

# Numerical Study of Forced Vibration Suppression by Parametric Anti-Resonance

Luděk PEŠEK, Petr ŠULC, Ladislav PŮST

*Institute of Thermomechanics AS CR, v.v.i.*

Dolejškova 5, 182 00 Prague 8, Czech Republic; e-mail: {pesek, sulc, pust}@it.cas.cz

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The parametric anti-resonance phenomenon as an active damping tool for suppression of externally excited resonant vibration is numerically studied herein. It is well known fact that the anti-resonance phenomenon, i.e. the stiffness periodic variation by subtractive, combination resonance frequency, brings stabilization and cancelling into self-excited vibrations. But this paper aims at a new possibility of its application, namely a damping of externally excited resonant vibration. For estimation of its effect we come both from a characteristic exponent of the analytical solution and numerical solution of forced vibration of 2DOF linear system with additional parametric excitation. The amplitude suppression owing to the parametric anti-resonance is studied on several parameters of the system: a depth of parametric excitation, mass ratio, damping coefficient and small frequency deviations from the parametric anti-resonance.

**Keywords:** active damping; parametric anti-resonance; external harmonic excitation.

## 1. Introduction

This contribution deals with next possibilities of an application of the additional parametric excitation (TONDL, 2012) as a tool for the suppression of vibration. By adding the parametric excitation, the equations of motion are governed by linear differential equations with periodic time variable coefficients. It is well known that these systems can be unstable due to the action of parametric excitation in the vicinity of the first kind and of combination parametric resonances (e.g. TONDL, 1959; HALANAY, 1966; METTLER, 1965; ŁUCZKO, CZERWIŃSKI, 2015). The combination resonances can be additive or subtractive. It was, however, discovered and mathematically proven (TONDL, 1998a; 1998b; 2000b; TONDL, ECKER, 1999; TONDL *et al.*, 2000) that due to the stiffness periodic variation only for additive, combination resonance the instability interval can exist and that the excitation by subtractive, combination resonance frequencies on the contrary brings stabilization and cancelling (also called quenching) into self-excited vibrations. This phenomenon is called *parametric anti-resonance*. There exists a numerous literature dealing with this problem where for different systems the influence on self-excited vibration

has been analyzed (TONDL, 2000a; 2013; 2008a; 2008b; ECKER *et al.*, 2002; TONDL, ECKER, 2003; TONDL, NABERGOJ, 2004; ECKER, TONDL, 2004; 2005; 2007; 2011; NABERGOJ *et al.*, 2006; 2007; DOHNAL, TONDL, 2009; ECKER, 2010; PUMHOESSEL *et al.*, 2011; TONDL, PUST, 2011; PEŠEK, TONDL, 2012; ECKER, PUMHOESSEL, 2012). Furthermore it was shown (TONDL, PŮST, 2010) that the subtractive, combination resonance can also reduce subharmonic oscillations in externally excited systems. From these findings it is obvious that the parametric anti-resonance excitation brings the additional damping into the system. Therefore we decided to deal herein with possibility to use parametric anti-resonance effect for suppression of externally excited resonant vibration since it can be a remedy for some structures that suffer by a resonant vibration as typical in many cases of rotordynamics (PEŠEK *et al.*, 2015).

## 2. Analytical solution of subtractive, combination resonance for stability analysis

Stability analysis of periodic vibrations of linear mechanical system with additional parametric excita-

tion by TONDL (1959) comes from quasi-normal form of homegenous equations of motion

$$\ddot{x}_s + \Omega_s^2 x_s = \varepsilon \left[ \sum_{k=1}^n (P_{sk}(t)\dot{x}_k + Q_{sk}(t)x_k) \right], \quad (s = 1, 2, \dots, n), \quad (1)$$

where  $P_{sk}(t)$ ,  $Q_{sk}(t)$  are periodic time functions with frequency  $\omega_p$ ,  $\varepsilon$  is a small parameter,  $\Omega_s$  are eigenfrequencies of the abbreviated linear (i.e. no coefficient periodicity) system of  $n$  DOFs. Then the equations of motion Eq. (1) have resonances  $\omega_p = (\Omega_j \pm \Omega_k)/N$  ( $N = 1, 2, \dots$ ). The resonances are called of the first kind and  $N$ -th order for a case  $k = j$  and sign plus between the eigenfrequencies. If  $k \neq j$  then the resonances are called of the second kind or combination parametric (additive or subtractive) resonances and  $N$ -th order.

