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DOROTA PAWLUS[1]

DYNAMIC RESPONSE OF THREE-LAYER ANNULAR PLATE WITH DAMAGED COMPOSITE FACINGS

The paper presents the dynamic behaviour of three-layer annular plates with damaged facings. The plate is composed of thin laminated, fibre-reinforced composite facings and thicker, foam core. Failure of the plate facings is modelled as fibre or matrix cracks. The plate loaded in the plane of facings with quickly increasing radially compressed forces loses its dynamic stability. Evaluation of the critical state of the plate with failures was carried out using both analytical and numerical solutions. The comparison of results between plates with material properties treated as isotropic, quasi-isotropic and composite has been conducted. Presented tables and figures create the image of dynamic responses of examined composite plates with structure failures.

1. Introduction

The composite structures of plates on account of their extensive, common use and increasing application requirements are the object of constant, scientific researches. The subject of the important analyses is the evaluation of work and behaviours of perfect structure correctly technically made without any imperfections. However, in the wide understood engineering practice the perceptive observation of the element including the forecasted defects is essential. The theoretical, numerical investigations of the influence of structure defects on values of evaluated parameters for some characteristic group of composites, whose are the layered composites popular called the sandwich ones, are presented in this paper.

The complex structure of sandwich plate built of thin, outer laminated composite layers and soft, foam core loaded with time-dependent forces can be subjected to the different forms of local or global failures and micro or macroscopic damages, which are presented in work [1]. The damage of the outer carrying layers of the

[1]*Faculty of Mechanical Engineering and Computer Science, University of Bielsko-Biala, Willowa 2, 43-309 Bielsko-Biala, Poland. Email: doro@ath.bielsko.pl*

sandwich structure changes its mechanical properties and capacity for further plate working under increasing in time load acting in the facings plane. The compound of the global form of plate failure, whose is the loss of dynamic stability with the microscopic defects of plate's facings, formulates the complex problem of dynamic plate response, which is difficult to predict. Partial, local damage of lamina in the form of fibre or matrix cracks does not eliminate the whole plate structure in carrying the outer loading. The parameters of the critical state, particularly the important value of the critical dynamic load of plate, change.

The problem of the evaluation of the dynamic stability behaviours of the plate with defects located in its facings is presented in this paper. The three-layer plate with the annular shape is the examined object loaded in the plane with the forces increasing in time. Such a kind of loading enables one to evaluate the practically important dynamic response of the structure subjected to the loss of stability. The area of the possible applications of the annular plates with various layered and homogeneous or heterogeneous structures is extensive, for example, in: aerospace industry, mechanical and nuclear engineering, civil engineering or miniature mechanical systems [2–4]. Papers [2] and [3] present the problem of axisymmetric, dynamic stability of additionally rotating sandwich plates with viscoelastic core under periodic radial stress. The dynamic problem of vibrations of annular plate with a characteristic microstructure and functionally graded properties is presented in work [4].

The analysed problem is the multi-parameter task, which does not only depend on the material, geometrical and loading parameters but also on accepted solution in modelling of examined annular plate object and on accepted forms of failure of the plate facings. The undertaken examination develops the analysed in paper [5] problem by showing the dynamic response of the sandwich plates with damaged laminated, fibre-reinforced composite facings. In the presented paper, the author considered both axisymmetrical and asymmetrical buckling deformations of the plate widen the generalization of proposed solution and its practical universality.

The problem of the axisymmetric buckling of laminated composite circular and annular plates is presented in works [6, 7]. The three-dimensional axisymmetric buckling of the laminated annular plates consisting of transversely isotropic layers has been presented.

The three-dimensional theory of elasticity was also used in analysis of axisymmetric deformation of a laminated transversely isotropic annular plate. The exact solution corresponding to specified boundary conditions and plate dimensions was presented in paper [8]. The axisymmetric vibrations of annular sandwich plates with isotropic core and composite facings studied using the harmonic quadrature method is presented in paper [9]. The effect of the shear deformation and rotatory inertia in the core has been taken into account.

The quasi-isotropic composite circular plate under quasi-static lateral load and low velocity impact tests is presented in paper [10]. The analytical analysis was performed for the non-linear approximation method and the large-deflection plate theory. Analytical and finite element results are compared to experimental measurements. Results show that the low velocity impact responses are close to the quasi-static plate behaviours. The fibre damage image with the damage propagation from the centre of plate to the edge is presented for plates with different thickness.

Accepted in analysis, presented in this paper, way of description of the damage of laminate facings of the plate uses the mathematical formulae proposed in work [1], which modify the elements of stiffness matrix of composite structure. The fibre or matrix of single lamina or in the extreme case of all laminate layers are subjected to the damage. The parameters of the critical state of the plate are analysed in detail. Presented results, like values of the dynamic, critical loads and the forms of the loss of stability of plates show the grade of structure sensitivity on dynamic load acting. The use of two different finite element plate models and the model obtained through the analytical and numerical problem solution for plates with facings treated also as the quasi-isotropic ones broadens the range of analysis and increases the certainly that calculations, which are based on approximated methods, are carried out correctly. The special arranged example of four-layer laminate of facing fulfilling the conditions of quasi-isotropic composite has been accepted in analyses. Such a choice improves the recognition of the phenomenon of structure response. It also enables us to think about the possibilities to conduct the simpler calculations using the engineering constants for quasi-isotropic composite. Presented results are the obvious attempt to evaluate the theoretically complex and practically meaningful problem of use of damaged composite layered structure. The results direct the attention to the differences of behaviours of healthy and damaged plates, fluctuations of values of the critical loads, modelling of the structure also for approximated case, when the composite properties of undamaged facings are described by engineering numbers of quasi-isotropic composite material. Then, the information meaning of results of the presented analysis seems to be extensive and practically important.

2. Problem formulation

The composite, three-layer annular plate is the subject of the analysis. The plate structure is composed of thin, laminated fibre-reinforced composite facings and thicker foam core. Material and geometrical symmetry of plate cross-structure occurs. The plate is loaded in the plane of facings with uniformly distributed on its perimeter stress quickly increasing in time according to the formula

$$p = st, \tag{1}$$

where: p – compressive stress, s – rate of loading growth, t – time.

Edges of the plate are slideably clamped. The scheme of analysed plate is presented in Fig. 1.

Accepted means of plate support is the chosen, possible one, which for analysed problem makes it possible to observe the basic buckling modes of the plate. The

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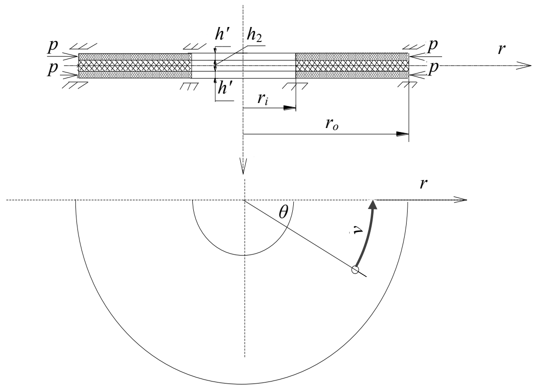

Fig. 1. Scheme of analysed plate

influence of the support system on the final results is not analysed. The examined object – the clamped, annular, three-layered plate – corresponds with analysed and presented one in earlier works, for example in [5, 11–14], thus ensuring some continuation of researches.

