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A geometric approach to evaluating the results of Polish copper ores beneficiation

Introduction

The primary task of mineral processing is ore partition into components differing in terms of quality of products. The mineral ability to carry out such a partition is called upgradeability beneficiation and its evaluation is the basic purpose of studies evaluating the prospects of the used partition techniques and technologies related to this. Separation techniques are based on utilizing differences in physical or physicochemical properties of particles including particles density, their size, settling velocity, wettability, etc. Depending on the utilized property during the partition, there are different types of beneficiation, i.e. gravitational, flotation, oil agglomeration etc.

The evaluation of separation results can be conducted on the basis of products grade. In the case of copper ores it is the content of copper in the feed (α), concentrate (β) and tailings (ϑ). Partition, as a rule, is not ideal. Therefore, the general balance equation, where a significant role is played by a partition ratio between products, that is, the so-called yield (γ), is applicable to it. In this situation, the correct evaluation of ore upgradeability must be multi-dimensional (three parameters). Yield is preferable because it is a significant feature in the economic evaluation of the beneficiation process (Yianatos et al. 2003; Tumidajski et al. 2007; 2012).

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It is clear that values of α , β , ϑ and γ depend on the ore structure, its preparation and processing. In the case of copper ores, due to differences in mineralogical rock structure, differences in mineralization and sizes of inclusions of copper minerals, beneficiation abilities of separate lithological fractions are different as are the processes of the feed preparation to beneficiation including comminution and classification.

The ore structure and manner of its preparation to the beneficiation process significantly affect the evaluation of its beneficiation and the separation results evaluation methodology should take this into account. It is important to reflect the nature of the process consisting in satisfactory, resulting from the potential possibilities of ore, concentration of the useful component in the product directed for further operations with unconditional regard to losses. The purpose of the article is to discuss the ways to analyze ores' beneficiation ability by geometric methods in space (β , ϑ , w), where w is the function of the process evaluation.

1. Formal bases of the proposed methods of describing the ore beneficiation ability

The basic mass balance of separation is

$$\alpha = \gamma\beta + (1 - \gamma)\vartheta \quad (1)$$

where α , β , ϑ are average contents of the useful component, accordingly in the feed, concentrate and tailings. The plot is a single-layer hyperboloid (Tumidajski et al. 2007, 2012), whose points represent the conducted partition.

Assuming that $\gamma = k = \frac{\beta - \vartheta}{\alpha - \vartheta} \cdot 100$, we prepare the horizontal cross-section of the hyper-

boloid, which projected on plane (β , ϑ) presents a line

$$\beta = \left(\frac{k}{100} - 1 \right) \vartheta - \frac{k}{100} \alpha \quad (2)$$

The selection of particular values (β , ϑ , k) is due to the ore structure, that is, the occurrence of particular copper minerals, a share of middlings in concentrate, and is basic information about the ore partition ability.

Due to the fact that the ideal separation practically does not exist in nature (transfer of every and only every particle of the useful component to concentrate) it both values β and ϑ must always be taken into account during the evaluation of the technological quality of the partition. This means that partition factors will be based on two dimensions, that is, β and ϑ , with a potential use of α .

The ore characteristics (its upgradeability) manifests itself, primarily, in the course of sections of a polygonal chain on a plane (β, ϑ) (projections of the trajectory sections on a surface (β, ϑ, w) , where w denotes any process evaluation factor (yield, recovery, losses, etc.), whose lengths, tilt angles to the axis are information about the ore structure and preparation for the process. The analysis of the course of a polygonal chain on a plane (β, ϑ) (Fig. 1) can determine the aggregates' ability to transfer to products and be a basis for the evaluation of a partition's effectiveness. Decreases $\beta_i - \beta_{i+1}$, $\vartheta_i - \vartheta_{i+1}$ referred (standardized) to value α are a measure of changes in the quality of concentrate and tailings and the additional characteristics of the ore quality, its upgradeability (i – the subsequent flotation results).

