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On dynamical identification of control in a system with time delay

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The problem of identification of a control through results of observations of phase states of dynamical systems described by differential equations with time delay is discussed. The paper presents an algorithm based on the method of feedback control with a model. The algorithm is stable with respect to informational noises and computational errors.

Key words: time-delay system, identification, feedback control

1. Introduction and problem formulation

A motion of dynamical system proceeds on a given interval $[0,T]$ and it is characterized by an N-vector x. The motion is supposed to be described by a differential equation with time delay

$$\dot{x}(t) = f(x(t), x(t-\nu)) + Bu(t), \tag{1}$$

$$t \in [0,T], \quad x(s) = x_0(s), \quad s \in [-\nu, 0].$$

It depends on a control $u = u(t) \in \mathbb{R}^n$ which varies in time t. Here, $\nu = \text{const} > 0$ is a time delay; B is an $N \times n$ matrix; f is a given $N \times N$ matrix function satisfying the Lipschitz condition:

$$|f(x_1,x_2) - f(y_1,y_2)|_N \leqslant L\{|x_1 - y_1|_N + |x_2 - y_2|_N\} \quad \forall x_1, x_2, y_1, y_2 \in \mathbb{R}^N;$$

$|\cdot|_N$ is the Euclidean norm in $\mathbb{R}^N$; $x_0(s)$ is a given continuous function (the initial state of the system).

Let $u = u(t)$ be the input for a real process; corresponding real motion is a function of time and is denoted by $x(t)$. The problem is to identify in "real time" a priori unknown control $u(t)$ through results of measurements of $x(t)$. All information on $u(t)$ given in

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advance is the following: this function is Lebesque measurable in $[0,T]$ and is a square integrable, i.e.,

$$u(\cdot) \in L_2([0,T];\mathbb{R}^n). \tag{2}$$

At discrete, frequent enough, time moments $\tau_i \in [0,T]$ the components of the state vector $x(\tau_i)$ are measured. The results of inaccurate measurements are vectors $\xi_i^h \in \mathbb{R}^N$ satisfying the following condition

$$\xi_i^h = x(\tau_i) + z_i, \quad |z_i|_N \leqslant h.$$

The value of the level of informational noise h is supposed to be small. Since the measurements are inaccurate, it is in general impossible to identify $u(t)$ precisely, therefore the problem is to approximate the input by some function $v^h(t)$.

Further, we specify an identification algorithm functioning in real time for approximating the function $v^h(t)$. This algorithm allows us to calculate the value of $v^h(t)$ at an arbitrary time instant $t \in [0,T]$ till this moment. The initial state $x_0(s)$ of system (1) is supposed to be known and given.

The suggested solution outline is the following [1–4, 10–13]. An auxiliary control system (a model) described by some equation of the form

$$\dot{w}(t) = F(t, \xi^h_{-\nu,t}(\cdot), v^h(t), \omega^h(t)), \tag{3}$$

$$w(0) = w_0, \quad t \in [0,T]$$

is associated with real dynamical system (1). Here the symbol $\xi^h_{-\nu,t}(\cdot)$ denotes a function $\xi^h(r)$, $-\nu \leqslant r \leqslant t$. A vector $w \in \mathbb{R}^N$ characterizes the state of the model, the form of function F is corrected below, vectors v^h and ω^h are control actions. The process of feedback control of the model is realized on the time interval $[0,T]$. One takes a uniform net $\Delta = \{\tau_i\}_{i=0}^m$, $\tau_{i+1} = \tau_i + \delta$, $\delta > 0$, $i \in [0:m]$, $\tau_0 = 0$, $\tau_m = T$ with some step δ. On the interval $t \in [\tau_i, \tau_{i+1})$ the model is acted upon the controls

$$v_i^h = V_1(\xi_i^h, w(\tau_i)), \quad \omega_i^h = V_2(\xi_i^h, w(\tau_i)) \tag{4}$$

calculated at the moment τ_i by the use of some rule, which hereinafter we shall identify with mappings V_1 and V_2. Thus, the controls in the model are realized by the method of feedback control. Their values on the interval $[\tau_i, \tau_{i+1}]$ depend on the results $\xi^h(\tau_i)$ of measuring the phase state $x(\tau_i)$ of system (1) and the state $w(\tau_i)$ of model (3). The described process forms the piece-wise constant functions

$$v^h(t) = v_i^h, \quad \omega^h(t) = \omega_i^h, \qquad t \in [\tau_i, \tau_{i+1}) \tag{5}$$

in “real time” synchro with the motion of real system (1). Those functions are control actions in the model, $v^h(t)$ is taken as an approximation of the function $u(t)$.