The mechanics of the fibrous composite is based on the classical lamination theory, which uses the following expressions [1, 15, 16]:

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k (z_k - z_{k-1}), \tag{2}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right), \tag{3}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right), \tag{4}$$

where: A_{ij}, B_{ij}, D_{ij} – extensional, coupling and bending stiffnesses, respectively, $\overline{Q}_{ij}$ – transformed reduced stiffness matrix of lamina, N – number of layers.

Adopted exemplary laminas configuration in each composite facing is expressed by the code $[0/\text{-}45/45/90]_T$ (for example: the fibres of lamina 1 are arranged with angle 0°). Each facing consists of four laminas. The configuration of laminas fulfils the conditions of the composite called as quasi-isotropic one. The elastic, engineering constants E, G, ν can be calculated using the expressions [15]:

$$E = 2\frac{A_{66}}{t}\left(1 + \frac{A_{12}}{A_{11}}\right), \qquad G = \frac{A_{66}}{t}, \qquad \nu = \frac{A_{12}}{A_{11}}, \tag{5}$$

where: E – Young's modulus, G – Kirchhoff's modulus, ν – Poisson's ratio, t – thickness of the laminate, A_{11}, A_{12}, A_{66} – extensional stiffnesses A_{ij} $(i, j = 1, 2, 6)$.

The geometry of laminate cross-section is presented in Fig. 2.

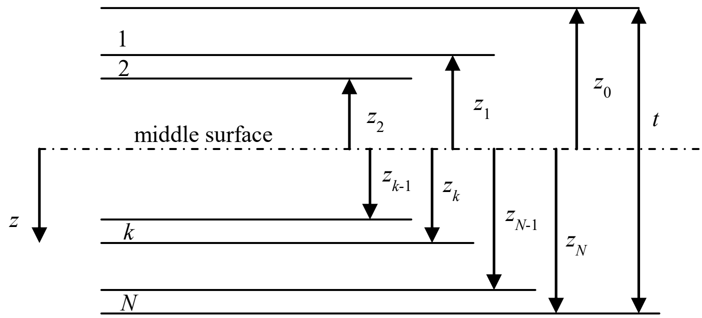

Fig. 2. Geometry of laminate cross-section

Accepted model of the composite degradation is based on the theory of correction parameter method presented in work [1]. The matrix or fibre cracks change the mechanical properties of laminate. The matrix crack causes the rigidity elimination in transverse direction to the fibres. It is expressed by the correction parameter η. Mathematically, it is described by the modification of the stiffness matrix. Its form for undamaged lamina is following [1]:

$$\begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}, \tag{6}$$

where:

$$C_{11} = \frac{E_1}{(1 - \nu_{12}\nu_{21})}, \quad C_{22} = \frac{E_2}{(1 - \nu_{12}\nu_{21})}, \quad C_{12} = \frac{E_1 \nu_{21}}{(1 - \nu_{12}\nu_{21}}, \quad C_{66} = G_{12}.$$

For lamina with matrix crack, the elements C_{11}, C_{12}, C_{22} take the following new values: $\mathbf{C}_{11} = \eta \cdot C_{11}$, $\mathbf{C}_{12} = \mathbf{C}_{22} = 0$ but when the fibre crack occurs: $\mathbf{C}_{11} = C_{22}$ [1].

As a criterion of the loss of the plate dynamic stability the criterion presented by Volmir in work [17] was adopted. According to this criterion, the loss of plate stability occurs at the moment of time, when the speed of the plate point of maximum deflection reaches the first maximum value. Fig. 3 explains the approach to determination of the parameters characterizing the plate critical state: critical time t_{cr} and critical, additional deflection w_{dcr}, which are related with non-dimensional quantities (8). The value of critical dynamic loads p_{crdyn} is calculated using the equation (1) for determined value of critical time t_{cr} and assumed value of the rate of loading growth s (data are presented in the section 5 of paper).

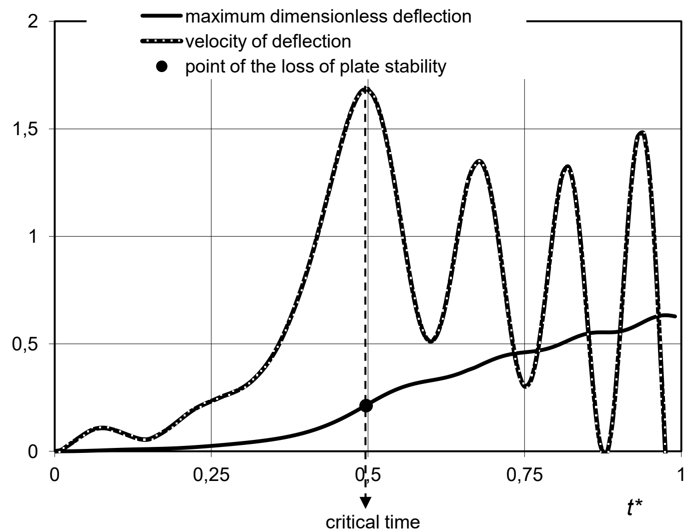

Fig. 3. Time histories of deflection and velocity of deflection for exemplary FDM plate model with determination of the critical time

The analysed problem of plates with quasi-isotropic composite facings was solved analytically an numerically using the orthogonalization method and finite difference method (FDM) and only numerically using the finite element method (FEM). FEM method enables the observation of plates with quasi-isotropic composite facings and exactly composite facings with damages. The examinations were led for plate models with damage facings in forms of fibre or matrix cracks of single lamina or all laminas together.

3. FDM plate model – analytical and numerical problem solution

The classical theory of sandwich plates was used to express the structure of plates with steel or quasi-isotropic composite facings. The broken line hypothesis and the distribution of stresses into normal and shear loading, with respect to the plate facings and the core, have been accepted. Description of the solution to the analysed, dynamic problem is presented in works [5, 11–13] in detail.

The main steps of the problem solution are the following:

- formulation of the dynamic equilibrium equations for each plate layer,
- description of the core deformation in radial and circumferential directions accepting that preliminary and additional deflections, are equal for each plate layer,
- application of the linear physical relations of Hooke's law for the facings and the core,

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- establishing the sectional forces and moments in facings using the equations of the non-linear Kármán's plate and formulation of the resultant, transverse radial and circumferential forces,
- formulation of the resultant membrane forces expressed by the introduced stress function Φ,
- determination of the initial loading and boundary conditions and conditions connected with the slideably clamped both inner and outer plate edges,
- determination of the basic differential equation expressing the deflections of the analysed plate

$$
\begin{aligned}
k_1 w_{d,rrrr} &+ \frac{2k_1}{r} w_{d,rrr} - \frac{k_1}{r^2} w_{d,rr} + \frac{k_1}{r^3} w_{d,r} \\
&+ \frac{k_1}{r^4} w_{d,\theta\theta\theta\theta} + \frac{2(k_1+k_2)}{r^4} w_{d,\theta\theta} + \frac{2k_2}{r^2} w_{d,rr\theta\theta} - \frac{2k_2}{r^3} w_{d,r\theta\theta} \\
&- G_2 \frac{H'}{h_2} \frac{1}{r} \left(\gamma_{,\theta} + \delta + r\delta_{,r} + H' \frac{1}{r} w_{d,\theta\theta} + H' w_{d,r} + rH' w_{d,rr} \right) \\
&= \frac{2h'}{r} \left(\frac{2}{r^2} \Phi_{,\theta} w_{,r\theta} - \frac{2}{r} \Phi_{,\theta r} w_{,\theta r} + \frac{2}{r^2} w_{,\theta} \Phi_{,\theta r} - \frac{2}{r^3} \Phi_{,\theta} w_{,\theta} \right. \\
&\quad \left. + w_{,r} \Phi_{,rr} + \Phi_{,r} w_{,rr} + \frac{1}{r} \Phi_{,\theta\theta} w_{,rr} + \frac{1}{r} \Phi_{,rr} w_{,\theta\theta} \right) - M w_{d,tt}\,,
\end{aligned}
\tag{7}
$$

