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Evaluation of phenomenological model parameters using density dependent laws for prediction of mechanical response of cellular materials

H.S.U. BUTT[1], P. XUE[1,2,*], and B. HOU[1]

[1] School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, P.R. China
[2] State Key Laboratory of Explosion Science and Technology, 100081 Beijing, P.R. China

Abstract. Cellular materials have found wide-spread attention in structural applications involving impact energy absorption. The choice of the most suitable density of a cellular material, for a particular impact application, is based on its mechanical response, which may be obtained through experimental tests and/or models. A current study is focused on prediction of a mechanical response of a wide range of densities of a cellular material using available experimental data of very few densities. Best fitting-parameters of four selected phenomenological models, to fit the available experimental response of three distinct aluminum foam densities, are evaluated. The relationship between the best-fitting parameters and density of the foam is established by using two types of functions. The first function is based on a power law relationship between each parameter and foam density ρ, while the second function assumes each parameter as a linear combination of ρ^n and ρ, where n is any real number. The former function is found reasonable in the cases of both parameter interpolation and extrapolation while the latter is found reasonable for a parameter interpolation only. The findings of a current study emphasize for a conscious approach during selection of density dependent laws for phenomenological model parameters to avoid any erroneous or misleading design decision.

Key words: cellular materials, aluminum foam, phenomenological models, energy absorption.

1. Introduction

Cellular materials have been widely used in structural as well as non-structural applications due to their outstanding features, such as light weight, high energy absorption, and good noise/vibration isolation/attenuation characteristics [1]. Among the cellular materials, aluminum foam is widely used for such applications. Aluminum foams have been found as promising materials for applications regarding the impact energy absorption [2]. To improve energy absorption during crushing of foam filled thin walled structures, aluminum foam has been adopted as one of new filler materials [3]. Typical mechanical behavior of aluminum foam under compression can be described as possessing a plasticity like stress plateau while undergoing a large compressive deformation. Because of this characteristic, they are suitable for cushioning or buffering applications.

Accurate modeling of the response of a material has many long term advantages because it may help to avoid large number of laboratory tests, since the experimental setup may not only complicate, expensive and laborious but also infeasible keeping in view the cost. The parameters involved in the model may be evaluated using a set of experimental data, and then these may be utilized to obtain the response of un-tested foams. Liu *et al.* [4] assumed that the parameters are dependent on initial density and claimed that the procedure may be used for generating stress-strain curves at any desired initial foam density as well as for developing "crushability maps" suitable for applications in design and analysis. Moreover, modeling allows evaluating the energy vs. stress, energy vs. density, stress vs. density, efficiency vs. stress and efficiency vs. density curves for any maximum level of stress or absorbed energy [5]. These all curves are being used in selecting optimum foam density for a specific impact or crashworthiness application. And these are in turn totally dependent on the modeled or predicted stress vs. strain curve.

Basically there are two types of models available to represent the behavior of cellular materials, i.e. micro-mechanical model, and phenomenological model. The micro-mechanical models, which are based on the micro-structural level deformation phenomenon, consequently may be difficult to manipulate because of the need of an analysis of actual foam structure. While, most of the models used for numerical simulations are phenomenological models and their parameter identification is based on calibration of available experimental data [5]. Furthermore, development of accurate density dependent laws for the parameters of a model may help to predict the mechanical response of larger range of densities by utilizing the available experiential data of foam with very few densities.

As an example, the importance of such density dependent laws may be found in the case of numerical simulations of functionally graded foam materials (FGFM). A FGFM may be visualized as foam for which micro-structural characteristics vary in a defined manner across the continuum of the foam [6]. Hence such foam may be considered as composed of large number of different density foams, and in conducting

*e-mail: p.xue@nwpu.edu.cn

numerical or analytical analyses, same number of stress-strain curves are required as input. The need of accurate, efficient, less sensitive and robust models becomes inevitable. In such scenarios, a phenomenological model may help to produce the stress-strain curves of un-tested foam densities. As the parameters of a phenomenological model are assumed to be dependent on density of the foam [4, 5], hence parameter-density relationship may be established based on some assumed density dependent law. Afterwards, parameters for un-tested density foam response may be estimated through interpolation or extrapolation of evaluated relationship. This procedure has been shown in Fig. 1 for explanation. However, this interpolation or extrapolation may develop some error, depending on the type of law used to establish relationship between parameters and density. If the predicted parameter is not exactly the best-fit parameter, then it may cause a noticeable error in the output of the model.

