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On parameters determination of multi-port equivalent scheme for multi-winding traction transformers

TADEUSZ J. SOBCZYK[1], JOSEPH EL HAYEK[2]

[1]*Cracow University of Technology*
Institute on Electromechanical Energy Conversion
Warszawska 24, 31-155 Kraków, Poland
tel./fax: +48 12 6282044
e-mail: pesobczy@cyf-kr.edu.pl

[2]*University of Applied Sciences and Arts Western Switzerland*
School of Engineering
Route du Rawyl 47, CP, 1950 Sion 2, Switzerland
e-mail: joseph.elhayek@hevs.ch

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Abstract: This paper aims to present a new equivalent scheme of multi-windings traction transformers, based on multiport purely inductive circuit. The mathematical background of this equivalent scheme is described. The determination of the different scheme elements is made through a finite-elements calculation of both main and leakage inductances, for the case of a four-winding transformer. A procedure is defined, which allows to estimate the values of these elements from some measurements on the transformer at no-load and short-circuit operations. A specific strategy of short-circuit tests is described, allowing to determine all parameters in a rather simple way.

Key words: multi-winding transformer, equivalent scheme, multi-port circuit, traction transformers

1. Introduction

The equivalent scheme of a transformer is a basic tool in electrical engineering of alternating currents. Its "classical" form is used everywhere when magnetic coupling exists. Commonly an equivalent scheme of T-type with one vertical magnetizing branch is used. It is well known that a transformer having more than three magnetically coupled windings cannot be represented uniquely by a T-type equivalent scheme. In order to be described correctly, three coupled windings need three self- and three mutual inductances. An equivalent scheme has to include the same number of independent parameters. The T-type equivalent scheme for three windings needs accordingly six parameters: three leakage inductances, one common magnetizing inductance and two winding ratios, recalculating the different parameters to a reference winding. So, six independent inductances can be uniquely represented by six parameters of the equivalent scheme. In case of four coupled windings there are ten independent quantities: four self-inductances and six mutual ones, but the T-type equivalent scheme with one magnetizing branch presents only eight parameters: four leakage inductances, the common magnetizing inductance and three winding ratios. It is therefore impossible to represent in a unique way more than three magnetically coupled windings by the equivalent scheme of T-type [1].

In [2] the multi-port circuit has been proposed as an equivalent circuit of set with an arbitrary number (N) of coils. In that representation the number of inductive elements is always equal to the number of independent self and mutual inductances. In [3, 4, 7] this approach has been applied as an equivalent scheme of a multi-winding traction transformer. Traction transformers have many windings with very different tasks within the locomotive. There are windings connecting the line catenary to the traction supply system, some ones supplying power electronics drives and others feeding the auxiliary systems of the train. A proper representation of traction transformers is of a great interest for designers as well as for users.

In [6] an equivalent scheme of a laboratory model traction transformer with four windings has been developed and a procedure of determining its parameters by field computation has been described, including magnetic non-linearity of the transformer core. This paper aims to present a procedure, which allows estimating these parameters from some measurements on the transformer by no-load and short-circuit operations.

2. Background for the multiport equivalent scheme

A set of magnetically coupled coils is modelled by relations between coils flux linkages and coils currents. Assuming magnetic linearity, this is described by relation (1)

$$\Psi = \mathbf{L}\,\mathbf{i}, \tag{1}$$

where: Ψ – is the vector of flux linkages

$$\Psi^T = [\psi_1 \ \psi_2 \ \cdots \ \psi_N],$$

i – is the vector of winding currents

$$\mathbf{i}^T = [i_1 \ i_2 \ \cdots \ i_\mathrm{N}],$$

L – is the inductance matrix

$$\mathbf{L} = \begin{bmatrix} \mathrm{L}_{11} & \mathrm{L}_{12} & \cdots & \mathrm{L}_{1\mathrm{N}} \\ & \mathrm{L}_{22} & \cdots & \mathrm{L}_{2\mathrm{N}} \\ & & \ddots & \vdots \\ (sym) & & & \mathrm{L}_{\mathrm{NN}} \end{bmatrix}.$$

