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RADAR Signal Parameters Estimation in the MTD Tasks

Igor G. Prokopenko, Igor P. Omelchuk, and Yury D. Chyrka

***Abstract*—The MTD method adaptive to current target speed, in which suboptimal iterative algorithms for the reflected signal parameters estimation are synthesized, is suggested. This method allows to detect a slowly moving targets with radial speed 3-4 times less, than for pulse-pair subtraction (PPS).**

***Keywords*—RADAR, detection, moving targets, frequency estimation, adaptive algorithms, Bayesian empirical approach.**

I. Introduction

MOVING target detection (MTD) on the background of interferences is one of basic functions of different modern detection systems: medical, hydro acoustic, guard, air traffic controls, etc. The MTD radars use the Doppler Effect of the current frequency shift of the radio signal reflected from the target. This shift depends on target speed. Spectral methods, which are based on fast Fourier transformation (FFT), are most often used for measuring the Doppler-beat frequency and moving target detection.

Development and use of new methods and algorithms of signal processing lead to increase the MTD digital systems efficiency. There are a plenty of scientific and technical researches dedicated to this subject.

However, estimation of the slowly moving target speed as well as its detection, when observation time of radar is commensurable with the period of the reflected signal Doppler-beat frequency, still is a problem. The slow motion target criterion will be conditionally set as:

$$T_{\mathrm{ob}} < 2\,/\,f_{\mathrm{d}},$$

where T_{ob} is the target observation time, which is in this case called “limited”, f_{d} is Doppler-beat frequency which is called “infra-low”. Obviously, this criterion depends not only on the actual target radial speed but also on the radar parameters; term “speed” always implies a radial speed.

Classic approaches, including FFT methods, in this case become useless, because considerable observation time $T_{\mathrm{ob}} >> 1\,/\,f_{\mathrm{d}}$ is required. Therefore, there is a requirement for creation of the new special methods of signal processing in accordance with the applied essence of tasks.

II. Adaptive MTD Method

Prospective directions to increase MTD methods practical fitness must be based on deepening statistical analysis of all available radar data with the purpose of adaptation to the current parameters of targets, interferences and noise. Thus, from the different methods of overcoming a priori uncertainty the most results were obtained by empiric Bayesian approach according to which unknown parameters in the optimal detection algorithm are replaced with grounded estimations.

Under this statement the detection of one target with constant speed by radar is related to the area of detection of unknown digitized complex harmonic signal with Doppler-beat frequency on the output of transceiver quadrature detector: $\dot{s}_{\mathrm{i}} = s_{\mathrm{xi}} + j s_{\mathrm{yi}} = \rho \cos\left[\gamma\,(i-1)+\varphi_1\right] + j\,\rho \sin\left[\gamma\,(i-1)+\varphi_1\right],\ i = \overline{1,N}$, where ρ is amplitude, $\gamma = 2\pi\, f_{\mathrm{d}}/f_{\tau}$ – the normalised frequency which depends on sampling frequency f_{τ}, φ_1 is an initial phase of sample, N is a size of sample. In future, under a term “frequency” we always imply exactly normalised frequency.

The additive model of quadrature detector samples set counts is accepted for further researches:

$$\dot{u}_{\mathrm{i}} = \dot{s}_{\mathrm{i}} + \dot{\eta}_{\mathrm{i}} + \dot{\xi}_{\mathrm{i}} = (s_{\mathrm{xi}} + \eta_{\mathrm{xi}} + \xi_{\mathrm{xi}}) + j(s_{\mathrm{yi}} + \eta_{\mathrm{yi}} + \xi_{\mathrm{yi}}) = = x_{\mathrm{i}} + j y_{\mathrm{i}}\,, \quad i = \overline{1,N},$$

where $\dot{\eta}_{\mathrm{i}} = \eta_{\mathrm{xi}} + j\eta_{\mathrm{yi}}$, $\xi_{\mathrm{i}} = \xi_{\mathrm{xi}} + j\xi_{\mathrm{yi}}$ are complex constituents of the uncorrelated Gaussian noise and correlated interference in quadratures $x_{\mathrm{i}}\,, y_{\mathrm{i}}$ of i-th sample.