The scheme described above was investigated in detail in [2–4, 10, 11] under the additional assumption on the control $u(t)$. A convex bounded and closed set $P \subset U =$

$\mathbb{R}^n$ ($u(t) \in P \quad \forall t \in [0,T]$) is known. In this paper, we have no such information. A real control $u(t)$ acting upon system (1) may in general be unbounded (we know only that $u(t) \in L_2([0,T];U)$). This circumstance complicates the investigation of discussed problem and does not allow us to use known algorithms [2–4, 10, 11].

In [12, 13] algorithms for solving the problem for equation (1) were suggested. In the first paper, the function $u(\cdot)$ was assumed to be bounded ($u(\cdot) \in L_\infty$); in the second one, the case when the control dimension is less than the phase vector dimension ($n \leqslant N$) and the matrix B is of the form: $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, I is the identity $n \times n$ matrix, was considered. In the present paper, we modify these algorithms. Namely, we correct rules of choosing a control in a model in such a way as to take into account the general case when the fuction $u(\cdot)$ is not bounded (see (2)) and the matrix B has an arbitrary structure.

Before realization of the scheme above, we put the following remark. It is easily seen that for the chosen procedure of construction of the functions $v^h(t)$ and $\omega^h(t)$ the complexity consists in an appropriate choice of model (2) and functions V_1 and V_2 (according to the terminology of the theory of positional control [1] these functions are often named by strategies). In the process, the strategies V_1 and V_2 are chosen in such a way as to provide stabilization of some estimating Lyapunov functional. It should be noted that a model is some artificial construction and a control process is a numerical algorithm to be realized on computers in real time mode. The described below algorithm of identification of a control is stable with respect to errors of calculation, for example, of model states.

2. Algorithm of identification

Let us specify a family of partitions $\Delta_h = \{\tau_{h,i}\}_{i=0}^{m_h}$, $\tau_{h,0} = 0$, $\tau_{h,m_h} = T$, $\tau_{h,i+1} = \tau_{h,i} + \delta(h)$ of the time interval $[0,T]$ with some step $\delta(h)$. Fix a function $\alpha(h)$ (a regularizator). Let the functions $\delta(h) \in (0,1)$ and $\alpha(h) \in (0,1)$ be such that the following conditions

$$\delta(h) \to 0, \quad \alpha(h) \to 0, \quad h/\alpha(h) \to 0, \tag{6}$$

$$\delta(h)/\alpha^2(h) \leqslant 1, \quad h/\delta(h) \leqslant 1 \quad \text{as} \quad h \to 0$$

are fulfilled. Then we introduce an auxiliary control system (a model) of the form

$$\dot{w}(t) = f(\xi_i^h, \xi_{i-k_h}^h) + Bv^h(t) + \omega^h(t), \tag{7}$$

$$t \in [\tau_i, \tau_{i+1}), \quad \tau_i = \tau_{h,i}$$

with the initial state $w(0) = x_0(0)$. Hereinafter we set for simplicity $k_h = \delta/m_h$. We assume that the initial condition $x_0(s)$ of system (1) is known. Hence, if $k - k_h < 0$ then we put $\xi_{k-k_h}^h = x_0(\delta(k-k_h))$. So, the right-hand side of equation of model (3) has the form

$$F(t, \xi_{-\nu,t}^h(\cdot), v^h, \omega^h) = f(\xi_i^h, \xi_{i-k_h}^h) + Bv^h + \omega^h, \quad t \in [\tau_i, \tau_{i+1}).$$