where: w, w_d – total and additional plate deflection, respectively, δ, γ – differences of radial and circumferential displacements of the points in middle surfaces of facings, $H' = h' + h_2$, $k_1 = 2D$, $k_2 = 4D_{r\theta} + \nu k_1$, $D = \dfrac{Eh'^3}{12(1-\nu^2)}$, $D_{r\theta} = \dfrac{Gh'^3}{12}$ – flexural rigidity of the outer layers, $M = 2h'\mu + h_2\mu_2$, E, G, ν – Young's and Kirchhoff's moduli and Poisson's ratio of the facings material, respectively, G_2 – core Kirchhoff's modulus, μ, μ_2 – facing and core mass density, respectively, h', h_2 – thickness of facing and core thickness, respectively.

The solution is based on the dimensionless quantities, expressions and shape functions [11, 18]. Some of them, important in this description, are following:

$$
\zeta = \frac{w}{h}, \qquad \zeta_1 = \frac{w_d}{h}, \qquad \rho = \frac{r}{r_o}, \qquad t^* = t \cdot \mathrm{K7}, \tag{8}
$$

$$
\zeta_1(\rho,\theta,t) = X_1(\rho,t)\cos(m\theta), \tag{9}
$$

$$
\zeta_o(\rho,\theta) = \xi_1\eta_o(\rho) + \xi_2\eta_o(\rho)\cos(m\theta), \tag{10}
$$

$$
F(\rho,\theta,t) = F_a(\rho,t) + F_b(\rho,t)\cos(m\theta) + F_c(\rho,t)\cos(2m\theta), \tag{11}
$$

where: $\zeta_1(\rho,\theta,t)$ – shape function of the additional plate deflection, $\zeta_o(\rho,\theta)$ – shape function of the preliminary plate deflection, $F(\rho,\theta,t)$ – shape function of the stress function, h – total thickness of plate, r_o – outer radius of the annular plate,

m – number of circumferential waves corresponding to the form of plate buckling, ξ_1, ξ_2 – calibrating numbers, $\eta_o(\rho) = \rho^4 + A_1\rho^2 + A_2\rho^2 \ln \rho + A_3 \ln \rho + A_4$, A_i – quantities fulfilling the conditions of clamped edges by the function $\eta_o(\rho)$, $i = 1$, 2, 3, 4, t^* – dimensionless time, K7 – the quotient of the rate of loading growth s to the value of the critical static load of plate with steel facings calculated solving the eigenvalue problem [11–13].

Using the orthogonalization method and the finite difference method the system of the differential equations was obtained. The presented system of equations is the solution to the task of dynamic deflections of the three-layered plate with elastic layers:

$$\mathbf{P}_{\mathrm{L}}\mathbf{U} + \mathbf{Q}_{\mathrm{L}} = K\ddot{\mathbf{U}}, \tag{12}$$

$$\mathbf{M}_{\mathrm{Y(V,Z)}}\mathbf{Y}(\mathbf{V},\mathbf{Z}) = \mathbf{Q}_{\mathrm{Y(V,Z)}}, \tag{13}$$

$$\mathbf{M}_{\mathrm{D}}\mathbf{D} = \mathbf{M}_{\mathrm{U}}\mathbf{U} + \mathbf{M}_{\mathrm{G}}\mathbf{G}, \tag{14}$$

$$\mathbf{M}_{\mathrm{GG}}\mathbf{G} = \mathbf{M}_{\mathrm{GU}}\mathbf{U} + \mathbf{M}_{\mathrm{GD}}\mathbf{D}, \tag{15}$$

where: $K = \mathrm{K7}^2\mathrm{WK5WK8}$, $\mathrm{WK5} = \dfrac{h'}{h}$, $\mathrm{WK8} = r_o\dfrac{h_2}{G_2}M$, $\mathbf{U}$, $\mathbf{Y}$, $\mathbf{V}$, $\mathbf{Z}$ – vectors of plate additional deflections and components F_a, F_b, F_c of the stress function $F_{a,\rho} = y$, $F_b = v$, $F_c = z$, respectively, $\mathbf{Q}_{\mathrm{L}}$, $\mathbf{Q}_{\mathrm{Y}}$, $\mathbf{Q}_{\mathrm{V}}$, $\mathbf{Q}_{\mathrm{Z}}$ – vectors of expressions composed of the initial and additional deflections, geometric and material parameters, components of the stress function, radius ρ, quantity b (b – length of the interval in the finite difference method), coefficients δ, γ and number m, $\mathbf{P}_{\mathrm{L}}$, $\mathbf{M}_{\mathrm{D}}$, $\mathbf{M}_{\mathrm{U}}$, $\mathbf{M}_{\mathrm{G}}$, $\mathbf{M}_{\mathrm{GG}}$, $\mathbf{M}_{\mathrm{GU}}$, $\mathbf{M}_{\mathrm{GD}}$, $\mathbf{M}_{\mathrm{Y}}$, $\mathbf{M}_{\mathrm{V}}$, $\mathbf{M}_{\mathrm{Z}}$ – matrices of elements composed of plate parameters, quantity b and number m, $\mathbf{D}$, $\mathbf{G}$ – vectors of expressions composed of coefficients δ and γ, respectively.

The system of Equations (12)–(15) was solved using for the initial state of the plate Runge-Kutta-Gill's integration method of forth order with the choice of integration step, whose nominal value is equal to 0.001 s.

4. FEM plate models

The plate models built of the finite elements were calculated using the ABAQUS system. The calculations were carried out at the Academic Computer Center CYFRONET-CRACOW: KBN/SGI_ORIGIN_2000/PŁódzka/030/1999. Two kinds of plate models were built: in circular, symmetrical, annular form called as basic model and in simplified form built of axisymmetrical elements called as simplistic model. The scheme of models is presented in Fig. 4. Facings are built of 9-node shell elements or 3-node axisymmetrical shell elements called in ABAQUS system as: S9R5 or SAX2, respectively. The core is built of 27-node solid elements or 8-node axisymmetric solid elements expressed as: C3D27 or CAX8, respectively.

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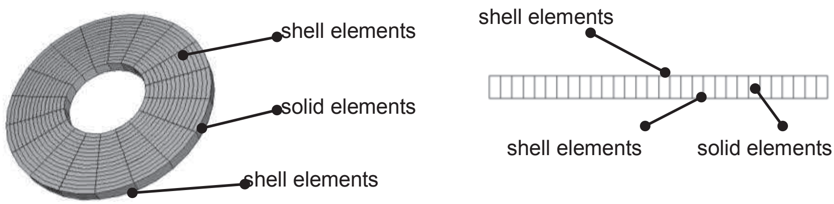


(a) (b)

Fig. 4. FEM plate models: a) basic, b) simplistic

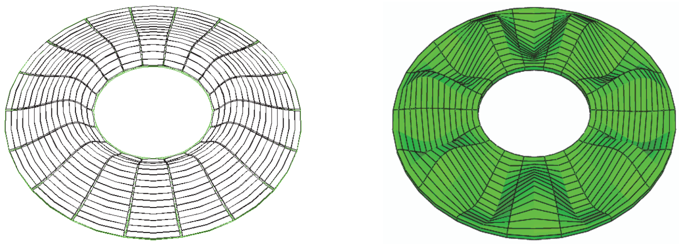
The outer surfaces of facing and core mesh elements are tied using the program option expressed as surface contact interaction.