In order to obtain the multi-port equivalent scheme, one has to express the coil currents in function of the flux linkages [2]. This means that, instead of the classical relation,

$$\psi_n = \mathrm{L}_{n,1}\, i_1 + \mathrm{L}_{n,2}\, i_2 + \cdots + \mathrm{L}_{n,\mathrm{N}}\, i_\mathrm{N}, \tag{2}$$

we should apply the expression

$$i_n = W_{n,1}\,\psi_1 + W_{n,2}\,\psi_2 + \cdots + W_{n,N}\,\psi_N \tag{3}$$

for $n = 1, 2, ..., N$, in which the current of the winding 'n' depends on each flux linked to each winding. Because the windings magnetic coupling is not ideal, the matrix of inductances **L** is not singular and the relations (2) exist. Using an analogy to a description of purely resistive circuit by the node potential method, relations (3) can be written as (4)

$$i_n = \frac{1}{L^e_{n,1}}\;(\psi_n - \psi_1) + \cdots + \frac{1}{L^e_n}\;\psi_n + \cdots + \frac{1}{L^e_{n,N}}\;(\psi_n - \psi_N). \tag{4}$$

In this expression the flux linkages replace the node potentials and the inductances $L^e_{n,k}$ and L_n replace the respective resistances. Actually, this substitution is possible, since the relations between flux linkages and winding currents are algebraic, like the relations between potentials and currents in a resistive network. Therefore one can write the relation

$$\mathbf{i} = \mathbf{W}\,\Psi, \tag{5}$$

where

$$\mathbf{W} = \mathbf{L}^{-1} = \begin{bmatrix} W_{1,1} & W_{1,2} & \cdots & W_{1,N} \\ & W_{2,2} & \cdots & W_{2,N} \\ & & \ddots & \vdots \\ (sym) & & & W_{N,N} \end{bmatrix}.$$

Taken into account (4), the matrix **W** can be written in the following form

$$\mathbf{W} = \begin{bmatrix} \frac{1}{L^e_1} + \sum\limits_{\substack{k=1 \\ k \neq 1}}^{N} \frac{1}{L^e_{1,k}} & -\frac{1}{L^e_{1,2}} & \cdots & -\frac{1}{L^e_{1,N}} \\ & \frac{1}{L^e_2} + \sum\limits_{\substack{k=1 \\ k \neq 2}}^{N} \frac{1}{L^e_{2,k}} & \cdots & -\frac{1}{L^e_{2,N}} \\ & & \ddots & \vdots \\ (\text{sym}) & & & \frac{1}{L^e_N} + \sum\limits_{\substack{k=1 \\ k \neq N}}^{N} \frac{1}{L^e_{N,k}} \end{bmatrix} \tag{6}$$

The inductances in this matrix with the superscript 'e' are related to the elements of the matrix **W** according to the following expressions:

$$\frac{1}{L^e_{n,k}} = -W_{n,k}\,, \text{ for } k \neq n, \tag{7}$$

$$\frac{1}{\mathrm{L}_n^{\mathrm{e}}} = \sum_{k=1}^{\mathrm{N}} \mathrm{W}_{n,k} . \tag{8}$$

Treating the matrix **W** in the form (6) as a conductance matrix **G** of purely resistive circuit described by the relations **i** = **Gv** (where **v** is a vector of node potentials), the relations **i** = **W**Ψ can be interpreted as an N-port, purely inductive circuit with 'N' nodes, providing the flux linkages $\psi_1, \psi_2, ..., \psi_{\mathrm{N}}$. The nodes are connected to each other by the inductances $\mathrm{L}_{n,k}^{\mathrm{e}}$ and to a reference node by the inductances $\mathrm{L}_n^{\mathrm{e}}$. The winding currents $i_1, i_2, ..., i_{\mathrm{N}}$ supply respective nodes. To fulfil the voltage equations

$$u_n = \mathrm{R}_n\, i_n + \frac{\mathrm{d}\psi_n}{\mathrm{d}t}, \text{ for } n = 1, 2, ..., \mathrm{N}, \tag{9}$$

the winding resistance R_n has to be added to each port.

A typical multi-port equivalent scheme for a four-winding transformer is shown in Figure 1, in which the inductances $\mathrm{L}_n^{\mathrm{e}}$ lay in vertical branches connecting all nodes to the reference one and the inductances $\mathrm{L}_{n,k}^{\mathrm{e}}$ connect all nodes constituting the N-polygon.

