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DYNAMIC RESPONSE CONTROL OF THREE-LAYERED ANNULAR PLATE DUE TO VARIOUS PARAMETRES OF ELECTRORHEOLOGICAL CORE

The paper presents dynamic responses of annular plate composed of three layers. The middle layer of the plate has electrorheological properties expressed by the Bingham body model. The plate is loaded in the plane of facings with time-dependent forces. The electrorheological effect is observed in the area of supercritical plate behaviour. The influence of both material properties and geometrical dimensions of the core on plate behaviour is examined. The problem is solved analytically and numerically using the orthogonalization method and the finite difference method. Comparison of the results obtained using the finite difference and the finite element methods for a plate in critical state is shown. The numerical calculations are carried out for axisymmetric and asymmetric plate modes. The presented diagrams show the plate reaction to the changes in values of plate parameters and indicate that the supercritical control of plate work is possible.

Main notations

r_i, r_o	– inner and outer radii of the annular plate
r, θ, z	– cylindrical coordinates
r, ρ	– plate radius and dimensionless radius
u, v	– displacements in radial and circumferential directions, respectively
$\delta, \gamma, \bar{\delta}, \bar{\gamma}$	– differences of radial and circumferential displacements of the points in middle surfaces of facings and dimensionless differences, respectively
$h_1 = h_3 = h', h_2$	– equal thickness of facings and core thickness, respectively

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$h = h_1 + h_2 + h_3$	– total thickness of plate
t, t^*	– time and dimensionless time
$p = f(t)$	– compressive stress
s	– rate of loading growth
p_{cr}, p_{crdyn}	– critical, static and dynamic stress
m	– number of circumferential waves corresponding to the form of plate buckling,
$\tau_{rz2}, \tau_{\theta z2}$	– core shear stresses in radial and circumferential directions, respectively
$\tau_{rmax}, \tau_{\theta max}$	– core static yield stress in radial and circumferential directions, respectively
$\dot{\gamma}_{rz2}, \dot{\gamma}_{\theta z2}$	– core shear rate in radial and circumferential directions, respectively
w, w_d, w_o	– total, additional and preliminary plate deflection, respectively
ζ, ζ_1, ζ_o	– dimensionless total, additional and preliminary plate deflection
E, G, ν	– Young's and Kirchhoff's moduli and Poisson's ratio of the facings material, respectively
G_2, η	– core Kirchhoff's modulus and viscosity constant
μ, μ_2	– facing and core mass density, respectively
Q_r, Q_θ	– radial and circumferential transverse resultant forces
$\sigma_r, \tau_{r\theta}$	– radial and shear stress in facings
Φ, F	– stress function and dimensionless stress function
$\eta_o = f(\rho)$	– form of plate predeflection
$t_{cr}, w_{dcr}, \tau_{cr}, \tau_{crmax}$	– critical time, additional deflection, core shear stress and maximum shear stress determined at the moment of the loss of plate stability
$()_{,x}$	– partial derivative

1. Introduction

The possibility of controlling the behaviour of construction element subjected to dynamic loads is an important issue of expanding application range. The problem of dynamic response of a smart structure is still the subject to investigations. It is a complex task, which physical knowledge and ability to effectively solve are still current issues. Creating a layered structure with the electrorheological (ER) layer is one of many ways for controlling the element response. The possibility of using the electrical field strength to change electrorheological material properties from elastic solid body to rheological, viscous suspension is a well known phenomenon. The basic physical model, which describes the electrorheological effect also called the Winslow effect [1] is the model of the Bingham body. The change in material properties between an elastic body and the properties of a viscous fluid depended on electrical field intensity takes place for a determined level of shear stress.

The values of static shear stresses and viscosity constants express the physical relation of the Bingham model.

The influence of changes in viscosity constant describing the properties of rheological suspension of the Bingham model as the material of the plate core on the plate behaviour has been evaluated in this paper. The supercritical response of an annular plate composed of three layers symmetrically arranged and loaded in the plane of facings has been specially highlighted. The possibility of controlling supercritical work of such a structure has inspired the author to undertake a numerical analysis. The investigations of dynamic behaviours of sandwich annular plates with ER fluid core are presented in works by Yeh [2, 3]. The effects of electric fields on the frequencies of the sandwich annular plates with different thickness of ER layer and with different radius ratio are shown in work [2]. Work [3] presents the instability regions changed with various thickness of electrorheological plate layer. The results show the possibility of active dynamic control. The parametric resonance problems of axisymmetric sandwich annular plate with an ER fluid core are presented in work [4]. The stability and instability regions for plates with different structure parameters, intensity of electric fields, thickness of ER layer were analysed there. Also, in work [5] the effect of ER fluid on the stability of a rectangular, sandwich plate is found. The ER materials have rheological properties such as viscosity, plasticity and elasticity which change with electric field. The authors noticed that the natural frequencies of sandwich plate decrease with an increase in thickness of ER layer. The control of the structural damping was confirmed. A rectangular elastic plate is the subject of the analysis in work [6], too. It is an interesting investigation where ER fluid with constraining layer don't cover the plate. They are in the form of a patch, whose dimensions and location change. The effects of electric field intensity and ER patch parameters on frequency response and modal loss factors are examined. The investigations of sandwich elements with ER core presented in works [7, 8, 9] are worth attention. The problem of sandwich plates with ER fluid core subjected to supersonic airflow is analysed in [7]. A sandwich beam with ER core resting on Winkler's elastic foundation under harmonic axial loads and a laminated composite beam with ER layer expressed by Bingham's model are analysed in works [8] and [9], respectively. Among other things, there was shown the significance of ER layer, which affects damping properties of structures, increases the critical loads and improves the dynamic stability. It was also noticed that the results depend on the ratios between thicknesses of plate layers.

The authors of work [10] have noticed that the applications of ER materials have many limitations. In spite of them, the ER technology exists in electronics, automatics, medicine, building engineering, nanotechnology and

control systems. The main aims of application of ER materials are vibration control and damping of structure response. The same applies to annular layered plates which can be used, for example, in: mechanical and nuclear engineering or aerospace industry.