To solve Eq. (1) analytically, we transform Eq. (1) to equations of the 1th order

$$\begin{aligned} \xi'_s &= i\Omega_s \xi_s \\ &+ \frac{1}{2}\varepsilon \left[ \sum_{k=1}^n \left( Q_{sk}(\xi_k + \eta_k) + \frac{1}{i\Omega_k} P_{sk}(\xi_k - \eta_k) \right) \right], \\ \eta'_s &= -i\Omega_s \eta_s \\ &+ \frac{1}{2}\varepsilon \left[ \sum_{k=1}^n \left( Q_{sk}(\xi_k + \eta_k) + \frac{1}{i\Omega_k} P_{sk}(\xi_k - \eta_k) \right) \right]. \end{aligned} \quad (2)$$

Floquet's theorem designs a solution of Eqs. (2) as functions  $\xi_s = e^{\mu_s \tau} u_s(\tau)$ ,  $\eta_s = e^{\mu_s \tau} v_s(\tau)$ , where  $u_s$ ,  $v_s$  are periodic functions and

$$\mu_s = i(\Omega_s + \kappa_s) \quad (3)$$

is a characteristic exponent of the solution. The exponent consists of the corresponding eigenfrequency  $\Omega_s$  of the abbreviated system and a complex number  $\kappa_s$ . The real part of  $\kappa_s$  contributes to the eigenfrequency and imaginary part to the damping coefficient of the exponent. According to the asymptotic stability criteria for small perturbations of equilibrium, the solution is unstable when at least for one  $\kappa_s$  ( $s = 1, 2, \dots, n$ ) holds that  $\text{Re}(i\kappa_s) > 0$ . In the paper (TONDL, 1959) there are solved the exponents  $\kappa_s$  and analyzed the instability intervals both for resonances of the first kind and combination resonances. Since the paper deals with the possibility and conditions for vibration suppression by the subtractive, combination parametric resonance  $|\Omega_j - \Omega_k|$ , we aim furthermore at the solution and analysis of this resonance.

The analytical solution of the exponent  $\kappa_s$  for a two degree of freedom (2DOF) vibrating linear system with periodic changes in stiffness coefficients is developed

in (TONDL, 1959). The equations of motion of such a system can be written in quasi-normal form as

$$\begin{aligned} \ddot{x}_1 + \Omega_1^2 x_1 &= \varepsilon (-2\delta_1 \dot{x}_1 + (q_1 \cos \omega_p t)x_1 \\ &+ (q_2 \cos \omega_p t)x_2), \\ \ddot{x}_2 + \Omega_2^2 x_2 &= \varepsilon (-2\delta_2 \dot{x}_2 + (Q_1 \cos \omega_p t)x_1 \\ &+ (Q_2 \cos \omega_p t)x_2), \end{aligned} \quad (4)$$

where  $\delta_1$ ,  $\delta_2$  are modal damping constants and  $q_i$ ,  $Q_i$  ( $i = 1, 2$ ) are modal depths of parametric modulation.

For the first approximation of the solution of (4), it is analytically shown in (TONDL, 1959) that when  $\Omega_2 > \Omega_1$ ,  $\delta_1 > 0$ ,  $\delta_2 > 0$  and  $\Omega_1/\Omega_2$  is not a rational number then the exponents  $\kappa_s$  in the vicinity ( $\pm\alpha_0$ ) of the resonance of the second kind and the 1th order, i.e.  $(\Omega_2 - \Omega_1) \pm \alpha_0$ , leads to the solution of the quadratic characteristic equation

$$\begin{aligned} \kappa^2 + \left( \alpha_0 - i \frac{\delta_1 + \delta_2}{\omega_p} \right) \kappa - \frac{q_2 Q_1}{16\omega_p^2 \Omega_1 \Omega_2} \\ - \frac{\delta_1 \delta_2}{\omega_p^2} - i\alpha_0 \frac{\delta_1}{\omega_p} = 0. \end{aligned} \quad (5)$$

From this equation it is clear that the exponent  $\kappa_s$  is dependent on a size of depth of parametric modulation, on damping constants and on the parametric excitation frequency  $\omega_p$ .

### 3. Computational model of forced vibration with additional parametric excitation

Therefore the parametric "anti-resonance" effect is studied on the 2DOF mechanical system. The equation of motion of the 2DOF system with parametric excitations can be expressed as

$$\begin{aligned} m_1 \ddot{y}_1 + b_1 \dot{y}_1 + k_1 (y_1 - y_2) &= F \cos \bar{\omega} t, \\ m_2 \ddot{y}_2 + b_2 \dot{y}_2 + k_2(t) y_2 - k_1 (y_1 - y_2) &= 0, \end{aligned} \quad (6)$$

where  $F$  is amplitude of harmonic excitation force of  $m_1$  with frequency  $\bar{\omega}$ ,  $m_i$ ,  $b_i$  ( $i = 1, 2$ ) are masses and damping coefficients, respectively.  $k_1$ ,  $k_2(t) = k_{20}(1 +$

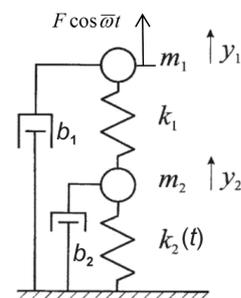


Fig. 1. 2DOF system with external force.

$\alpha_2 \cos \overline{\omega_p t}$ ) are constant and time-variable spring parameters.  $\overline{\omega_p}$  is a frequency of parametric excitation and  $\alpha_2$  is a depth of parametric modulation and substitute here the small parameter  $\varepsilon$  of the analytical model.