Using the shell option in modelling of the structure of plate facings, one introduced the calculated elements of stiffnesses A_{ij}, B_{ij}, D_{ij} (2)–(4). Description of the facing structure stiffness was modified according to the accepted form of damages of facing laminas.

5. Numerical analyses and result discussion

The exemplary results are for plates with the values of inner radius equal to: $r_i = 0.2$ m and outer radius $r_o = 0.5$ m. Each lamina in composite facings arranged as $[0/\text{-}45/45/90]_\mathrm{T}$ has thickness $h'' = 0.125$ mm. The total thickness of composite and also steel facing is equal to $h' = 0.5$ mm. The material parameters of laminated fibre-reinforced composite which correspond to the glass/epoxy composite are the following: $E_1 = 53.781$ GPa, $E_2 = 17.927$ GPa, $G_{12} = 8.964$ GPa, $\nu_{12} = 0.25$, $\mu = 2900$ kg/m^3 [19]. The calculated engineering constants E, G, ν of quasi-isotropic glass/epoxy composite are as follows: $E = 31.1$ GPa, $G = 12.5$ GPa, $\nu = 0.24$. The steel facings have the parameters: Young's modulus $E = 2.1 \cdot 10^5$ MPa, Poisson's ratio $\nu = 0.3$, mass density $\mu = 7850$ kg/m^3. The value of the quotient K7 is equal to K7 = 20 1/s (8) and the value of the rate of loading growth s (1) is equal to: $s = 4346$ MPa/s for plates loaded on inner edge, $s = 931$ MPa/s for plates loaded on outer edge.

The plate core is made of polyurethane foam with the thickness equal to: $h_2 = 5$ mm and values of Kirchhoff's modulus $G_2 = 5$ MPa, Poisson's ratio $\nu_2 = 0.3$ and mass density $\mu_2 = 64$ kg/m^3 [20]. Young's modulus was calculated treating the core as isotropic. The correction parameter η in accepted damage theory of composite facings is equal to $\eta = 0.1$ [1]. The exemplary results are shown for the plates compressed on inner or outer edges The modes of plate buckling: axisymmetrical $m = 0$ and with number $m = 7$ circumferential waves were analysed in detail. Axisymmetrical $m = 0$ form of buckling is those one which corresponds to the minimal values of critical dynamic loads p_{crdyn} of plates radially compressed on inner edge. Fig. 5 shows the exemplary global buckling modes: $m = 0$ and $m = 7$.



(a) (b)

Fig. 5. Global buckling plate modes: a) axisymmetrical (m = 0), b) circumferentially waved (m = 7)

Table 1 shows the values of the critical dynamic loads p_{crdyn} of plates with steel and undamaged quasi-isotropic composite facings. The calculations were carried out using the finite difference method (FDM) for number $N = 14$ of discrete points, which fulfils the accuracy condition expressed by the technical error up to 5% [11]. Plate is compressed on outer edge.

Table 1.

Values of critical, dynamic loads p_{crdyn} with corresponding plate modes m

plate mode m		0	1	2	3	4	5	6	7	8	9
p_{crdyn} [MPa]	steel facings	43.78	44.25	40.52	32.61	27.48	24.69	22.82	24.22	22.36	25.62
	quasi-isotropic composite facings	36.33	34.47	30.28	26.08	21.43	20.03	19.10	18.63	19.10	20.03

Table 2 presents the values of critical time t_{cr} and deflection w_{dcr} of FEM plate simplistic model with quasi-isotropic composite facings compressed on outer edge. The number and the accepted kind of elements do not influence the values of critical time. Insignificant differences are observed in values of critical deflection of plate node, whose deflection is maximum. The calculations were carried out for 30-element plate model built of SAX2 and CAX8 elements.

Table 2.

Values of critical time and deflection depending on number and kind of mesh axisymmetic elements

kind of elements	SAX1 and CAX4			SAX2 and CAX8	
number of elements	15	30	60	15	30
critical time t_{cr} [s]	0.038	0.038	0.038	0.038	0.038
critical deflection w_{dcr} [mm]	11.73	12.20	12.37	12.37	12.43

5.1. Plates with steel or quasi-isotropic facings

The calculation results of plates with steel facings can be the important referring point to the analyses of plates with the composite facings. Fig. 6 shows the time histories of deflections of the plates compressed on outer edge. The minimal value

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of critical dynamic load p_{crdyn}, which corresponds with buckling mode $m = 8$, is equal to $p_{crdyn} = 22.36$ MPa. The exemplary comparison of values of critical dynamic loads p_{crdyn} of axisymmetrical mode ($m = 0$) for plate models: FDM – $p_{crdyn} = 43.78$ MPa and FEM simplistic – $p_{crdyn} = 40.06$ MPa presents their good compatibility. Fig. 7, which shows the time histories of deflections and velocities of deflections for FEM simplistic plate model confirms observed for plates with steel facings correctness of plate dynamic response.

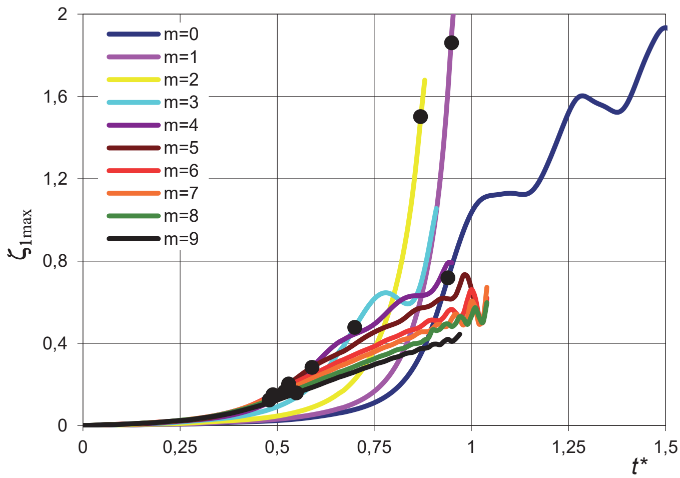


Fig. 6. Time histories of deflections of FDM plate model compressed on outer edge with steel facing

