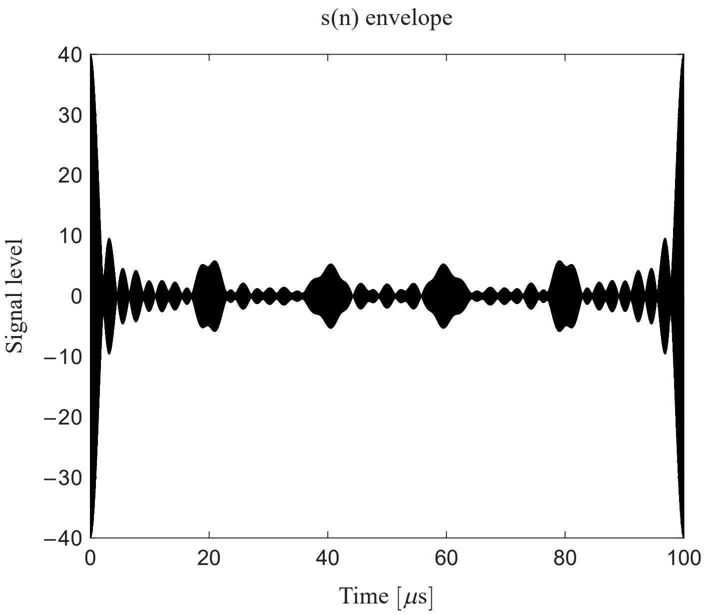

Fig. 8. Signal envelope for all 40 channel phases equal

Figure 9 presents signal envelope for the best found solution (case 2, $pp = 17.6$). Table 8 contains the best found phases for case 2.

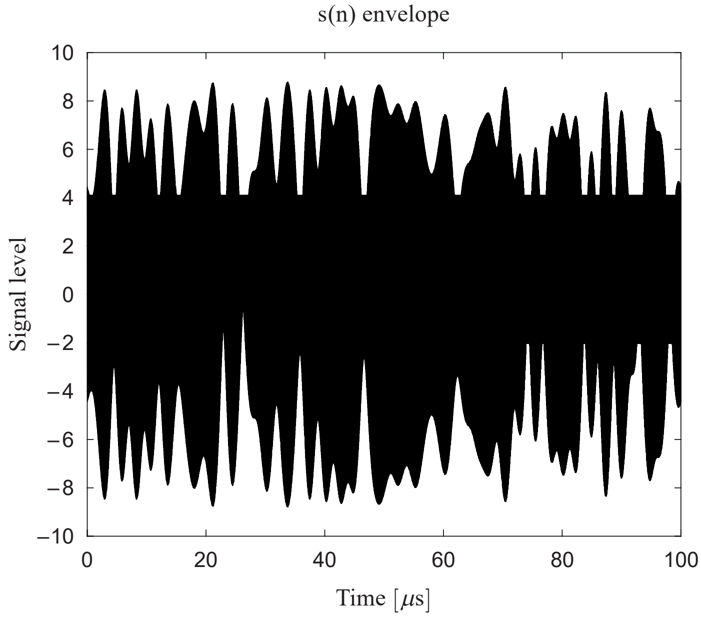

Fig. 9. Signal envelope for the best found solution (example 1, case 2)

Table 8
Found phase shifts [°] for case 2

φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8
0	0	90	−90	−135	−135	180	45
φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}
135	90	45	−90	90	135	45	−45
φ_{17}	φ_{18}	φ_{19}	φ_{20}	φ_{21}	φ_{22}	φ_{23}	φ_{24}
90	180	90	0	−135	45	180	−90
φ_{25}	φ_{26}	φ_{27}	φ_{28}	φ_{29}	φ_{30}	φ_{31}	φ_{32}
0	0	−135	45	0	−90	90	0
φ_{33}	φ_{34}	φ_{35}	φ_{36}	φ_{37}	φ_{38}	φ_{39}	φ_{40}
180	0	180	45	−90	90	180	180

Average computation time for single individual took ca. 50 ms (PC-class computer) and has been dominated by evaluation: approx. 99.8 %. Total operation time for 1000 iterations took ∼5000 s ≡ 1 h 23 m, while time required for exhaustive search is completely unpractical (Table 7). Figure 10 presents progress of the GA.

Solution found in case 3 ($\Delta\varphi \approx \pm 5°$) is surprisingly not better than solution found in case 2 ($\Delta\varphi \approx \pm 45°$). Probably, used GA parameters (e.g. 100 individuals, up to 1000 iterations) seem to be insufficient to more effectively explore such complex problem.