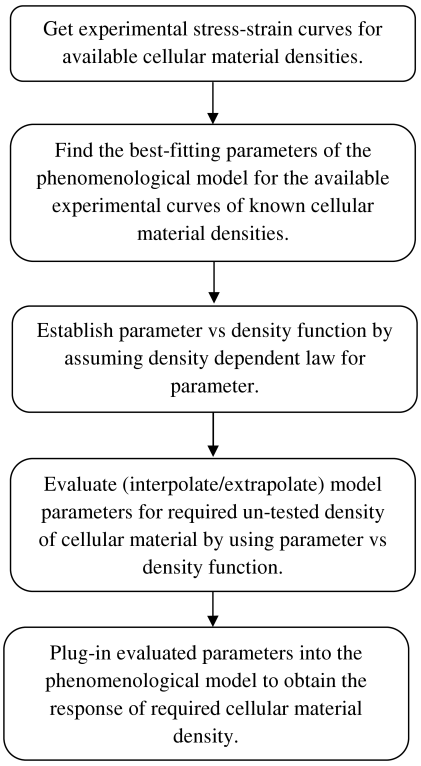


Fig. 1. Flow chart for cellular material response prediction using phenomenological model parameters evaluated through density dependent law

Various phenomenological models have been proposed in the literature to describe the stress-strain relationship of foams. They differ in terms of number of parameters, limitations and accuracy, flexibility and sensitivity. For presentation purpose only four different phenomenological models, already available in literature, are compared with respect to their accuracy to predict the compressive behavior of aluminum foam. Furthermore, two types of laws are used to establish parameter-density functions namely, Type-1 law which is a power law and Type-2 law being linear combination of ρ^n and ρ, where ρ and n are foam density and any real number, respectively. These both laws have been chosen for expressing parameter dependence on density of foam in various studies e.g. [5, 6]. However, the effect of error in these parameters, due to quality or limitations of curve fitting procedures, on the model output has not been quantified or discussed. Here an attempt is made to highlight such type of errors on the model outputs. And findings of the study emphasize for a conscious approach for the choice of density dependent laws for phenomenological model parameters to avoid any erroneous or misleading design decision. The same strategy may be used for evaluating other models not being discussed in current study.

2. Description of selected models

Four models selected in this study are being employed in the sense of phenomenological models. They are Schraad model, Liu model, Avalle model and Wang model. Avalle and Liu models [4, 5] have been found suitable for manipulation of parameters, by taking parameters as functions of density. Schraad mode [7] has already been used to model and manipulate the stress-strain curves for wide range of densities in the recent research regarding functionally graded foams [6, 8, 9]. The phenomenological model proposed by Wang *et al.* [10] can model the stress-strain response of a foam material at different loading rates. This model has capability of taking effect of loading rate; therefore, it is a suitable option for simulations of foams under crash scenarios, where both density and strain-rate dependent properties are required. The models studied are tabulated in Table 1.

Table 1

Investigated models and respective evaluated parameters

No.	Model proposed by	Named in current study	Model parameters selected for evaluation
1	Schraad *et el.* [7]	Schraad Model	ε_y, ε_d, A_o, A_1, A_2
2	Liu *et el.* [1]	Liu Model	A, α, β, C, γ
3	Avalle *et el.* [4]	Avalle Model	A, E, m, B, n
4	Wang *et el.* [10]	Wang Model	A, α, β, C, n

Schraad model. Schraad constitutive model [7] is a continuum-scale constitutive model that relates the macroscopic stress rate in the cellular material to the macroscopic strain rate.