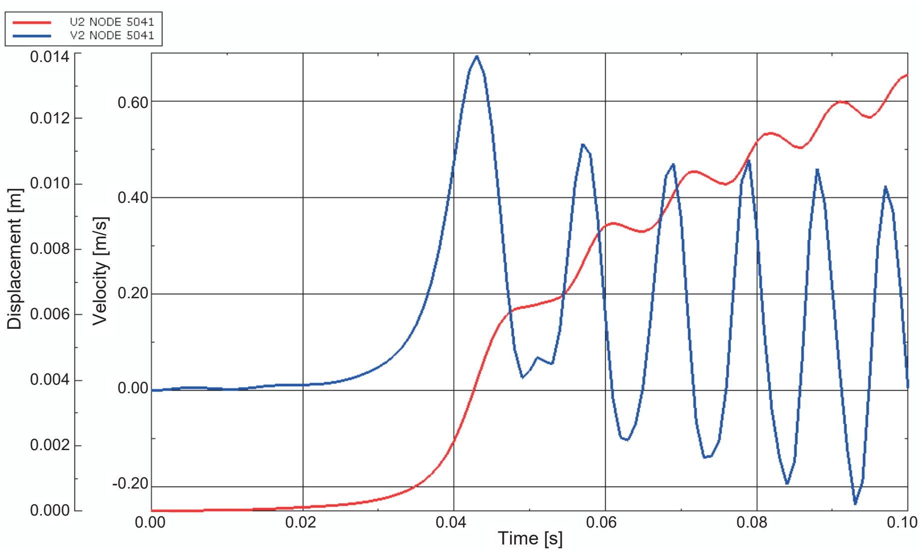
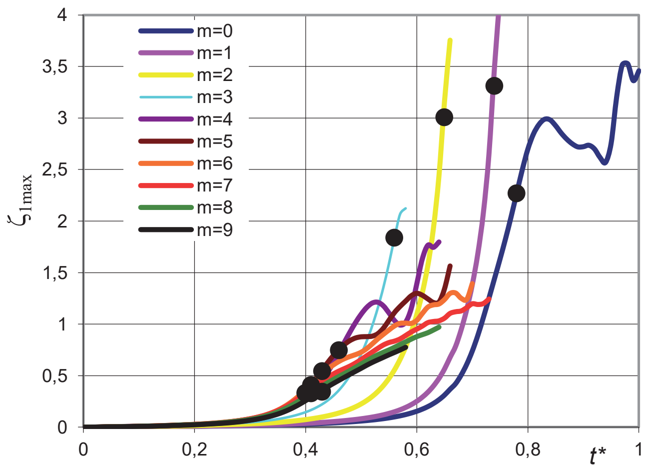


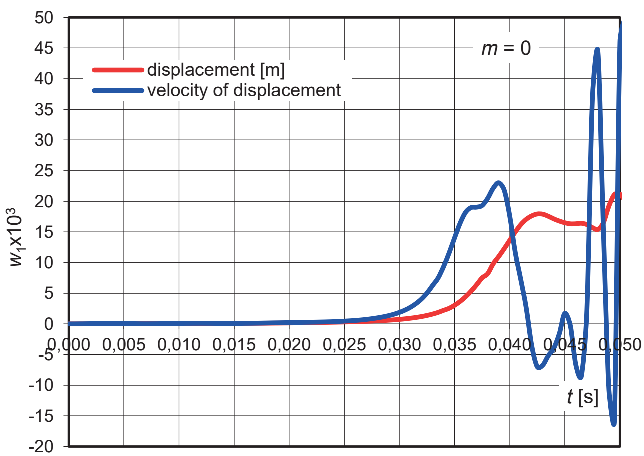
Fig. 7. Time histories of deflection and velocity of deflection of axisymmetric ($m = 0$) FEM simplistic plate model compressed on outer edge with steel facings

The exemplary diagram (Fig. 7) and others similar are presented in the form, which shows two curves: the time history of deflection and time history of velocity of deflection for the node of plate mesh with maximum deflection or two nodes from the critical zone of plate deformation. Simultaneous presentation of both curves allows for the observation of supercritical plate work and the evaluation of the moment of the loss of plate dynamic stability according to the explanation shown in Fig. 3 and presented in section 2 of this paper.

Similar comparison for FDM and FEM plate models with quasi-isotropic composite facings is presented in Figs 8 and 9. Additionally, time history of deflections



(a)



(b)

Fig. 8. Time histories of deflections of FDM plate model compressed on outer edge with quasi-isotropic composite facings: a) plates with different number m, b) axisymmetrical plate model $m = 0$

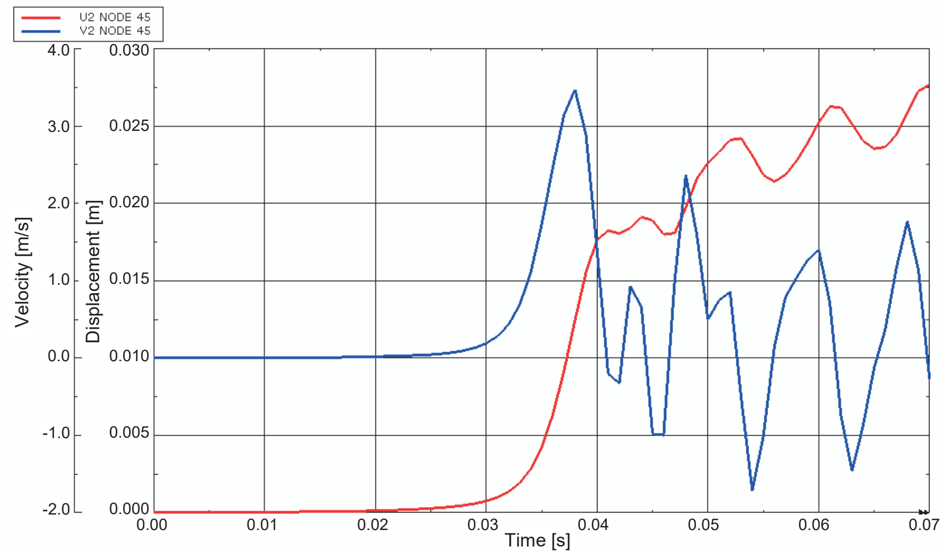


Fig. 9. Time histories of deflections of axisymmetric (m = 0) FEM simplistic plate model compressed on outer edge with quasi-isotropic composite facings

and velocity of deflections for axisymmetrical ($m = 0$) case of plate mode are presented in Fig. 8b. The runs of curves are shown in the range of dimension time up to $t = 0.5$ s corresponding with the dimensionless time $t^* = 1$ in Fig. 8a. The scale of deflections is real, too. The run of time history of velocity of deflection is not graduated, because it only serves to evaluate the moment of the loss of plate dynamic stability. This presentation of diagram of plate deflections and velocity of deflections allows for the comparison of dynamic responses of plate two FDM and FEM models – Fig. 8b and Fig. 9. The calculated values of critical dynamic loads p_{crdyn} for these plate models are equal to: p_{crdyn} = 36.33 MPa and p_{crdyn} = 35.37 MPa, respectively. The minimal value of critical dynamic load of examined plate compressed on outer edge is equal to p_{crdyn} = 18.63 MPa and corresponds with mode $m = 7$. The runs of curves of the plates with steel facings and quasi-isotropic composite ones show the similar dynamic behaviours of the plates with smaller and greater number of buckling circumferential waves. The relevant decrease in values of critical deflections and the tendency to supercritical linear growth of plate deflections with increasing load exist for plates with number m greater than $m > 3$.