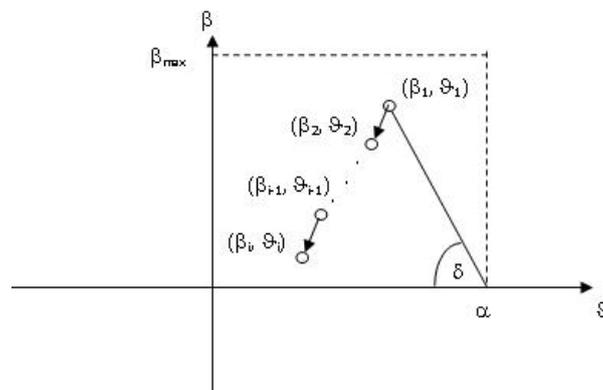


Fig. 1. Elements of the analysis of ore upgradeability, where $tg\delta = \frac{\gamma}{100} - 1$,
 β_{\max} is the useful component content in its richest mineral, $\alpha = \text{const}$, $\gamma = \frac{\alpha - \vartheta}{\beta - \vartheta} \cdot 100\%$

Rys. 1. Elementy analizy wzbogalności rudy, gdzie $tg\delta = \frac{\gamma}{100} - 1$,
 β_{\max} oznacza zawartość składnika użytecznego w najbogatszym mineralu, $\alpha = \text{const}$, $\gamma = \frac{\alpha - \vartheta}{\beta - \vartheta} \cdot 100\%$

The realized analysis of upgradeability based on the investigation of the mutual relations between β and ϑ must be subordinated to the main purpose, that is, the evaluation factor – recovery or other, which measures the level of the useful component recovery and takes the quality of the concentrate into account. The notion of upgradeability is qualitative and has not been interpreted quantitatively so far. In general, the obtained results of the analysis of ore (mineral) beneficiation ability will be therefore presented as points of the trajectory located on the surface (β, ϑ, w) , where $w = f(\beta, \vartheta)$ is taken into account as the aspect of upgradeability. Beneficiation factors should also take the effects of the mineral's mineralogical evaluations into account. Factor w , specifying ore beneficiation degree, that is, the degree of partition into concentrate and tailings, can be determined on the basis of the obtained flotation results. Let us assume that $w = \frac{\beta - \vartheta}{\beta}$ can be written as $w = 1 - \frac{\vartheta}{\beta}$. If beneficiation is ideal (thorough

separation), then $\vartheta = 0$ and $w = 1$. If $\vartheta = \beta \equiv \alpha$, then $w = 0$, that is, the beneficiation process does not occur. Factor w is thus a natural, intuitive meter of the beneficiation ability. If we have the mineralogical analyses' results, the following formulas can be proposed as factors w

$$w = \frac{\beta - \vartheta}{\beta_{\max} - \alpha} \quad \text{or} \quad w = 1 - \frac{\vartheta}{\beta_{\max} - \alpha} \quad (3)$$

where β_{\max} is the useful component content in its richest mineral. Difference $\beta_{\max} - \alpha$ is the maximum difference in values β .

Mineralogy decides primarily about the quality of concentrate, i.e. the content of the useful component in it. It is the useful component content in the mineral, which its potential maximum content in the concentrate depends on. It is the so-called LTC – the limit theoretical content of the useful component in the product. The content of other substances or the nature of mineralization and sizes of minerals also play a significant role in determining the assumed technological solutions related to beneficiation just as the feed particle size in the beneficiation process or a particular beneficiation method. The assumed solutions to the process and its effectiveness will determine both the level of the useful component recovery and the level of achieving the potential maximum content in concentrate, LTC – this level cannot be exceeded without the introduction of the hydrometallurgical process. The analysis of beneficiation results regarding the selected aspect (the analysis of the trajectory course) can lead to the creation of an interesting factor of beneficiation (its evaluation). The location of the empirical trajectory of beneficiation (multi-product flotation) results on the surface of factor w , its projection on a plane (β, ϑ) is the characteristics of the ore structure and its potential ability to partition into products having average values β and ϑ . The trajectory maximum is the information about the transfer of the richest copper minerals to the concentrate. The trajectory directions between subsequent points are the source of information about variable conditions of partition between concentrate and tailings – about the presence of aggregates and particles transferred into concentrate and contaminating it. It depends on the ore mineralization, share of lithological layers and components demonstrating properties similar to particles of useful minerals in terms of the process.