In this work, the method of analytical and numerical solution to the problem is presented. Many calculation results of plates with various structure geometry have been shown. The influence of geometrical dimensions of electrorheological core layer on sensitivity of plate dynamic responses has been examined. The obtained results indicate that the control of supercritical plate work is possible. The structure exhibits high sensitivity to small value fluctuations of plate material parameters and geometrical parameters. Some observations were also presented in [11].

2. Problem formulation

The three-layered, annular plate is the subject of the consideration. The plate structure is symmetrically arranged. It is composed of thin, elastic facings and thin, middle layer with electrorheological properties treated as the plate core. The core material is expressed by the physical relations of the Bingham material (see, Eq. (2)). The plate is loaded on facings with the stress uniformly distributed on perimeter, increasing in time according to the linear formula:

$$p = st. \quad (1)$$

The plate edges are slideably clamped. The scheme of the plate is presented in Fig.1.

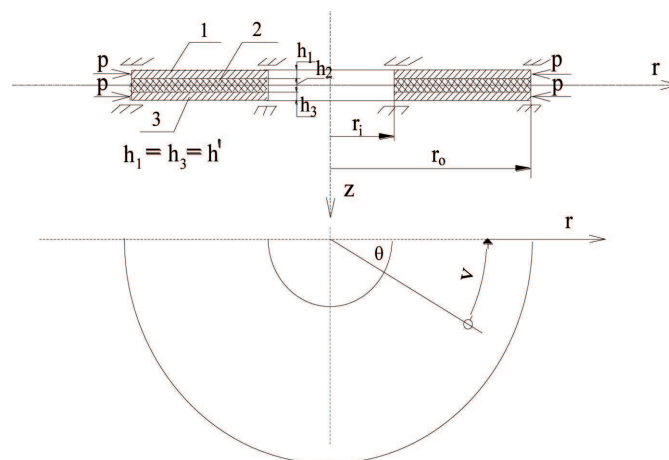


Fig. 1. Scheme of analysed plate

As a criterion of the loss of the plate dynamic stability the criterion presented in work [12] by Volmir was adopted. According to this criterion, the loss of plate stability occurs at the moment of time, when the speed of the plate point of maximum deflection reaches the first maximum value.

At the moment of the loss of plate stability, expressed by critical time t_{cr} , the change in core material behaviour reveals via switching from the properties of elastic material to properties of electrorheological, Bingham material. The approach to the problem enables the evaluation of the supercritical behaviour of a electrorheological structure. The analyses are focused on the influence of the geometry, particularly thickness of a plate core and the values of rheological quantities (value of viscosity constant η – see, Eq.(2)) on the plate dynamic reactions. The second components in the equations of the Bingham body (see, Eq.(2)) i.e. the shear stresses $\tau_{r\max}$, $\tau_{\theta\max}$ have been accepted as equal to values of critical radial and circumferential stresses τ_{cr} calculated at the moment of the loss of plate dynamic stability.

3. Problem solution

In analytical and numerical solution the classical theory of sandwich plates with the broken line hypothesis [13] has been used. The distribution of stresses into normal and shear loading the plate facings and the core has been accepted, respectively. The proposed method of solution, which is based on the solution to the dynamic deflection task of three-layered, annular plate with elastic and viscoelastic core is presented in works [14,15].

The main steps of the solution to the discussed problem are the following:

- formulation of the dynamic equilibrium equations for each plate layer,
- description of the core deformation in radial and circumferential directions accepting the rule: preliminary and additional deflections are equal for each plate layer,
- application of the linear physical relations of the Hooke's law for facings and linear elastic and linear viscous flow relations expressed by the Bingham body equation (2) of the plate core material. Physical relations of the Bingham core material are expressed by equations [1,16]:

$$\tau_{rz_2} = \eta \cdot \dot{\gamma}_{rz_2} + \tau_{r\max}, \quad \tau_{\theta z_2} = \eta \cdot \dot{\gamma}_{\theta z_2} + \tau_{\theta\max}, \quad (2)$$

- establishing the sectional forces and moments in facings using the equations of the nonlinear Kármán's plate and formulation of the resultant, transverse radial Q_r and circumferential Q_θ forces. The resultant forces Q_r , Q_θ as the sums of the individual layer forces $Q_{r_{1(2,3)}}$, $Q_{\theta_{1(2,3)}}$ found based upon the equilibrium equations have the following form:

$$Q_r = -k_1 w_{d,rrr} - \frac{k_1}{r} w_{d,rr} + \frac{k_1}{r^2} w_{d,r} - \frac{k_2}{r^2} w_{d,r\theta\theta} + \frac{k_1 + k_2}{r^3} w_{d,\theta\theta} + \tilde{G}_2 (\delta + H' w_{d,r}) \frac{H'}{h_2} + \tau_{r \max} H', \quad (3)$$

$$Q_\theta = -\frac{k_1}{r^3} w_{d,\theta\theta\theta} - \frac{k_1}{r^2} w_{d,\theta r} - \frac{k_2}{r} w_{d,\theta rr} + \tilde{G}_2 \left(\gamma + H' \frac{1}{r} w_{d,\theta} \right) \frac{H'}{h_2} + \tau_{\theta \max} H'$$

where:

$$H' = h' + h_2$$

$$k_1 = 2D, \quad k_2 = 4D_{r\theta} + \nu k_1,$$

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad D_{r\theta} = \frac{Gh^3}{12} - \text{flexural rigidity of the outer layers,}$$

\tilde{G}_2 – quantity expressed as: $\tilde{G}_2 = \eta \frac{\partial}{\partial t}$, $\frac{\partial}{\partial t}$ – differential operator,

$\tilde{G}_2 = G_2$ – for the elastic core,

- formulation of the resultant membrane forces expressed by the introduced stress function Φ ,
- determination of the initial loading and boundary conditions and conditions connected with the slideably clamped both inner and outer plate edges:

$$w|_{t=0} = w_o, \quad w_{,t}|_{t=0} = 0, \quad w_d|_{t=0} = 0, \quad w_{d,t}|_{t=0} = 0,$$

$$\sigma_r|_{r=r_i(r_o)} = -p(t) d_1(2), \quad \sigma_{r,t}|_{r=r_i(r_o)} = -(p(t))_{,t} d_1(2), \quad \tau_{r\theta}|_{r=r_i(r_o)} = 0 \quad (4)$$

$$w|_{r=r_i(r_o)} = 0, \quad w_{,r}|_{r=r_i(r_o)} = 0, \quad \delta = \gamma|_{r=r_i(r_o)} = 0, \quad \delta_{,r}|_{r=r_i(r_o)} = 0$$

where:

d_1, d_2 – quantities, equalled to 0 or 1, determining the loading of the inner or/and outer plate perimeter,

