

# A Verification Technique for Multiple Soft Fault Diagnosis of Linear Analog Circuits

Michał Tadeusiewicz and Marek Ossowski

**Abstract**—The paper deals with multiple soft fault diagnosis of linear analog circuits. A fault verification method is developed that allows estimating the values of a set of the parameters considered as potentially faulty. The method exploits the transmittance of the circuit and is based on a diagnostic test leading to output signal in discrete form. Applying Z-transform a diagnostic equation is written which is next reproduced. The obtained system of equations consisting of larger number of equations than the number of the parameters is solved using appropriate numerical approach. The method is adapted to real circumstances taking into account scattering of the fault-free parameters within their tolerance ranges and some errors produced by the method. In consequence, the results provided by the method have the form of ranges including the values of the tested parameters. To illustrate the method two examples of real electronic circuits are given.

**Keywords**—analog circuits, fault diagnosis, linear circuits, multiple soft faults, verification technique

## I. INTRODUCTION

**F**AULT diagnosis of analog circuits plays a key role in electronic circuit design [9], [13]. Unlike digital circuits, methodology for fault diagnosis and testing of the analog ones remains relatively underdeveloped due to numerous limitations, e.g. technological variations of the parameters within their tolerance ranges, a limited number of nodes accessible for measurement, and nonlinear nature of the test equations. For these reasons in many cases analog testing relies on a black-box approach where specifications of the circuitry are verified without paying attention to the structure. In consequence, fault diagnosis of analog circuits has considerable interest, leading to numerous publications, during the last years e.g. [1], [5], [7], [10], [12], [17-18], [21], [23].

If a circuit parameter is drifted from its tolerance range but does not produce any topological changes the fault is called soft or parametric. If a fault is open or short it is called hard or catastrophic. The fault diagnosis including detecting, locating and estimating of the values of the faulty elements can be performed using different diagnostic methods. If most of the circuit simulations take place after any testing, the diagnostic method is classified as the simulation after test (SAT) approach, otherwise it is classified as the simulation before test (SBT) approach. In the case of soft fault diagnosis SAT

approach is preferred. During the last decades a wide variety of methods, concepts and techniques have been adapted to soft fault diagnosis, e.g. the Woodbury formula in matrix theory [24-25], support vector machine [15], [20], linear programming [26], homotopy method [27], fuzzy approach [3], wavelet transform [1-2], neural networks [1], [12], [16], frequency response function [11], [17], V-transform of polynomial coefficients [21], evolutionary algorithm [12], Volterra series [7]. Recently several papers have been focused on multiple soft fault diagnosis in analog integrated circuits designed in micrometer and submicrometer technology, e.g. [23], [28-29].

Numerous results in the diagnosis area relate to the circuits with single defects. Multiple fault diagnosis is more complex and insufficiently resolved. This paper is devoted to multiple soft fault diagnosis of analog linear electronic circuits. Usually not all parameters in the circuit can be tested whatever diagnostic algorithm employed. Information on how many and which parameters can be unambiguously diagnosed is provided by testability analysis, e.g. [8].

Many methods for fault diagnosis of analog circuits exploit SAT approach and a fault verification concept. The fault verification technique is based on the hypothesis that some parameters can be faulty and the others are nominal or within their tolerance ranges. In consequence, the number of unknown variables is reduced. Next the hypothesis is verified on the basis of performed measurements. The number of the measurement data points can be greater than the number of parameters. In such a case overdetermined set of diagnostic equations is produced.

Fault verification techniques have been applied for soft and hard fault diagnosis of analog circuits in the last decades, e.g. [4], [6], [19], [22-23], [27], [29-31]. Different methods and computational techniques have been developed to solve the corresponding diagnostic equations based on measurement test performed in DC, AC, or transient state. Most of them exploit DC or AC test. In real nonlinear circuits the diagnostic equations cannot be presented in an explicit analytical form. For example, the transistors of modern CMOS circuits are characterized by BSIM 4.6 model which is described by several hundred equations, mostly nonlinear. In such a case no explicit analytical representation of the diagnostic equations exists, even in the case of small sized circuits, and some sophisticated methods are needed to perform soft fault diagnosis, based on verification concept, e.g. [28-29]. Linear circuits can be described in symbolic form in DC, AC, or transient state, using node method or state approach, e.g. [5-6], [22]. Also transmittance which characterizes linear time invariant circuit in the frequency domain can be exploited for soft fault diagnosis, e.g. [4]. Some diagnostic methods are used

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Michał Tadeusiewicz is with Department of Electrical, Electronic, Computer and Control Engineering, Łódź University of Technology, Poland, (e-mail: [michal.tadeusiewicz@p.lodz.pl](mailto:michal.tadeusiewicz@p.lodz.pl)).