Using the optimal coherent algorithm of the determined signal detection for the known dispersion of noise σ^2

$$\sum_{i=1}^{N} x_i \cdot s_i/\sigma > C,$$

where C is a decision-making threshold, and in accordance with empiric Bayesian approach the adaptive MTD algorithm will be written as:

$$\frac{\sum_{i=1}^{N}\left(x_i - \xi_x^*\right)\cdot s_{xi}^*}{\left(\sigma_x^*\right)^2} + \frac{\sum_{i=1}^{N}\left(y_i - \xi_y^*\right)\cdot s_{yi}^*}{\left(\sigma_y^*\right)^2} > C_c, \quad (1)$$

where $\left(\sigma_x^*\right)^2$, $\left(\sigma_y^*\right)^2$ are estimations of noise dispersions in quadrature channels, other estimations are also marked *.

Thus, we consider interference as constants $\xi_{\mathrm{xi}} = \xi_{\mathrm{x}}\,,\ \ \xi_{\mathrm{yi}} = \xi_{\mathrm{y}}\,,\ \ \forall i$, that allow to simplify the signal processing procedures, dividing them into independent by quadrature. Therefore the further synthesis of the frequency estimation algorithms is made for the scalar harmonic signal of only one quadrature; the index of quadrature is not specified.

III. Analysis of Likelihood Function

It is obvious, that the harmonic signal estimations of a formula (1) must be calculated on the basis of its parameters estimations

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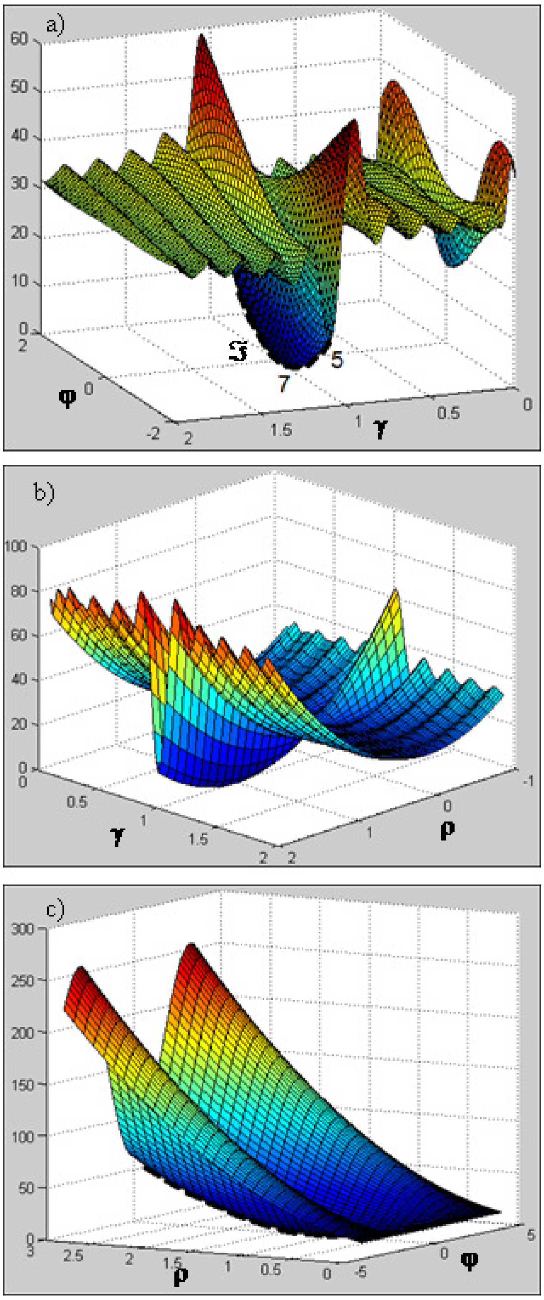


Fig. 1. Projections of likelihood function in the co-ordinates: a) frequency – the initial phase; b) frequency – amplitude; c) amplitude – the initial phase.

$$s_i^* = s\,(\,\rho^*, \varphi_1^*, \gamma^*) \tag{2}$$

We write the logarithm of harmonic signal likelihood function as

$$\begin{aligned} &\ln L(\rho, \gamma, \varphi, \sigma, \xi | x_0,\ \dots,\ x_{N-1}) = \\ &= \ln \frac{\sigma^{-N}}{(2\pi)^{N/2}} - \frac{\Lambda(\rho, \gamma, \varphi, \xi | x_0,\ \dots,\ x_{N-1})}{2\sigma^2}\ , \end{aligned} \tag{3}$$

where sufficient statistics are marked as

$$\Lambda(\rho, \gamma, \varphi, \xi | x_0,\ \dots,\ x_{N-1}) = \sum_{i=0}^{N-1} (x_i - \rho \sin(\gamma i + \varphi) - \xi)^2. \tag{4}$$