2. A review of approaches to the beneficiation evaluation used to-date

During the presentation of results of beneficiation conducted especially in laboratory conditions, three types of upgrading curves have been used so far: Henry curves (the only ones allowing direct reading of the parameters values), Halbich curves and Fuerstenau curves (Ding et al. 2015; Drzymala 2007; Drzymala and Ahmed 2005; Drzymala et al. 2013; Foszcz 2013; Foszcz et al. 2010; 2016; Kelly and Spottiswood 1982; Madej 1978; Mayer and Craig 2010; Niedoba 2013; Yang et al. 2015). Henry's curves are usually used (for technical

reasons – the possibility to partition into density fractions) for coal. The Halbich curve presenting dependence $\varepsilon = f(\beta)$ and the Fuerstenau curve presenting dependence $\varepsilon_r = f(\varepsilon)$, where

$$\varepsilon = \frac{\alpha - \vartheta}{\beta - \vartheta} \cdot \frac{\beta}{\alpha} \cdot 100 \quad \text{and} \quad \varepsilon_r = \left(100 - \frac{\alpha - \vartheta}{\beta - \vartheta} \cdot 100 \right) \cdot \frac{100 - \vartheta}{100 - \alpha} \quad (4)$$

are usually used during the ore flotation or coal cleaning evaluation. The Halbich and Fuerstenau curves in most cases do not have extreme points and therefore determining the optimal values of beneficiation factors on their basis is complicated. It is possible to attempt at determining points of a starting decrease of a dependent variable, that is, ε or ε_r or points with the greatest curvature for these curves.

Looking at Fig. 1 many 3D projections on plane (ϑ, β) can give the same graph, but they can be a result of various values of α and can provide various trajectories in space. Therefore, this can be a result of various courses of a process. In approaching the issue of beneficiation ability two-dimensionally (in relation to plane (ϑ, β)), we should pay attention to the sign and

value of the directional derivative $\frac{\partial \varepsilon}{\partial s}$, where s is the direction of increments β and ϑ . In

accordance with the formula we have

$$\frac{\partial \varepsilon}{\partial s} = \left(\frac{\partial \varepsilon}{\partial \vartheta} \right)_{P=P_0} \cos x + \left(\frac{\partial \varepsilon}{\partial \beta} \right)_{P=P_0} \cos y \quad (5)$$

↪ $P = P_i(\vartheta_i, \beta_i)$ – the point of the i -th fractionated flotation,
 $\cos x$ and $\cos y$ – directional cosines of a section $(\vartheta_i - \vartheta_{i+1}, \beta_i - \beta_{i+1})$.

As it is known from the analysis of derivatives of recovery, value ϑ has the greatest effect on its value – a small change of this value noticeably changes the value of recovery. Directional cosines $\cos x$ are close to zero and $\cos y$ are close to one. Changes in recovery at some stage of fractionated flotation become of the order of one or smaller, which is information about achieving the flotation optimum. It is worth noting that the approach to the study of beneficiation by methods of a directional derivative draws attention to both significant variables β and ϑ .

For the Halbich curve, we can acknowledge that the point of the greatest convexity (the greatest curvature), that is, point f_H , is the highlighted point of the fractionated flotation (Fig. 2). In accordance with the formula specifying the curvature radius, it is the section determined on the basis of the second derivative of the recovery calculated in relation to the content in concentrate, that is, its minimum. According to Kelly and Spottiswood (Kelly and Spottiswood 1982), this point can be determined on the basis of the maximum value

of parameter $f = \frac{\varepsilon \beta}{100}$, for which we obtain a curve tangent to the Halbich curve, in the

example from Fig. 2, this occurs for value $f=54\%$. At this point, the dynamics of the non-useful component supply to concentrate begins to exceed the dynamics of the useful component supply. The optimal point coordinates can be obtained, in approximation, by an experimental method with the very careful conduct of flotation.

Similarly, as in the case of the Halbich curve, a point of the optimal ore efficiency for Fuerstenau curve can be defined as a point of the greatest convexity. It is the point at which more non-useful waste components begin to enter into concentrate.

According to Drzymala and Ahmed (Drzymala and Ahmed 2005), it is possible to determine this point at the intersection of the actual beneficiation curve with a diagonal of recovery identity of the useful component in concentrate and the remaining components in tailings ($\varepsilon = \varepsilon_{reszt}$).