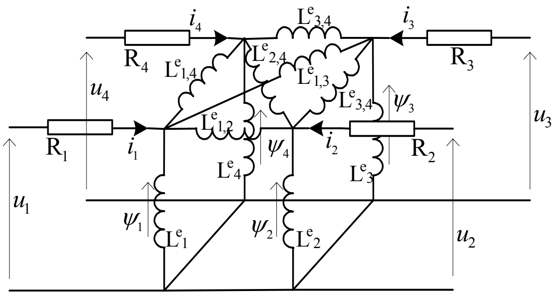

Fig. 1. Equivalent scheme of a four-winding transformer

In the new multi-port equivalent scheme there are many 'vertical inductances' instead of a single common 'magnetizing inductance' in the "classical" T-type one. The total number of equivalent inductances in that multi-port equivalent scheme is exactly equal to the number of independent elements of the inductances matrix. So, the multi-port equivalent scheme can represent precisely any multi-winding transformer, even without recalculating the parameters to one reference side. Inductances appearing in the multiport equivalent scheme can be collected into the matrix (10).

$$\mathbf{L}_{\mathrm{e}} = \begin{bmatrix} \mathrm{L}_1^{\mathrm{e}} & \mathrm{L}_{1,2}^{\mathrm{e}} & \cdots & \mathrm{L}_{1,\mathrm{N}}^{\mathrm{e}} \\ & \mathrm{L}_2^{\mathrm{e}} & \cdots & \mathrm{L}_{2,\mathrm{N}}^{\mathrm{e}} \\ & & \ddots & \vdots \\ (sym) & & & \mathrm{L}_{\mathrm{N}}^{\mathrm{e}} \end{bmatrix}. \tag{10}$$

The elements of that matrix characterize all inductances of the multi-port equivalent scheme.

3. Equivalent scheme of traction transformers

A traction transformer is located in the train. In this sense, it does not belong to a static supply station. Its main use is to bring the single-phase catenary's voltage level to values, which are appropriate to the power electronics and the traction motors. Static converters are fed through low voltage sources, which are the secondary windings of the transformer. Whenever more than two single-phase power sources are needed, we have recourse to a transformer with several secondary coils in order to save place and weight. Generally such transformers have also other low voltage windings used for supplying different auxiliary devices within the train, such as lights, heaters or air conditioners for example. Moreover, especially in Europe where the railways supply networks operate at different voltages and frequencies, transformers should be able to work under diverse systems; this leads to use even more coils in order to achieve a wide range of operations. Typically, a transformer can involve up to 24 different winding parts.

In [4] the multiport equivalent scheme for a model transformer has been created and investigated. This transformer operates at two network's frequencies: 16.7 Hz and 50 Hz. It consists of a two-limb magnetic core, and several windings dispatched as in Figure 2:

– on the primary side:
  4 × high-voltage windings (**HT**) connected in parallel in normal configuration,
– on the secondary side:
  4 × traction windings (**Tr**) which supply the locomotive motors through static converters,
  4 × filter windings (**Fi**) designed for filtering harmonic currents on the transformer primary side and further to the supply network.

In order to insure the dual frequency operation, one winding per block had to be divided into two parts. This leads to the representation of each windings group by four windings, instead of three, like in Figure 2.

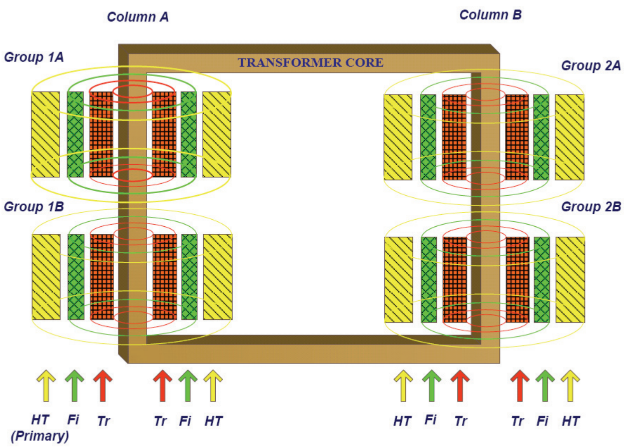

Fig. 2. Layout of a traction transformer

In [4] a group of four windings has been modelled and the equivalent scheme has been determined. Such windings are magnetically coupled through a common flux in the iron core. However, windings are magnetically coupled in the air also, which differs for each pair of windings. So, the inductance matrix can be divided into two parts: the matrix of main magnetizing inductances, due to the coupling through the iron, and the matrix of leakage inductances representing the coupling in the air.