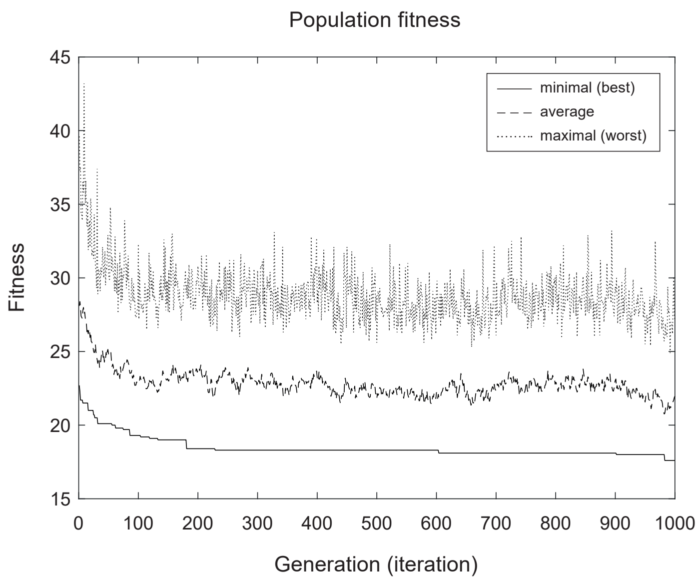

Fig. 10. Fitness progress of GA algorithm (example 1, case 2)

6. Example 2 – 103 VHF FM channels

Another real-life example covers simultaneous emergency transmission using all channels of FM broadcasting band. In most of the world, continuous range 87.5–108 MHz is used with station bandwidth of 200 kHz (CCIR standard), thus resulting in 103 available channels.

The signal $s(n)$ is calculated according to (5). Sampling frequency $f_s = 3$ GHz (30 samples per period) and repetition period (observation window) equals 5 μs.

Parameters of the GA and fitness calculation have been the same as in example 1.

Phase shift of the first tone (channel 1) equals 0° (reference). There have been analysed 3 cases for phase shifts of the other 102 tones. The GA has been run 3 times for each case.

6.1. Case 1: $\Delta\varphi = \pm 90°$. In Case 1, phase shifts of remaining 102 tones can take four possible values: –90°, 0°, +90° or 180° (with respect to the channel 1). Chromosome (problem) coding is the same as in the demonstration (Fig. 2, Table 2). However now, 102 phase shifts × 2 bits = 204 bits, thus total number of combinations is:

$$4^{102} = 2^{204} \cong 2{,}6 \cdot 10^{61}. \tag{10}$$

The minimal found value of peak-to-peak and summary for case 1 are shown in the Table 9.

6.2. Case 2: $\Delta\varphi = \pm 45°$. In the second case, phase shifts of remaining 102 tones can take eight possible values in range [–135°, 180°]. Problem coding requires 3 bits/phase, so chromosome length is 102 phase shifts × 3 bits = 306 bits. The coding is the same as in the example 1 (Table 6). Size of search space equals:

Table 9
Results for 103 VHF FM channels

Case	1	2	3
Peak-to-peak upper boundary	206		
Peak-to-peak for all phases equal	204.5		
Minimal found peak-to-peak	33.1	32.8	33.9
Relative peak-to-peak decrease	6.18	6.23	6.03
Average single evaluation time [ms]	2.7		
Time of exhaustive search [years]	$2.2 \cdot 10^{51}$	$1.1 \cdot 10^{83}$	$2.2 \cdot 10^{170}$

$$8^{102} = 2^{306} \cong 1{,}3 \cdot 10^{92}. \tag{11}$$

The minimal found value of peak-to-peak and summary for this case are shown in the Table 9.

6.3. Case 3: $\Delta\varphi = \pm 5.625°$. In the last case, phase shifts of remaining 102 tones can take 64 possible values with 5.625° of phase shift step (with respect to channel 1). Problem coding requires 6 bits/phase, so chromosome length is 102 phase shifts × 6 bits = 612 bits. The coding is the same as in the example 1 (case 3). Size of search space equals then:

$$64^{102} = 2^{612} \cong 2{,}6 \cdot 10^{180}. \tag{12}$$

The minimal found value of peak-to-peak and summary for the last case are shown in Table 9.

