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New approach to designing input-output decoupling controllers for mobile manipulators

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Abstract. Main topic of the paper is a problem of designing the input-output decoupling controllers for nonholonomic mobile manipulators. We propose a selection of output functions in much more general form than in [1,2]. Regularity conditions guaranteeing the existence of the input-output decoupling control law are presented. Theoretical considerations are illustrated with simulations for mobile manipulator consisting of RTR robotic arm mounted atop of a unicycle which moves in 3D-space.

Key words: nonholonomic constraint, mobile manipulator, input-output decoupling controller.

1. Introduction

In this article we address the input-output decoupling problem for mobile manipulators. By a mobile manipulator we mean a rigid robotic manipulator mounted on a wheeled mobile platform, which is often called in the literature 'a wheeled mobile robot'. Such a combined system is able to perform manipulation tasks in a much larger workspace than a fixed-base manipulator, but its analysis introduces new issues that are not present in the analysis of each subsystem considered separately. First, the dynamics of the combined system are much more complicated because they include dynamic interactions between mobile platform and manipulator, which can be substantial. Second, due to complex structure of the mobile manipulator, the constraints which are valid only for one subsystem will also hold for the whole mobile manipulator. In other words, if we assume that the motion of wheeled mobile platform is pure rolling, without slippage of platform wheels, it implies the nonholonomic character of the whole mobile manipulator. The third issue is a problem concerned with the definition of desired trajectory [3]. The desired task in the workspace can be realized by the motion of the mobile platform only, by the motion of the onboard manipulator only or by the coordinated motion of both subsystems. In this paper we focus on the third case.

Although a control problem of mobile manipulators has attracted more attention over last decade, see e.g. [4–6], yet an approach exploiting input-output decoupling controller can be found only in works of few authors: Yamamoto and Yun [1,2] and Mazur [7,8]. Such approach needs full knowledge about the kinematics and the dynamics of nonholonomic mobile manipulator. As output functions which are necessary to define an input-output decoupling controller Yamamoto and Yun have chosen coordinates of the end-effector, if the onboard manipulator stays in constant configuration relative to the mobile platform. It means that the desired task formulated for the whole mobile manipulator can be realized by the motion of the mobile platform only. The goal of this work is an attempt at extending of tasks which can be realized by the mobile manipulator from constant to changing configurations of the onboard manipulator using much general form of output functions. The proposed output functions exploit both maneuvers of the nonholonomic mobile platform and the motion of the onboard manipulator.

The paper is organized as follows. Section 2 is devoted to mathematical description of the mobile manipulator with nonholonomic constraints. In Section 3 necessary conditions for existence of the input-output decoupling control law are presented. Section 4 contains some practical hints dedicated to the design of such controller for mobile manipulators. Section 5 presents computer simulations for a special object, namely an RTR manipulator mounted atop of the mobile platform with restricted mobility belonging to $(2, 0)$ class. Conclusions are formulated in Section 6.

2. Mathematical model of mobile manipulator

We consider the mobile manipulator consisting of two subsystems, namely of holonomic rigid manipulator and nonholonomic wheeled mobile platform. Because the mobile platform has to satisfy nonholonomic constraints, it means that these nonholonomic constraints apply to the whole mobile manipulator. Therefore we have to consider the nonholonomic structure of such combined object.

2.1. Nonholonomic constraints. Let the motion of the wheeled mobile platform be described by n generalized coordinates $q_m \in R^n$ and generalized velocities $\dot{q}_m \in R^n$. The mobile platform should move without slippage of its

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wheels. It means that the relative velocity at the contact point between each wheel and the ground is 0. This assumption implies the existence of l ($l < n$) independent nonholonomic constraints in the so-called Pfaff form which establish the following relationship between generalized coordinates and velocities of the platform [9]

$$A(q_m)\dot{q}_m = 0, \tag{1}$$

where $A(q_m)$ is an $l \times n$ matrix of full rank. From (1) we know that the platform velocity stays in the null space of $A(q_m)$. It is always possible to find a vector of auxiliary velocities $\eta \in R^m$, $m = n - l$, such that

$$\dot{q}_m = G(q_m)\eta, \tag{2}$$

where $G(q_m)$ is a special $n \times m$ full rank matrix satisfying the following relationship

$$A(q_m)G(q_m) = 0. \tag{3}$$

We call the Equation (2) the kinematics of the mobile platform.