Before the initial moment the value h and the partition $\Delta = \Delta_h$ with the diameter $\delta = \delta(h)$ are fixed. The work of the algorithm starting at the moment $t = 0$ is decomposed into $m_h - 1$ steps. At the i-th step carried out during the time interval $\delta_i = [\tau_i, \tau_{i+1})$, the following actions are fulfilled. First, at the moment τ_i vectors v_i^h and ω_i^h are calculated by the following formulas

$$v_i^h = \frac{1}{\alpha} B'(\xi_i^h - w(\tau_i)), \quad \omega_i^h = \frac{c\delta}{\alpha^2}(\xi_i^h - w(\tau_i)), \tag{8}$$

i. e., the strategies V_1 and V_2 (see (4)) have the form:

$$V_1(\xi_i^h, w(\tau_i)) = \frac{1}{\alpha} B'(\xi_i^h - w(\tau_i)), \quad V_2(\xi_i^h, w(\tau_i)) = \frac{c\delta}{\alpha^2}(\xi_i^h - w(\tau_i)).$$

Here and below $c = \text{const} > 8b^4$, where $b = |B|$ is the norm of the matrix B, prime stands for transposition. Then controls (5) are fed onto the input of the model. After that, we transform the state $w(\tau_i)$ of the model into $w(\tau_{i+1})$. The procedure stops at the time T.

The following theorem is true.

Theorem 1 *Let relations* (6) *between parameters of the algorithm hold. Then the following convergence takes place:*

$$v^h(\cdot) \to u_*(\cdot) \quad in \quad L_2([0,T];\mathbb{R}^n) \quad as \quad h \to 0,$$

i.e., $\int_0^T |v^h(t) - u_*(t)|_n^2 \, dt \to 0$ *as* $h \to 0$.

Here $u_*(\cdot) = u_*(\cdot; x(\cdot))$ is an element from $U(x(\cdot))$ with the minimal $L_2([0,T];\mathbb{R}^n)$-norm, $U(x(\cdot))$ is the set of all controls $u(\cdot) \in L_2([0,T];\mathbb{R}^n)$ compatible with the output $x(\cdot)$.

Note that $U(x(\cdot))$ is a convex and closed set from the space $L_2([0,T];\mathbb{R}^n)$. In virtue of this fact the element $u_*(\cdot; x(\cdot))$ is defined uniquely.

The proof of Theorem 1 is performed by the standard scheme (see, for example, [2–4, 10–13]) and is based on the lemma below. The proof of convergence of the algorithm is founded on a procedure of stabilizing an appropriate functional of Lyapunov type:

$$\mu(t) = |w_h(t) - x(t)|_N^2 + \alpha(h) \int_0^t [|v^h(s)|_n^2 - |u_*(s)|_n^2] \, ds, \tag{9}$$

where $w_h(\cdot) = w(\cdot; x_0(0), v^h(\cdot), \omega^h(\cdot))$ is a phase trajectory of model (7).

Lemma 1 *Let the conditions of Theorem* 1 *hold. Then there exist a number* h_* *and constants* d_0, d_1, *and* d_2 *such that for all* $h \in (0, h_*)$ *the inequalities*

$$|x(\tau_i) - w_h(\tau_i)|_N^2 \leqslant d_0(h + \delta + \alpha), \tag{10}$$

$$\int_0^T |v^h(\tau)|_n^2 \, d\tau \leqslant \int_0^T |u_*(\tau)|_n^2 \, d\tau + d_1(h+\delta)\alpha^{-1} + d_2\alpha \tag{11}$$

hold. Here $\alpha = \alpha(h)$, $\delta = \delta(h)$.

Proof. Introduce the value

$$\varepsilon(t) = |x(t) - w_h(t)|_N^2.$$

For simplicity we assume $\delta(h)/\alpha(h) \leqslant 1$. It is easily seen that for a. a. $t \in \delta_i = [\tau_i, \tau_{i+1})$ the equality

$$\frac{1}{2}\dot{\varepsilon}(t) = \Big(x_i - w_i + \int_{\tau_i}^t \{\pi_i(\tau; x, \xi^h) + B(u_*(\tau) - v_i^h) - \omega_i^h\}\, d\tau,$$

$$\pi_i(t; x, \xi^h) + B(u_*(t) - v_i^h) - \omega_i^h\Big)$$

is true.Here the symbol $(\cdot,\cdot)$ denotes the scalar product in the corresponding Euclidean space, $\varepsilon_i = \varepsilon(\tau_i)$, $x_i = x(\tau_i)$, $\quad w_i = w_h(\tau_i)$,

$$\pi_i(\tau; x, \xi^h) = f(x(\tau), x(\tau - \nu)) - f(\xi_i^h, \xi_{i-k_h}^h).$$