Thus we have

$$\frac{\alpha - \vartheta}{\beta - \vartheta} \cdot \frac{\beta}{\alpha} \cdot 100 = \left(100 - \frac{\alpha - \vartheta}{\beta - \vartheta} \cdot 100 \right) \cdot \frac{100 - \vartheta}{100 - \alpha} \quad (6)$$

After the transformation we obtain

$$\vartheta = \frac{100 \cdot \alpha \cdot (\beta - \alpha) - \alpha \cdot \beta \cdot (100 - \alpha)}{(\beta - \alpha) \cdot \alpha - \beta \cdot (100 - \alpha)} \quad (7)$$

Assuming that $\alpha = \text{const}$, values β and ϑ depend only on the manner of ore preparation (comminution), that is, the exposure of useful minerals of their recovery, which is decisive in terms of losses expressed through ϑ .

The properties of the Madej curve in system $(\beta - \alpha, \text{unit losses})$, where unit losses

$G_w = \frac{\eta}{\beta - \alpha} = \frac{100 - \varepsilon}{\beta - \alpha}$ are shown in Fig. 2c. It is a rather complex separation curve. The fig-

ure presents ways of the correct determination of the optimal separation point (the greatest curvature and the so-called central point) on the basis of beneficiation products. It should be noted that according to Madej (Madej 1978), it is also possible to determine a different

optimal point meeting different criteria from the separation curve $\frac{\eta}{\beta - \alpha} = f(\beta - \alpha)$. It is the

point obtained as a result of the approximation of real beneficiation points by a parabola and the determination of a minimum point (Fig. 3)

The use of function $G_w(\beta - \alpha)$ is in accordance with the proposed concept of the beneficiation evaluation through factors analyzing benefits resulting from the applied beneficiation method in the sense of an increment or decrease of amount of metal in concentrate or tailings.

The copper ore beneficiation results were obtained within the research work (Research Project O/ZWR 2011). Investigations were conducted with a view of identifying the effect of

undergrinding and overgrinding on technological factors of flotation. These investigations were carried out both in terms of specifying minerals' liberation and their particle size distribution through mineralogical tests of products and through specifying beneficiation characteristics by means of laboratory flotation (Research Project O/ZWR 2011; Foszcz 2013). The effect of grinding on the beneficiation characteristics was investigated during the realization of this paper for the product from the technological process. Investigations presented in this article were conducted for the regrinding mill product- a sample marked as Feed-1 and for product ground below 0.075 mm (marked Feed-2) and 0.045 mm (marked Feed-3).