2.2. Dynamic model of mobile manipulator. Let a vector of generalized coordinates of the mobile manipulator be denoted as

$$q = \begin{pmatrix} q_m \\ q_r \end{pmatrix} \in R^{n+p},$$

where $q_m \in R^n$ is a vector of generalized coordinates for the mobile platform and $q_r \in R^p$ denotes a vector of joint coordinates of the onboard manipulator. Because of the nonholonomic character of constraints, to obtain the dynamic model of mobile manipulator, the d'Alembert Principle should be used

$$Q(q)\ddot{q} + Q_m(q)\ddot{q} + C(q,\dot{q})\dot{q} + C_m(q,\dot{q})\dot{q} + D(q) = A^T(q)\lambda + B(q)u. \tag{4}$$

The model of dynamics (4) can be expressed in more detail as

$$\begin{bmatrix} Q_{11}(q) + Q_m(q_m) & Q_{12}(q) \\ Q_{21}(q) & Q_{22}(q_r) \end{bmatrix} \begin{pmatrix} \ddot{q}_m \\ \ddot{q}_r \end{pmatrix} + \begin{bmatrix} C_{11}(q,\dot{q}) + C_m(q_m,\dot{q}_m) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q_r,\dot{q}_r) \end{bmatrix} \begin{pmatrix} \dot{q}_m \\ \dot{q}_r \end{pmatrix} + \begin{pmatrix} 0 \\ D(q_r) \end{pmatrix} = \begin{pmatrix} A^T(q_m)\lambda \\ 0 \end{pmatrix} + \begin{pmatrix} B(q_m)u_m \\ u_r \end{pmatrix} \tag{5}$$

where:

$Q(q) = \begin{bmatrix} Q_{11}(q) & Q_{12}(q) \\ Q_{21}(q) & Q_{22}(q_r) \end{bmatrix}$ – inertia matrix of the onboard manipulator,

$Q_m(q) = \begin{bmatrix} Q_m(q_m) & 0 \\ 0 & 0 \end{bmatrix}$ – inertia matrix of the mobile platform,

$C(q,\dot{q}) = \begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q_r,\dot{q}_r) \end{bmatrix}$ – matrix of Coriolis and centrifugal forces of the onboard manipulator,

$C_m(q,\dot{q}) = \begin{bmatrix} C_m(q_m,\dot{q}_m) & 0 \\ 0 & 0 \end{bmatrix}$ – matrix of Coriolis and centrifugal forces of the mobile platform,

$D(q) = \begin{pmatrix} 0 \\ D(q_r) \end{pmatrix}$ – vector of gravity of the mobile manipulator,

$\lambda \in R^l$ – vector of Lagrange multipliers,

$B(q) = \begin{bmatrix} B(q_m) & 0 \\ 0 & I_p \end{bmatrix}$ – input matrix,

$u = \begin{pmatrix} u_m \\ u_r \end{pmatrix}$ – vector of controls.

The input matrix $B(q_m) \in R^{n \times m}$ defines coordinates of the mobile platform directly driven by the actuators. The model of dynamics (5) is often called a model in generalized coordinates. Now we want to express the model of dynamics using auxiliary velocities (2) for the mobile platform. We compute

$$\ddot{q}_m = G(q_m)\dot{\eta} + \dot{G}(q_m)\eta,$$

and eliminate from the model (5) the vector of Lagrange multipliers (using the condition $G^T(q_m)A^T(q_m) = 0$) by left-sided multiplying the mobile platform equations by $G^T(q_m)$ matrix. After substituting for $\dot{q}_m$ and $\ddot{q}_m$ we get

$$\begin{bmatrix} G^T(Q_{11} + Q_m)G & G^TQ_{12} \\ Q_{21}G & Q_{22} \end{bmatrix} \begin{pmatrix} \dot{\eta} \\ \ddot{q}_r \end{pmatrix} + \begin{bmatrix} G^T\left((C_{11} + C_m)G + (Q_{11} + Q_m)\dot{G}\right) & G^TC_{12} \\ C_{21}G & C_{22} \end{bmatrix} \begin{pmatrix} \eta \\ \dot{q}_r \end{pmatrix} + \begin{pmatrix} 0 \\ D \end{pmatrix} = \begin{bmatrix} G^TB & 0 \\ 0 & I \end{bmatrix} \begin{pmatrix} u_m \\ u_r \end{pmatrix}, \tag{6}$$

or, in more compact form,

$$Q^*\dot{z} + C^*z + D^* = B^*u, \qquad z = \begin{pmatrix} \eta \\ \dot{q}_r \end{pmatrix}, \tag{7}$$

where the arguments of matrices and vectors are omitted for the sake of simplicity. We will refer to the model (7) as the model of dynamics in auxiliary variables.