Marek Ossowski is with Department of Electrical, Electronic, Computer and Control Engineering, Łódź University of Technology, Poland, (e-mail: [marek.ossowski@p.lodz.pl](mailto:marek.ossowski@p.lodz.pl)).

for self-testing of analog parts of the mixed-signal systems, e.g. [5], [31].

The main achievement of this paper is a new method, belonging to the class of fault verification techniques, described in Sections II and III. The method can be applied to middle sized linear and time-invariant analog circuits with faulty parameters, which according to testability analysis can be unambiguously diagnosed. Usually the method requires one accessible input node and one accessible output node only. It asks for transmittance in a symbolic form. The method is effective, easy to implement and considers conditions occurring in real circumstances.

## II. THE DIAGNOSTIC METHOD

Let us consider a linear time-invariant dynamic circuit. Let  $p_1, \dots, p_n$  be the parameters which according to the information provided by the testability analysis, can be tested. We want to estimate values of the parameters using the SAT approach and a fault verification technique. These parameters, whose values exceed the tolerance ranges are considered as faulty. The circuit parameters that are to be diagnosed  $p_1, \dots, p_n$  will be considered as elements of the vector  $\mathbf{p} = [p_1 \dots p_n]^T$ , where T denotes transposition. Let us consider the circuit as a linear time invariant (LTI) two-port shown in Fig. 1. Assume temporarily that the circuit parameters, except  $p_1, \dots, p_n$ , have nominal values. We find the transmittance  $Y(s)/X(s)$  of this two-port in a symbolic form. This transmittance labeled  $H(s, \mathbf{p})$  depends on the unknown parameters  $p_1, \dots, p_n$  and the complex variable  $s$ . Let  $x(t)$  be a rectangular pulse signal with the height  $A$  and the duration time  $T_0$ . Its magnitude spectrum is labeled  $|X_F(j\omega)|$ , whereas the magnitude spectrum of the transmittance, describing the circuit with nominal parameters is  $|H(j\omega, \mathbf{p}^{\text{nom}})|$ , where  $\mathbf{p}^{\text{nom}} = [p_1^{\text{nom}} \dots p_n^{\text{nom}}]^T$ .

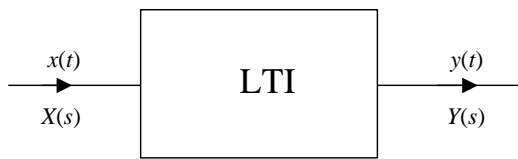


Fig. 1. Linear time invariant two-port

We find the transmittance  $\hat{H}(z, \mathbf{p})$  in Z-transform scenario using the equation [14, p. 619]

$$\hat{H}(z, \mathbf{p}) = H(s, \mathbf{p}) \Big|_{s = \frac{1}{T_s} \ln z}, \quad (1)$$

where  $T_s$  is the sample spacing. To determine  $T_s$  we evaluate the highest frequencies  $\tilde{\omega}_0$  and  $\tilde{\tilde{\omega}}_0$  of the infinite amplitude spectra  $|H(j\omega, \mathbf{p}^{\text{nom}})|$  and  $|X_F(j\omega)|$  so that above them the spectra are sufficiently small. Next we find  $\omega_0 = \max \{ \tilde{\omega}_0, \tilde{\tilde{\omega}}_0 \}$ . The sampling frequency  $f_s$  is specified by the equation

$f_s = M \frac{\omega_0}{2\pi}$ , where  $M > 2$ . Hence, the sample spacing  $T_s$  is equal to  $\frac{1}{f_s}$ .