In their model, the finite compressive strain is related to stress by a Young's modulus $E(\varepsilon)$. $E(\varepsilon)$ is itself dependent on the base material stiffness E_s, relative density $\phi(\varepsilon)$ of the foam, and a shape factor $A(\varepsilon)$, and is given by:

$$E(\varepsilon) = A(\varepsilon)E_s[\phi(\varepsilon)]^2, \tag{1}$$

where

$$At(\varepsilon)=\begin{cases} A_0, & \varepsilon_y+\Delta\varepsilon_y \le \varepsilon \le 0, \\ \dfrac{(A_0-A_1)\varepsilon - A_0(\varepsilon_y-\Delta\varepsilon_y)+A_1(\varepsilon_y+\Delta\varepsilon_y)}{2\Delta\varepsilon_y}, & \varepsilon_y-\Delta\varepsilon_y \le \varepsilon \le \varepsilon_y+\Delta\varepsilon_y, \\ A_1, & \varepsilon_d+\Delta\varepsilon_d \le \varepsilon \le \varepsilon_y-\Delta\varepsilon_y, \\ \dfrac{(A_1-A_2)\varepsilon - A_1(\varepsilon_d-\Delta\varepsilon_d)+A_2(\varepsilon_d+\Delta\varepsilon_d)}{2\Delta\varepsilon_d}, & \varepsilon_d-\Delta\varepsilon_d \le \varepsilon \le \varepsilon_d+\Delta\varepsilon_d, \\ A_2 & -1 \le \varepsilon \le \varepsilon_d-\Delta\varepsilon_d. \end{cases} \tag{2}$$

where A_0, A_1 and A_2 are material dependent coefficients for a cellular material. ε_y and ε_d are the strains corresponding to onset of yield and densification, respectively. Transitions at elastic state to plateau range, and plateau range to densification occur through a range of strain ($2\Delta\varepsilon_y$ and $2\Delta\varepsilon_d$, respectively). The value of $\Delta\varepsilon_y$ and $\Delta\varepsilon_d$ is fixed as 0.5% and 3%, respectively, in current study for each density.

Liu model. The model proposed by Liu [1] is continuously differentiable and defined in such a way that both the compressive and the tensile stress-strain curves can be characterized. This model includes six parameters and takes the form:

$$\sigma = A\frac{e^{\alpha\varepsilon}+1}{B+e^{\beta\varepsilon}} + e^{C}(e^{\gamma\varepsilon}+1). \tag{3}$$

In the equation above, the first term is for elastic and plateau regions and the second term accounts for densification region. This function is continuously differentiable and passes through the origin of the stress-strain axes. Parameter A plays the role of scaling factor for yield stress in compression and helps determine the maximum (or the asymptote) if it exists. Parameters, α and β, help to define the softening or hardening like behavior. The parameter Bplays a role of shifting the lower asymptote of the function and may be used as an indicator of tensile yield strength of the foam. The value of parameter B is set to unity for simplicity as proposed by Liu [1]. The expression $e^{C}(e^{\gamma\varepsilon}+1)$ defines the rapid increase in stress during the densification stages of compressive deformation.

Avalle model. The objective of Avalle model [5] is to obtain improvements in terms of reduced weighted sum of the squared errors.

$$\sigma = A(1-e^{(-E/A)\varepsilon(1-\varepsilon)^m}) + B\left(\frac{\varepsilon}{1-\varepsilon}\right)^n, \tag{4}$$

where A and E define the yield stress and elastic modulus respectively. The second addendum is a modification of the second one of the Rusch model [11] and has been introduced in order to have a vertical asymptote corresponding to the physical limit of compression i.e. full densification (ε=1). It may be noted that the second term is similar to the third part of the Gibson model [12] i.e. densification region. Each parameter influences a particular region of the model as in Gibson model, but the formulation is unique and not composed of a separate formula for each region.