After time transformation  $\omega_1 t = \tau$  ( $\omega_1^2 = k_1/m_1$ ) and modifications of Eq. (3) we get non-dimensional form of equations of motion:

$$\begin{aligned} y_1'' + (y_1 - y_2) + \kappa_{10} y_1' - \alpha_{1F} \cos \omega \tau &= 0, \\ y_2'' - M(y_1 - y_2) + q_{20}(1 + \alpha_2 \cos \omega_p \tau) y_2 + \kappa_{20} y_2' &= 0, \end{aligned} \quad (7)$$

where

$$\begin{aligned} M &= \frac{m_1}{m_2}, & q_{20} &= \frac{\omega_2^2}{\omega_1^2}, \\ \omega &= \frac{\overline{\omega}}{\omega_1}, & \omega_p &= \frac{\overline{\omega_p}}{\omega_1}, \\ \kappa_{10} &= \frac{b_1}{m_1 \omega_1}, & \kappa_{20} &= \frac{b_2}{m_2 \omega_1}, \\ \alpha_{1F} &= \frac{F}{m_1 \omega_1^2}, & \omega_2^2 &= \frac{k_{20}}{m_2}. \end{aligned}$$

For comparison with the analytical solution, the Eqs. (4) will be transformed into quasi-normal form by the method of modal decomposition. The Eqs. (4) will be first rewritten into a matrix form

$$(\mathbf{K} + \mathbf{K}_p \alpha_2 \cos \omega_p \tau) \mathbf{y} + \mathbf{B} \mathbf{y}' + \mathbf{y}'' = \mathbf{f}_b \cos \omega \tau, \quad (8)$$

where  $\mathbf{K}$ ,  $\mathbf{K}_p$  and  $\mathbf{B}$  are stiffness, parametric stiffness and damping matrices,  $\mathbf{y}$ ,  $\mathbf{y}'$  and  $\mathbf{y}''$  are the vectors of displacement, velocity and acceleration, respectively. Vector  $\mathbf{f}_b$  represents force vector. By use of the left  $\mathbf{X}_L$  and right  $\mathbf{X}_R$  eigenvector matrices, i.e. substituting  $\mathbf{y} = \mathbf{X}_R \mathbf{y}_N$ ,  $\mathbf{y}' = \mathbf{X}_R \mathbf{y}'_N$ ,  $\mathbf{y}'' = \mathbf{X}_R \mathbf{y}''_N$  and multiplying the Eq. (8) from the left by  ${}^T \mathbf{X}_L$ , we get the quasi-normal form

$$(\mathbf{k}_N + \mathbf{k}_{pN} \alpha_2 \cos \omega_p \tau) \mathbf{y}_N + \mathbf{b}_N \mathbf{y}'_N + \mathbf{m}_N \mathbf{y}''_N = \mathbf{f}_{bN} \cos \omega \tau, \quad (9)$$

where  $\mathbf{k}_N = {}^T \mathbf{X}_L \mathbf{K} \mathbf{X}_R$ ,  $\mathbf{k}_{pN} = {}^T \mathbf{X}_L \mathbf{K}_p \mathbf{X}_R$ ,  $\mathbf{m}_N = {}^T \mathbf{X}_L \mathbf{X}_R$ ,  $\mathbf{b}_N = {}^T \mathbf{X}_L \mathbf{B} \mathbf{X}_R$ ,  $\mathbf{f}_{bN} = {}^T \mathbf{X}_L \mathbf{f}_b$ . If the matrices  $\mathbf{X}_L$ ,  $\mathbf{X}_R$  are orthonormalized to the unit matrix ( ${}^T \mathbf{X}_L \mathbf{X}_R = \mathbf{I}$ ) then Eq. (9) can be expressed as Eqs. (4) and its coefficients used for analytical solution of the exponent  $\kappa_s$  given by (5).

#### 4. Numerical solution of forced vibration with additional parametric excitation

The vibration suppression is presented herein for 4 cases of 2DOF model parameters with the additional parametric excitation:

- a)  $M = 2$ ,  $q_{20} = 1$ ,  $\kappa_{10} = 0.001$ ,  $\kappa_{20} = 0.05$ ,  
 $\alpha_2 = 0 \div 0.5$ ,  $\alpha_{1F} = 0.1$ ,  $\alpha_0 = 0$ ,

- b)  $M = 2$ ,  $q_{20} = 1$ ,  $\kappa_{10} = 0.05$ ,  $\kappa_{20} = 0.05$ ,  
 $\alpha_2 = 0 \div 0.5$ ,  $\alpha_{1F} = 0.1$ ,  $\alpha_0 = 0$ ,  
c)  $M = 1 \div 7$ ,  $q_{20} = 1$ ,  $\kappa_{10} = 0.001$ ,  $\kappa_{20} = 0.05$ ,  
 $\alpha_2 = 0.2$ ,  $\alpha_{1F} = 0.1$ ,  $\alpha_0 = 0$ ,  
d)  $M = 2$ ,  $q_{20} = 1$ ,  $\kappa_{10} = 0.001$ ,  $\kappa_{20} = 0.05$ ,  
 $\alpha_2 = 0.2$ ,  $\alpha_{1F} = 0.1$ ,  $\alpha_0 = -0.28 \div 0.28$ .