Let $\mu_i^{(1)}(t) = (t - \tau_i)|\omega_i^h|_N^2$,

$$\mu_i^{(0)}(t) = -\Big(\int_{\tau_i}^t B(u_*(\tau) - v_i^h)\, d\tau, \omega_i^h\Big).$$

Note that the estimations

$$|v_i^h|_n = |B'(w_i - \xi_i^h)\alpha^{-1}|_N \leqslant \alpha^{-1}(h + \varepsilon_i^{1/2})b,$$

$$|\omega_i^h|_N \leqslant \frac{c\delta}{\alpha^2}(h + \varepsilon_i^{1/2}), \quad \alpha = \alpha(h) \tag{12}$$

are valid. Besides, we have

$$\int_{\tau_i}^t \mu_i^{(0)}(\tau)\, d\tau \leqslant 4cb(\delta\alpha^{-1})^3\varepsilon_i + \delta c^{-1} b \int_{\tau_i}^{\tau_{i+1}} |u_*(\tau)|_n^2\, d\tau + 4ch^2 b. \tag{13}$$

From (12) it follows that the inequality

$$\int_{\tau_i}^t \mu_i^{(1)}(\tau)\, d\tau \leqslant 2c^2(h^2 + (\delta\alpha^{-1})^3\varepsilon_i), \quad t \in \delta_i \tag{14}$$

is fulfilled. By the Lipschitz property of the function f, we obtain

$$\int_{\tau_i}^{t} \mu_i^{(2)}(\tau)\,d\tau \leqslant 0,25c(\delta\alpha^{-1})^2\varepsilon_i + k_*(c)\alpha^2(\delta + h^2), \quad t \in \delta_i. \tag{15}$$

Here $\mu_i^{(2)}(\tau) = (x_i - w_i, \pi_i(\tau; x, \xi^h))$. It is not difficult to find a constant k_1 such that

$$\int_{\tau_i}^{t} \mu_i^{(3)}(\tau)\,d\tau \leqslant k_1\delta^2, \tag{16}$$

$$\mu_i^{(3)}(t) = \Big(\int_{\tau_i}^{t} \pi_i(\tau; x, \xi^h)\,d\tau, \pi_i(t; x, \xi^h)\Big), \quad t \in \delta_i.$$

Further, from the following estimation

$$|\pi_i(t; x, \xi^h)|_N \leqslant k_0(h + \delta^{1/2}), \tag{17}$$

we have

$$\int_{\tau_i}^{t} \mu_i^{(4)}(\tau)\,d\tau \leqslant 0.5(\delta\alpha^{-1})^3\varepsilon_i + k_2(c)\delta(h^2 + \delta + h\delta^{1/2}). \tag{18}$$

An analogous estimation is valid for $\int_{\tau_i}^{t} \mu_i^{(5)}(\tau)\,d\tau$. Here

$$\mu_i^{(4)}(t) = -\Big(\int_{\tau_i}^{t} \pi_i(\tau; x, \xi^h)\,d\tau,\ \omega_i^h\Big),$$

$$\mu_i^{(5)}(t) = -(t - \tau_i)(\omega_i^h, \pi_i(t; x, \xi^h)), \quad t \in \delta_i.$$

Let $\mu_i^{(6)}(t) = -(t - \tau_i)(\omega_i^h, B(u_*(t) - v_i^h))$. It is easy to see that

$$\Big|\int_{\tau_i}^{t} \mu_i^{(6)}(\tau)\,d\tau\Big| \leqslant \delta b\alpha^{-1}(h + \varepsilon_i^{1/2}) + cb^2(\delta\alpha^{-1})^2(h + \varepsilon_i^{1/2}) \int_{\tau_i}^{\tau_{i+1}} |u_*(\tau)|_n\,d\tau. \tag{19}$$