Obviously, the likelihood function (3) gets the maximum on the set of harmonic signal parameter's estimates while providing minimum sufficient statistics

$$\{\,\rho^*,\ \gamma^*, \varphi^*, \xi^*\,\} = \underset{\rho,\,\gamma,\varphi,\xi}{\arg\min}\, \Lambda(\rho, \gamma, \varphi, \xi | x_0,\ \dots,\ x_{N-1}),$$

which further use as function of parameters $\rho, \gamma, \varphi, \xi$ with known samples $x_0,\ \dots,\ x_{N-1}$.

Thus, the problem belongs to a class of multidimensional and multiextremal minimization [1]. To obtain the solution the practically numerical methods are used. This work is devoted to the development and research of the parameter estimation search algorithm, adapted to the properties of harmonic signal likelihood function.

Since the likelihood function (4) of harmonic signal depends on four parameters, the analysis of its structure performed by few two-parameter projections for possible pairwise combinations of parameters. Fig. 1 shows an example of such projections, which are built with environment MatLab. For some set of 32 samples, the signal / noise ratio $SNR = 100$ and parameters of the harmonic signal: $\rho = 1, \gamma = 1,\ \varphi = 0$**.**

We consider the likelihood function as function of two parameters (frequency and phase). In this case amplitude is fixed. This function has a pronounced recurrent character. Because the structure of the likelihood function is repeated with the phase period 2π. The graph shows only the area around its true value.

The dependence on frequency is more complicated. It is the alternation of different minima depth (gutters) that has "resonance" nature.

As revealed by simulation, the length of intervals between minima is depended on a sample size:

$$\Delta\gamma \approx 2\,\pi\,/N. \tag{5}$$

Local minima gradually deepen toward the true frequency, and near the point of zero phase and frequency deviation of observed two-dimensional global minimum.

Likelihood function also has some distinctive characteristics. All gutters are placed under the same angle to the axis of phase, which, as shown with simulation results, approximately are: $\alpha \approx -\arctan\{\,1.1/N\}$. A near zero frequency is another deep false minimum.

While the power of noise increases the global minimum of likelihood function may be further away from the point of true values. Effective parameters measurement are achieved only under conditions of significant excess of signal over noise. Some minor differences do not significantly alter the overall structure of the likelihood function. So the example is sufficient to qualitative determination of its basic properties.

The likelihood function projection in coordinates frequency – amplitude highlights its resonance frequency character. It is similar to the previous version, with the difference that the gutters are placed parallel to the axis amplitude.

The latest projection shows the dependence of the likelihood function on the pair amplitude – phase is also non-monotonic, and its sensitivity to phase changes slightly larger than the amplitude.

Additional projection for the constant amplitude is shown in Fig. 2. The function is unimodal by these parameters.

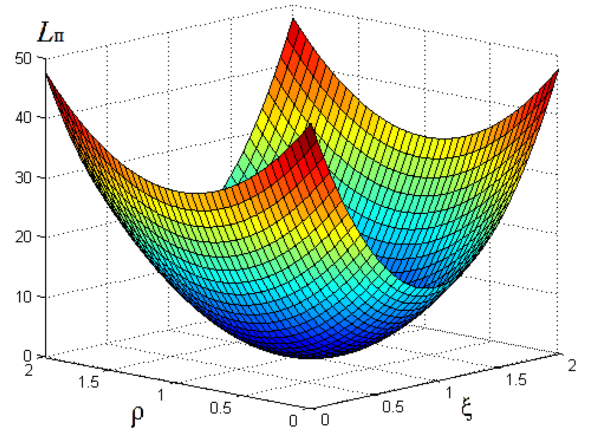
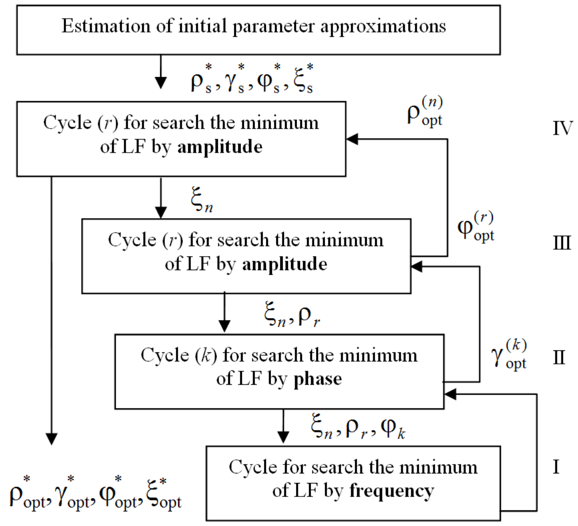