Wang model. The Wang model [10] can model the tensile and compressive response of foam, while taking into account the strain-rate effects.

$$\sigma = \left(A\frac{e^{\alpha\varepsilon}+1}{B+e^{\beta\varepsilon}} + C\left(\frac{\varepsilon}{1-\varepsilon}\right)^n\right)\left(1+D\log\left(\frac{\varepsilon_{r,c}}{\varepsilon_{r,o}}\right)\right). \tag{5}$$

The first term models the elastic, plateau and densification regimes, while the second term updates the response for strain rate effects. $\varepsilon_{r,c}$ and $\varepsilon_{r,o}$ are the current and reference strain rates. As the experiments in current study are performed at quasi-static state, so the second term of the model is ignored i.e. $\varepsilon_{r,c}=\varepsilon_{r,o}$ makes the second term equal to unity. A defines the yield limit while parameters α and β capture the inelastic response in the same fashion as in the Liu model, as discussed earlier. The parameters C and n can capture the initiation and the rate of the densification, respectively. B is an indicator of yield limit in tension and is taken as unity in this study for simplification.

Five parameters are selected for each model, assumed as dependent on density of the foam, to fit the experimental curves and are tabulated in Table 1.

3. Models calibration

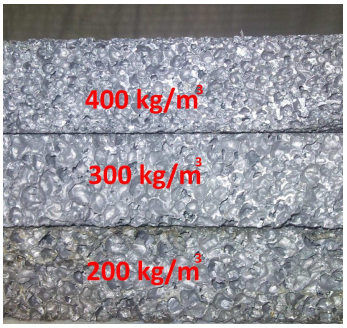
Different cellular materials possess quite different mechanical characteristics, that are consequences of several factors such as type and mechanical properties of the parent material, manufacturing/foaming process, porosity (or relative density) and micro-structural details [12]. However, the most important factor that influences the foam behavior is the relative density [12]. The response of cellular material i.e. elastic and plateau regions and densification is highly dependent on its relative density.

In order to assess the capability of the selected models, experimental data of compression tests of some foam material was obtained. Moreover, in order to explore the applicability and robustness of these models for a wide range of densities, foam samples having same base material (i.e. aluminum) but possessing different densities were used. Experimental program to achieve these goals is described in detail in following paragraphs.

Experimental data. Aluminum foam studied is a closed cell foam with composition of cell walls as Al+Ca 13+ C 3 (wt.%). The foams with three different densities were selected, i.e. the average density for each density group was found around 200, 300 and 400 kg/m^3, respectively. All specimens were prepared in cylindrical geometry with 25 mm diameter and height of 25 mm. Each specimen was previously weighted and measured.

Static uni-axial compression tests were performed by a hydraulic universal testing machine. The foam specimen was compressed between two rigid steel plates at a constant relative velocity of 3 mm/min. The displacement and the force were measured at an appropriate sampling frequency. For each nominal density of foam, at least three repetitions of the compression test were conducted. The photograph of foam sheets and averaged experimental curves for each density are plotted in Fig. 2.

a)



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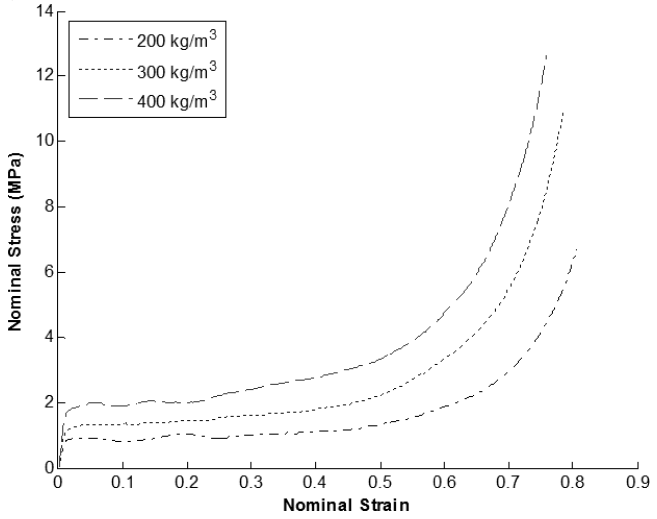


Fig. 2. a) Sheets of aluminum foam used to obtain the test samples, b) compressive stress-strain response of different density aluminum foams

This set of experimental results reveals a quite typical mechanical behavior of the cellular foams with three distinct regimes i.e. elastic, plateau and densification regions.