Let  $x(n)$  be the sampled signal  $x(t)$  and the measured discrete output signal be  $y(n)$ , respectively. Their Z-transforms will be labeled  $X(z)$  and  $Y(z)$ . We consider the truncated formula  $\hat{Y}(z)$  consisting of  $N+1$  first terms of  $Y(z)$

$$\hat{Y}(z) = \sum_{k=0}^N y(k) z^{-k}, \quad (2)$$

where the number  $N$  is chosen as described in Section III. Replacing  $Y(z)$  by the approximate formula (2), yields

$$\hat{Y}(z) = \hat{H}(z, \mathbf{p}) X(z). \quad (3)$$

Equation 3 will be reproduced using  $m > n$  values of  $z$ . As a result the system of  $m$  equations is generated

$$\begin{aligned} \hat{Y}(z_1) &= \hat{H}(z_1, \mathbf{p}) X(z_1) \\ \hat{Y}(z_2) &= \hat{H}(z_2, \mathbf{p}) X(z_2) \\ &\vdots \\ \hat{Y}(z_m) &= \hat{H}(z_m, \mathbf{p}) X(z_m), \end{aligned} \quad (4)$$

with  $n$  unknown variables  $p_1, \dots, p_n$ . Under the denotation

$$f_j(\mathbf{p}) = \hat{Y}(z_j) - \hat{H}(z_j, \mathbf{p}) X(z_j) \quad j = 1, \dots, m, \quad (5)$$

the set of equations (4) can be rewritten in the compact form

$$\mathbf{f}(\mathbf{p}) = \mathbf{0}, \quad (6)$$

where  $\mathbf{f}(\mathbf{p}) = [f_1(\mathbf{p}) \dots f_m(\mathbf{p})]^T$  and  $\mathbf{0} = [0 \dots 0]^T$ . Let us expand  $\mathbf{f}(\mathbf{p})$  into the Taylor series about  $\mathbf{p}^{(0)} = \mathbf{p}^{\text{nom}}$  and neglect the higher order terms

$$\mathbf{f}(\mathbf{p}) = \mathbf{f}(\mathbf{p}^{(0)}) + \left[ \frac{d\mathbf{f}}{d\mathbf{p}}(\mathbf{p}^{(0)}) \right] (\mathbf{p} - \mathbf{p}^{(0)}). \quad (7)$$

Substituting (7) into (6) and performing simple rearrangements yields

$$\mathbf{F}^{(1)} \mathbf{p} = \mathbf{b}^{(1)}, \quad (8)$$

where  $\mathbf{F}^{(1)} = \frac{d\mathbf{f}}{d\mathbf{p}}(\mathbf{p}^{(0)})$  is  $m \times n$  Jacoby matrix and  $\mathbf{b}^{(1)} = \mathbf{F}^{(1)} \mathbf{p}^{(0)} - \mathbf{f}(\mathbf{p}^{(0)})$  is an  $m$ -vector. We solve equation (8) using the method of normal equation [32]

$$\left( \mathbf{F}^{(1)} \right)^T \mathbf{F}^{(1)} \mathbf{p} = \left( \mathbf{F}^{(1)} \right)^T \mathbf{b}^{(1)}, \quad (9)$$

where  $\left( \mathbf{F}^{(1)} \right)^T \mathbf{F}^{(1)}$  is an  $n \times n$  matrix and  $\left( \mathbf{F}^{(1)} \right)^T \mathbf{b}^{(1)}$  is an  $n$ -vector. Let us denote  $\mathbf{D}^{(1)} = \left( \mathbf{F}^{(1)} \right)^T \mathbf{F}^{(1)}$  and  $\mathbf{c}^{(1)} = \left( \mathbf{F}^{(1)} \right)^T \mathbf{b}^{(1)}$ , then the solution vector  $\mathbf{p}$ , labeled  $\mathbf{p}^{(1)}$ , meets the equation

$$\mathbf{D}^{(1)} \mathbf{p}^{(1)} = \mathbf{c}^{(1)}. \quad (10)$$

Equation (10) is a compact representation of  $n$  individual equations with  $n$  unknown variables  $p_1^{(1)}, \dots, p_n^{(1)}$ . To

determine  $\mathbf{p}^{(1)}$  the Gaussian elimination procedure can be used. Next we replace  $\mathbf{p}^{(0)}$  with  $\mathbf{p}^{(1)}$  and repeat the above described procedure, finding  $\mathbf{p}^{(2)}$  that meets the equation

$$\mathbf{D}^{(2)}\mathbf{p}^{(2)} = \mathbf{c}^{(2)}, \quad (11)$$

where  $\mathbf{D}^{(2)} = (\mathbf{F}^{(2)})^T \mathbf{F}^{(2)}$ ,  $\mathbf{c}^{(2)} = (\mathbf{F}^{(2)})^T \mathbf{b}^{(2)}$ ,  $\mathbf{F}^{(2)} = \frac{d\mathbf{f}}{d\mathbf{p}}(\mathbf{p}^{(1)})$ ,  $\mathbf{b}^{(2)} = \mathbf{F}^{(2)}\mathbf{p}^{(1)} - \mathbf{f}(\mathbf{p}^{(1)})$ . This iteration process is continued generating the sequence  $\mathbf{p}^{(1)}, \mathbf{p}^{(2)}, \mathbf{p}^{(3)}, \dots$ , until