Besides,

$$\mu_i^{(7)}(\tau_{i+1}) = \int_{\tau_i}^{\tau_{i+1}} (x_i^h - w_i, B(u_*(\tau) - v_i^h))\,d\tau \leqslant \int_{\tau_i}^{\tau_{i+1}} (\xi_i^h - w_i, B(u_*(\tau) - v_i^h))\,d\tau + \tag{20}$$

$$+hb\int_{\tau_i}^{\tau_{i+1}}|u_*(\tau)|_n\,d\tau+b^2(1+b^2)h^2+0.5(\delta\alpha^{-1})^2\varepsilon_i.$$

The following inequalities

$$\omega_i^{1,t}=\Big(\int_{\tau_i}^{t}\pi_i(\tau;x,\xi^h)\,d\tau,\int_{\tau_i}^{t}B(u_*(\tau)-v_i^h)\,d\tau\Big)\leqslant \tag{21}$$

$$\leqslant 0.5\frac{\delta^2}{\alpha^2}\varepsilon_i+k_4\delta(h+\delta^{1/2})(h+\delta^{1/2}+\int_{\tau_i}^{\tau_{i+1}}|u_*(\tau)|_n\,d\tau),\quad t\in\delta_i,$$

$$\omega_i^{2,t}=\Big|\int_{\tau_i}^{t}B(u_*(\tau)-v_i^h)\,d\tau\Big|_N^2\leqslant 4b^4(\frac{\delta}{\alpha})^2\varepsilon_i+4h^2b^4+2\delta b^2\int_{\tau_i}^{\tau_{i+1}}|u_*(\tau)|_n^2\,d\tau \tag{22}$$

are true. Here k_4 does not depend on c. Further, we have

$$\varepsilon(t)\leqslant\varepsilon(\tau_i)+\mu_i^{(7)}(t)+\omega_i^{1,t}+\omega_i^{2,t}+\int_{\tau_i}^{t}\{-(x_i-w_i,\omega_i^h)+\sum_{j=0}^{6}\mu_i^{(j)}(\tau)\}\,d\tau, \tag{23}$$

$$-\int_{\tau_i}^{\tau_{i+1}}(x_i-w_i,\omega_i^h)\,d\tau\leqslant -c\Big(\frac{\delta}{\alpha}\Big)^2\varepsilon_i+0.5c(\delta\alpha^{-1})^3\varepsilon_i+0.5c^2h^2\delta\alpha. \tag{24}$$

Introduce a value $\mu_i=\mu(\tau_i)$. (The function $\mu(t),t\in[0,T]$ is defined according to (9).) Combining (13)–(24), we obtain

$$\mu_{i+1}\leqslant d_1(c)\delta\int_{\tau_i}^{\tau_{i+1}}|u_*(\tau)|_n^2\,d\tau+d_2(h+\delta^{3/2})\int_{\tau_i}^{\tau_{i+1}}|u_*(\tau)|_n\,d\tau+ \tag{25}$$

$$+d_3(c)(1+\delta\alpha+\alpha^2)h^2+d_4\delta^2+d_5(c)(\alpha^2+h^2+\delta)\delta+\mu_i+$$

$$+2\int_{\tau_i}^{\tau_{i+1}}(\xi_i^h-w_i,B(u_*(\tau)-v_i^h))\,d\tau+$$

$$+\alpha\int_{\tau_i}^{\tau_{i+1}}\{|v_i^h|_n^2-|u_*(\tau)|_n^2\}\,d\tau+\lambda(b,c)\Big(\frac{\delta}{\alpha}\Big)^3\varepsilon_i+(4b^4-0.75c)\Big(\frac{\delta}{\alpha}\Big)^2\varepsilon_i,$$

where $\lambda(b,c)$ is a constant which can be written in an explicit form. We choose $h=h_1>0$ such that for $h\in(0,h_1)$ the inequality $4\lambda(b,c)\delta(h)/\alpha(h)\leqslant c$ is valid. In this case for such h we obtain

$$\lambda(b,c)\delta(h)/\alpha(h)+(4b^4-0.75c)\leqslant 0. \tag{26}$$

Note that (see (5), (8)) we choose $v^h(\cdot)$ by the rule

$$v^h_{|_{[\tau_i,\tau_{i+1})}}(\cdot) = \arg\min\Big\{ \int\limits_{\tau_i}^{\tau_{i+1}} \{2(w(\tau_i)-\xi_i^h, Bv(s)) + $$

$$+\alpha|v(s)|_n^2\}\,ds\colon\, v(\cdot)\in L_2([\tau_i,\tau_{i+1}];\mathbb{R}^n)\Big\}. \tag{27}$$