- determination of the basic differential equation expressing the deflections of the analysed sandwich plate with ER core material:

$$\begin{aligned} & k_1 w_{d,rrrr} + \frac{2k_1}{r} w_{d,rrr} - \frac{k_1}{r^2} w_{d,rr} + \frac{k_1}{r^3} w_{d,r} + \frac{k_1}{r^4} w_{d,\theta\theta\theta\theta} + \\ & \frac{2(k_1 + k_2)}{r^4} w_{d,\theta\theta} + \frac{2k_2}{r^2} w_{d,rr\theta\theta} - \frac{2k_2}{r^3} w_{d,r\theta\theta} + \\ & - \tilde{G}_2 \frac{H'}{h_2} \frac{1}{r} \left(\gamma_{,\theta} + \delta + r\delta_{,r} + H' \frac{1}{r} w_{d,\theta\theta} + H' w_{d,r} + H' r w_{d,rr} \right) + \\ & - \frac{H'}{h_2} \frac{1}{r} \left(r \tilde{Q}_{r^2 r} + \tilde{Q}_{r^2} + \tilde{Q}_{\theta^2 \theta} \right) = \frac{2h'}{r} \left(\frac{2}{r^2} \Phi_{,\theta} w_{,r\theta} - \frac{2}{r} \Phi_{,r\theta} w_{,r\theta} + \right. \\ & + \frac{2}{r^2} w_{,\theta} \Phi_{,r\theta} - \frac{2}{r^3} w_{,\theta} \Phi_{,\theta} + w_{,r} \Phi_{,rr} + \Phi_{,r} w_{,rr} + \\ & \left. + \frac{1}{r} \Phi_{,\theta\theta} w_{,rr} + \frac{1}{r} \Phi_{,rr} w_{,\theta\theta} \right) - M w_{d,tt} \end{aligned} \quad (5)$$

where:

$M = 2h_1\mu + h_2\mu_2$, \tilde{Q}_{r2} , $\tilde{Q}_{\theta 2}$ – transverse forces of ER core in radial and circumferential direction, respectively.

In the solution, the following dimensionless quantities, expressions and shape functions have been accepted [14,17]:

$$\zeta = \frac{w}{h}, \quad \zeta_1 = \frac{w_d}{h}, \quad \zeta_o = \frac{w_o}{h}, \quad F = \frac{\Phi}{Eh^2}, \quad \rho = \frac{r}{r_o},$$

$$\bar{\delta} = \frac{\delta}{h}, \quad \bar{\gamma} = \frac{\gamma}{h}, \quad t^* = t \cdot K7, \quad K7 = \frac{s}{p_{cr}}, \quad (6)$$

$$\zeta_1(\rho, \theta, t) = X_1(\rho, t) \cos(m\theta),$$

$$\zeta_o(\rho, \theta) = X_a(\rho) + X_b(\rho) \cos(m\theta),$$

$$X_a(\rho) = \xi_1 \eta_o(\rho), \quad X_b(\rho) = \xi_2 \eta_o(\rho), \quad (7)$$

$$\eta_o(\rho) = \rho^4 + A_1 \rho^2 + A_2 \rho^2 \ln \rho + A_3 \ln \rho + A_4,$$

$$\zeta = \zeta_1 + \zeta_o,$$

$$F(\rho, \theta, t) = F_a(\rho, t) + F_b(\rho, t) \cos(m\theta) + F_c(\rho, t) \cos(2m\theta),$$

$$\bar{\delta}(\rho, \theta, t) = \bar{\delta}(\rho, t) \cos(m\theta), \quad \bar{\gamma}(\rho, \theta, t) = \bar{\gamma}(\rho, t) \sin(m\theta), \quad (8)$$

$$\tau_{r \max}(\rho, \theta) = \tau_{r \max}(\rho) \cos(m\theta), \quad \tau_{\theta \max}(\rho, \theta) = \tau_{\theta \max}(\rho) \sin(m\theta)$$

where:

ξ_1, ξ_2 – calibrating numbers,

A_i – quantities fulfilling the conditions of clamped edges by the function $\eta(\rho)$, $i = 1, 2, 3, 4$.

Using the orthogonalization method and the finite difference method after several consecutive transformations and addition of some new equations to calculate the quantities $\bar{\delta}, \bar{\gamma}$ the form of the system of differential equations expressing the deflections of the three-layered, annular plate with ER core was obtained:

$$\mathbf{P}\mathbf{U} + \mathbf{Q} + \mathbf{P}_L \dot{\mathbf{U}} + \mathbf{Q}_L + \mathbf{Q}_E = K_E \ddot{\mathbf{U}}, \quad (9)$$

$$\mathbf{M}_{Y(V,Z)} \mathbf{Y}(V, Z) = \mathbf{Q}_{Y(V,Z)}, \quad (10)$$

$$\mathbf{M}_{Y(V,Z)} \dot{\mathbf{Y}}(\dot{V}, \dot{Z}) = \dot{\mathbf{Q}}_{Y(V,Z)}, \quad (11)$$

$$\mathbf{M}_{DL} \dot{\mathbf{D}} = \mathbf{M}_D \mathbf{D} + \mathbf{M}_U \mathbf{U} + \mathbf{M}_{UL} \dot{\mathbf{U}} + \mathbf{M}_G \mathbf{G} + \mathbf{M}_{GL} \dot{\mathbf{G}} + \mathbf{E}\mathbf{R}, \quad (12)$$

$$\mathbf{M}_{GGL} \mathbf{G} = \mathbf{M}_{GG} \mathbf{G} + \mathbf{M}_{GU} \mathbf{U} + \mathbf{M}_{GUL} \dot{\mathbf{U}} + \mathbf{M}_{GD} \mathbf{D} + \mathbf{M}_{GDL} \dot{\mathbf{D}} + \mathbf{E}\mathbf{O}, \quad (13)$$

where:

K_E – expression, $K_E = K7^2 WK5 WK8'$,

$WK5, WK8'$ – expressions: $WK5 = \frac{h'}{h}$, $WK8' = r_o h_2 M$,

$\mathbf{U}, \mathbf{Y}, \mathbf{V}, \mathbf{Z}, \dot{\mathbf{U}}, \ddot{\mathbf{U}}, \dot{\mathbf{U}}, \dot{\mathbf{Y}}, \dot{\mathbf{V}}, \dot{\mathbf{Z}}$ – vectors of plate additional deflections and components of the stress function,

$\mathbf{Q}, \mathbf{Q}_L, \mathbf{Q}_Y, \mathbf{Q}_V, \mathbf{Q}_Z, \dot{\mathbf{Q}}_Y, \dot{\mathbf{Q}}_V, \dot{\mathbf{Q}}_Z, \mathbf{D}, \mathbf{G}, \dot{\mathbf{D}}, \dot{\mathbf{G}}$ – vectors of expressions composed of plate model parameters,

\mathbf{Q}_E – vector of elements expressed as: $\frac{H'}{h_2 h} \text{WK5} \left(\frac{m}{\rho_i} \tau_{\theta \max_i} + \frac{1}{\rho_i} \tau_{r \max_i} + \tau_{r \max_i \rho} \right)$
(i – discretization points),

$\mathbf{ER}, \mathbf{E\Theta}$ – vectors of elements expressed as: $ER = \frac{r_o}{h_2 h H'} \tau_{r \max_i}$,

$E\Theta = \frac{2r_o \rho_i}{h_2^2 h} \tau_{\theta \max_i}$, respectively,

$\mathbf{M}_D, \mathbf{M}_G, \mathbf{M}_{GG}, \mathbf{M}_{GD}, \dot{\mathbf{M}}_{DL}, \dot{\mathbf{M}}_{GL}, \dot{\mathbf{M}}_{GGL}, \dot{\mathbf{M}}_{GDL}, \mathbf{M}_U, \dot{\mathbf{M}}_U, \mathbf{M}_{GU}, \dot{\mathbf{M}}_{GUL}$ – matrices of elements composed of plate parameters,

$\mathbf{M}_Y, \mathbf{M}_V, \mathbf{M}_Z$ – matrices of elements composed of the plate radius, value of the length of the interval in the finite difference method and number of buckling mode m .

The system of Equations (9)-(13) was solved using the Runge-Kutta's integration method for the initial state of the plate.

Critical static stress p_{cr} has been calculated solving the eigenproblem for the problem of the disk state neglecting the inertial components and nonlinear expressions [14, 15].

4. Plate model in Finite Element Method

The plate model called as a simplified model is built of axisymmetrical elements. The scheme is presented in Fig. 2.

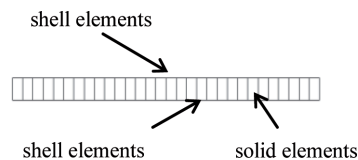


Fig. 2. Simplified model of plate

The facings are built of shell elements, but the core mesh is built of solid elements. The grids of facings elements are tied with the grid of core elements using the surface contact interaction. The calculations were carried out at the Academic Computer Center CYFRONET-CRACOW (KBN/SGL_ORIGIN_2000/PŁódzka/030/1999) using the ABAQUS system.

5. Numerical results

The exemplary results show the dynamic response of the plate loaded on inner or outer edge. The subject of analyses are the plates, whose quotient of inner radius to outer one is equal to: $\rho_i = 0.4$ or 0.5 . The plate is loaded with linear, quickly increasing stress (see, Eq.(1)). The rate of loading growth s is equal to: $s \approx 4346$ MPa/s and $s \approx 932$ MPa/s for plates loaded on inner or outer perimeter of facings, respectively. The plate facings made of steel with parameters: Young's modulus $E = 2.1 \cdot 10^5$ MPa, Poisson's ratio $\nu = 0.3$, have thickness equal to $h' = 0.001$ m. The plate core material in elastic field behaviour has a value of Kirchhoff's modulus G_2 equal to $G_2 = 2.5$ MPa but for rheological fluid properties the values of viscosity constants η are in range of $\eta = 1.7$ Pa·s to $\eta = 17000$ Pa·s. The adopted value of the mass density is equal to $\mu = 64$ kg/m³. The core thickness is equal to: $h_2 = 0.001, 0.002, 0.003$ m. The plate cases, for which the loss of dynamic stability occurs for the minimal value of critical load, have been examined. It is observed for axisymmetrical buckling form $m = 0$ of the plate loaded on inner edge and for a circumferentially wavy form of the plate compressed on outer edge. One can see the confirmation of these results in works [14, 15]. The examined plate loaded on outer edge, whose geometry and material are determined by the following parameters: inner radius $r_i = 0.2$ m, outer radius $r_o = 0.5$ m, core thickness $h_2 = 0.002$ m, Kirchhoff's modulus $G_2 = 2.5$ MPa, loses its dynamic stability in the buckling form with $m = 7$ circumferential waves for the minimal value of critical load p_{crdyn} . The distribution of values of dynamic critical loads p_{crdyn} for several plate modes is presented in Table 1.

Table 1.
Critical dynamic loads p_{crdyn} and corresponding buckling modes m of plate compressed on outer edge

buckling mode m	0	3	5	6	7	8	9	10
critical dynamic load p_{crdyn} [MPa]	26.97	18.77	14.95	14.30	14.11	14.77	15.23	16.72

The cases of plates loaded on inner edge with different core thickness $h_2 = 0.001, 0.002$ and 0.003 m, in which the size of inner dimensionless radius ρ_i is equal to $\rho_i = 0.4$ are shown in Figs. 3, 4, 5. Fig. 3b presents the character of the plate behaviour for the value of Kirchhoff's modulus of elastic core equal to $G_2 = 2.5$ MPa and additionally for $G_2 = 0.5$ MPa and $G_2 = 5.0$ MPa. The plate with core material parameter, in which Kirchhoff's modulus is equal to $G_2 = 2.5$ MPa is subjected to the detailed analysis. The results for ER core properties and the marked point of the loss of plate dynamic stability are presented. The plate with value of viscosity constant above $\eta(\eta) = 17$ Pa·s behaves as the plate with elastic core with supercritical vibrations

initiated by increasing load. The limitation of the supercritical plate dynamic response is possible for electrorheological core parameters, whose value of the viscosity constant is about $\eta = 10 \text{ Pa}\cdot\text{s}$ (see, Fig. 3b). The decrease in the core thickness to value of $h_2 = 0.001 \text{ m}$ shows that for smaller values of core viscosity constant (below the value of $\eta = 10 \text{ Pa}\cdot\text{s}$) the control of the plate behaviour is possible. Figure 4 presents these observations. The increase in core thickness to the value $h_2 = 0.003 \text{ m}$ causes that influence of ER core properties on the plate response exists for higher values of viscosity constant in the range of $\eta = 20 \text{ Pa}\cdot\text{s}$ (see, Fig. 5). The results show that significance of ER core thickness is important for the plates loaded on inner edge. The controlling supercritical plate behaviour depends on both value of rheological core parameters and core thickness.