The eigenfrequencies of the undamped system ( $M = 2$ ,  $q_{20} = 1$ ,  $\kappa_{10} = \kappa_{20} = 0$ ) are  $\Omega_1 = 0.5176$ ,  $\Omega_2 = 1.9319$  and then the anti-resonance frequency  $\omega_p = \eta = \Omega_2 - \Omega_1 = 1.4142$ . The results after time integration of Eqs. (7) for sweep excitation ( $\dot{\omega} = 2e-4$  in frequency range  $\omega = \langle 0.2, 1 \rangle$ ) and for different values of a selected parameter (underlined for each case) are summarized graphically in Figs. 2 up to 5. The maximal vibration amplitudes were first determined from all successive amplitudes during the sweep excitation. Since the maximal amplitudes fluctuate, the maximal amplitudes were smoothed next by moving average method over ten exciting cycles for better graphical contour mapping.

- a) The contour lines of the maximal amplitudes (Fig. 2, left) show that amplitudes of resonant vibration gradually decrease by increasing amplification factor of additional parametric excitation up to  $\alpha_2 = 0.075$ . Then after exceeding the level  $\alpha_2 = 0.075$  the resonant frequency split into two frequencies. It is in accordance with the analytical solution of the characteristic exponent  $i\kappa$  (Fig. 2, right). Imaginary part describing the change of eigenfrequency  $\Omega_1$  splits after  $\alpha_2 = 0.075$ , too. Adding the imaginary parts to  $\Omega_1$  ( $\Omega_1 + \text{imag}(i\kappa)$ ) we get the resulting values of the exponent (white lines in Fig. 2, left). Real part of the exponents, real ( $i\kappa$ ), (Fig. 2, bottom right) that define the level of damping behave otherwise to the imaginary part. For lowest values of  $\alpha_2$ , the two branches of the characteristics have the maximal splitting. Increasing value of  $\alpha_2$  a level of damping of upper branch gradually increases and after exceeding value 0.075 split branches fuse together in a constant value line. This behavior is in accordance with the observation of the maximal amplitudes of contour diagram.
- b) The contour lines of maximal amplitudes (Fig. 3, left) and characteristic exponent diagram (Fig. 3, right) are qualitatively very similar to the previous case. The only difference is that the moment of the splitting arises for higher values of  $\alpha_2$  due to higher damping coefficient  $\kappa_{10} = 0.05$  than in previous case. Analytical solution gives higher estimation of this value and it shows that analytical solution is more sensitive on the value of damping. White lines in the contours of maximal amplitudes (Fig. 2, left top) were obtained again like in the previous case by sum of the imaginary parts of the exponent  $i\kappa$  and the eigenfrequency  $\Omega_1$ .