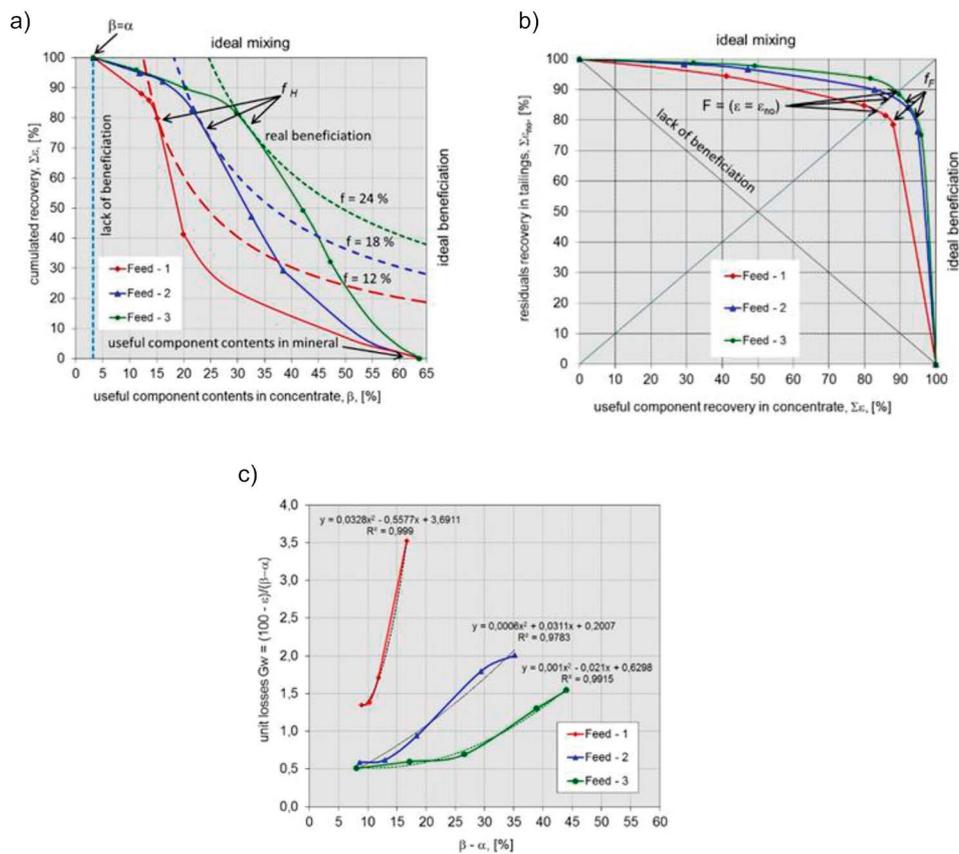


Fig. 2. Upgrading curves and optimal points determined on their basis
 a – the Halbach curve, b – the Fuerstenau curve, c – the Madej curve and their polynomial approximations;
 Feed-1 – underflow of regrinding mill without grinding;
 Feed-2 – underflow of regrinding mill comminuted below 0.075 mm;
 Feed-3 – underflow of regrinding mill comminuted below 0.045 mm

Rys. 2. Krzywe wzbogalności i punkty optymalne określone na podstawie
 a – krzywej Halbicha, b – krzywej Fuerstenau'a, c – krzywej Madeja oraz ich aproksymacji;
 Feed-1 – wylew młyna domielającego bez mielenia;
 Feed-2 – wylew młyna domielającego rozdrobniony do rozmiaru poniżej 0,075 mm;
 Feed-3 – wylew młyna domielającego rozdrobniony poniżej 0,045 mm

3. The beneficiation analysis based on the sum of recovery of the useful component in concentrate and recovery of the remaining components in tailings

Utilizing formulas (4) we can obtain a quadratic form

$$S = \frac{\alpha - \vartheta}{\beta - \vartheta} \frac{\beta}{\alpha} \cdot 100 + \left(100 - \frac{\alpha - \vartheta}{\beta - \vartheta} \cdot 100 \right) \frac{100 - \vartheta}{100 - \alpha} - 100 \quad (8)$$

$$S = -\varepsilon^2 \frac{(100 - \vartheta)(\beta - \vartheta)}{100(100 - \alpha)(\alpha - \vartheta)} + \varepsilon + 100 \frac{100 - \vartheta}{100 - \alpha} - 100 \quad (9)$$

where S is a sum of recovery and residuals recovery in tailings.

Trajectories obtained for formulas (8) and (9) are presented in Figs. 3 and 4.

Curves $S = f(\beta)$ and $S = f(\varepsilon)$ are the flat interpretation of trajectories determined by the flotation on surface $S = f(\beta, \vartheta)$ (Figs. 4 and 5). Curve $S = f(\beta)$ has a maximum which can become a plateau with the progressing comminution of the feed whose coordinates depend on the ore mineralogical and lithological structure (Fig. from theoretical calculations). A plateau appears as a result of equal increases in the recovery of non-useful parts in tailings and decreases in the recovery of the useful component in concentrate, that is, the appearance of

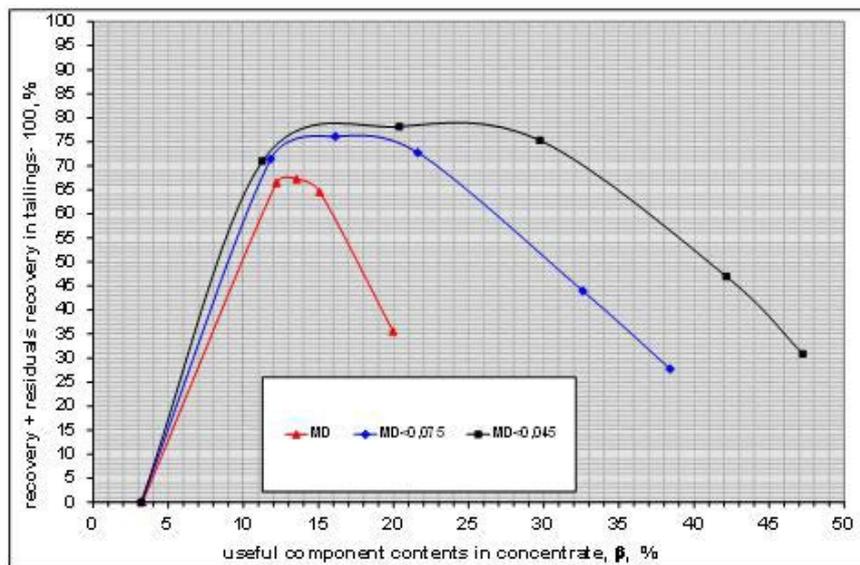


Fig. 3. Projections of trajectories obtained in the multi-product flotations onto a plane (β, S)