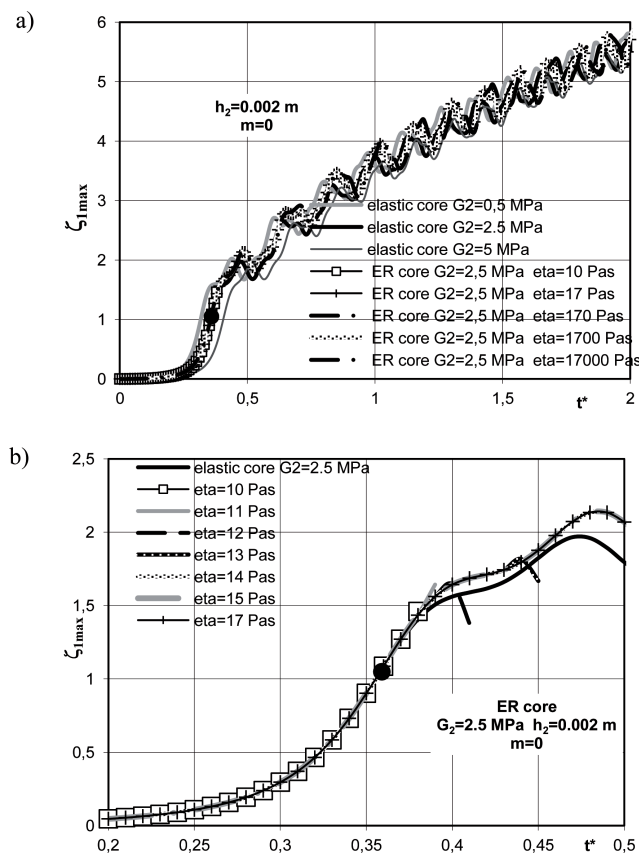


Fig. 3. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.002 \text{ m}$) loaded on inner edge and with value of inner radius equal to $\rho_i = 0.4$: a) in full examined range, b) in range of time $t^* = 0.2 \div 0.5$ and for values of $\eta(\eta) = 10 \div 17 \text{ Pa}\cdot\text{s}$

The comparison of results presented in Fig. 3b for the plate with core thickness $h_2 = 0.002 \text{ m}$ and the value of plate inner radius $\rho_i = 0.4$ with

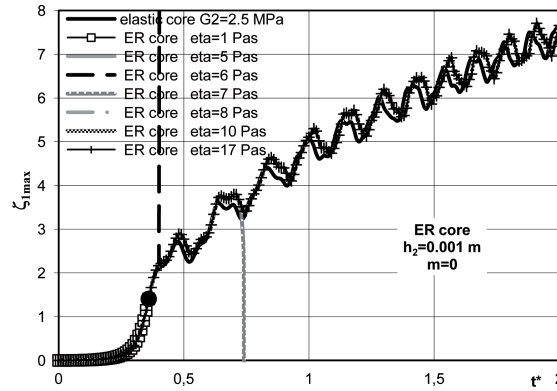


Fig. 4. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.001$ m) loaded on inner edge and with value of inner radius equal to $\rho_i = 0.4$

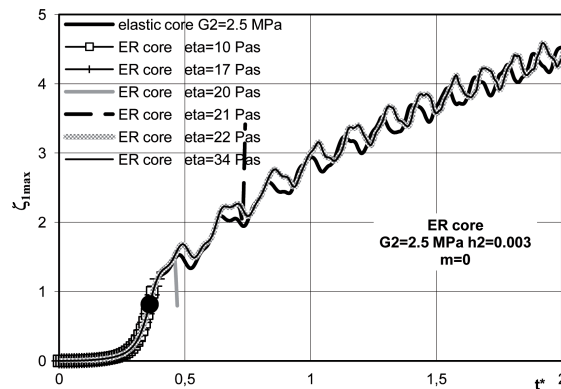


Fig. 5. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.003$ m) loaded on inner edge and with value of inner radius equal to $\rho_i = 0.4$

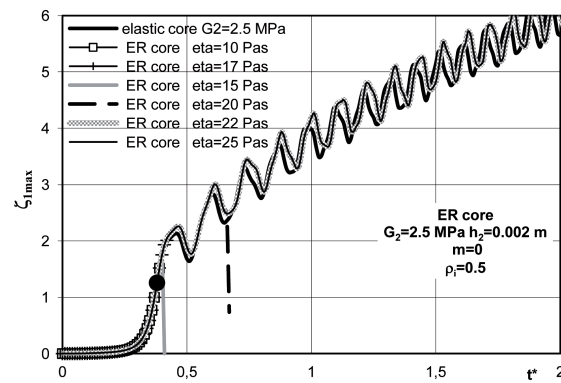


Fig. 6. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.002$ m) loaded on inner edge and with value of inner radius equal to $\rho_i = 0.5$