Now we want to recall a few properties of this dynamic model.

PROPERTY 1. The inertia matrix of the whole mobile manipulator is symmetric and positive definite

$$Q^*(q) = Q^{*T}(q) > 0 \qquad \forall q \in R^{m+p}.$$

This property is obvious because the inertia matrix is the matrix, which is defined by kinetic energy of the mechanical system, independently of a choice of coordinates.

PROPERTY 2 [10]. For a mobile manipulator with a wheeled nonholonomic mobile platform a skew-symmetry between inertia matrix Q^* and the matrix of Coriolis and centrifugal forces C^* does not hold anymore. To regain the skew-symmetry, a special correction matrix C_K should such that be added

$$\frac{d}{dt}Q^* = (C^* + C_K) + (C^* + C_K)^T. \tag{8}$$

The correction matrix C_K, which should be calculated before regulation process, can be introduced to the model as a preliminary feedback term

$$u = -(B^*)^{-1}C_K z + v,$$

where v is a new input signal.

3. Necessary conditions for existence of the input-output decoupling controller for nonlinear affine control systems

It is well known, see e.g. Ref. 11, that smooth affine nonlinear control system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i, \qquad x \in R^n, \tag{9}$$

with outputs

$$y = h(x), \qquad y \in R^p, \tag{10}$$

can be input-output decoupled, if the following conditions hold

- $p = m$,
- there exist finite nonnegative integers $\rho_1, \ldots, \rho_m$ such that

$$\begin{cases} L_{g_i} L_f^k h_i \equiv 0, \;\; k = 0, 1, \ldots, \rho_i - 1, \\ L_{g_i} L_f^{\rho_i} h_i \neq 0, \; i = 1, \ldots, m. \end{cases} \tag{11}$$

The above conditions mean, that $(\rho_i + 1)$ time derivative of ith output can be controlled by ith input, and has a form

$$y_i^{\rho_i+1} = L_f^{\rho_i+1} h_i + L_{g_i} L_f^{\rho_i} h_i u_i, \quad i = 1, \ldots, m, \tag{12}$$

where $L_{g_i} h_j$ and $L_f h_j$ are the Lie derivatives of a function h_j along smooth vector fields g_i and f, respectively.

If we want to obtain input-output decoupled system, it is necessary to use the following control law, namely

$$u_i = (L_{g_i} L_f^{\rho_i} h_i)^{-1}[-L_f^{\rho_i+1} h_i + \zeta_i]. \tag{13}$$

The next step is to apply the feedback law (13) to the system (12) such that each of the m outputs will be controlled by one of the new defined inputs ζ_i

$$y_i^{\rho_i+1} = \zeta_i, \qquad i = 1, \ldots, m.$$

4. Design of the decoupling controller for nonholonomic mobile manipulator

In order to design an input-output decoupling controller for mobile manipulator, we need to apply two different feedback loops to the system

- inner loop-linearization of the input-state equations,
- outer loop-linearization of the input-output description.

The input-state transformation can be fully linearized by a control law from a class of the 'computed-torque algorithms', namely exact linearization algorithm as follows

$$u = (B^*)^{-1}[C^* z + D^* + Q^* v]. \tag{14}$$

It is easy to show, see e.g. Ref. 9, that for any chosen class of mobile platforms there exists a proper selection of actuators preserving the invertibility of the input matrix $B^*(q_m) = G^T(q_m)B(q_m)$.

A substitution of (14) into (7) results in the system

$$Q^* \dot{z} = Q^* v,$$

or, equivalently,

$$\dot{z} = v, \tag{15}$$

where the positive definiteness of Q^* is used.

The second step of the design process is the linearization of the full model of the mobile manipulator – simultaneously the input-state and the state-output transformation. For this reason we introduce so-called 'output functions', which describe the behaviour of the end-effector of the mobile manipulator.