Rys. 3. Rzuty trajektorii otrzymanych podczas flotacji wieloproduktowej na powierzchni (β, S)

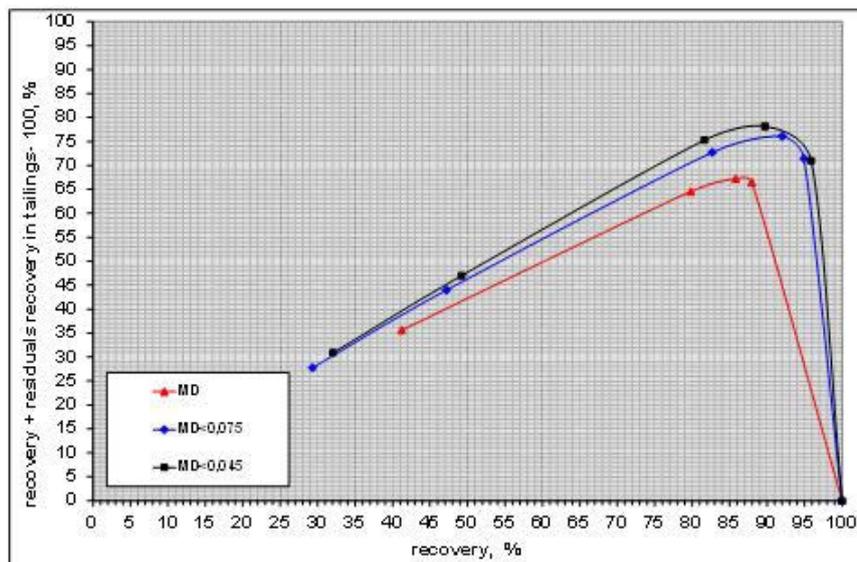


Fig. 4. Projections of trajectories obtained in the multi-product flotations onto a plane (ϵ, S)

Rys. 4. Rzuty trajektorii otrzymanych podczas flotacji wieloproduktowej na powierzchnię (ϵ, S)

particles circulating between concentrate and tailings in appropriate proportions. These are intergrowth particles, fine particles of a useful mineral or additionally activated particles, e.g. of slate, coal. The appearance of a plateau is evidence to the additional contamination of concentrate and should be taken into account during the organization of flotation (selection of the reagents' amount, flotation time).

Concluding considerations – summary

Summing up the presented considerations, it must be underlined once more that the notion of beneficiation is primarily qualitative in nature and considering it in terms of quantity is largely based on arbitrariness related to the criterion or evaluation coefficient selection. Trajectories obtained during flotations depend on the sample preparation for analyzing and selection of factor w and their location on the factor surface depends on the ore nature (structure). The selection of the characteristic point is also arbitrary in nature, there is practically no single convincing justification for the selection. A defined criterion is needed and the characteristic point will be the result of the criterion.

Figures related to the sum of recovery clearly expose interpretive ambiguity of quantitative results – what the particular results characterizing extrema mean (Fig. 5), which are more beneficial, and how they are translated into the applicable technology. It is clear that the results of fractionated flotation have much valuable information which is a basis for planning

the system of processes realizing the planned beneficiation technology, yields, number of cleaning steps, etc. Also, very rough forecasting of the beneficiation time effects is possible. The accuracy of forecasting depends on the precise analysis of components of vector (β_i, ϑ_i) and its changes during experimental flotation.

The paper is a result of realizing statutory project No. 11.11.100.276.