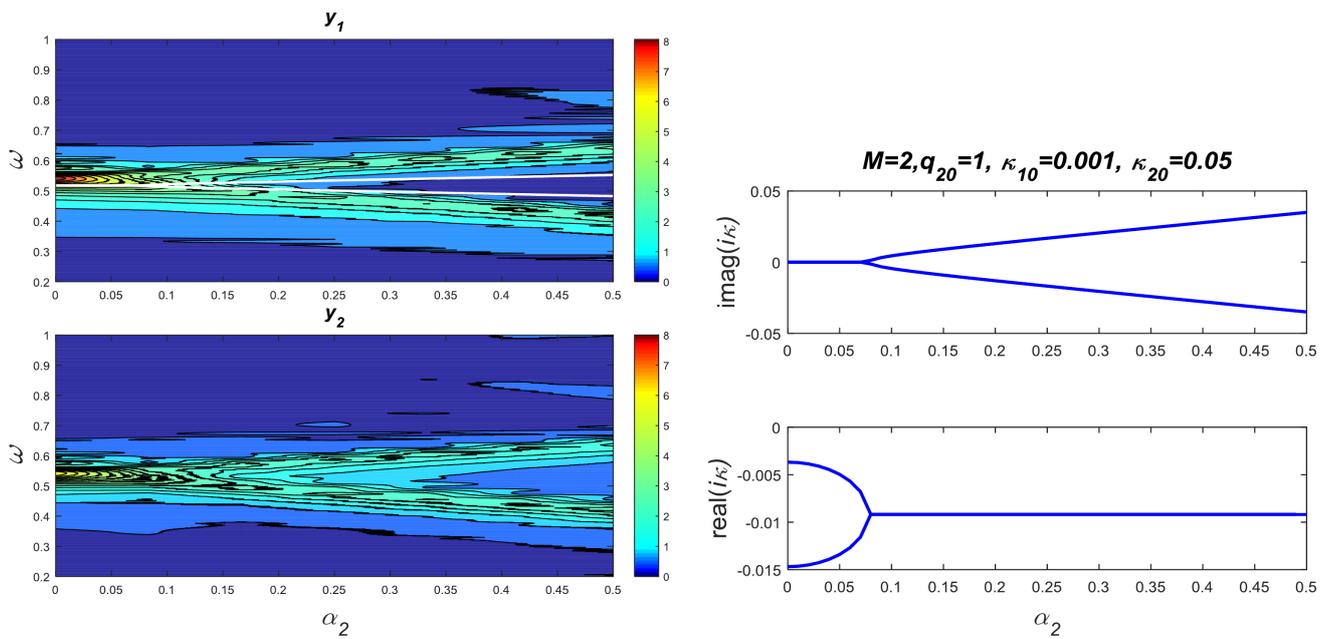


Fig. 2. Contours of maximal amplitudes of displacements (left) and characteristic exponent  $i\kappa$  (right) for different values of  $\alpha_2 = 0 \div 0.5$ ,  $M = 2$ ,  $q_{20} = 1$ ,  $\kappa_{10} = 0.001$ ,  $\kappa_{20} = 0.05$ ,  $\alpha_0 = 0$  of 2DOF model.

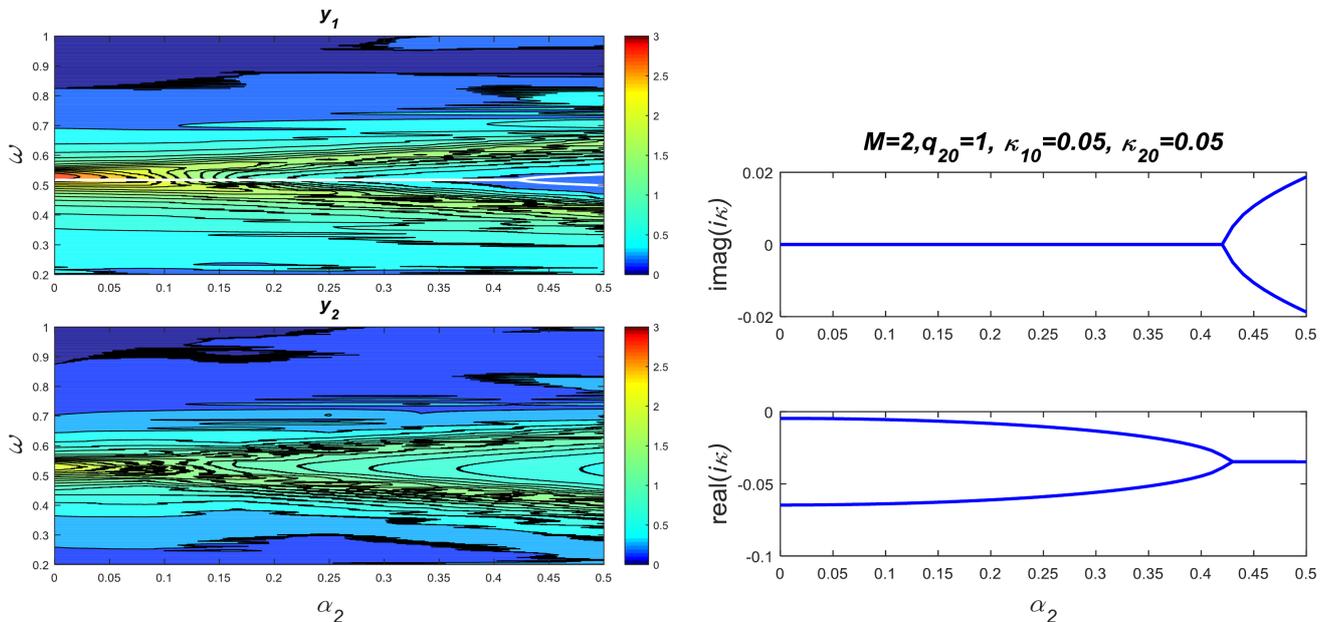


Fig. 3. Contours of maximal amplitudes of displacements (left) and characteristic exponent  $i\kappa$  (right) for different values of  $\alpha_2 = 0 \div 0.5$  and  $M = 2$ ,  $q_{20} = 1$ ,  $\kappa_{10} = 0.05$ ,  $\kappa_{20} = 0.05$ ,  $\alpha_0 = 0$  of 2DOF model.

c) The dependence of the splitting on different values of the parameter  $M$  for a depth of parametric modulation  $\alpha_2 = 0.2$  and for value of  $\kappa_{10} = 0.001$  is shown in Fig. 4, respectively. Again both contours of maximal amplitudes and characteristic exponent diagram are presented. The size of splitting is diminishing with the size of parameter  $M$  and damping value is decreasing at the same time according to the analytical solution. As to the damping the results of numerical simulations are in accor-

dance with this analytical prediction, however, the size of splitting is *vice versa* growing with the size of  $M$ .

d) The influence of small deviation of parametric excitation frequency from the parametric anti-resonance on the amplitude suppression was studied on the last study case. We considered the same parameters as in the case a) and the deviation parameter  $\alpha_0$  from the interval  $(\pm 0.28)$  was added to the parametric excitation  $\omega_p = \Omega_2 - \Omega_1 \pm \alpha_0$ .