$$\|\mathbf{p}^{(l)} - \mathbf{p}^{(l-1)}\|_2 = \sqrt{\sum_{j=1}^n (p_j^{(l)} - p_j^{(l-1)})^2} < \delta_1, \quad (12)$$

and

$$\|\mathbf{f}(\mathbf{p}^{(l)})\|_2 = \sqrt{f_1^2(\mathbf{p}^{(l)}) + \dots + f_m^2(\mathbf{p}^{(l)})} < \delta_2, \quad (13)$$

where  $\delta_1$  and  $\delta_2$  are accepted errors. Then,  $\mathbf{p}^{(l)}$  is considered as a solution of equation (6).

Note

Creating  $m > n$  equations (4) in order to find  $n$  unknown parameters needs an explanation. If  $n$  equations (4) had been formed then  $\mathbf{f}(\mathbf{p})$  would have been an  $n$ -vector function and  $\mathbf{F}^{(l)}$  an  $n \times n$  matrix. In such a case the equation (8) could be directly solved to find  $\mathbf{p}^{(l)}$ . Unfortunately, if the rank of the matrix  $\mathbf{F}^{(l)}$  is smaller than  $n$ , this equation does not have a unique solution. Even if the matrix is nearly full rank the solution is unstable and the iteration method fails. However, if we create  $m > n$  equations (4) there is much more probable that there are  $n$  among  $m$  rows of matrix  $\mathbf{F}^{(l)}$  that are linearly independent and the rank of this matrix is equal to  $n$ . In such a case matrix  $\mathbf{D}^{(l)} = (\mathbf{F}^{(l)})^T \mathbf{F}^{(l)}$  is nonsingular and the equation (10) has a unique solution. The same discussion relates to matrices  $\mathbf{F}^{(2)}, \mathbf{F}^{(3)}, \dots$ . In consequence, the possibility of failure of the iteration method is considerably reduced and its efficiency increases. Many numerical experiments, performed with  $m = 2n$ , fully support this statement.

### III. IMPROVEMENT OF THE METHOD

The above developed diagnosis method works in idealized frameworks and omits some conditions occurring in real circumstances. The main of them are listed below.

- (i) The parameters considered as fault-free do not have nominal values but are scattered within their tolerance ranges.
- (ii) A truncation error occurs due to replacing  $Y(z)$  by  $\hat{Y}(z)$  consisting of finite number of terms.
- (iii) An error is produced by the measurement equipment leading to distortion of  $y(n)$ .

To adapt the method to a realistic framework  $\tilde{M}$  sets of the values of the parameters considered as fault-free are created

by random selection from their tolerance ranges, assuming uniform distribution. For each of the sets the diagnosis method is performed. As a result any of the diagnosed parameters is specified by range  $[p_j^-, p_j^+]$ ,  $j=1, \dots, n$ , of its possible values rather than by a single value. To estimate  $p_j$  the average value  $p_j^{\text{av}}$  over the range  $[p_j^-, p_j^+]$  can be used.

To minimize the truncation error in equation (2) a proper number  $N$  must be chosen together with the values of the variable  $z: z_1, \dots, z_m$ . They are picked at the preliminary stage of the method. For this purpose the circuit with nominal parameters is considered and the following procedure is applied.