After recalculating all inductances to a reference number of turns, for example the winding denoted '1', the inductance matrix takes the form

$$\mathbf{L} = \mathrm{L}_{\mu}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & 1 \\ 1 & 1 & \cdots & 1 \end{bmatrix} + \begin{bmatrix} \mathrm{L}_1^{\sigma} & \mathrm{L'}_{1,2}^{\sigma} & \cdots & \mathrm{L'}_{1,N}^{\sigma} \\ & \mathrm{L'}_2^{\sigma} & \cdots & \mathrm{L'}_{2,N}^{\sigma} \\ & & \ddots & \vdots \\ \text{(sym)} & & & \mathrm{L'}_{N,N}^{\sigma} \end{bmatrix}. \tag{11}$$

In [7] calculation results of inductances in those matrices are presented, using FLUX2D as finite elements software. It has been shown that whereas the L_{μ} value changed with the saturation of the iron core, the leakage inductances in the second matrix remain constant. For unsaturated state of the transformer the following data have been obtained:

– the matrix of main inductances

$$\mathbf{L}_{\mu} = 4600\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}[\mathrm{H}]. \tag{12}$$

– the matrix of leakage inductances

$$\mathbf{L}_{\sigma} = \begin{bmatrix} 53.1 & 1.1 & 49.7 & 52.2 \\ 1.1 & 53.1 & 52.3 & 49.8 \\ 49.7 & 52.3 & 53.1 & 1.1 \\ 52.2 & 49.8 & 1.1 & 53.1 \end{bmatrix}[\mathrm{H}]. \tag{13}$$

The equivalent scheme of the modelled group of windings is exactly the same as in Figure 1. The inductances appearing in this equivalent scheme, obtained by the procedure described in the previous sections and arranged as in (10), are summarized in (14):

$$\mathbf{L}_{\mathrm{e}} = \begin{bmatrix} 18521 & 67.3 & -176 & -210 \\ 67.3 & 18592 & -209 & -175 \\ -176 & -209 & 18555 & 67.3 \\ -210 & -175 & 67.3 & 18558 \end{bmatrix}[\mathrm{H}]. \tag{14}$$

These results show consequently, that the procedure based on field computation, allow the determination of the multi-port equivalent scheme.

4. Determination of the multiport equivalent scheme parameters by measurements

The multiport equivalent circuit of N-winding transformer has N(N+1)/2 independent parameters. In general the determination of those parameters is much more complicated as for T-type equivalent scheme. In the considered case of four windings traction transformer, 10 equivalent inductances should be found. It means that at least 10 equations have to be stated to calculate them. However, the estimation of parameters values can be relatively easily done as it will be presented in this chapter.

Usually the scheme parameters of a transformer are determined from no-load and short-circuit tests under sinusoidal voltage supply. The elements of the multi-port equivalent scheme can be also established from such tests, but carried out in a slightly different manner [4]. For the no-load tests the individual windings should be supplied successively, keeping the other windings open. For the short-circuit test the same is done, but all non-supplied windings are short-circuited.

4.1. The no-load tests

For the no-load operation, when only the winding '1' is supplied by a sinusoidal voltage with a pulsation Ω, and all the other windings are open, the transformer is described by the Equations (15)

$$\mathbf{U}_{1,o} = \left(\mathbf{R} + j\Omega(\mathbf{L}_{\mu} + \mathbf{L}_{\sigma}\right)\mathbf{I}_{1,o} \tag{15}$$

in which: $\mathbf{I}_{1,o}{}^{T} = [\mathrm{I}_{1,o}\ 0\ 0\ 0]$, $\mathbf{U}_{1,o}{}^{T} = [\mathrm{U}_1\ \mathrm{U}'_2\ \mathrm{U}'_3\ \mathrm{U}'_4]$, where $\mathrm{I}_{1,o}$ is a measured current, U_1 is the supplied voltage and $\mathrm{U}'_2, \mathrm{U}'_3, \mathrm{U}'_4$ are measured voltages on the open windings, recalculating to the winding '1'. In order to link the measured values to the parameters of the multiport equivalent scheme, Equation (15) should be rewritten as

$$\mathbf{I}_{1,o} = \mathbf{Y}\left(\mathbf{U}_{1,o} - \mathbf{R}\,\mathbf{I}_{1,o}\right), \tag{16}$$