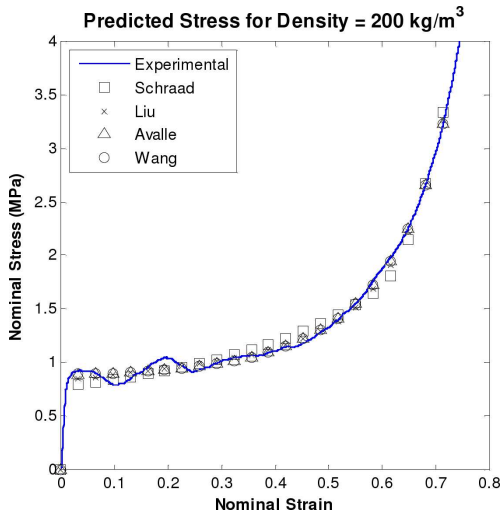
Model parameters vs. experimental data. Liu and Subhash [1] presented a detailed procedure on determining the model parameters through a nonlinear fitting function, *nlinfit* in Matlab®. This function adopts the Gauss-Newton algorithm for iterative adjustment of parameters so as to minimize the mean squared error (MSE) between the experimental data and the prediction of the nonlinear function from a given set of initial parameters. The MSE can be calculated as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\sigma_{i,predicted} - \sigma_{i,\exp erimental})^2, \qquad (6)$$

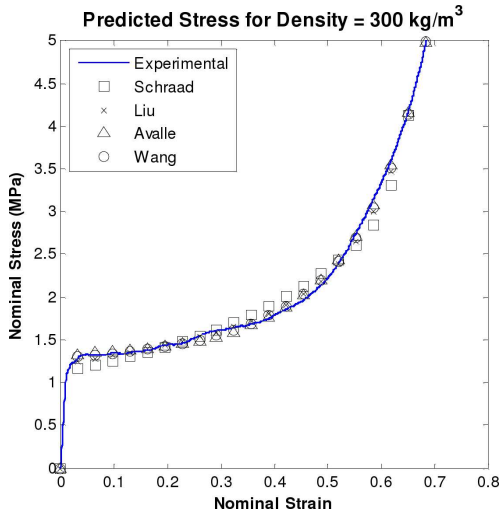
where $\sigma_{i,predicted}$ is the stress predicted by the model and $\sigma_{i,\exp erimental}$ is the experimental stress corresponding to *ith* value of strain. The same scheme was used for each model to fit the experimental data in this study.

The function *nlinfit* outputs the values of the parameters best fitting the experimental data with minimum MSE. These parameters are termed as '*best-fitting parameters*' henceforth in current study. The mean squared error was used to assess the accuracy of each model. The predicted curves using different phenomenological models are plotted against the respective experimental curves for each density in Fig. 3 The corresponding MSEs' are tabulated in Table 2.

a)



b)



c)

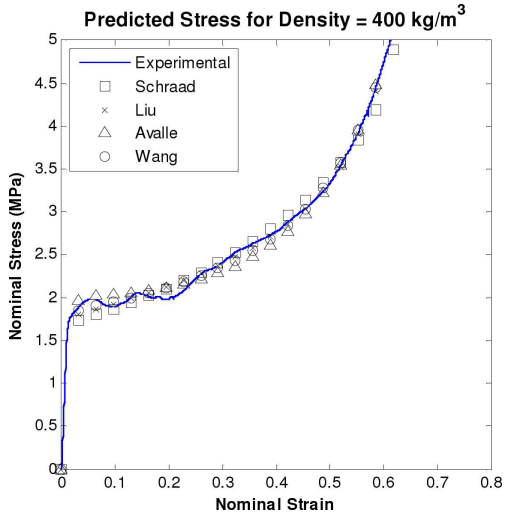


Fig. 3. Fitting curves using best fit parameters of different models: a) for 200 kg/m^3, b) for 300 kg/m^3, and c) for 400 kg/m^3 density foam