Differently to Yamamoto and Yun [1], we want to extend the output functions to the more general form, namely

$$y(q) = \begin{pmatrix} y_1(q_m, q_r) \\ y_2(q_r) \end{pmatrix}$$

where $y_1(q)$ describes the selected coordinates of the end-effector relative to the basic (inertial) frame $0X_0Y_0$ (see Fig. 1), and $y_2(q_r)$ represents a position and an orientation of the onboard manipulator relative to moving local frame $0X_pY_p$ associated with the mass center of the mobile platform. Such a choice of the output functions makes possible to control the location of the end-effector and the joint coordinates of the onboard manipulator. It means that we do not directly control the position of the platform, but we can do it indirectly by the compensation of the drift using the outputs y_2.

In order to obtain a decoupling controller, first we have to compute the time derivative of the output functions y as follows

$$\dot{y}(q) = \begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} \dfrac{\partial y_1}{\partial q_m}\dot{q}_m + \dfrac{\partial y_1}{\partial q_r}\dot{q}_r \\ \dfrac{\partial y_2}{\partial q_r}\dot{q}_r \end{pmatrix}$$

$$= \begin{bmatrix} \dfrac{\partial y_1}{\partial q_m}G & \dfrac{\partial y_1}{\partial q_r} \\ 0 & \dfrac{\partial y_2}{\partial q_r} \end{bmatrix} \begin{pmatrix} \eta \\ \dot{q}_r \end{pmatrix} = \begin{bmatrix} \Phi_m & \Phi_{mr} \\ 0 & \Phi_r \end{bmatrix} z = \Phi(q)z.$$

We can see that the first time derivative of the output function is equal to

$$\dot{y} = \Phi(q)z \tag{16}$$

and it does not depend on the input signal. After the second differentiating we get

$$\ddot{y} = \dot{\Phi}z + \Phi\dot{z}.$$

Having substituted the linearized dynamical equations (15) into the above equation, we obtain

$$\ddot{y} = \dot{\Phi}z + \Phi v. \tag{17}$$

Now we see that for the proper input-output decoupling and linearization, the following feedback control is needed

$$v = \Phi^{-1}\left(-\dot{\Phi}z + \zeta\right), \tag{18}$$

where ζ is a new input for the decoupled system. The proposed control law can be realized only if the matrix Φ is invertible.

Now we want to consider the conditions, which guarantee the invertibility of the Φ matrix. First, we know that

$$\Phi^{-1} = \begin{bmatrix} \Phi_m^{-1} & -\Phi_m^{-1}\Phi_{mr}\Phi_r^{-1} \\ 0 & \Phi_r^{-1} \end{bmatrix}.$$

The regularity conditions preserving the existence of the input-output decoupling controller will mean that the matrix Φ is nonsingular and because

$$\det \Phi = \det \Phi_m \cdot \det \Phi_r,$$

it is equivalent to the conditions

$$\det \Phi_m \neq 0 \quad \text{and} \quad \det \Phi_r \neq 0.$$

The first element of the control law which has to be invertible is $\Phi_m(q)$ matrix. This matrix is quadrat matrix because from (11) we know that a number of inputs m has to be equal to a number of outputs p for any decoupled system. A number of coordinates of the manipulator, which can be tracked, depends on the class the nonholonomic wheeled mobile platform. If the platform belongs to $(2,0)$ or $(1,1)$ class, it is possible to track only two coordinates of the manipulator. If the platform is from $(1,2)$ or $(2,1)$ class, it is possible to track three coordinates of the manipulator.

The matrix $\Phi_m(q)$ is invertible, if its determinant is not equal to 0. Because $\det \Phi_m$ is a function of coordinates of the mobile manipulator, it is necessary to exclude the configurations q which lead to singular $\Phi_m(q)$.

The second element of the control law which has to be invertible is Φ_r matrix. We observe that the matrix Φ_r is in fact the Jacobi matrix for the kinematics of the onboard manipulator. This matrix is invertible, if the configuration of the manipulator is nonsingular.

Next problem concerned with Φ_r^{-1} matrix is that during the control action every configuration of the manipulator has to be 'far enough' from any singularity. This fact requires that the start position of the manipulator relative to its base has to be nonsingular and the gains of the controller used for the regulation process in the decoupled system have to be properly chosen – they have to preserve the convergence of the manipulator to the desired configuration without any overshoot.

5. Simulation study

In further considerations we focus on a specific mobile manipulator that consists of an RTR robotic arm mounted on the unicycle presented in Fig. 1.