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**A GEOMETRIC APPROACH TO EVALUATING THE RESULTS
OF POLISH COPPER ORES BENEFICIATION****Abstract**

The separation or beneficiation processes are conducted in many devices and concern many various types of minerals and raw materials. The aim of conducting these processes is always to achieve the best possible results allowing as much of the useful component as possible to be obtained by maintaining reasonable costs of the process. Therefore, it is important to have the possibility to monitor the process effects and to have efficient tools to evaluate the course of it. Generally, the ore's ability to partition into concentrate and tailings is called its efficiency, upgradeability etc. It can be said that there is no unambiguous measure of upgradeability and there are many factors in use which enable to evaluate it qualitatively. Among them are such commonly known parameters as: recovery, losses, yield, upgrading ratio and many others. They are based on three principal parameters that is the average content of the useful component α , the contents of this component in concentrate β and the contents of this component in tailings ϑ . For a given ore (assuming that $\alpha = \text{constant}$), the multi-product separation results can be treated as points of a trajectory located on the surface of factor w in a three dimensional space (β, ϑ, w). The course of the trajectory depends on the ore petrographic and mineralogical properties preparation for the process. For these reasons, searching for optimal (potential) possibilities of the ore is relative, which is presented in the example of Halbich, Fuerstenau and Madej upgrading curves. Such curves are efficient tools to evaluate the course of a separation (beneficiation) process and each of their types allow the effects to be shown in different perspective. Apart from this, they allow also the optimal feed conditions to conduct a certain process with aim of achieving the expected results to be found. Furthermore, the effect of the ore preparation on the flotation results, on the sum of recoveries of the useful component in concentrate and residual recovery in tailings is presented in the paper. The results indicated that any additional contamination of concentrate should be taken into account during the organization of the flotation process. In this way, the results of fractionated flotation have much valuable information to establish the course of the process.

Key words: copper upgradeability, copper ore, upgrading curves, optimal beneficiation

GEOMETRYCZNA OCENA WYNIKÓW WZBOGACANIA POLSKICH RUD MIEDZI**Streszczenie**

Procesy rozdziału czy wzbogacania prowadzone są za pomocą wielu typów urządzeń i dotyczą wielu różnorodnych minerałów i surowców. Celem prowadzenia tych procesów jest zawsze uzyskanie możliwie najlepszych wyników, które umożliwią uzysk tak dużej ilości składnika użytecznego, jak tylko jest to możliwe, przy utrzymaniu rozsądnych kosztów prowadzenia procesu. Zatem istotne jest, aby istniała możliwość monitorowania efektów procesu oraz aby dysponować efektywnymi narzę-

dziami oceny jego przebiegu. Ogólnie, zdolność rudy do rozdziału na produkty, którymi są koncentrat i odpad nazywa się jego wzbogalnością. Można powiedzieć, że nie istnieje jedna uniwersalna miara wzbogalności, a w użyciu jest wiele wskaźników, które umożliwiają jej jakościową ocenę. Między nimi są tak powszechnie znane wskaźniki, jak uzysk, straty, wychód, wskaźnik wzbogacania oraz wiele innych. Bazują one na trzech głównych parametrach, którymi są średnia zawartość składnika użytecznego w nadawie α , zawartość tego składnika w koncentracie β oraz zawartość tego składnika w odpadzie ϑ . Dla konkretnej rudy (przy przyjęciu, że $\alpha = \text{constant}$) wyniki rozdziału na wiele produktów można traktować jako punkty na trajektorii, zlokalizowane na powierzchni wskaźnika w w trójwymiarowej przestrzeni (β , ϑ , w). Przebieg trajektorii zależy od przygotowania właściwości petrograficznych i mineralogicznych rudy do procesu. Z tych powodów poszukiwanie optymalnych (potencjalnych) możliwości wzbogacania rudy jest relatywne, co można zaobserwować na przykładzie krzywych wzbogalności Halbicha, Fuerstenau'a i Madeja. Takie krzywe są efektywnymi narzędziami oceny przebiegu procesu rozdziału (wzbogacania) i każdy z ich typów pozwala na przedstawienie efektów z innej perspektywy. Ponadto, pozwalają one również na znalezienie optymalnych warunków nadawy do prowadzenia danego procesu z celem osiągnięcia oczekiwanych wyników. Co więcej, wpływ przygotowania rudy na wyniki flotacji, sumę uzysków składnika użytecznego w koncentracie oraz uzysk reszt w odpadach zostały zaprezentowane w artykule.

Słowa kluczowe: wzbogalność rudy miedzi, ruda miedzi, krzywe wzbogalności, optymalne wzbogacanie