1. In the circuit with nominal parameters, driven by the input signal, find the output signal  $y(t)$  and estimate the duration time  $\tilde{T}$  of this signal.
2. Pick minimum value  $z^- > 1$  and maximum value  $z^+$  of  $z$ .
3. Find the preliminary number  $N$ , labeled  $N_o$ , of the terms in equation (2),  $N_o = \text{round}\left(\frac{\tilde{T}}{T_s}\right)$ .
4. Form the sequence  $\hat{Y}(z^-, N_j)$  for  $j=1, 2, \dots$ , where  $N_j = N_o + j10$ , and choose  $\min\{j\}$  for which  $|\hat{Y}(z^-, N_j) - \hat{Y}(z^-, N_{j-1})| < \varepsilon_N$ , where  $\varepsilon_N$  is a small positive number.
5. Set  $N = N_j$ .
6. Pick a positive value  $\Delta z$  and find  $m$  values of  $z$  so that  $z_{k+1} = z_k + \Delta z$ ,  $k=1, \dots, m-1$ , where  $z_1 \geq z^-$ ,  $z_m \leq z^+$  and  $\left| \frac{\hat{Y}(z_k) - \hat{H}(z_k, \mathbf{p}^{\text{nom}})X(z_k)}{\hat{H}(z_k, \mathbf{p}^{\text{nom}})X(z_k)} \right| < \varepsilon_z$ ,  $k=1, \dots, m$ , where  $\varepsilon_z$  is a small positive number. To satisfy these constraints several trials of selecting  $\Delta z$  and  $z_1$  may be necessary.

### IV. NUMERICAL EXAMPLES

To illustrate the method described in Sections II and III we consider two numerical examples. The diagnosed circuits are shown in Figures 2, and 3, where nominal values of the parameters are indicated. All the operational amplifiers included in the circuits are characterized by the ideal model. The computations were executed on a PC with the processor Intel (R) Xeon (R) E 3-1230 using MATLAB R2012a with Symbolic Toolbox.

#### Example 1

Let us consider the Sallen-Key bandpass filter shown in Fig. 2. We take into account ten sets of  $n=3$  parameters:  $\{R_1, R_2, R_5\}$ ,  $\{R_3, R_5, C_2\}$ ,  $\{R_2, R_3, R_5\}$ ,  $\{R_2, R_3, R_4\}$ ,  $\{R_1, R_2, C_1\}$ ,  $\{R_1, R_3, R_5\}$ ,  $\{R_1, R_3, C_1\}$ ,  $\{R_1, R_4, C_1\}$ ,  $\{R_1, R_5, C_1\}$ ,  $\{R_3, R_4, C_2\}$ , which according to the testability analysis [8] can be unambiguously diagnosed. For each of the sets three combinations of the parameter values are assumed. Thus, the total number of the diagnoses is 30.

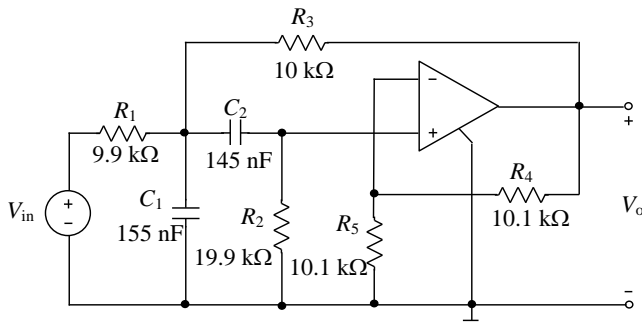


Fig. 2. The Sallen-Key bandpass filter.

At the preliminary stage the transmittance  $\frac{V_o(s)}{V_{in}(s)}$  in symbolic

form is determined. In any case the rectangular pulse input voltage is set with the height 1 V and the duration time  $T_o = 0.2$  ms. The tolerance  $\varepsilon$  of the parameters is 1% for resistors and 3% for capacitors.

All the above defined 30 cases were laboratory tested using the measurement system consisting of Tektronix DPO4032 digital oscilloscope with P2100 passive probes and Tektronix AFG3022 function generator. To estimate the parameter values the method proposed in this paper is used with the constants:  $m = 2n = 6$ ,  $\varepsilon_N = 3 \cdot 10^{-7}$ ,  $\varepsilon_z = 3 \cdot 10^{-3}$ ,  $f_s = 10^5$  Hz. In each of the cases  $\tilde{M} = 100$  sets of the values of the parameters considered as fault-free are created by random selection from their tolerance ranges, assuming uniform distribution. Every time the diagnosis method is applied leading to the ranges  $[p_j^-, p_j^+]$  of the three parameters included in the diagnosed set.

Next the average value  $p_j^{av}$  of the values belonging to  $[p_j^-, p_j^+]$  is calculated and the relative error

$$\eta_j = \left| \frac{p_j^{actual} - p_j^{av}}{p_j^{actual}} \right| 100\%$$

is determined.