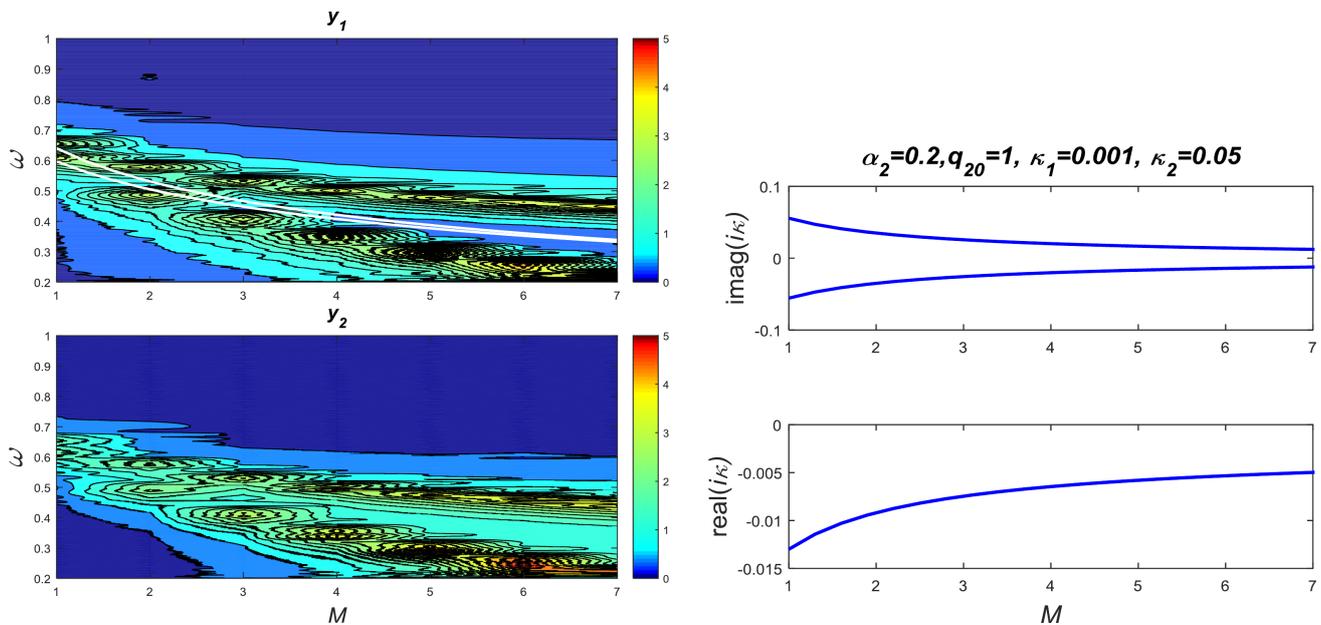


Fig. 4. Contours of maximal amplitudes of displacements (left) and characteristic exponent  $\kappa$  (right) for different values of  $M = 1 \div 7$  and  $\alpha_2 = 0.2, q_{20} = 1, \kappa_{10} = 0.001, \kappa_{20} = 0.05, \alpha_0 = 0$  of 2DOF model.