Taking into account (25)–(27), we derive

$$\mu_i \leqslant \mu_0 + a_4(\delta + h + \alpha^2). \tag{28}$$

From (28) in virtue of conditions (6), follows that there exists a number $h_* > 0$ such that for $h \in (0, h_*)$ inequalities (10), (11) are true. The lemma is proved. □

3. Estimation of convergence rate

Let in equation (1) $B = I$ (the identity matrix), i.e., $n = N$.

Lemma 2 *Let the function $u_*(\cdot) = u_*(\cdot; x(\cdot))$ be a function of bounded variation. Then the following estimate of algorithm convergence rate takes place:*

$$\int\limits_0^T |v^h(\tau) - u_*(\tau)|_n^2\,d\tau \leqslant c_1\alpha^{-1}(h+\delta) + c_2(h+\alpha+\delta)^{1/2}.$$

Here c_1 and c_2 are some constants which can be explicitly written.
Proof. We have

$$g_h(t) \equiv \Big|\int\limits_0^t (v^h(\tau) - u_*(\tau))\,d\tau\Big|_N \leqslant$$

$$\leqslant \sum_{i=0}^{i(t)} \int\limits_{\tau_i}^{\tau_{i+1}} |\pi_i(\tau; x, \xi^h)|_N\,d\tau + c\delta \sum_{i=0}^{i(t)} |\xi_i^h - w_n(\tau_i)|_N.$$

Here the symbol $i(t)$ denotes the integer part of a number t. From (10) and (17), it follows that the inequality

$$g_h(t) \leqslant k_0(h+\alpha+\delta)^{1/2} \tag{29}$$

is true. Taking into account (11), (29), we deduce that

$$\int\limits_0^t |v^h(\tau) - u_*(\tau)|_n^2\,d\tau =$$

$$= \int_0^t |v^h(\tau)|_n^2 d\tau - 2\int_0^t (v^h(\tau), u_*(\tau))\, d\tau + \int_0^t |u_*(\tau)|_n^2 d\tau \leqslant$$

$$\leqslant 2\int_0^t (u_*(\tau) - v^h(\tau), u_*(\tau))\, d\tau + d_1(h+\delta)\alpha^{-1} + d_2\alpha.$$

From this inequality and results of [3] we have

$$\int_0^t |v^h(\tau) - u_*(\tau)|_n^2 d\tau \leqslant d_0(h+\alpha+\delta)^{1/2} + d_1(h+\delta)\alpha^{-1}.$$

The lemma is proved. □

4. Example

The algorithm was tested by a model example. The following system was considered on the time interval $T = [0,2]$:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t) + a\sin(x_2(t-\nu)) + u_1(t) \\ \dot{x}_2(t) &= b\cos(x_1(t-\nu)) + x_2(t) + u_2(t). \end{aligned}$$

It was assumed that $x_1(t) = 1+t$, $x_2(t) = -2\cos(t)$, for $t \in [-\nu, 0]$, $u_1(t) = t^2$, $u_2(t) = 5\sin(4t)+1$. At the moments τ_i the values $\xi^h_{1i} = x_1(\tau_i) + h\sin(M\tau_i)$, $\xi^h_{2i} = x_2(\tau_i) + h\cos(M_1\tau_i)$ were measured. As a model, we took the system

$$\dot{w}_1(t) = \xi_1^h(t) + a\sin(\xi_2^h(t-\nu)) + v^{h1}(t) + \omega^{h1}(t)$$

$$\dot{w}_2(t) = b\cos(\xi_1^h(t-\nu)) + \xi_2(t)^h + v^{h2}(t) + \omega^{h2}(t)$$