Fig. 2. Projection of likelihood function in the co-ordinates constant – amplitude.



$$\text{IV:}\quad \xi_{\text{opt}} = \arg\min \Lambda(\xi / \bar{x}, \rho = \rho_{\text{opt}}, \varphi = \varphi_{\text{opt}}, \gamma = \gamma_{\text{opt}})$$

$$\text{III:}\quad \rho^{(n)}_{\text{opt}} = \arg\min \Lambda(\rho / \xi_n, \bar{x}, \varphi = \varphi_{\text{opt}}, \gamma = \gamma_{\text{opt}})$$

$$\text{II:}\quad \varphi^{(r)}_{\text{opt}} = \arg\min \Lambda(\varphi / \xi_n, \rho_r, \bar{x}, \gamma = \gamma_{\text{opt}})$$

$$\text{I:}\quad \gamma^{(k)}_{\text{opt}} = \arg\min \Lambda(\gamma / \xi_n, \rho_r, \varphi_k, \bar{x})$$

Fig. 3. General block diagram of the harmonic signal parameter estimation algorithm.

IV. ITERATIVE ALGORITHM OF PARAMETERS ESTIMATION

Although the problem of harmonic signal parameters estimation are four-dimensional, the set above "resonance" character of likelihood function requires the allocation procedure of frequency estimation as a separate one-dimensional. It is because the multivariate simultaneous search can lead to uncontrolled jumps between local minima. Higher sensitivity of the likelihood function to frequency deviations than to the phase, amplitude and constant interference deviation defines the procedure of optimal frequency estimation as the primary parameter estimation algorithm. Further we conditionally define the meaning of the term "optimal" parameter. It means the point of likelihood function minimum for this parameter.

Given the monotonic relief likelihood function for constant, amplitude and phase should also separate the search for their optimum values for three one-dimensional procedure. And the priority is the search for the phase sensitivity as to the likelihood function it is slightly more than the amplitude and constant interference. Thus, as a result of the reduction [1] the algorithm of four-parameter minimization is built as a four-tier hierarchical structure (Fig. 3).

At each level the minimization is performed iterative for the different harmonic signal parameter: at the bottom I – frequency; II – phase; III – amplitude; at the top IV – constant. A characteristic of this nested search algorithm is that some top-level procedure always uses the locally-optimal estimation of other parameters that is defined as a result of the previous iterative search on the lower levels.

Note that the additional research has shown the greater stability of the proposed algorithm for parameter estimation with respect to such algorithms that are used for simultaneous multivariate optimization methods – step and gradient. This is the result of significant likelihood function polymodality.

The search algorithm starts with the initial estimation of all harmonic signal parameters. Taking into account [2]–[5], the known value of frequency allows us easily calculate two other parameters. The frequency estimation will continue in accordance with the algorithm that was synthesized in [3], as

$$\gamma^*_{\text{s}} = \arccos\ (\alpha/2)\,, \tag{6}$$

where

$$\alpha = (-B + \sqrt{B^2 - 4AC})/2A$$

is allowable root of square equation $A\alpha^2 - B\alpha - C = 0$, where a coefficients A, B, C are calculated by the formulas:

$$A = \sum_{i=3}^{N-1} \left[\left(x_{\mathrm{i}-2} - x_{\mathrm{i}-1} \right)^2 - \left(x_{\mathrm{i}-2} - x_{\mathrm{i}-1} \right) \times \right.$$
$$\left. \times \left(x_{\mathrm{i}} - x_{\mathrm{i}-1} + x_{\mathrm{i}-2} - x_{\mathrm{i}-3} \right) \right]$$
$$B = \sum_{i=3}^{N-1} \left[2 \left(x_{\mathrm{i}-2} - x_{\mathrm{i}-1} \right)^2 - \right.$$
$$\left. - \left(x_{\mathrm{i}} - x_{\mathrm{i}-1} + x_{\mathrm{i}-2} - x_{\mathrm{i}-3} \right)^2 \right]$$
$$C = \sum_{i=3}^{N-1} \left[2 \left(x_{\mathrm{i}-2} - x_{\mathrm{i}-1} \right) \left(x_{\mathrm{i}} - x_{\mathrm{i}-1} + x_{\mathrm{i}-2} - x_{\mathrm{i}-3} \right) - \right.$$
$$\left. - \left(x_{\mathrm{i}} - x_{\mathrm{i}-1} + x_{\mathrm{i}-2} - x_{\mathrm{i}-3} \right)^2 \right]$$

When the initial value of harmonic signal frequency is known, the initial values of its amplitude and phase will be found by the method of maximum likelihood from equations which obtained as differentiation of likelihood function (4) for variables ρ, ϕ, ξ:

$$\begin{cases} \frac{\partial\Lambda}{\partial\rho} = -2\sum\limits_{i=0}^{N-1}\sin(\gamma i+\phi)\left[x_i-\rho\sin(\gamma i+\phi)-\xi\right]=0; \\ \frac{\partial\Lambda}{\partial\phi} = -2\rho\sum\limits_{i=0}^{N-1}\cos(\gamma i+\phi)\left[x_i-\rho\sin(\gamma i+\phi)-\xi\right]=0; \\ \frac{\partial\Lambda}{\partial\xi} = -2\sum\limits_{i=0}^{N-1}\left[x_i-\rho\sin(\gamma i+\phi)-\xi\right]=0. \end{cases}$$

We use new variables:

$$A_x = \rho\cos\varphi\,, \quad A_y = \rho\sin\varphi, \quad A_z = \xi.$$

Then the system of equations can be transformed relatively new variables in a linear form similar to [[3]]:

$$\begin{cases} A_x\sum_{i=0}^{N-1}\sin^2(\gamma i)+A_y\sum_{i=0}^{N-1}\sin(\gamma i)\cos(\gamma i)+ \\ +A_z\sum_{i=0}^{N-1}\sin(\gamma i)=\sum_{i=0}^{N-1}x_i\sin(\gamma i); \\ A_x\sum_{i=0}^{N-1}\cos(\gamma i)\sin(\gamma i)+A_y\sum_{i=0}^{N-1}\cos^2(\gamma i)+ \\ +A_z\sum_{i=0}^{N-1}\cos(\gamma i)=\sum_{i=0}^{N-1}x_i\cos(\gamma i); \\ A_x\sum_{i=0}^{N-1}\sin(\gamma i)+A_y\sum_{i=0}^{N-1}\cos(\gamma i)+ \\ +A_z\cdot N=\sum_{i=0}^{N-1}x_i. \end{cases}$$

The calculated initial estimates of amplitude and phase for its solution:

$$\rho_s^* = \sqrt{A_x^2+A_y^2}, \varphi_s^* = \arctan\left(A_y/A_x\right), \xi_s^* = A_z$$

which together with initial estimates of frequency determine the three-dimensional starting point for further search of likelihood function global minimum. According to the hierarchical algorithm the procedure of minimization of the likelihood function for the frequency has highest priority. Because the values of phase and amplitude at this level are some fixed parameters, which "launched" from the upper levels of minimization, the retrieved global minimum of functions of one variable $F(\gamma)$, is formed as the cross-section of likelihood function values for these two parameters. For example Fig. 4 shows the frequency – phase projection section for initial amplitude estimate $\rho_s^* = 1$ (see Fig. 1, *a*) and the initial phase estimate $\varphi_s^* = -2$ rad. Note that the existence of global minimum shift function $F(\gamma)$ regarding the true frequency ($\gamma = 1$ Rad) errors resulting from the initial estimates are not principle at this level of minimization.

An essential feature of the obtained one-dimensional functions is polymodality. But the typical features previously expressed likelihood function as a monotonous decrease the values of local minima toward the global minimum and alternation all around with a constant period (5) – enable to build appropriate procedure multiextremal minimization. One can consider it with example, when the initial frequency estimate $\gamma_s^* = 1.45$ gets right into the second zone of local minimum – point 1 (Fig. 4).