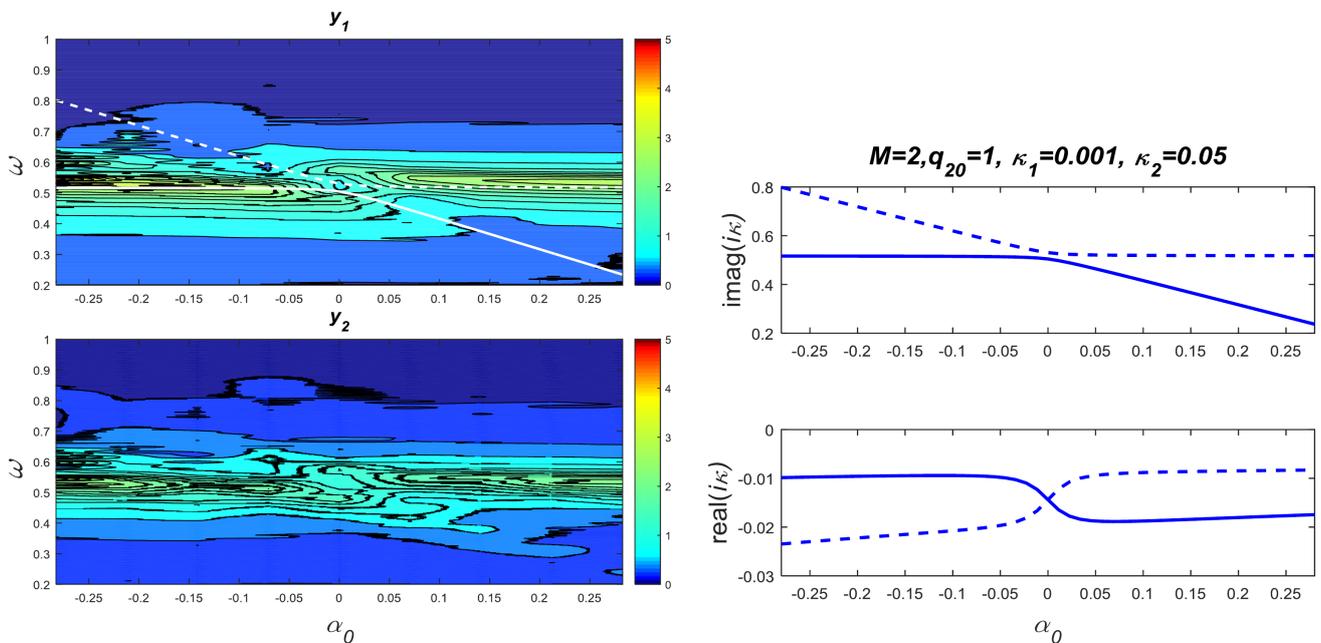


Fig. 5. Contours of maximal amplitudes of displacements (left) and characteristic exponent  $i\kappa$  (right) for different values of  $\alpha_0 = -0.28 \div 0.28$  and  $\alpha_2 = 0.2, q_{20} = 1, \kappa_{10} = 0.05, \kappa_{20} = 0.05, M = 2$  of 2DOF model.

Again both contours of maximal amplitudes and characteristic exponent diagram are presented. It can be seen that the maximal suppression of maximal amplitudes occurs in the close vicinity  $|\alpha_0| < 0.05$  of the anti-resonance frequency. Due to lower parametric modulation  $\alpha_2 = 0.2$ , a splitting of eigenfrequency is very weak and damping value is highest in this interval. Above the absolute value 0.05 of the parameter  $\alpha_0$  a splitting increases and an effectiveness of damping rapidly decreases.

### 5. Conclusion

The results of 2DOF system with additional parametric excitation show that parametric anti-resonance effect is characterized by two factors: a) additional damping coming out from the parametric excitation; b) splitting of the first eigenfrequency of the system. Both these factors are influenced by the mass ratio, mutual mass/stiffness ratios  $(q_{20} = \frac{k_{20}}{m_2} / \frac{k_1}{m_1})$  and damping coefficients.

Numerical simulations for parameters  $M = 2$ ,  $q_{20} = 1$  and different values of the depths of parametric excitation  $\alpha_2$  (case a, b) showed that at lower values of the depth ( $\alpha_2 < 0.075$ ) the decrease of the maximal amplitudes with no eigenfrequency splitting occur. The splitting arises at higher values of the depth. So, by the value of the depth the parametric anti-resonance effect can be set up with respect to a nature of the system conceived for suppression of externally excited resonant vibration

Numerical simulations for parameters  $\alpha_2 = 0.2$ ,  $q_{20} = 1$  and different  $M$  (case c) showed that high efficiency of anti-resonance effect for maximum amplitude suppression of the 2DOF system can be expected for lower value of parameter  $M = 1, 2$ . For higher value of  $M > 5$  the efficiency is quite poor even though the parametric excitation is amplified since the damping is decreasing.

The influence of small deviation of parametric excitation frequency from the parametric anti-resonance on the amplitude suppression was studied for the depth of parametric modulation  $\alpha_2 = 0.2$  (case d). The maximal suppression of maximal amplitudes occurs in the close vicinity  $|\alpha_0| < 0.05$  of the anti-resonance frequency. Above the absolute value 0.05 of the parameter  $\alpha_0$ , effectiveness of damping rapidly decreases.

The approximate analytical solution of the characteristic exponents can be very helpful to estimate the efficiency of the parametric anti-resonance or to design parameters of the mechanical system with the parametric anti-resonance excitation, however, for more accurate description of its dynamic behavior, the direct integration of the equations of motion is recommended.