Table 2
Calculated MSEs (MPa^2)

Model	Density		
	200 kg/m^3	300 kg/m^3	400 kg/m^3
Schraad Model	0.77	2.20	1.51
Liu Model	0.22	0.43	0.34
Avalle Model	0.16	0.14	0.81
Wang Model	0.15	0.18	0.28

It is found in comparative analyses that MSE of all the models is $\leq$2.2 MPa^2, which indicates that they are sufficiently capable of fitting the experimental dada of each density.

4. Parameter-density functions using density dependent laws

Once best-fitting parameters of a phenomenological model are evaluated for known foam densities, and then these parameters may be related to the density of the foam by assuming a predefined density dependent law. Once the law is defined in the form of a function then coefficients of the relevant function may be evaluated (or calibrated) by the same non-linear curve fitting tool *nlinfit*, available in Matlab®. Two types of functions are evaluated by assuming two laws between each parameter of a model and only the density of foam ρ.

Power law:

$$\textit{Parameter} = f(\rho) = C_{11} \times (\rho)^{C_{12}}. \tag{7}$$

Linear combination of ρ^n and ρ (n is a real number):

$$\textit{Parameter} = f(\rho) = C_{21} \times (\rho)^{C_{22}} + C_{23} \times (\rho), \tag{8}$$

where C_{ij} represents the unknown *j*-th coefficient of *i*-th function (type-1 and type-2). Two coefficients are to be evaluated in case of power law and three in the second case. The coefficients of the two assumed laws as output from non-linear curve fitting tool *nlinfit*, are tabulated in following Table 3.

Aforementioned procedure incorporates a fitting error due to the nature of algorithm of curve fitting tool and the selected law. The absolute fitting error is defined here to show the degree of variation of each parameter estimated using parameter-density relationship, with respect to the best-fitting parameter values of known densities (as found in Subsec. 3.2).

$$\text{Abs. fitting error} = abs\left(\frac{Pfit - Pbest}{Pbest}\right) \times 100. \tag{9}$$

Pfit is the parameter value estimated using parameter-density function based on a density dependent law. *Pbest* is the best-fitting parameter value of the corresponding parameters. The calculated absolute errors for each parameter are tabulated in Table 4.

It is evident from the absolute errors that evaluated coefficients for both of the functions are sufficiently accurate as in most of the cases the error is less than 15%. The second type of function is more capable of defining the parameter-density relationship keeping in view the resulting absolute errors. However, both the functions resulted in high error, 17% to 41%, in case of one parameter of Avalle model, i.e. m. However, the effect of this parameter deviation on the prediction of required foam density response may be quantified in the preceding section.

Table 3
Evaluated coefficients for assumed laws for parameter-density relationships

Model	Model Parameters	Type-1 function $P = f(\rho) = C_{11} \times (\rho)^{C_{12}}$		Type-2 function $P = f(\rho) = C_{21} \times (\rho)^{C_{22}} + C_{23} \times (\rho)$		
		C_{11}	C_{12}	C_{21}	C_{22}	C_{23}
Schraad	ε_y	−0.002	0.259	−0.0004	0.885	0.0002
	ε_d	−0.864	−0.062	−2.039	−0.243	−0.0003
	A_o	394.554	−1.237	3.848E+04	−2.125	0.0004
	A_1	0.049	−0.483	0.003	0.131	0.0000
	A_2	0.263	−0.595	0.013	0.039	0.0000
Liu	A	0.002	1.108	−0.013	0.956	0.014
	α	9.498	−0.235	1.046E+04	−1.627	0.005
	β	9.629	−0.238	1.065E+04	−1.631	0.004
	C	−14.119	−0.142	−1.655E+03	−1.104	−0.010
	γ	0.069	0.077	2.193	−0.644	0.0002
Avalle	A	0.001	1.235	−0.027	0.947	0.025
	E	3.723	0.689	8.654E+08	−3.214	0.576
	m	9.367	−0.234	0.353	0.951	−0.258
	B	0.000	1.400	−7.443E+03	−1.969	0.003
	n	1.535	0.030	0.239	0.468	−0.005
Wang	A	0.003	1.054	−0.008	0.961	0.010
	α	448.720	−0.128	4.021E+07	−2.374	0.483
	β	460.106	−0.132	3.726E+07	−2.359	0.480
	C	0.008	0.787	0.022	0.970	−0.016
	n	0.448	0.257	144.167	−0.944	0.004