5.2. Plate models with undamaged facings

The presented results show the dynamic response of FEM basic plate models with composite facings. Fig. 10 presents the form of deflection of plate models loaded on inner edge. The FEM simplistic plate model (see, Fig. 10b) shows the

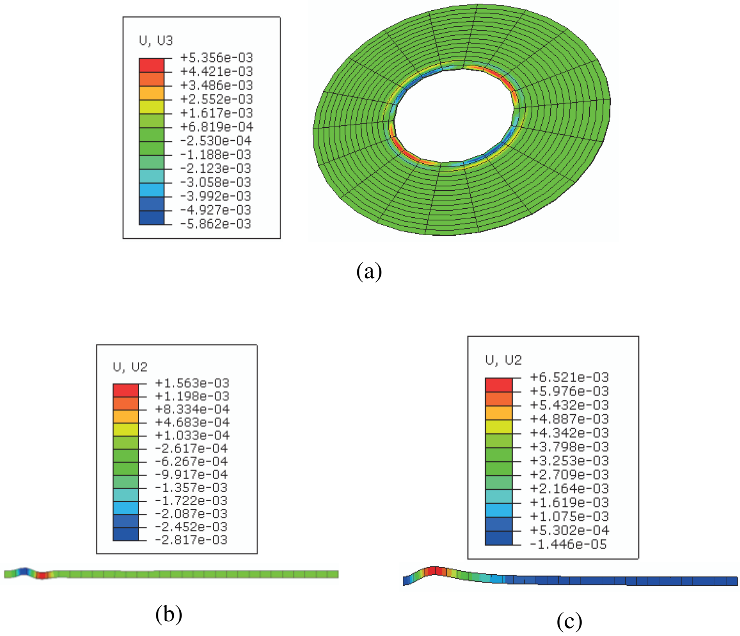


Fig. 10. Buckling forms of plate models with undamaged composite facings loaded on inner edge: a) FEM basic model, b) FEM simplistic model, c) FEM simplistic model with quasi-isotropic facings

axisymmetrical form of buckling with the radial waves close to the loaded plate edge. The value of critical dynamic load p_{crdyn} is equal to p_{crdyn} = 147.76 MPa. It is a little greater value than obtained for FEM basic model (see, Fig. 10a). In Fig. 10a the radial and circumferential waves are observed only in the zone of loaded plate perimeter. The critical dynamic load p_{crdyn} is equal to p_{crdyn} = 140.81 MPa. Values of p_{crdyn} calculated for FDM and FEM simplistic plate models with quasi-isotropic composite are significantly smaller. They are equal to p_{crdyn} = 54.33 MPa and p_{crdyn} = 56.50 MPa, respectively. The buckling form of FDM plate model is global, quasi-eulerian one (see, the form shown in Fig. 10c for FEM simplistic plate model). Similar dynamic responses are observed for plate models loaded on outer edge (see, Fig. 11). The values of critical dynamic loads p_{crdyn} are the following: p_{crdyn} = 77.26 MPa for FEM simplistic model with composite facings (see, Fig. 11b); p_{crdyn} = 71.21 MPa for FEM basic model with composite facings (see, Fig. 11a); p_{crdyn} = 35.37 MPa and p_{crdyn} = 36.33 MPa for FEM simplistic model (see, Fig. 11c) and FDM model of plate with quasi-isotropic facings, respectively. Fig. 12 shows the response of the plate loaded on outer edge modelled with m = 7 circumferential waves expressing the plate predeflection form. The critical form of deformation is more complex. Circumferential deformations in the plate area and close to the loaded outer edge are observed.

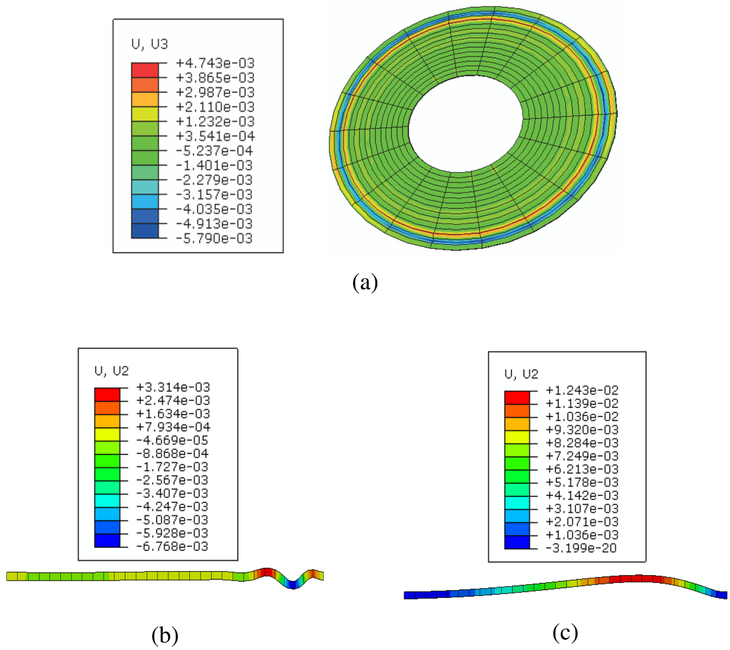


Fig. 11. Buckling forms of plate models with undamaged composite facings loaded on outer edge: a) FEM basic model, b) FEM simplistic model, c) FEM simplistic model with quasi-isotropic facings

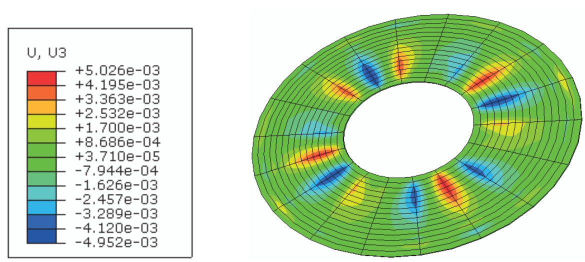
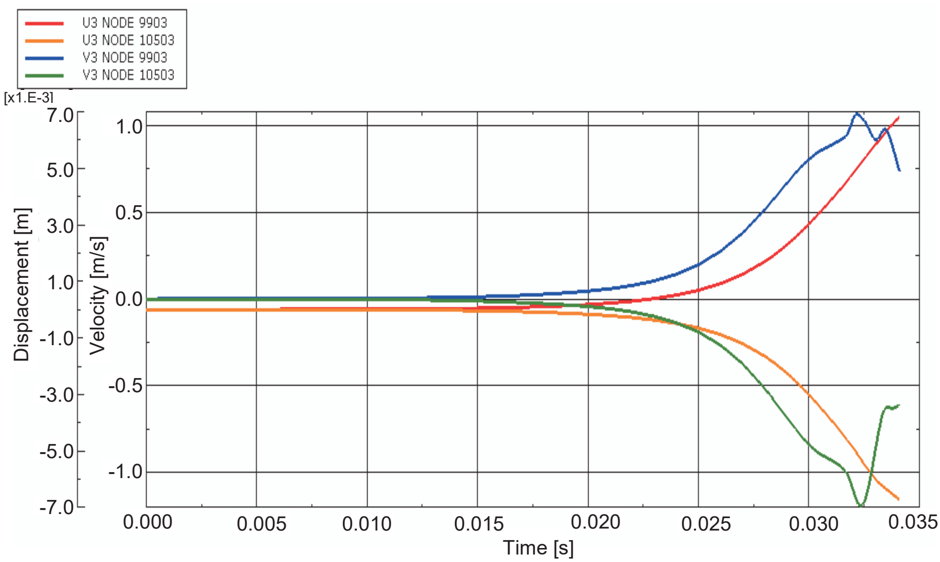


Fig. 12. Asymmetric buckling form (m = 7) of FEM basic model with undamaged composite facings loaded on outer edge

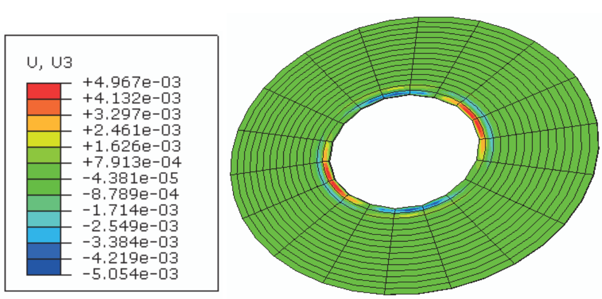
The analyses of plates with composite facings and comparisons of their dynamic behaviours are presented in work [5] in more detail.