The procedure for finding the global minimum consists of the following stages:

1) for a point 1 provides search near a local minimum (point 2), for example, using the classical method of golden section;
2) for a known period of alternating minima (5) with respect to the two neighboring zones is determined right and left local minima;

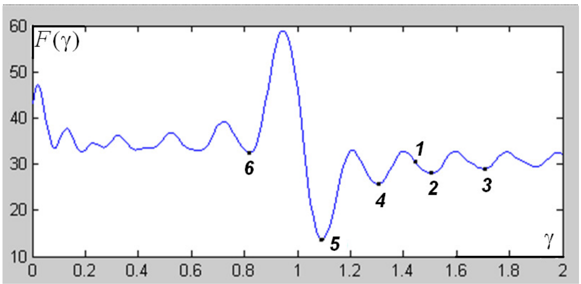


Fig. 4. Section of likelihood function for constant values of phase and amplitude.

3) the same stage a) specifies the coordinates of these minima (points 3 and 4) further constructed and analyzed bypass function $F(\gamma)$, formed with the points of its minima;
4) three found local minima determine the direction for reducing them, if at least at the middle point (point 2) will be minimal, it is defined as the point of global minimum and then search stops;
5) then within the period (5) determine zone of following minimum;
6) the same stage a) specifies a local minimum (point 5);
7) further search is carried out cyclically within stages 4), 5) and 6), which leads to a global minimum – a point 5 relative to points 4 and 6.

Thus, the signal frequency is found to minimize the likelihood function for fixed values of amplitude ρ_r and initial phase φ_k:

$$\gamma_{\text{opt}}^{(k)} = \arg\min_{\gamma}\Lambda(\gamma/\xi_n,\rho_r,\varphi_k,\overline{x}).$$

It is obvious that the found optimal frequency point located at the bottom of the main gutter $\Im$ of projection frequency – phase, which is indicated with dots in Fig. 1 as well. This fact, and unimodality of this gutter by phase in the zone which limited $\pm\pi$ from its true value, enables to search the optimal estimate of phase at level II by one-dimensional minimization procedure: $\varphi_{\text{opt}}^{(r)} = \arg\min_{\varphi}\Lambda(\varphi/\xi_n,\rho_r,\overline{x},\ \gamma=\gamma_{\text{opt}})$. For this purpose, for example, the stated before a local minimization procedure at point 2 can be used. At level II calculation of the current value of likelihood function is not done, and always used the result of the lower level.

Obviously, every step of the level II means a formation of the appropriate section relatively level 1 by the current frequency, so the gradual convergence of phase to the optimal value also helps to reduce deviation of global minimum function $F(\gamma)$, processed at that level 1. The result of a second level optimization of likelihood function by two parameters is minimum located at the point 7.

Since the likelihood function for the amplitude and constant interference is unimodal (see Fig. 1, *b*), *c*); Fig. 2), the minimization on the third and fourth level is similar to the level II with the difference that has used the results of nested likelihood function optimization procedures for other parameters. After completion of level IV we have the final estimates of three unknown parameters $\rho_{\text{opt}}^*, \gamma_{\text{opt}}^*, \varphi_{\text{opt}}^*, \xi_{\text{opt}}^*$.

V. Iterative Algorithm Efficiency

Multifaceted search procedures of proposed harmonic signal parameter estimation algorithm contribute to the fact that the process of handling a particular sample can be completed in several events that have different characteristic.

If the process of calculations appear clearly incorrect current results, a decision on the existence of a "failure of the algorithm", resulting in lack of parameter estimates should be taken. One can select two events of failures. One of these – "Event A" (or initial failure) – can appear at the stage of calculating the initial frequency estimation when the arc cosine argument (6) is outside the limits of existence [-1, 1]. The feature of the second failure – "Event B" (or a final failure) – this algorithm after receiving some negative value of frequency, indicating the inability to reliably estimate of the signal under the given conditions. This happens when the search procedure finds itself in some of the likelihood function gutters side, which has monotonous slope. A sufficient numerical criterion for both events is the failure probability of their occurrence.

Another characteristic event – "an event D", as shown by statistical studies – is stop of the frequency evaluation procedure in the local minimum, i.e. we have shift of the frequency estimation. The parameters estimation algorithm can't recognize such an event and a rough estimate can be communicated without control to other algorithms for data processing of the measurement system. On the stage of a general analysis of the properties of parameter estimation algorithm one should separately examine characteristics of such event. The probability of appearance is the general criterion; more thorough description is the histogram with calculated estimates of the moments, for example, mathematical expectation and dispersion.