where

$$\mathbf{Y} = \frac{1}{j\Omega}\mathbf{W}.$$

The sum of all Equations (16) gives the equation

$$\mathrm{I}_{1,o} = \mathrm{Y}_1^{e}\,\mathrm{U}'_1 + \mathrm{Y}_2^{e}\,\mathrm{U}'_2 + \mathrm{Y}_3^{e}\,\mathrm{U}'_3 + \mathrm{Y}_4^{e}\,\mathrm{U}'_4, \tag{17}$$

where $\mathrm{U}'_1 = \mathrm{U}_1 - \mathrm{R}_1\mathrm{I}_{1,o}$ and $\mathrm{Y}_1^{e} = 1/j\Omega\mathrm{L}_1^{e}$. Repeating tests for the windings '2','3' and '4' the equation set could be written

$$\begin{bmatrix} I_{1,o} \\ I_{2,o} \\ I_{3,o} \\ I_{4,o} \end{bmatrix} = \begin{bmatrix} U'_{1,1} & U'_{2,1} & U'_{3,1} & U'_{4,1} \\ U'_{1,2} & U'_{2,2} & U'_{3,2} & U'_{4,2} \\ U'_{1,3} & U'_{2,3} & U'_{3,3} & U'_{4,3} \\ U'_{1,3} & U'_{2,3} & U'_{2,3} & U'_{2,3} \end{bmatrix} \begin{bmatrix} Y_1^e \\ Y_2^e \\ Y_3^e \\ Y_4^e \end{bmatrix}, \tag{18}$$

in which the second subscript in $U'_{2,n}$ denotes the number of the test. It means that values of vertical branches in multi-port scheme can be determined separately from the values of reactances in the upper polygonal.

Traction transformer is a single-phase multi-windings one, and the main magnetic flux is common for all windings, which led to the matrix (12). The voltages in (18) have very close values, and they should therefore be measured precisely. Assuming that they are equal, the equations (18) reduces to only one equation

$$I_{1,o} = (Y_1^e + Y_2^e + Y_3^e + Y_4^e)\, U_1 \tag{19}$$

from which the values of individual parameters cannot be found. The multi-port equivalent scheme under such assumption becomes as in Figure 3 [8].

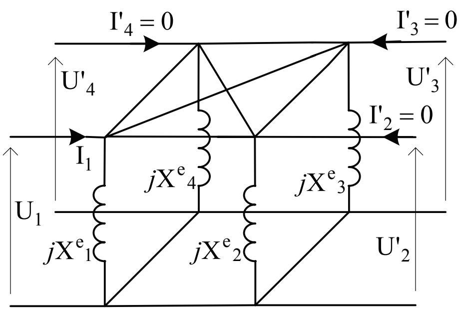

Fig. 3. Equivalent scheme of a four-winding one-phase transformer for the no-load condition

Calculations of the multiport scheme parameters from design data in previous chapter showed that vertical elements have almost the same values $L_1^e = L_2^e = L_3^e = L_4^e$. On the other hand, the first estimation from (15), if resistances and leakage inductances are neglected, leads to $U_1 = j\,X_\mu\, I_1$. So, in general case the vertical inductances in the multiport equivalent scheme of single-phase transformers can be estimated as

$$L^e{}_1 = L^e{}_2 = \cdots = L^e{}_N = N\, L_\mu . \tag{20}$$

However, for three-phase power transformers such estimation is not valid [6].

4.2. The short-circuit tests

The classical short-circuit test for the T-type equivalent circuit is not useful for the multi-port one. In [4] it has been shown that the following strategy is very effective: one winding is supplied by a limited voltage having all other windings short-circuited; and this is successively repeated for all windings.