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### References

1. ECKER H., PUMHOESSEL T., TONDL A. (2002), *A study on parametric excitation for suppressing self-excited rotor vibrations*, Proceedings of the 6th Int. Conference on Rotor Dynamics (IFToMM), Sydney.
2. ECKER H., TONDL A. (2004), *Stabilization of a rigid rotor by a time-varying stiffness of the bearing mounts*, Proceedings of the 8th Int. Conference on Vibrations in Rotating Machinery, Swansea, Professional Engineering Publishing Limited for the Institution of Mechanical Engineers, Berry St. Edmunds and London, pp. 45–54.
3. ECKER H., TONDL A. (2005), *Increasing the stability threshold of a rotor by open-loop control of the bearing mount stiffness*, Proceedings of Int. Symposium on Stability Control of Rotating Machinery, Ohio, pp. 351–361.
4. ECKER H., TONDL A. (2007), *Suppression of self-excited vibrations in valve systems*, Proceedings of Colloquium Dynamics of Machines 2007, Institute of Thermomechanics AS CR, Prague, pp. 55–62.
5. ECKER H. (2010), *Exploring the Use of Parametric Excitation*, Proceedings of the 10th Int. Conference on Recent Advances in Structural Dynamics, The Institute of Sound and Vibration, UK.
6. ECKER H., TONDL A. (2011), *On the Suppression of Flow-Generated Self-Excited Vibrations of a Valve*, International Conference on Vibration Problems (ICOVP), Springer Proceedings in Physics 139, Springer, pp. 793–799.
7. ECKER H., PUMHOESSEL T. (2012), *Vibration suppression and energy transfer by parametric excitation in drive systems*, Journal of Mechanical Engineering Science, **226**, 8, 2000–2014.
8. DOHNAL F., TONDL A. (2009), *Suppressing Flutter Vibrations by Parametric Inertia Excitation*, Journal of Applied Mechanics, **76**, 7.
9. HALANAY A. (1966), *Differential equations – stability, oscillations, time lags*, New York, Academy press.
10. ŁUCZKO J., CZERWIŃSKI A. (2015), *Parametric vibrations of flexible hoses excited by a pulsating fluid flow, Part I: Modelling, solution method and simulation*, Journal of Fluids and Structures, **55**, 155–173.
11. NABERGOJ R., TONDL A., ECKER H. (2006), *To the problem of self-excited systems having several unstable vibration modes*, Proceedings of the Colloquium Dynamics of Machines 2006, Institute of Thermomechanics AS CR, Prague, pp. 71–78.
12. NABERGOJ R., TONDL A., ECKER H. (2007), *Additional contribution to the problem of self-excited systems having several unstable vibration modes*, Engineering Mechanics, **14**, 1/2, 1–12.
13. METTLER E. (1965), *Schwingungs- und stabilitätsprobleme bei mechanischen systemen mit harmonischer erregung*, ZAMM – Zeitschrift für Angewandte Mathematik und Mechanik, **45**, 7–8, 475–484.
14. PEŠEK L., TONDL A. (2012), *Contribution to application of parametric ‘anti-resonance’ for autoparametric systems*, Engineering Mechanics, **19**, 5, 1–9.
15. PEŠEK L., PŮST L., BULA V., CIBULKA J. (2015), *Application of piezofilms for excitation and active damping of blade flexural vibration*, Archives of Acoustics, **40**, 1, 59–69.
16. PUMHOESSEL T., ECKER H., TONDL A. (2011), *Zum Einfluss einer periodischen axialen Kraft auf die lateralen Schwingungen eines Laval-Rotors*, 9 Tagung Schwingungen in rotierenden Maschinen (SIRM), Darmstadt, pp. 21–23.
17. TONDL A. (1959), *Metoda k určení intervalů nestability quasi-harmonických kmitů [The method for the determination of instability intervals of quasi-harmonic vibration systems]*, Aplikace Matematiky, **4**, 4, 278–289.

18. TONDL A. (1998a), *To the problem of quenching self-excited vibrations*, Acta Technica ČSAV, **43**, 109–116.
19. TONDL A. (1998b), *Vibration quenching of an externally excited system by means of dynamic absorber*, Acta Technica ČSAV, **43**, 301–309.
20. TONDL A., ECKER H. (1999), *Cancelling of self-excited vibrations by means of parametric excitation*, Proceedings of ASME Design Engineering Technical Conference, Las Vegas 1999, DETC/VIB-8071.
21. TONDL A., RUIJGROK T., VERHULST F., NABERGOJ R. (2000), *Autoparametric Resonance in Mechanical Systems*, Cambridge University Press.
22. TONDL A. (2000a), *Suppressing self-excited vibration by means of parametric excitation*, Proceedings of Colloquium Dynamics of Machines 2000, Institute of Thermomechanics AS CR, Prague, pp. 225–230.
23. TONDL A. (2000b), *Self-excited vibration quenching in a rotor system by means of parametric excitation*, Acta Technica ČSAV, **45**, 199–211.
24. TONDL A., ECKER H. (2003), *On the problem of self-excited vibration quenching by means of parametric excitation*, Archive of Applied Mechanics, **72**, 923–932.
25. TONDL A. (2003), *Combination resonance and anti-resonances in systems parametrically excited by harmonic variation of linear damping coefficients*, Acta Technica ČSAV, **48**, 239–248.
26. TONDL A., NABERGOJ R. (2004), *The effect of parametric excitation on a self-excited three-mass system*, International Journal of Non-linear Mechanics, **39**, 821–832.
27. TONDL A. (2008a), *Passive and active means for self-excited vibration suppressing: Two-mass model*, Engineering Mechanics, **15**, 2, 133–138.
28. TONDL A. (2008b), *To the problem of self-excited vibration suppression*, Engineering Mechanics, **15**, 4, 297–308.
29. TONDL A., PŮST L. (2010), *To the Parametric Anti-Resonance Application*, Engineering Mechanics, **17**, 2, 135–144.
30. TONDL A., PUST L. (2011), *Further application of parametric anti-resonance*, Proceedings of the Colloquium Dynamics of Machines 2011, Institute of Thermomechanics AS CR, Prague, pp. 97–104.
31. TONDL A., PUST L. (2012), *On the phenomenon “Parametric anti-resonance”*, Strojnický Časopis, **63**, 3, 125–137.