Similarly to the example 1, there can be observed significant (at least 6 times) reduction of peak-to-peak values of the multi-tone signal, comparing to the case, where all phases are equal (Fig. 11, $pp = 204.5$). Resulting signal dynamics compression

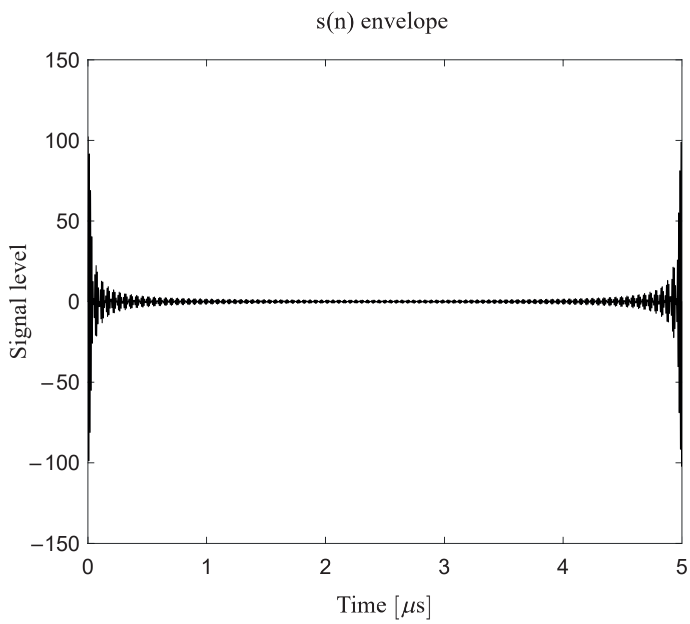

Fig. 11. Signal envelope for all 103 channel phases equal

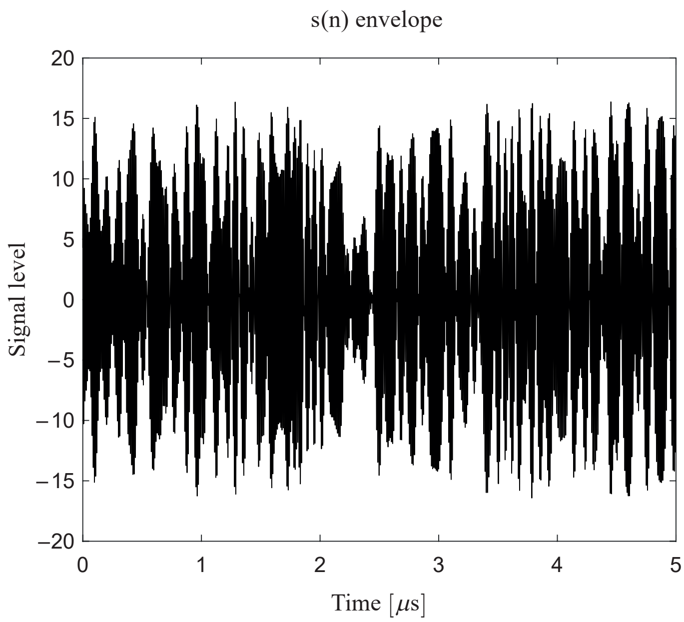

Fig. 12. Signal envelope for the best found solution (example 2, case 2)

is 15.6 ÷ 15.9 dB. Figure 12 presents signal envelope for the best found solution (case 2, $pp = 32.8$).

Average computation time for single individual took ca. 27 ms and has been dominated by evaluation: approx. 99.6 %. Total operation time for 1000 iterations took $\sim$2800 s $\equiv$ 47 m, while time required for exhaustive search is completely unpractical (Table 9). Figure 13 presents progress of the GA.

Similar conclusions can be stated for example 2. Slow GA progress (Fig. 13) and solution from case 2 ($\Delta\varphi \approx \pm 45°$) better than from Case 3 ($\Delta\varphi \approx \pm 5°$) suggests that used GA parameters (e.g. 100 individuals, up to 1000 iterations) seem to be insufficient to effectively explore such complex problem (Case 3).