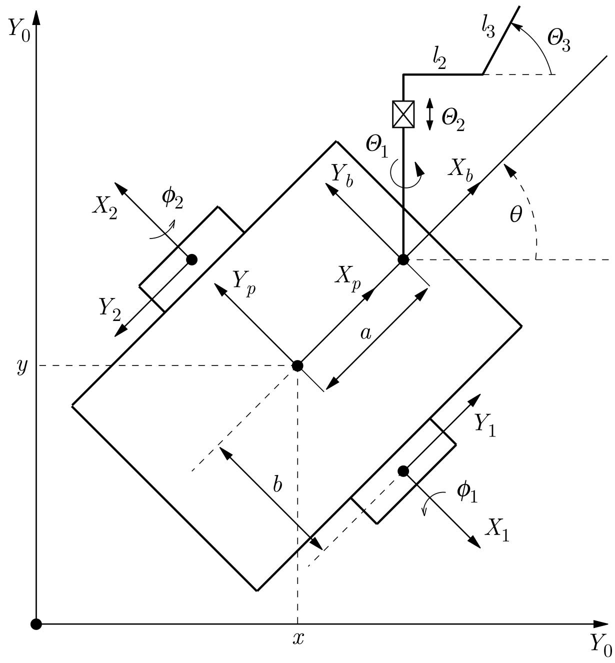

Fig. 1. Mobile manipulator consisting of RTR robotic arm and mobile platform of class $(2,0)$

Because the unicycle has only two control inputs, it means that only two coordinates of the end-effector of the mobile manipulator can be decoupled and linearized. We have chosen the x and y coordinates of the end-effector relative to the basic (inertial) frame $0X_0Y_0$ as follows

$$y_1 = \begin{pmatrix} x + a\cos\theta + l_2\cos_{01} + l_3\cos_{01}\cos\theta_3 \\ y + a\sin\theta + l_2\sin_{01} + l_3\sin_{01}\cos\theta_3 \end{pmatrix}$$

where $\cos_{01} = \cos(\theta + \theta_1)$ and $\sin_{01} = \sin(\theta + \theta_1)$. The orientation of the mobile platform is equal to θ, the joint coordinates of the RTR manipulator are denoted as θ_1, θ_2 and θ_3. Geometrical parameters (lengths) of RTR robotic arm are equal to l_2 and l_3, and (x, y) describe the position of the platform mass center relative to the basic frame $0X_0Y_0$. The base of the manipulator is fixed at the point $(a, 0)$ relative to the local frame OX_pY_p associated with the mass center of the platform.

The regularity conditions imply

$$\det \Phi_m \neq 0, \qquad \det \Phi_r \neq 0.$$

For the considered mobile manipulator we have computed

$$\det \Phi_m = -\frac{2}{b}[(l_2 + l_3\cos\theta_3)\cos\theta_1 + a],$$
$$\det \Phi_r = -l_3\sin\theta_3(l_2 + l_3\cos\theta_3).$$

Because we assume that the geometrical parameters of RTR manipulator are nontrivial $(l_2, l_3 > 0)$ and that the distance between wheels and the platform mass center is nonzero $(b > 0)$, we get the following conditions for the existence of the decoupling controller as follows

$$\begin{cases} (l_2 + l_3\cos\theta_3)\cos\theta_1 + a \neq 0, \\ \theta_3 \neq k\pi, \qquad k = 0, 1, \ldots, \\ l_2 + l_3\cos\theta_3 \neq 0. \end{cases}$$

First condition means that the end-effector cannot achieve any point lying on the axis joining wheels of the platform. In other words, the motion of the effector has to be limited only to one prescribed region relative to Y_p axis of the local frame (this region is limited to only a half of the workspace – it covers its front or its back part). The second and the third conditions mean that the singular configurations of the manipulator cannot be achieved.

We can see that the regularity conditions are similar to the regularity conditions for static feedback linearization of mobile platforms from $(2, 0)$ class.