The latest event in the full group is "Event C" (or accurate estimation), when frequency estimate gets into the area of global minimum. The width of this area is set equal to the minimum period of alternation (5). Event C probability is the probability which we define as reliability assessment. To estimate the conditional accuracy of reliable signal parameters estimation we use conditional histogram with the required moments.

Thus, a significant diversity of parameter estimation algorithm quality criteria allows us to detail its specific properties. This knowledge is used for a decision on the applicability of this algorithm in the technical measurement systems.

Analysis of the parameter estimation algorithm efficiency is based on the statistical simulation method. Calculated characteristics are presented in the table for different harmonic signal parameters and noise power. The size of the sample is N = 32, and the number of same statistical tests for one experiment was 500. Since research has shown that the accuracy of the initial phase estimation does not depend on its true value, the results are for only one of its values $\varphi = 0$.

Results of statistical researchers have found the next properties of harmonic signal parameter estimation algorithm. Its efficiency depends largely on the length of the interval of observation. If it process a few periods of the signal, the accuracy and precision of parameter estimates is sufficiently high.

TABLE I
Performance Indicators Parameter Estimates

No.	P_S/P_η	P_ξ/P_η	Freq.	Freq. est. shift, %	Event A: prob. of failure	Event B: prob. of failure
1	10	10	1.00	-0.10	0.00	0.00
2	5	10	1.00	-0.50	0.00	0.03
3	20	10	0.50	2.40	0.03	0.11
4	20	100	0.50	17.40	0.07	0.39
5	50	100	0.50	6.00	0.01	0.19
6	100	20	0.20	85.50	0.30	0.36
7	50	20	0.20	136.50	0.33	0.45

Continuation of table

No.	Event C – reliable estimates			Event D – wrong estimates	
	Prob. of Event C	Freq. est. shift, %	Freq. est. STD, %	Prob. of Event D	Freq. est. shift, %
1	1.00	-0.10	1.36	0.00	-100.00
2	0.96	-0.30	2.28	0.01	-99.50
3	0.80	-0.40	-0.13	0.06	5.80
4	0.43	-1.20	-1.88	0.11	-29.00
5	0.74	0.00	-4.08	0.06	-40.80
6	0.34	-2.50	3.67	0.00	-100.00
7	0.20	-3.00	4.45	0.02	205.50

When the signal is available for measurement only during the one period time interval , then the algorithm efficiency drops sharply, and for improving the estimates reliability it is necessary to ensure the signal / noise ratio in the amount of several tens of units.

Above all significantly worse is accuracy of the initial frequency estimation, and, consequently, increases the probability of failures and errors. It was found that parameter estimation algorithm is more sensitive to the positive error of the initial estimate from about 20%, and negative errors to -50% can still be adjusted for an iterative process of further estimation.

When parameter estimation algorithm still leads to accurate zones (event C), even at small signal power, parameter estimates bias is very low. Moreover, the relative error of the final frequency estimate is slightly lower than for the initial phase and amplitude.

Also we research failure of the algorithm when its output frequency has negative value. Established that this behavior is caused by a significant deviation from the true value of the initial evaluation phase more than in π , resulting in the incorrect search finish in the region of negative values. To get rid of this, the output of phase search cycle checked for negative value. If the condition is satisfied, a sign of the initial frequency estimate changes to the opposite, and the phase is shifted by $-\pi$. Studies have shown that this artificial procedure slightly improves the efficiency and accuracy of parameter estimation.

From the practical point of view, failure detection allows to protect user-information system from their influence. Much more unpleasant is the appearance of uncontrolled rough frequency estimate error, even with lower probability because its effects can lead to catastrophic consequences.

It is remarkable that the final frequency shift usually exceeds the period of alternation of local minima (5), so for protection against such events should be conducted censoring procedure for estimation results.