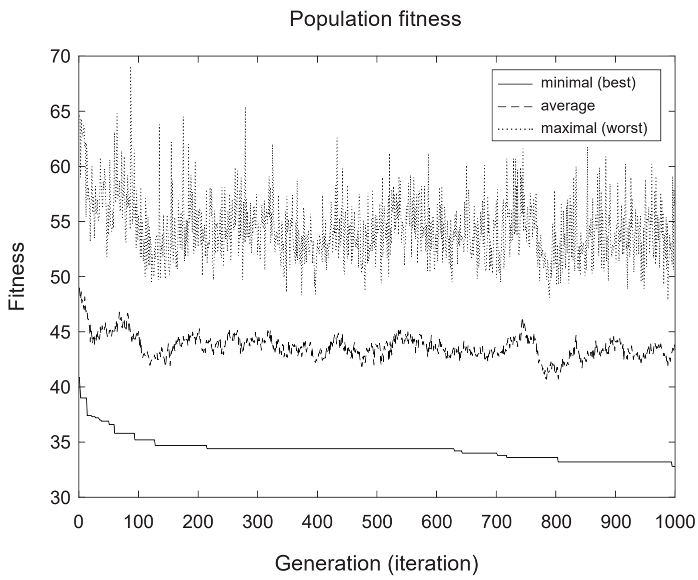

Fig. 13. Fitness progress of GA algorithm (example 2, case 2)

7. Example 3 – 206 VHF FM channels

Previous example can be further extended. In some densely populated areas, congestion of radio stations forced 100 kHz spacing (with reduction to mono quality) and number of available channels doubles to 206. In such case, complexity of the problem (search space size) is multiplied by the factor of 2 in exponent (thus increases quadratically). Even for such complex problems and intentionally unchanged parameters of the GA, peak-to-peak value of multi-tone signal can be significantly reduced (Table 10, Fig. 14).

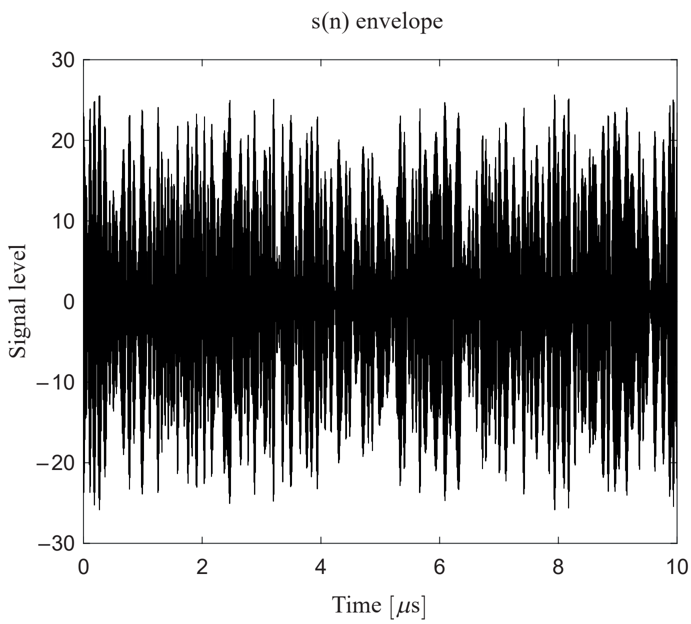

Fig. 14. Signal envelope for the best found solution (Example 3, Case 2)

Table 10
Results for 206 VHF FM channels

Case	1	2	3
Peak-to-peak upper boundary		412	
Peak-to-peak for all phases equal		409	
Minimal found peak-to-peak	51.6	51.5	52.9
Relative peak-to-peak reduction	7.93	7.94	7.73
Average single evaluation time [ms]		10	

Compression of signal dynamics is even higher than for example 2: 17.8 ÷ 18 dB. Figure 14 presents signal envelope for the best found solution (case 2, $pp = 51.5$).

8. Conclusions

Utilisation of genetic algorithm enables effective solution of problems with large combinatorial complexity. Even for the simplest presented example (40 channels with 4 possible phase shifts), exhaustive search is completely impractical: $4.8 \cdot 10^{11}$ *ages*! Even a computer a billion times faster would perform such calculation in unacceptable time. The genetic algorithm has been able to return acceptable solution after time of single hours, for this particular problem. Nevertheless, there is no certainty of optimality i.e. that discovered solution is a global one. This is, however, common disadvantage of stochastic, evolutionary-based optimisation algorithms.

The found combination of particular tone phases, allowed reduction of peak-to-peak level of a multi-tone signal by factors of 4 ÷ 8. This is equivalent to dynamics compression by 12 ÷ 18 dB. Possible effects can be: reduction of supply voltage (without risk of signal distortion) or increase of signal power (e.g. transmitter) without need of supply voltage modification.

Acknowledgements. This work was supported by the Polish Ministry of Science and Higher Education funding for statutory activities.

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