The goal of the simulation study is to examine the behaviour of the decoupled mobile manipulator. If we apply the control law (18) to the matrix Eq. (17), then we obtain a linear decoupled system in the form

$$\ddot{y} = \zeta.$$

To preserve the trajectory tracking for the decoupled mobile manipulator, it is sufficient to apply the PD-controller with correction as follows

$$\zeta = \ddot{y}_d - K_d(\dot{y} - \dot{y}_d) - K_p(y - y_d). \tag{19}$$

The desired trajectory for the mobile manipulator has been chosen as follows

$$\begin{pmatrix} y_{1d}(t) \\ y_{2d}(t) \\ y_{3d}(t) \\ y_{4d}(t) \\ y_{5d}(t) \end{pmatrix} = \begin{pmatrix} 2t \\ 0 \\ 0.5 \\ -0.3\cos(2t) \\ 0 \end{pmatrix}.$$

Two first output functions y_1, y_2 are the (x, y) coordinates of the end-effector expressed in basic (global) frame $0X_0Y_0$. The output functions y_3, y_4, y_5 are the (x, y, z) coordinates of the end-effector relative to local frame $0X_pY_p$. The total time of simulations has been taken as T=10 s. The initial position of the platform was equal to $(x(0), y(0), \theta(0)) = (0, 0, 0)$ and the initial position of the manipulator was equal to $(\theta_1(0), \theta_2(0), \theta_3(0)) = (\frac{\pi}{2}, l_3, -\frac{\pi}{2})$. The initial position of the end-effector of the mobile manipulator relative to the basic frame $0X_0Y_0$ was equal to (a, l_2). We have chosen the following values of link lengths of the RTR manipulator: $l_2 = 0.3$ m, $l_3 = 0.2$ m. The gains of PD-controller in the control algorithm for the dynamics (19) are equal to $K_d = 100$ and $K_p = 100$. It is worth to mention that it is not necessary to use such big values for PD-controller to obtain the tracking errors without overshoots. It will be enough, if we use lower values for K_d and K_p but they have to be chosen properly to obtain smooth trajectory tracking errors.

Simulations have been run with the MATLAB package and the SIMULINK toolbox[1]. Trajectory tracking errors for successive output functions $e_{yi} = y_i - y_{id}$ are depicted in Figs. 2–6. The trajectory tracking for the posture configuration of the mobile manipulator on the horizontal plane XY are presented in Fig. 7. The orthogonal projection of the posture of the RTR manipulator on the vertical plane has been shown in Fig. 8.

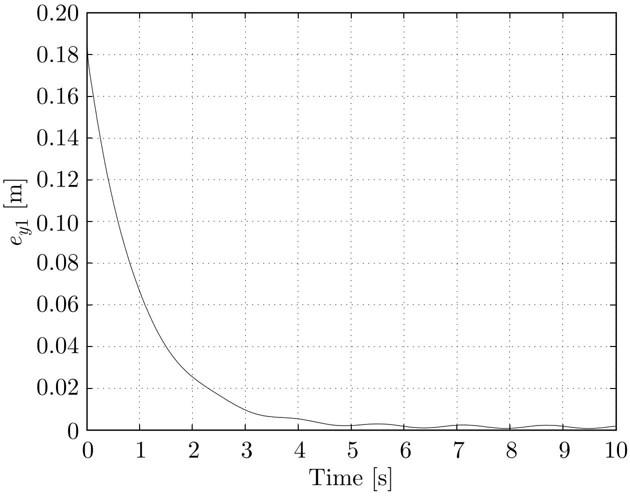

Fig. 2. Trajectory tracking error e_{y1} for the first output function of the RTR mobile manipulator

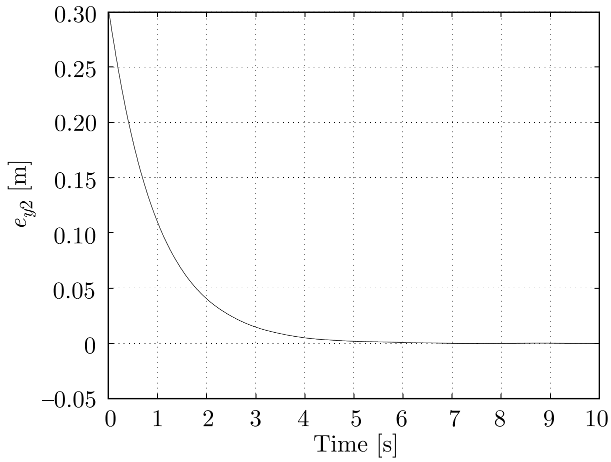

Fig. 3. Trajectory tracking error e_{y2} for the second output function of the RTR mobile manipulator