Also conducted research for improving efficiency of searching parameters estimation algorithm. As you know, theoretically the best method for finding an extremum of unimodal functions of one variable is the method of golden section, however, studies have shown that this method for finding extreme values of the parameters does not give the desired results because the gain in accuracy does not exceed a few percent of the estimations values received by regular step descent, and sometimes is absent. In addition, when using the golden section at all levels of optimization, amount of iterations is increased several times, which directly reflected in the performance of the algorithm as a whole. By increasing the initial finding step and reducing the localization interval of golden section procedure for reducing the number of iterations is also ineffective. This can be explained by several reasons:

– Initially, the extremum search area is unknown, so to find the zone in any case still applies stepped descent algorithm. And after the minimum point is found between two neighboring points, these points are considered as the extremum area bounds and are used in the golden section procedure. It leads to a redundancy in the number of search cycles.
– Increase the search step on the one hand promotes rapid detection of a local minimum area for term of the big error of initial estimation, on the other – extends it for term of accurate enough estimate. Therefore, statistically the number of iterations in this case even with maintaining accuracy is still higher than without the golden section.
– The main reason for the error of final frequency estimation is actually data distortion in samples of signal and noise mix. Therefore using of the golden section method leads only to a slight increase of accuracy.

At all known parameters estimations the estimation of noise dispersion is determined as

$$(\sigma^*)^2 = (N-1)^{-1} \sum_{1}^{N} (x_{\mathrm{i}} - s_{\mathrm{i}}^* - \xi_{\mathrm{i}}^*)^2 \tag{7}$$

VI. Comparative Analysis of the Adaptive MTD Method and Pulse-Pair Subtraction

Consequently the adaptive MTD method requires execution of such set of operations: frequency estimation, determination of other parameters, iterative search and calculation of signal values according to a formula (2), calculation of noise dispersion by equation (7) and decision-making in accordance to expression (1).

The practical value of the offered adaptive MTD method is proved by the results of statistical researches by comparison with the pulse-pair subtraction (PPS) method. The special attention was paid on the slowly moving target detection. The interference was modeled as random process with autocorrelation coefficient *r*.

From adduced, as an example, the speed detection characteristics of both methods in the Fig 5 are presented.

One can see that offered adaptive MTD algorithm enables to detect the slowly moving target in the frequencies range, where PPS procedure represses them fully.

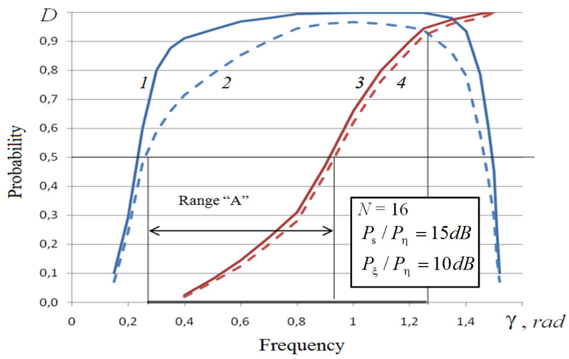


Fig. 5. Speed regulation characteristics of both methods: 1 – Adaptive, *r*=1; 2 – Adaptive, *r*=0.95; 3 – PPS, *r*=1; 4 – PPS, *r*=0.95.

VII. Conclusion

The use of empiric Bayesian approach allows us to create the adaptive MTD method.

The use of ARMA models allows us to synthesize effective harmonic signal frequency estimation, by which all other adaptation parameters are determined.

The proposed iterative algorithm, which synthesized with information about structure of likelihood function, allows improving efficiency and accuracy of parameter estimation in comparison with known methods.

The offered adaptive MTD method allows us to detect slowly moving targets, which have radial speed 3-4 times less, than possible for PPS procedure. The application of the synthesized algorithms will allow us to increase precision of other equipment for harmonic signals parameters measuring.

References

[1] R. G. Strongin, *Numerical methods in multiextremal problems*. Moscow, USSR: Nauka, 1978.
[2] M. Mints and V. Chinkov, "Operative method for harmonic signal parameters measuring," *Measuring Equipment*, no. 7, pp. 47–51, 1994.
[3] I. Prokopenko and I. Omelchuk, "The quasioptimal valuation of harmonic signal frequency on a limited observation interval," *Electronics and control systems*, no. 1 (19), pp. 39–45, 2009.
[4] A. Nikitin and S. Yushanov, "Measurement of instantaneous frequency broadband signals over a short interval of observation," *Measuring Equipment*, no. 2, pp. 50–54, 2008.
[5] N. Kayukov and V. Manelis, "Comparative analysis of different methods of signals frequency estimation," *Radioelectronics*, no. 7, pp. 42–56, 2006.