The equations, when the winding '1' is supplied, take the form

$$\mathbf{I}_{1,sc} = \mathbf{Y}\left(\mathbf{U}_{1,sc} - \mathbf{R}\,\mathbf{I}_{1,sc}\right), \tag{21}$$

where: $\mathbf{I}_{sc}{}^{T} = \left[\mathrm{I}_{1,sc}\ \ \mathrm{I'}_{2,sc}\ \ \mathrm{I'}_{3,sc}\ \ \mathrm{I'}_{4,sc}\right]$, $\mathbf{U}_{sc}^{T} = \left[\mathrm{U}_{1,sc}\ \ 0\ 0\ 0\right]$. All elements of these vectors are measurable from the short-circuit test. As it has been stated, the matrix **Y** has 10 independent elements. However, 4 of them – the vertical ones – can be determined from the no-load tests by the Equations (18). Values of the elements $\mathrm{Y}^{e}_{n,k} = 1/j\,\Omega\,\mathrm{L}^{e}_{n,k}$ in the upper polygonal fulfil equations

$$\begin{bmatrix} \mathrm{I}_{1,sc} - \mathrm{Y}^{e}_{1}\mathrm{U'}_{1} \\ \mathrm{I'}_{2,sc} - \mathrm{Y}^{e}_{2}\mathrm{U'}_{2} \\ \mathrm{I'}_{3,sc} - \mathrm{Y}^{e}_{3}\mathrm{U'}_{3} \\ \mathrm{I'}_{4,sc} - \mathrm{Y}^{e}_{4}\mathrm{U'}_{4} \end{bmatrix} = \left[\begin{bmatrix} \mathrm{U'}_1-\mathrm{U'}_2 & \mathrm{U'}_1-\mathrm{U'}_3 & \mathrm{U'}_1-\mathrm{U'}_4 & 0 & 0 & 0 \\ \mathrm{U'}_2-\mathrm{U'}_1 & 0 & 0 & \mathrm{U'}_2-\mathrm{U'}_3 & \mathrm{U'}_2-\mathrm{U'}_4 & 0 \\ 0 & \mathrm{U'}_3-\mathrm{U'}_1 & 0 & \mathrm{U'}_3-\mathrm{U'}_2 & 0 & \mathrm{U'}_3-\mathrm{U'}_4 \\ 0 & 0 & \mathrm{U'}_4-\mathrm{U'}_1 & 0 & \mathrm{U'}_4-\mathrm{U'}_2 & \mathrm{U'}_4-\mathrm{U'}_3 \end{bmatrix}\right] \left[\begin{bmatrix} \mathrm{Y}^{e}_{1,2} \\ \mathrm{Y}^{e}_{1,3} \\ \mathrm{Y}^{e}_{1,4} \\ \mathrm{Y}^{e}_{2,3} \\ \mathrm{Y}^{e}_{2,4} \\ \mathrm{Y}^{e}_{3,4} \end{bmatrix}\right] \tag{22}$$

or shortly, $\mathbf{I}^{1} = \mathbf{U}^{1}\,\mathbf{Y}$. Voltages in these equations are: $\mathrm{U'}_1 = \mathrm{U}_{1,sc} - \mathrm{R}_1\,\mathrm{I}_{1,sc}$, $\mathrm{U'}_2 = -\mathrm{R'}_2\,\mathrm{I'}_{2,sc}$, $\mathrm{U'}_3 = -\mathrm{R'}_3\,\mathrm{I'}_{3,sc}$, $\mathrm{U'}_4 = -\mathrm{R'}_4\,\mathrm{I'}_{4,sc}$. So, it is necessary to know winding resistances to write it.

The short-circuit test for the winding ‘1’ gives 4 equations but there are 6 unknown values. So, one short-circuit test is no enough to determine the upper parameters of the multiport scheme. Repeating the short-circuit test for the windings ‘2’, ‘3’ and ‘4’ the set of 16 equations with 6 unknown values can be written

$$\begin{bmatrix} \mathbf{I}^1 \\ \mathbf{I}^2 \\ \mathbf{I}^3 \\ \mathbf{I}^4 \end{bmatrix} = \begin{bmatrix} \mathbf{U}^1 \\ \mathbf{U}^2 \\ \mathbf{U}^3 \\ \mathbf{U}^4 \end{bmatrix}\mathbf{Y}. \tag{23}$$

Writing (23) in the form **I** = **U** **Y**, the vector **Y** can be found, applying a linear regression method, from formula [5]

$$\mathrm{Y} = (\mathbf{U}^{T}\,\mathrm{U})^{-1}\,(\mathbf{U}^{T}\,\mathbf{I}) \tag{24}$$

Such approach is rather complicated and not suitable for engineering application.