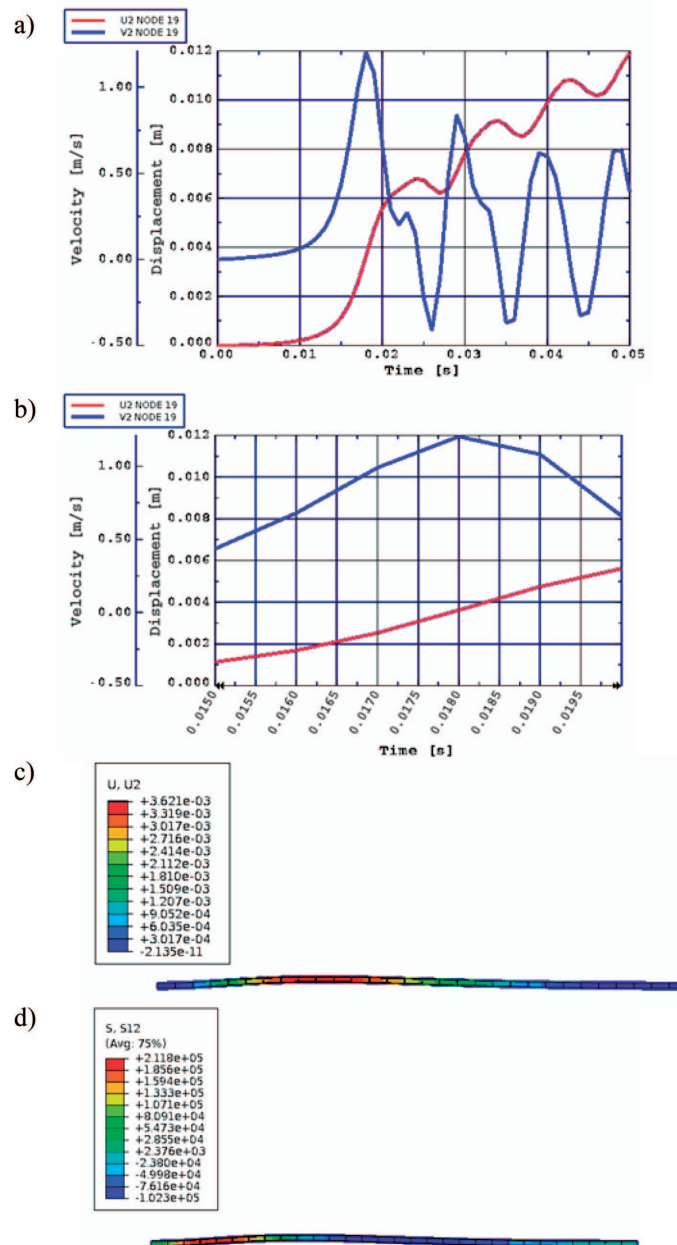


Fig. 7. Results for FEM plate model loaded on inner edge with parameters: $G_2 = 2.5$ MPa, $h_2 = 0.003$ m, $\rho_i = 0.4$: a) time histories of deflection and velocity of deflection in full analysed range and b) in range of time $t = 0.015 \div 0.025$ s, c) form of plate buckling, d) distribution of values of critical radial shear stress

the results shown in Fig. 6 for the plate with the same core thickness but with the value of radius ρ_i equal to: $\rho_i = 0.5$ indicates the meaning of the plate outer geometry. One can notice that with the increase in value of inner plate radius ($\rho_i = 0.5$) the ER effect is observed for higher values of viscosity constant, equal about $\eta = 20$ Pa·s.

The Fig. 7 shows the time histories of deflection and velocity of deflection of the plate model calculated using the Finite Element Method (FEM). The plate with the value of inner radius $\rho_i = 0.4$ is loaded on inner edge. The core thickness is equal to $h_2 = 0.003$ m. The Fig. 7a,b enables the establishing of the critical parameters: time t_{cr} and deflection w_{dcr} . Figure 7c and Fig. 7d show the plate deformation and the distribution of radial shear critical stress τ_{cr} , respectively. The values of critical parameters: t_{cr} , w_{dcr} , τ_{cr} are comparable with the ones calculated using the Finite Difference Method (FDM). The deflection curve $\zeta_{1 \max} = f(t^*)$ and the distribution of critical shear stress for FDM plate model are shown in Fig. 5 and Fig. 8, respectively. Figure 8 shows the distribution of critical values of shear stresses for the examined plate cases with different core thickness and inner radius ρ_i (ro). The results setting-up for other plate cases are presented in Table 2.

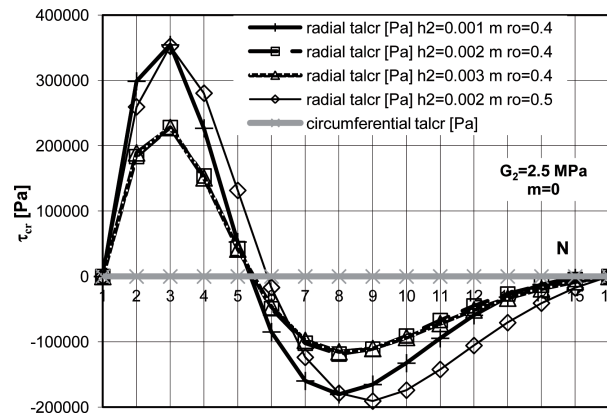


Fig. 8. Distribution of values of critical shear stress τ_{cr} (talcr) depending on discrete points N for plates loaded on inner edge

Table 2. The values of critical time t_{cr} and deflection w_{dcr} and maximum value of radial shear stress $\tau_{cr \max}$ for plate models calculated using FDM and FEM methods

h_2 [m] / ρ_i	FDM			FEM		
	t_{cr} [s]	w_{dcr} [m]	$\tau_{cr \max}$ [MPa]	t_{cr} [s]	w_{dcr} [m]	$\tau_{cr \max}$ [MPa]
0.001 / 0.4	0.018	0.0042	0.35	0.018	0.0037	0.33
0.002 / 0.4	0.018	0.0042	0.23	0.018	0.0038	0.25
0.003 / 0.4	0.018	0.0041	0.23	0.018	0.0036	0.21
0.002 / 0.5	0.019	0.005	0.35	0.019	0.0045	0.32

Figures 9, 10, 11 show the character of dynamic behaviour of the plates loaded on the outer edge. The presented critical deformation is in the form of several circumferential waves $m = 6, 7, 8$. Results corresponding to the plates with $m = 6, 8$ numbers of buckling waves enable the observation of plate behaviour particularly in the range of the critical state. The marked points, which mean the loss of plate stability, indicate the small differences of the critical values of time t_{cr} and deflection w_{dcr} between the plate modes $m = 6, 7, 8$. The plate with buckling mode equal to $m = 7$ is subjected to the detailed numerical analysis. The plate reactions for the Bingham effect in the formulated dynamic problem were examined. The evaluation of the results presented in Figs. 9, 10, 11 indicates that the core thickness doesn't influence dynamic response of plate with electrorheological effect. For each of the analysed cases of core thickness ($h_2 = 0.001, 0.002, 0.003$ m) the limitation of the supercritical plate behaviour is possible for the values of viscosity constant, which are in range of $\eta = 1.7 \text{ Pa}\cdot\text{s} \div 17 \text{ Pa}\cdot\text{s}$. For the higher values of viscosity constant the plate behaves as an elastic one with supercritical vibrations. One can notice that the sensitivity of waved plate on the fluctuation of the size of structure geometry is low. This observation could be interesting for some practical applications. One could try to formulate some conclusion that determined, fixed values of electrorheological fluid as the Bingham core material control the dynamic response of strongly circumferentially buckled plate with geometrically different structure.