For statistical analysis all the above defined sets of the parameters were considered, each one with three combinations of the parameter values. Thus, the total number of the diagnoses is 30, leading to 90 values of the diagnosed parameters. The method gives the following results. In 73.3% of the cases the relative error  $\eta_j$  does not exceed 6%, in 20% it is greater than 6% but smaller than 10%. In 6.7% the error exceeds 10%. The maximum error is 14.1%. To shed more light on the obtained results we consider in detail the diagnosis of the set of parameters  $\{R_1, R_4, C_1\}$  with three combinations of the parameter values. One of the combinations is very close to the nominal, whereas the parameters of the others are deviated from the nominal values up to 42%. The parameters of the fault-free elements are as follows:  $R_2 = 19.9$  kΩ,  $R_3 = 10.01$  kΩ,  $R_5 = 10.13$  kΩ,  $C_2 = 144$  nF. The results are summarized in Tables I-III.

The total error is produced by the proposed method and by the measurement system. To estimate the error produced by the method, the same diagnoses were performed using numerical tests. In such a case  $y(n)$  is found numerically assuming the reading error 1 mV. Similarly as in the laboratory experiments the diagnosis method is applied  $\tilde{M} = 100$  times in each of the 30 diagnoses and the average value  $p_j^{av}$  as well as the relative error  $\eta_j$  are calculated. The statistical results are as follows. In 82.2% of the cases the error  $\eta_j$  does not exceed 2%, in 13.3% it is greater than 2% but smaller than 4%. The maximum error is 5.4%. The numerically obtained results, relating to the same set  $\{R_1, R_4, C_1\}$  are summarized in Tables IV-VI, which correspond to the Tables I-III. The above results show that the error produced by the measurement system dominates the error of the method.

TABLE I  
DIAGNOSIS OF THE SET OF PARAMETERS  $\{R_1, R_4, C_1\}$  USING LABORATORY TEST

Parameters $p_j$	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_1$ [Ω]	9910.00	9365.67	10133.63	9742.59	1.69
$p_2 = R_4$ [Ω]	10080.00	9745.91	10421.07	10038.44	0.41
$p_3 = C_1$ [nF]	156.00	153.73	170.37	161.87	3.77

TABLE II  
DIAGNOSIS OF THE SET OF PARAMETERS  $\{R_1, R_4, C_1\}$  USING LABORATORY TEST

Parameters $p_j$	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_1$ [Ω]	7470.00	6494.78	7184.18	6819.17	8.71
$p_2 = R_4$ [Ω]	12930.00	11947.15	12970.71	12477.57	3.50
$p_3 = C_1$ [nF]	220.00	216.45	248.97	232.78	5.81

TABLE III  
DIAGNOSIS OF THE SET OF PARAMETERS  $\{R_1, R_4, C_1\}$  USING LABORATORY TEST

Parameters $p_j$	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_1$ [Ω]	7470.00	6695.22	7331.45	7032.95	5.85
$p_2 = R_4$ [Ω]	10080.00	9336.91	10119.51	9657.01	4.20
$p_3 = C_1$ [nF]	220.00	212.06	238.71	224.21	1.91

TABLE IV  
DIAGNOSIS OF THE SET OF THREE PARAMETERS  $\{R_1, R_4, C_1\}$  USING NUMERICAL TEST

Parameters $p_j$	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_1$ [Ω]	9910.00	9572.22	10384.71	10000.08	0.91
$p_2 = R_4$ [Ω]	10080.00	9608.03	10340.09	9971.20	1.08
$p_3 = C_1$ [nF]	156.00	146.34	163.13	154.06	1.25

TABLE V  
 DIAGNOSIS OF THE SET OF THREE PARAMETERS  $\{R_1, R_4, C_1\}$  USING  
 NUMERICAL TEST

Parameters $p_j$	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_1 [\Omega]$	7470.00	7208.82	7865.65	7546.89	1.03
$p_2 = R_4 [\Omega]$	12930.00	12354.11	13416.31	12784.79	1.12
$p_3 = C_1 [\text{nF}]$	220.00	204.66	231.65	216.82	1.45

TABLE VI  
 DIAGNOSIS OF THE SET OF THREE PARAMETERS  $\{R_1, R_4, C_1\}$  USING  
 NUMERICAL TEST

Parameters $p_j$	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_1 [\Omega]$	7470.00	7245.94	7884.28	7523.51	0.72
$p_2 = R_4 [\Omega]$	10080.00	9662.31	10382.29	9975.82	1.03
$p_3 = C_1 [\text{nF}]$	220.00	205.32	229.60	217.88	0.96

The average computation time consumed by the method for finding the values of the three parameters considered as potentially faulty, for given values of the parameters considered as fault-free is 1.5s. The time of the preliminary calculations, including mainly finding of the symbolic transmittance, is 2.1 s.