Assuming that all windings’ resistances can be omitted, voltages in (22) are: $\mathrm{U'}_1 = \mathrm{U}_{1,sc}$, $\mathrm{U'}_2 = 0$, $\mathrm{U'}_3 = 0$ and $\mathrm{U'}_4 = 0$. Additionally, for traction transformers vertical admittances are very high, which leads to very simple formulae for the currents in the short-circuited windings

$$\mathrm{I'}_{2,sc} = -\frac{\mathrm{U}_{1,sc}}{j\,\mathrm{X}^{e}_{1,2}}, \quad \mathrm{I'}_{3,sc} = -\frac{\mathrm{U}_{1,sc}}{j\,\mathrm{X}^{e}_{1,3}}, \quad \mathrm{I'}_{4,sc} = -\frac{\mathrm{U}_{1,sc}}{j\,\mathrm{X}^{e}_{1,4}} \tag{25}$$

and the current in the supplied winding is

$$\mathrm{I}_{1,sc} = -\mathrm{I'}_{2,sc} - \mathrm{I'}_{3,sc} - \mathrm{I'}_{4,sc} \tag{26}$$

i.e. it is just the sum of the currents circulating in the short-circuited windings. These results are evident from the equivalent scheme at a considered short-circuit condition, shown in Figure 4 [8].

This test allows to determine the inductances $\mathrm{L}^{e}_{1,2}$ $\mathrm{L}^{e}_{1,3}$ and $\mathrm{L}^{e}_{1,4}$. When the winding '2' is supplied and the other ones are short-circuited, the inductances $\mathrm{L}^{e}_{2,3}$ and $\mathrm{L}^{e}_{2,4}$ can be additionally found. From the test for winding '3' the last inductance $\mathrm{L}^{e}_{3,4}$ can be found. Then, from only three short-circuit tests all the inductances constituting the upper polygonal of the equivalent circuit can be determined very easily.

$$\mathrm{I'}_{3,sc} = -\frac{\mathrm{U}_{2,sc}}{j\,\mathrm{X}^{e}_{2,3}}, \quad \mathrm{I'}_{4,sc} = -\frac{\mathrm{U}_{2,sc}}{j\,\mathrm{X}^{e}_{2,4}}. \tag{27}$$

From the some test for winding '3' the last inductance $\mathrm{L}^{e}_{3,4}$ can be found

$$\mathrm{I'}_{4,sc} = -\frac{\mathrm{U}_{3,sc}}{j\,\mathrm{X}^{e}_{3,4}}. \tag{28}$$

Then, all inductances constituting the upper polygonal of the equivalent circuit can be very easy determined from only three short-circuit tests.

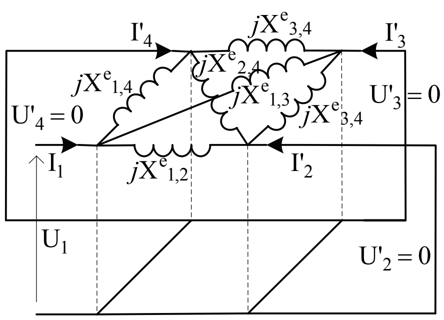

Fig. 4. Equivalent scheme of a four-winding transformer for the short-circuit condition when the winding '1' is supplied

However, it should be noticed that some values of some inductances $\mathrm{L}^{e}_{n,k}$ for the analysed traction transformer could be negative, as it can be seen in the matrix (14). So, when measuring the short circuit currents, one has to take, not only their rms values, but also their phases with respect to the phase of the supplied winding current.

5. Conclusions

In this paper, a new multi-port equivalent scheme of the multi-winding traction transformers is presented. It is universal because the number of its inductive elements is exactly equal to the number of self and mutual inductances of transformer's windings. Thanks to that, any traction transformer with an arbitrary high number of windings can be correctly represented by such an equivalent scheme. The centrally located magnetizing inductance in the T-type equivalent scheme is decentralized in the multi-port equivalent scheme to many magnetizing inductances distributed at each port respectively.

In this paper the procedures of parameters' determination of the multi-port equivalent scheme from calculations and measurements are presented, basing on field calculations as well as no-load and short-circuit tests.

Determining the parameters of the multiport equivalent scheme requires simulation of magnetic field distribution by the finite elements method, from which the self and mutual inductances of transformer's windings have to be calculated precisely, especially their coupling by leakage fluxes in the air.

Accurate determination of the multiport equivalent scheme parameters from measurements is rather complicated. However, their estimation from no-load and short-circuit tests is rather simple, similar as for the classical T-type scheme, which should be well accepted by a majority of engineers.

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