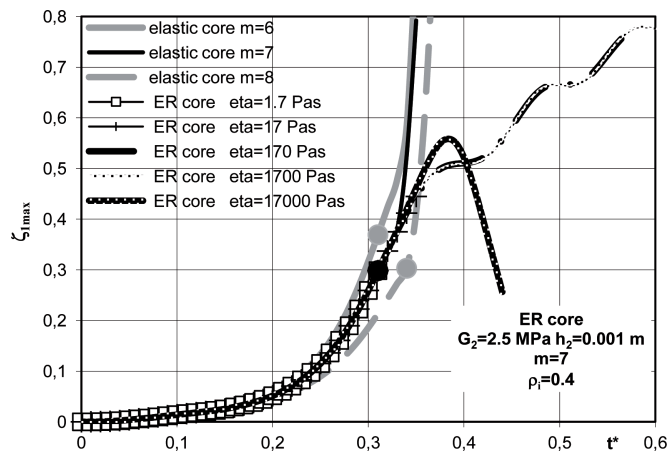


Fig. 9. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.001$ m) loaded on outer edge and with value of inner radius equal to $\rho_i = 0.4$

Additionally, the results shown in Fig. 12 for the plate with value of dimensionless inner radius equal to $\rho_i = 0.5$ confirm presented observations. The inner radius of each plate layer also the layer of the Bingham core is

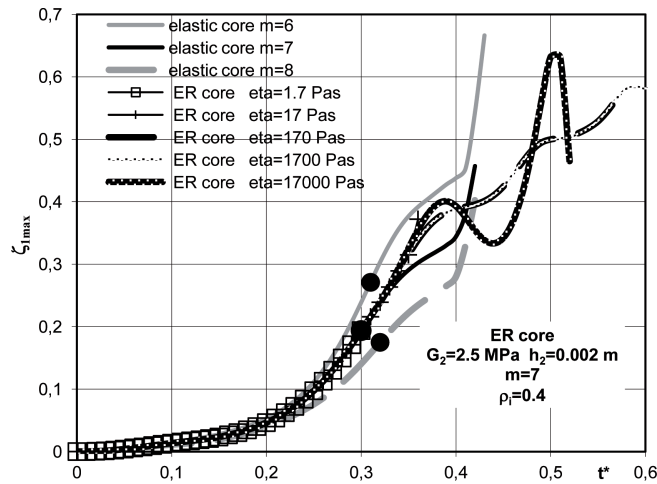


Fig. 10. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.002$ m) loaded on outer edge and with value of inner radius equal to $\rho_i = 0.4$

greater than for plates described above (see Figs. 9, 10, 11). The plate behaviour doesn't change. For low, fixed values of viscosity constants, supercritical zone of plate behaviour is limited. For higher values of viscosity constants, the vibrations are initiated. The observed fluctuations of values of critical parameters and the tendency to relegation of the minimal value of critical time t_{cr} to higher value of mode (from $m = 7$ to $m = 8$) are the result of characteristic changes in geometry of the plate with an elastic core [14].

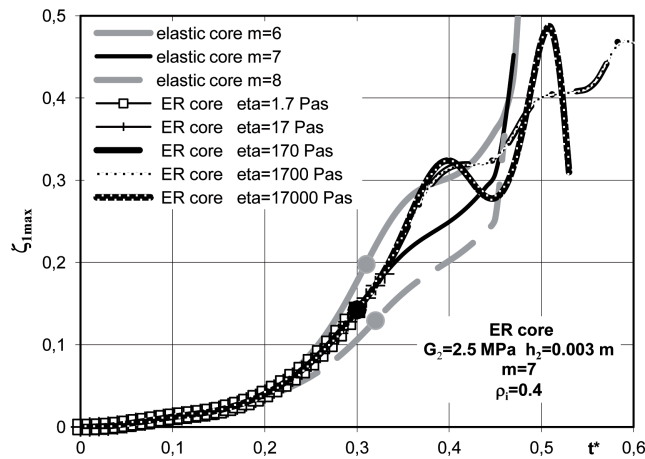


Fig. 11. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.003$ m) loaded on outer edge and with value of inner radius equal to $\rho_i = 0.4$

Figure 13 shows the distribution of radial and circumferential values of critical shear stresses for the plate compressed on outer edge with core

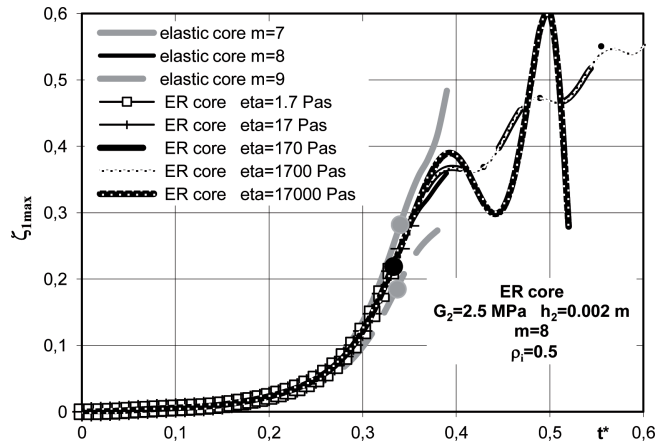


Fig. 12. Time histories of deflection of plates with elastic and ER core ($h_2 = 0.002$ m) loaded on outer edge and with value of inner radius equal to $\rho_i = 0.5$

thickness equal to $h_2 = 0.001$ m and number of buckling waves $m = 7$. The presented values for this example of the plate and also for the plate loaded on inner edge (values shown in Fig. 8) were accepted in description of the Bingham core material (see, Eq. (2)). These critical stresses control plate elastic behaviour, which exists below the plate critical state and rheological, supercritical response of the plate, which is expressed by physical relations of the Bingham body of core material.