with the initial state $w_1(0) = 1$, $w_2(0) = 2$. Here $\xi_1^h(t) = \xi^h_{1i}$, $\xi_2^h(t) = \xi^h_{2i}$, $t \in [\tau_i, \tau_{i+1})$. The controls v^h and ω^h at the moments τ_i were calculated by formulas (8). In figures 1–3 the results of calculations are presented for the case when $a = 5$, $b = 3$, $\nu = 1$, $\alpha = 0.01$, $c = 1$, $M = 10$, $M_1 = 50$. Fig. 1 corresponds to the case when $h = 0.001$, $\delta = 0.001$, Fig. 2— $h = 0.001$, $\delta = 0.005$, Fig. 3— $h = 0.1$, $\delta = 0.005$. In figures 1–3 the solid (dashed) lines represent the control $u(t)$ (the model controls $v^h(t)$).

The equation was solved by the Euler method with step δ. The results of numerical experiments show that the mean-square convergence takes place under “reduction” of parameters h and δ or of one of them.

5. Conclusions

We consider the problem of dynamical identification of a variable input of a nonlinear delay system on the basis of an inexact measurement of the phase vector. We present a solution algorithm on the basis of the method of auxiliary control models.

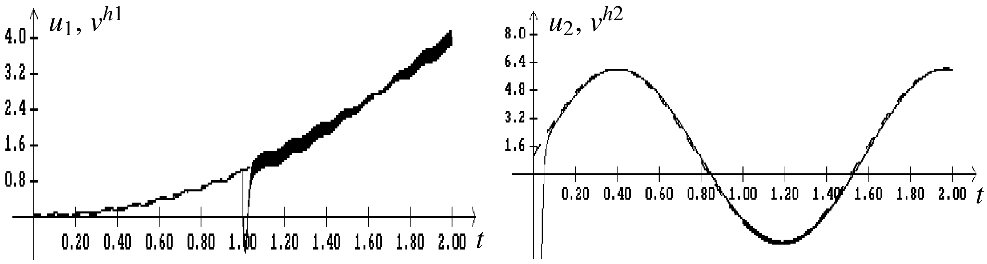

Figure 1. $h = 0.001$, $\delta = 0.001$

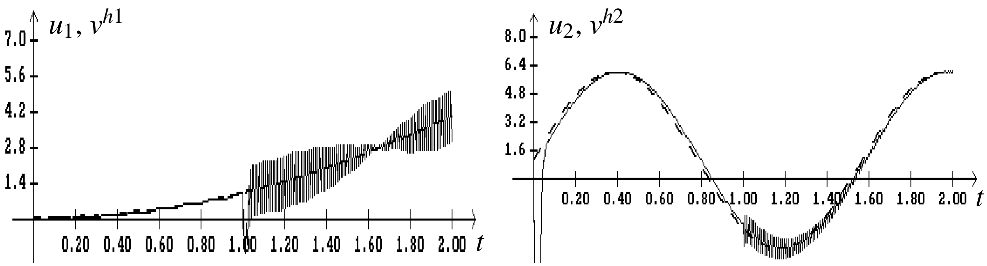

Figure 2. $h = 0.001$, $\delta = 0.005$

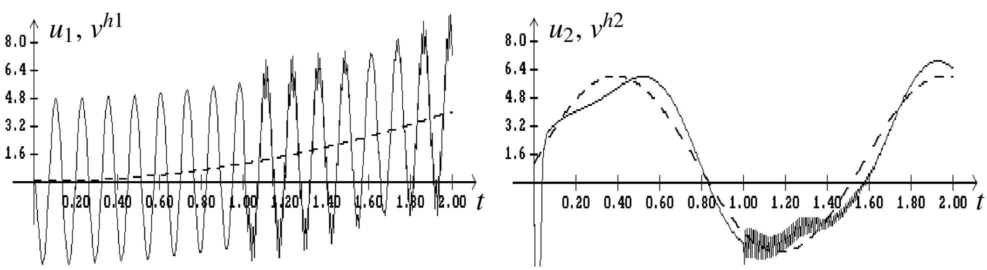

Figure 3. $h = 0.1$, $\delta = 0.005$

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