5.3. Plate models with damaged composite facings

The image of dynamic response of plates with damaged facings is shown by analyses of plates with all damaged laminas of composite expressed by the code $[0/\text{-}45/45/90]_T$ or for facings with damaged only one of layer – lamina 1, whose fibres are arranged with angle 0°. The failure concerns the fibre or matrix cracks of composite facings. Figs 13a and 14a show the time histories of deflections and velocity of deflections of the plate points, which are located close to the plate inner edge. This part of the plate is subjected to the most deformation field when the plate is loaded on inner edge. Fig. 13b shows the critical deformation of plate with damaged lamina 1 in the form of the fibre crack. The buckling mode is not global. The inner area of plate is radially and circumferentially deformed. The buckling form is similar to that observed for the plate with undamaged facings (see, Fig. 10a). The value of critical dynamic load is insignificantly smaller, equal to p_{crdyn} = 139.75 MPa. In the case when all laminas are damaged, the buckling deformation (see, Fig. 14b) has axisymmetrical, radially waved form.



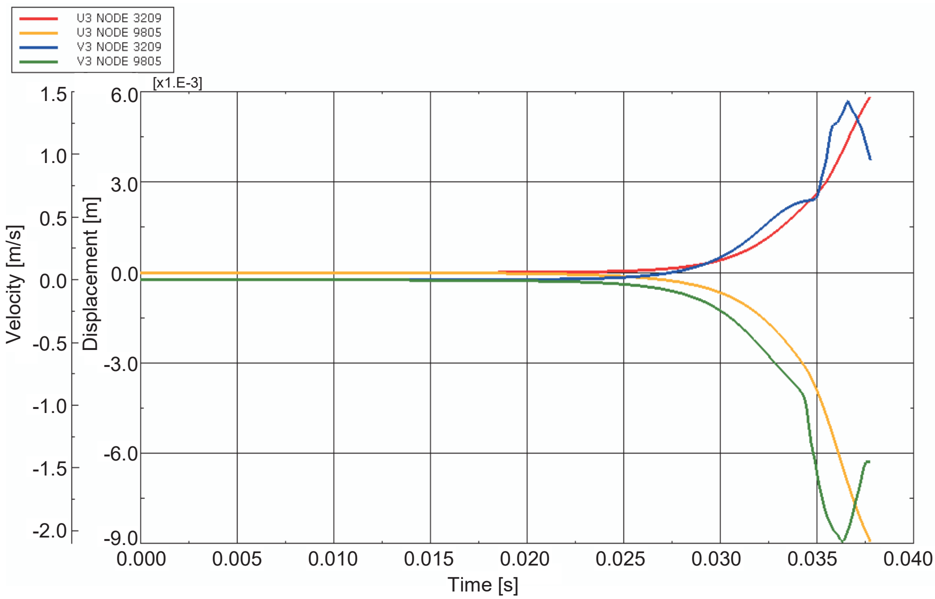
(a)



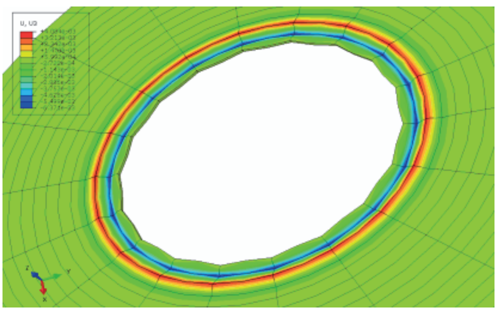
(b)

Fig. 13. FEM basic plate model compressed on inner edge with damaged fibres of lamina 1: a) time histories of deflections and velocities of deflections, b) buckling form

It corresponds with greater value of p_{crdyn} equal to $p_{crdyn} = 157.76$ MPa. When the matrix crack occurs the decrease in values of critical dynamic loads p_{crdyn} is clear.



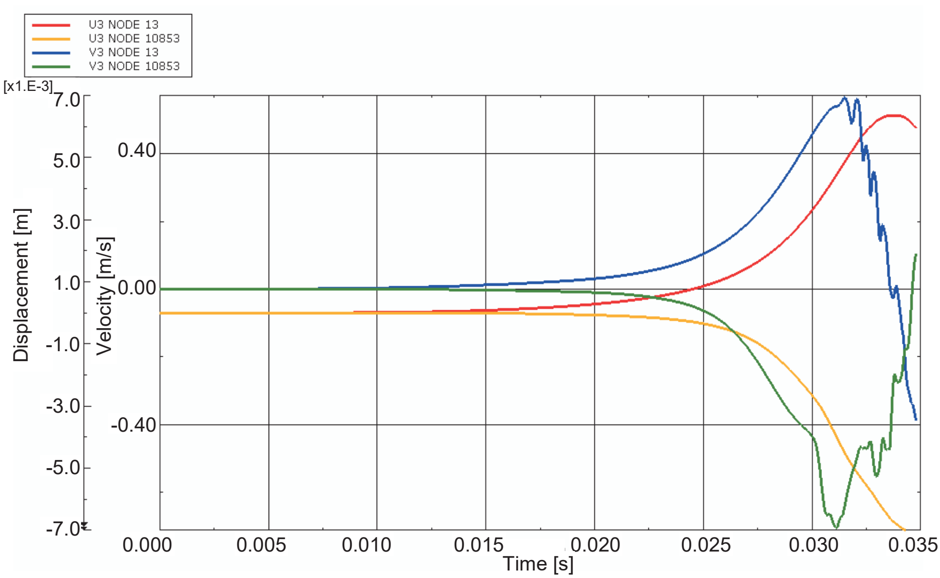
(a)



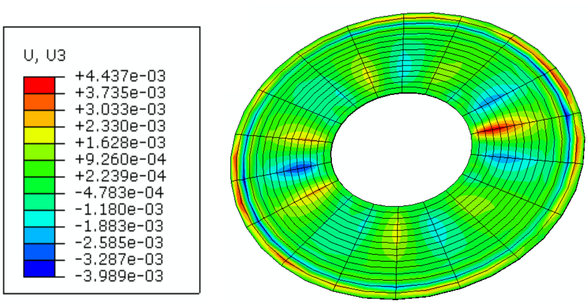
(b)

Fig. 14. FEM basic plate model compressed on inner edge with all fibres cracks: a) time histories of deflections and velocities of deflections, b) zoomed in buckling form

The buckling image of plates with matrix crack of lamina 1 and all facings laminas is presented in Figs 15 and 16. The plates are compressed on outer edge. The buckling form is not global, either. The regular circumferentially waved critical form with expected $m = 7$ waves is not observed. Particularly, for plate with all damaged facings the dynamic response of the plate deformed on outer, loaded edge is noticeable. Here, one should compare the Fig. 12 and Fig. 16 with undamaged and damaged facings, respectively. The decrease in values of critical dynamic loads p_{crdyn} exists. The value of p_{crdyn} for plate with damaged lamina 1 is equal to p_{crdyn} = 57.94 MPa, but for plate with damaged all laminas critical load is equal to p_{crdyn} = 52.59 MPa. In this example, one cannot observe the tendency to perform the circumferential waves, too.