The results presented in Table 3 can recap the numerical investigations undertaken in this work. The Table 3 shows the range of values of viscosity constant of ER core material for which the essential changes in supercritical plate behaviours have been observed. The existing rapid change in rheological properties influences the supercritical vibrations and dynamic response of the analysed plate. The vibrations are not initiated by increasing load or they suddenly disappear. Different plate reactions for fluctuations of ER

Table 3.

Range of values of viscosity constant η with observed ER effect for plates loaded on inner and outer edge with different ER core thickness h_2 and dimension of inner radius ρ_i

inner radius ρ_i	core thickness h_2 [m]	range of viscosity constant η [Pa·s] / buckling mode m	
		loaded edge	
		inner	outer
0.4	0.001	1 ÷ 7 / 0	1.7 ÷ 17 / 7
	0.002	10 ÷ 14 / 0	1.7 ÷ 17 / 7
	0.003	10 ÷ 21 / 0	1.7 ÷ 17 / 7
0.5	0.002	10 ÷ 20 / 0	1.7 ÷ 17 / 8

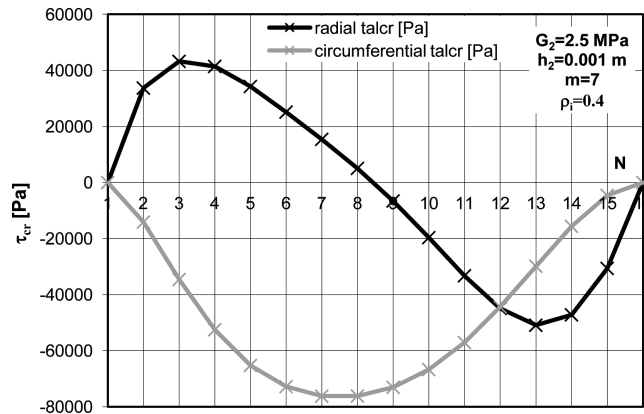


Fig. 13. Distribution of values of critical shear stress τ_{cr} (talcr) depending on discrete points N for the plate loaded on outer edge with parameters: $m = 7$, $h_2 = 0.001$ m, $\rho_i = 0.4$

core properties indicate the possibility of controlling plate behaviour. The geometry of ER core is significant for plates radially compressed on inner edge, whose buckling form is axisymmetrical.

6. Summary

The paper presents the dynamic response of an annular plate with the core layer with electrorheological properties expressed by the relations of the Bingham body. The main elements of problem solution with the presentation of the system of differential equations, which is the solution to the dynamic deflections of the examined plate, have been shown. The solution has been generalized on the plate modes circumferentially waved. The numerical results have been shown for the basic forms of plate buckling, which are important in plate dynamic stability problems: axisymmetrical for the plate loaded on inner edge and highly circumferentially waved for the plate compressed on outer edge.

The analysis was focused on the influence of core thickness and geometry size of the plate as well as the core layer on the final results. The behaviours of the plate structure for various values of viscosity constant, which is a parameter characterizing the Bingham body properties, have been presented. The results of numerical calculations have shown the possibility of controlling plate behaviour by changes in the electrical field intensity. Then, the changes of rheological properties of the core material can limit the zone of supercritical vibrations and influence on plate behaviour. Of course, the choice of specific values of the Bingham parameters requires carrying out of the perceptive experimental investigations. The presented plate seems to be an effective system which could control the dynamic behaviours of the other

objects containing such structure as a constructive element. It could be the main application of the analysed annular plate.

The numerical calculations indicate the narrow range of values of viscosity constant of the Bingham core material for the assumed level of values of shear stresses for which the searched plate responses are observed. The range of values of viscosity constant, which cause the limitations in supercritical plate behaviour, changes for different core thickness for plates loaded on inner edge. Much lower sensitivity to the thickness of ER core is observed for plates compressed on outer edge. The introduced change of inner plate radius has the similar influence on supercritical plate behaviour.

The presented image of dynamic behaviour of the analysed plate could be a helpful hint in designing and numerical modelling of “smart” layered structures working under dynamic conditions. However, the examined problem is a complex, multiparameter task, which in the case of aiming for its full evaluation requires further detailed analyses.

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Sterowanie dynamicznej odpowiedzi trójwarstwowej płyty pierścieniowej zmiennymi parametrami elektroeologicznego rdzenia

Streszczenie

W pracy przedstawiono dynamiczne odpowiedzi pierścieniowej trójwarstwowej płyty poddanej obciążeniom zmiennym w czasie. Struktura płyty złożona jest z trzech symetrycznych warstw: cienkich, stalowych okładzin i cienkiego rdzenia o właściwościach elektroeologicznych wyrażonych cechami cieczy Bingham. Płyta obciążona jest w płaszczyźnie okładzin siłami równomiernie rozłożonymi na obwodzie, liniowo szybko narastającymi w czasie. W momencie dynamicznej utraty stateczności płyty o sprężystych własnościach rdzenia pokrytyczne zachowanie płyty sterowane jest parametrami materiałowymi i geometrycznymi jej rdzenia o cechach elektroeologicznych. Zagadnienie dynamicznych ugięć płyty rozwiązano analitycznie i numerycznie wykorzystując metodę ortogonalizacyjną i metodę różnic skończonych. Ocenę wartości parametrów krytycznych płyty przeprowadzono wykorzystując także metodę elementów skończonych. Obliczeniom numerycznym poddano płyty, których utrata stateczności dynamicznej ma postać osiowosymetryczną, jak i płyty obwodowo pofalowane. Liczne wyniki obliczeń pokazują wrażliwość struktury badanej płyty na niewielkie wahania wartości parametrów materiałowych i geometrycznych rdzenia. Obserwacje potwierdzają możliwość sterowania pokrytyczną pracą płyty i jej wykorzystania jako elementu tzw. konstrukcji inteligentnej.