**Example 2**

In the low pass filter shown in Fig. 3 we consider two sets of the parameters  $\{R_4, R_7, R_8, C_1, C_3\}$ , and  $\{R_4, R_8, C_1, C_3, C_4\}$ . According to the testability analysis [8], they can be unambiguously diagnosed. For this purpose the method developed in this paper is applied using numerical tests. We assume the reading error 0.1 mV,  $\varepsilon = 0.1\%$  for the resistors and capacitors,  $\tilde{M} = 25$ ,  $m = 2n = 10$ ,  $f_s = 10^5$  Hz. The other constants are similar as in Example 1. The results are presented in Tables VII-VIII.

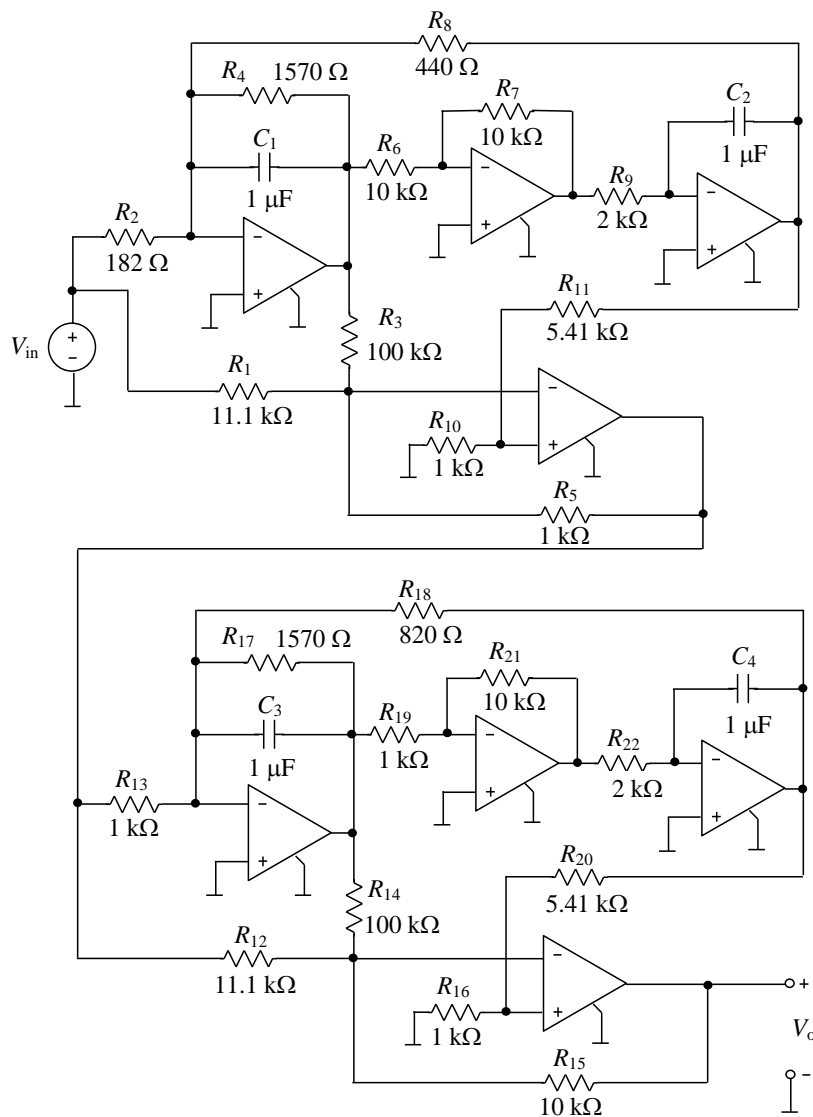


Fig. 3 A low-pass filter

TABLE VII  
DIAGNOSIS OF THE SET OF THREE PARAMETERS  $\{R_4, R_8, C_1, C_3, C_4\}$  USING

Parameters $p_j$	NUMERICAL TEST				
	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_4 [\Omega]$	1350.00	1348.65	1357.91	1352.52	0.19
$p_2 = R_8 [\Omega]$	380.00	379.18	381.07	380.18	0.05
$p_3 = C_1 [nF]$	850.00	841.93	851.72	847.68	0.27
$p_4 = C_3 [nF]$	900.00	826.80	999.83	923.56	2.62
$p_5 = C_4 [nF]$	1150.00	1130.62	1277.31	1176.59	2.31