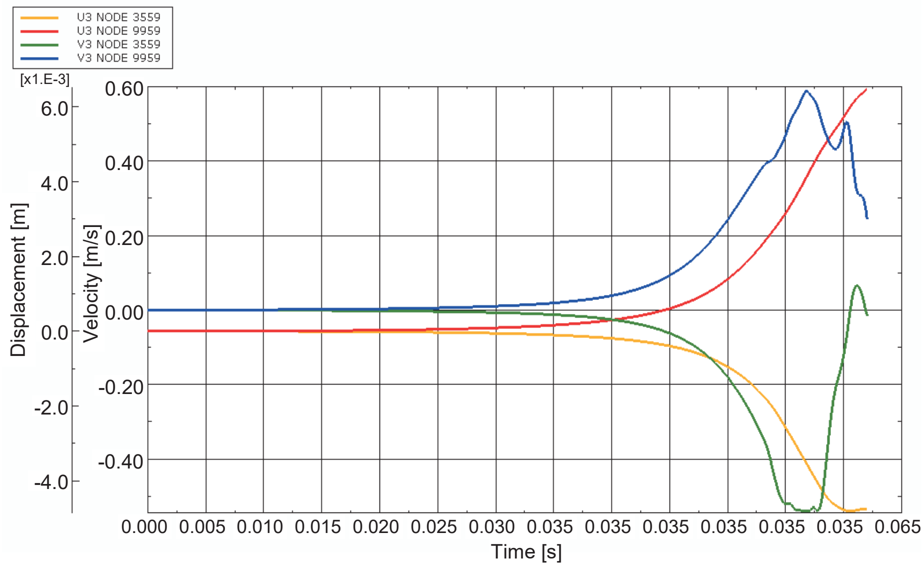


(a)

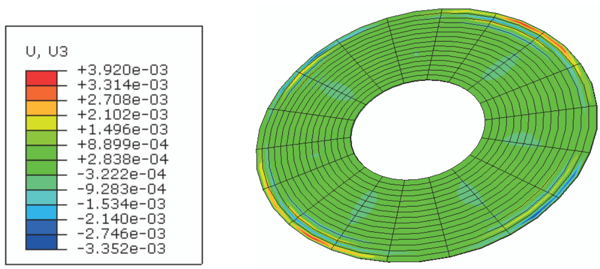


(b)

Fig. 15. FEM basic plate model compressed on outer edge with damaged matrix of lamina 1: a) time histories of deflections and velocities of deflections, b) buckling form



(a)



(b)

Fig. 16. FEM basic plate model compressed on outer edge with cracks of all matrices: a) time histories of deflections and velocities of deflections, b) buckling form

6. Recapitulation of analyses

Table 3 shows the values of critical dynamic loads calculated for plates with steel, quasi-isotropic composite facings and composite facings with undamaged and damaged laminas. It presents the values of critical dynamic loads with the information about the form of the loss of the plate stability, which is expressed by the number m for the regular, global form of the plate buckling or for other forms of critical deformation is described by numbers explained under the table.

The presented results concern two cases of FEM plate models and plate model solved using the finite difference method (FDM). Table 3 shows the results for the plates loaded on inner edge, whose preliminary plate deformation is in axisymmet-

Table 3.

Critical state parameters of plate models with undamaged and damaged facings

		dynamic critical load / mode p_{crdyn} [MPa] / m											
plate model		FEM simplistic	FEM basic	FEM simplistic	FEM basic	FEM simplistic	FEM basic	FEM simplistic	FEM basic	FEM simplistic	FEM basic	FDM	
damaged layers		matrix of lamina 1		fibre of lamina 1		all matrix		all fibres		undamaged facings			
										composite		quasi-isotropic	steel
outer edge loading	$m = 7$	–	57.94/[1]	–	59.57/[1]	–	52.59/[1]	–	65.16/[1]	–	64.23/[1]	18.63/7	22.36/8
	$m = 0$	68.66/[2]	58.17/[3]	71.67/[2]	64.69/[2]	53.99/[2]	53.06/[3]	71.67/[2]	69.81/[2]	77.26/[2] (35.37)	71.21/[3]	36.33/0	43.78/0
inner edge loading	$m = 0$	134.73/[2]	128.21/[3]	134.73/[2]	139.75/[3]	99.96/[2]	103.43/[3]	130.38/[2]	157.76/[2]	147.76/[2] (56.50)	140.81/[3]	54.33/0	89.32/0

[1] – not global form with $m = 7$ circumferential waves – examples are shown in Fig. 15b and Fig. 16b
[2] – circumferentially axisymmetrical form with radial waves – examples are shown in Fig. 14b and Fig. 11b
[3] – not global form, deformation close to the loading edge – example is shown in Fig. 13b
( ) – results for FEM simplistic plate model with quasi-isotropic composite facings

rical form ($m = 0$) and for plates loaded on outer edge with also axisymmetrical ($m = 0$) preliminary deformation and with number $m = 7$ circumferential waves. The following plate structures have been compared: with damages of matrix or fibre in single, outer facing lamina or in all laminas; with undamaged facings made of laminated composite or steel and also treated in problem solution as quasi-isotropic composites (in the case of FEM plate model results are shown in parentheses).

Axisymmetric and radially and circumferentially waved buckling forms are considered. Inserted results include the presented above plate cases and additional examples of both FEM simplistic and basic plate models.

Table 3 is some recap of undertaken attempts to evaluate of the critical state of examined plates whose laminated composite facings can be damaged in different way. The presented results clearly show higher values of critical loads of plate models with composite facings than those obtained for plates with steel facings or treated as quasi-isotropic ones. The comparison of dynamic responses of FEM simplistic and basic plate models draws the attention to the meaning of the modelling of plate FEM structure. The observations of the various forms of plate critical deformation indicate the basic FEM model as this one which enables more comprehensive evaluation of plate dynamic behaviours.

7. Conclusions

The presented solutions are the attempt to evaluate of the dynamic stability response of the composite plate with damaged facings. The plate structure is composed of thin laminated fibre-reinforced composite facings with fibre or matrix cracks and thicker foam core. The presented plate models are built of the finite elements using the ABAQUS system or they are the result of the analytical and numerical solution. The calculations were carried out for the plates with damaged facings and plates with undamaged, healthy steel or quasi-isotropic composite facings. Axisymmetrical and asymmetrical modes were considered. The kind of failure of laminated layers of facings influences the rate of changes of the critical state of analysed plates. Both critical dynamic loads and corresponding with them the buckling modes depend on the working structure of the plate.

The stability behaviour of the composite plates with damaged layers creates the problem difficult to formulate the general conclusions. The plates responses are ambiguous so the meaning of the effective solution increases. The failure of the matrix of facing laminated layers decreases the values of critical dynamic loads significantly. The forms of the loss of the plate stability cannot be global. Particularly, the local deformation close to the loaded edge is observed for the plates quickly dynamically loaded. The values of the critical dynamic loads are higher than calculated in static analysis (results for the plates statically loaded are presented in works [5, 14]). It shows certain resistance of composite plates to the dynamic loading process. Quasi-isotropic simplistic description of the composite plate facings does not show the real plate behaviour in dynamic issue. However, because the critical dynamic loads are smaller for plate models with quasi-isotropic facings, such simplification could be useful only in some approximation evaluations. The undertaken evaluation of the dynamic behaviour of sandwich plates with not ideal, defective composite facings should be supported by the analyses of some experimental investigations.

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