Table 4
Fitting errors in *manipulated* parameters predicted through density depended laws as compared to best-fitting parameters

Model	Model Parameters	Density (kg/m^3)					
		Type-1 function			Type-2 function		
		200	300	400	200	300	400
Schraad	ε_y	1.371128	2.574562	1.350162	0.527639	0.764388	0.335648
	ε_d	0.29513	0.755452	0.452391	3.2E-13	3.31E-13	3.33E-13
	A_o	1.015511	7.225601	7.433315	0	0	0
	A_1	1.042965	3.54835	2.924207	0	0	0
	A_2	1.057168	3.923462	3.481197	0	0	0
Liu	A	3.436018	2.074281	2.074822	3.316626	3.715315	1.147308
	α	3.616186	6.583542	6.602478	0	0	0
	β	1.067449	3.972832	3.989717	0	0	0
	C	3.436018	2.074281	2.074822	1.44E-13	0	0
	γ	3.616186	6.583542	6.602478	0	0	0
Avalle	A	4.322505	4.164339	1.135921	4.158901	4.504593	1.336977
	E	5.246183	8.257684	2.913699	1.066676	3.861981	1.856015
	m	17.35712	30.10437	40.48958	23.15294	24.8553	23.81005
	B	9.617054	6.361691	1.808303	1.289878	1.953496	0.85845
	n	0.723632	1.660797	0.980294	0	0	0
Wang	A	2.483792	2.680101	0.825583	2.422153	2.715199	0.859764
	α	3.115178	9.400891	5.132484	0	0	0
	β	3.093044	9.364356	5.128278	0	0	0
	C	12.32011	12.25092	5.47743	13.01767	11.96454	5.128212
	n	1.672235	3.440968	1.650611	0	0	0

The term *'manipulated parameters'* would be used henceforth in this study to represent the values of parameters for a required density of foam ρ, estimated through evaluated parameter-density functions (see Table 3).

5. Results and discussions

5.1. Response prediction for known density foams vs experimental data. Firstly, the prediction through models with manipulated parameters is compared with the available experimental data of known densities. The prediction is limited to 75% nominal strain, as densification has been achieved up to this limit strain for the case of densities discussed here. It is to be noted that two types of indicators are used to show the accuracy of the predicted response. The first one is the MSE, which shows the deviation of prediction with respect to known response. The second indicator is the standard deviation of the output of all models at every strain value at every strain value. The second indicator is assumed to show a virtual assessment of accuracy of the predicted response; the low standard deviation means the predicted response using all the models is sufficiently accurate. This assumption would help to assess the accuracy of the model predictions for those densities whose experimental data is not available. The calculated MSEs through models with manipulated parameters using two distinct parameter-density relations are tabulated in Table 5 and Table 6, respectively. While the respective predictions along with standard deviation of output of four models are graphically plotted in Figs. 4 and 5, respectively. Very low values of MSEs and standard deviation (abbreviated as Std-Dev in the legends of relevant figures) at whole span of strain show that for the experimental data, both of the functions for parameter manipulation are accurate enough.