TABLE VIII  
DIAGNOSIS OF THE SET OF THREE PARAMETERS  $\{R_4, R_7, R_8, C_1, C_3\}$  USING

Parameters $p_j$	NUMERICAL TEST				
	Actual value $p_j^{ac}$	Lower limit $p_j^-$	Upper limit $p_j^+$	Average value $p_j^{av}$	Relative error $\eta_j$
$p_1 = R_4 [\Omega]$	1570.00	1557.37	1595.15	1574.74	0.30
$p_2 = R_7 [\Omega]$	12000.00	11820.39	12082.28	11969.19	0.26
$p_3 = R_8 [\Omega]$	550.00	548.53	551.04	549.78	0.04
$p_4 = C_1 [nF]$	1100.00	1085.32	1108.37	1097.92	0.19
$p_5 = C_3 [nF]$	850.00	726.14	926.92	825.01	2.94

In this case  $\varepsilon = 0.1\%$ , because at less restrictive tolerance the large number of the parameters included in the circuit masks the faults of the diagnosed elements.

The average computation time consumed by the method for finding the values of the five parameters considered as potentially faulty, for given values of the parameters considered as fault-free is 12.3s. The time of the preliminary calculations, including mainly finding of the symbolic transmittance, is 17.7 s.

### V. COMPARISON RESULTS

From among numerous works devoted to multiple soft fault diagnosis reference [6] brings a sound fault verification approach based on the test in transient state, similarly as the method developed in this paper. For convenience the method proposed in this paper will be named method A whereas the method of reference [6] will be named method B. To compare the methods, method B has been implemented and tested using the circuit shown in Fig. 2. Based on the performed comparison the following conclusions can be drawn.

Method A requires transmittance in symbolic form and sampled input voltage and output voltage measured during the test phase. Usually it requires access to one input node and one output node only. The diagnostic equations are created using more data points than the number of unknown parameters. For this purpose an idea is proposed leading to overdetermined system of nonlinear algebraic equations. The developed method for solving this system of equations requires derivatives of some functions. Because they are in symbolic form it is a very simple task. The system of equations is reduced using a transformation called the method of normal equation. Thus, the method A is easy to implement and allows diagnosing different faults in the circuit automatically, without any user's intervention. It asks for standard performance measurement equipment. Moreover, the method takes into

account the tolerances of the parameters considered as fault-free.

Method B requires transient analysis and sensitivity analysis in the time domain, performed numerically, at each iteration. This is time consuming process which needs careful selection of the time step size and error control. On the basis of the sensitivity analysis the parameters and the data points are selected using QR factorization of the sensitivity matrix. The values of the selected parameters are up to dated in each iteration. Method B has been illustrated via a simple exemplary circuit. Unfortunately, for some sets of the parameters which can be unambiguously diagnosed using method A the sensitivity matrix exploited by method B is nearly singular and the convergence problem arises. More complicated circuits may ask for several output signals what complicates the implementation of the method. It does not take into account the tolerances of the parameters considered as fault-free.

Summarizing, the advantages of the method A are as follows. The method offers a new fault verification technique. It does not require sensitivity analyses in the time domain and is easy to implement. The method avoids the matrix singularity and takes into account the perturbations of the fault-free parameters within their tolerance ranges. The effectiveness of the method has been verified using several exemplary circuits.

### VI. CONCLUSION

The method developed in this paper is focused on a multiple soft fault diagnosis of linear analog circuits. It allows estimating of the values of a set of several parameters considered as potentially faulty in real circumstances, taking into account the variations of the fault-free parameters within the tolerance ranges. The method is simple and easy to implement. The results provided by the method have the form of ranges including the values of the tested parameters. In small and middle sized circuits the accuracy of the obtained results is satisfactory. The evaluated ranges effectively frame the actual values of the parameters and the average values are close to the actual ones. The comparison of laboratory and numerically experiments show that the main error is due to uncertainty in the measurement.

Computational cost of the method's preliminary stage, which consists mainly of the circuit transmittance calculation, is approximately 40% greater than computation cost of the main algorithm. The size of investigated circuits seems to be the main limitation of the proposed method because the available symbolic computation tools, used to find the transmittance, are not effective for larger sized circuits and some estimation algorithms are needed.

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