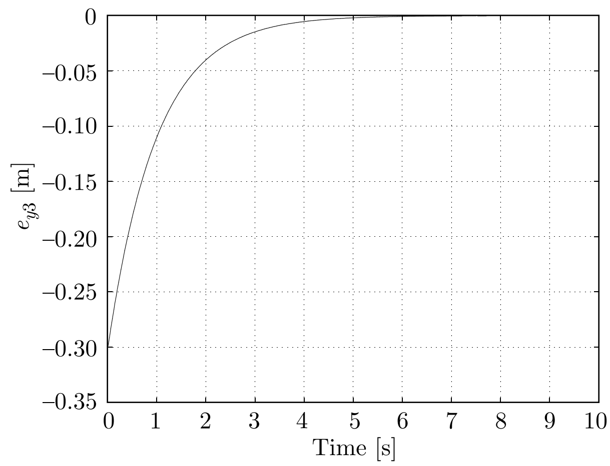

Fig. 4. Trajectory tracking error e_{y3} for the third output function of the RTR mobile manipulator

[1]MATLAB package and the SIMULINK toolbox were available thanks to Wrocław Centre of Networking and Supercomputing.

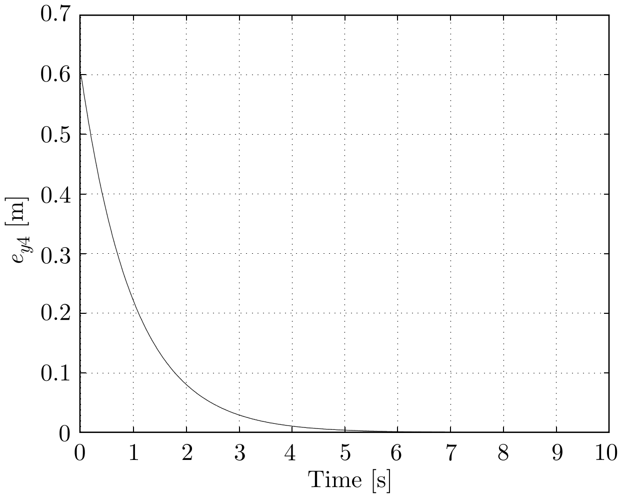

Fig. 5. Trajectory tracking error e_{y4} for the fourth output function of the RTR mobile manipulator

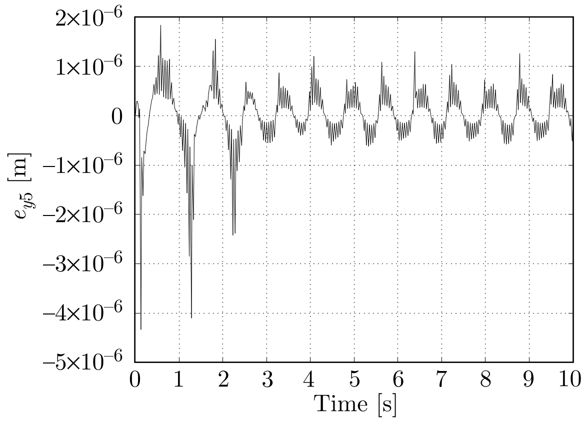

Fig. 6. Trajectory tracking error e_{y5} for the fifth output function of the RTR mobile manipulator

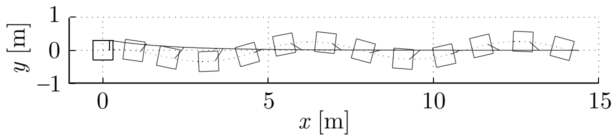

Fig. 7. Tracking of the desired trajectory (straight line) for the mobile manipulator consisting of RTR arm and the mobile platform of (2, 0) class – orthogonal projection on XY plane

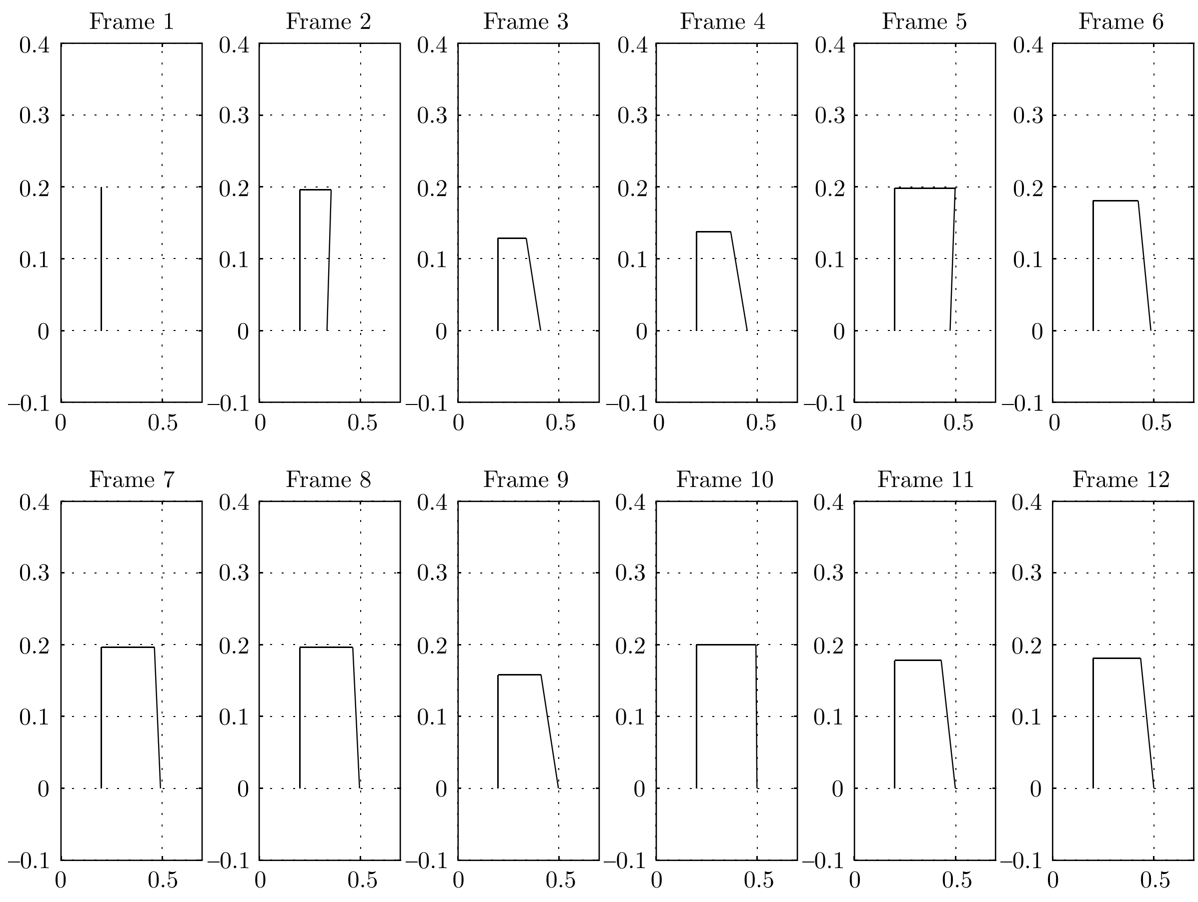

Fig. 8. Tracking of the desired trajectory (straight line) for the mobile manipulator consisting of RTR arm and the mobile platform of (2, 0) class – vertical projection on XZ plane.

6. Conclusions

In the paper we have considered a possibility of the design of input-output decoupling controller for nonholonomic mobile manipulators. We have proposed a selection of output functions in much more general form than in [1]. We have observed that our choice of the output functions allows to realize more complicated motions and maneuvers of the mobile platform for the nonholonomic mobile manipulator than the output functions in [1]. The price that should be paid are more restrictive regularity conditions (the motion of the effector is limited only to one prescribed region relative to Y_p axis of the local frame – only a front or back half of the workspace) than regularity conditions obtained in [1] (only a position of the mass center is forbidden). This conclusion is interesting because by the proper selection of the output functions some trade-off between the simplicity of the designed decoupling control law and the possibility to realize more complicated tasks in the workspace can be achieved.

In turn, we have shown that by a selection of the output coordinates of the mobile manipulator end-effector as (x, y) we obtain similar regularity conditions as in the case of the static feedback linearization for the mobile platform with restricted mobility.

As an illustration we have chosen RTR manipulator mounted on the unicycle. Till now such mobile manipulator moving in 3D-space has not been considered in the literature. We have shown that although it was impossible to control z-coordinate of RTR mobile manipulator directly, it could be realized by the proper choice of desired configuration of the onboard manipulator (choice of desired output vector $y_{2d}(t)$).

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