Table 5
Calculated MSEs (MPa^2) of foam response through models with manipulated parameters using Type-1 function

Model	Density		
	200 kg/m^3	300kg/m^3	400 kg/m^3
Schraad Model	0.0079	0.0194	0.0309
Liu Model	0.0032	0.0065	0.0109
Avalle Model	0.0078	0.0032	0.0143
Wang Model	0.0056	0.0058	0.0036

Table 6
Calculated MSEs (MPa^2) of foam response through models with manipulated parameters using Type-2 function

Model	Density		
	200 kg/m^3	300kg/m^3	400 kg/m^3
Schraad Model	0.0076	0.0199	0.0148
Liu Model	0.0034	0.0081	0.0041
Avalle Model	0.0028	0.0193	0.0087
Wang Model	0.0132	0.0349	0.0132

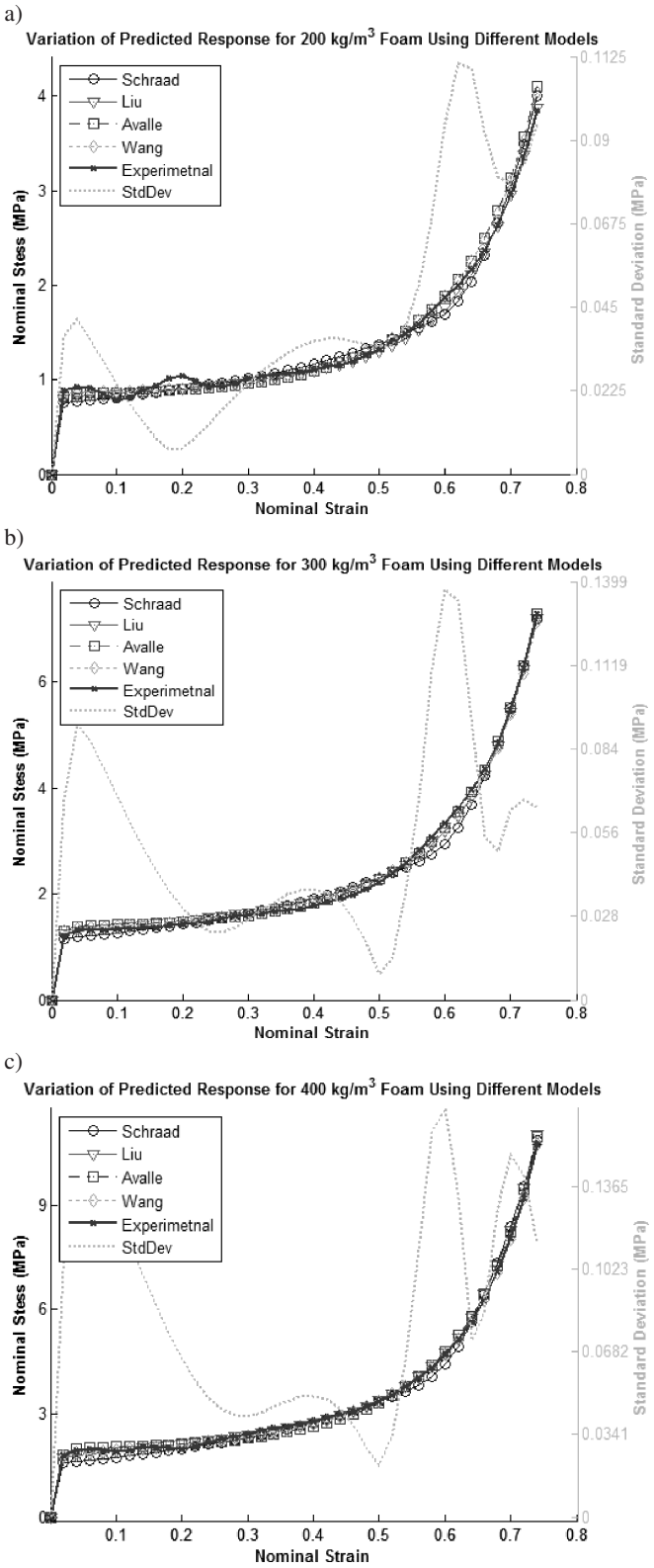


Fig. 4. Predicted foam response through models with manipulated parameters using Type-1 function: a) for 200 kg/m^3, b) for 300 kg/m^3, and c) for 400 kg/m^3 density foam

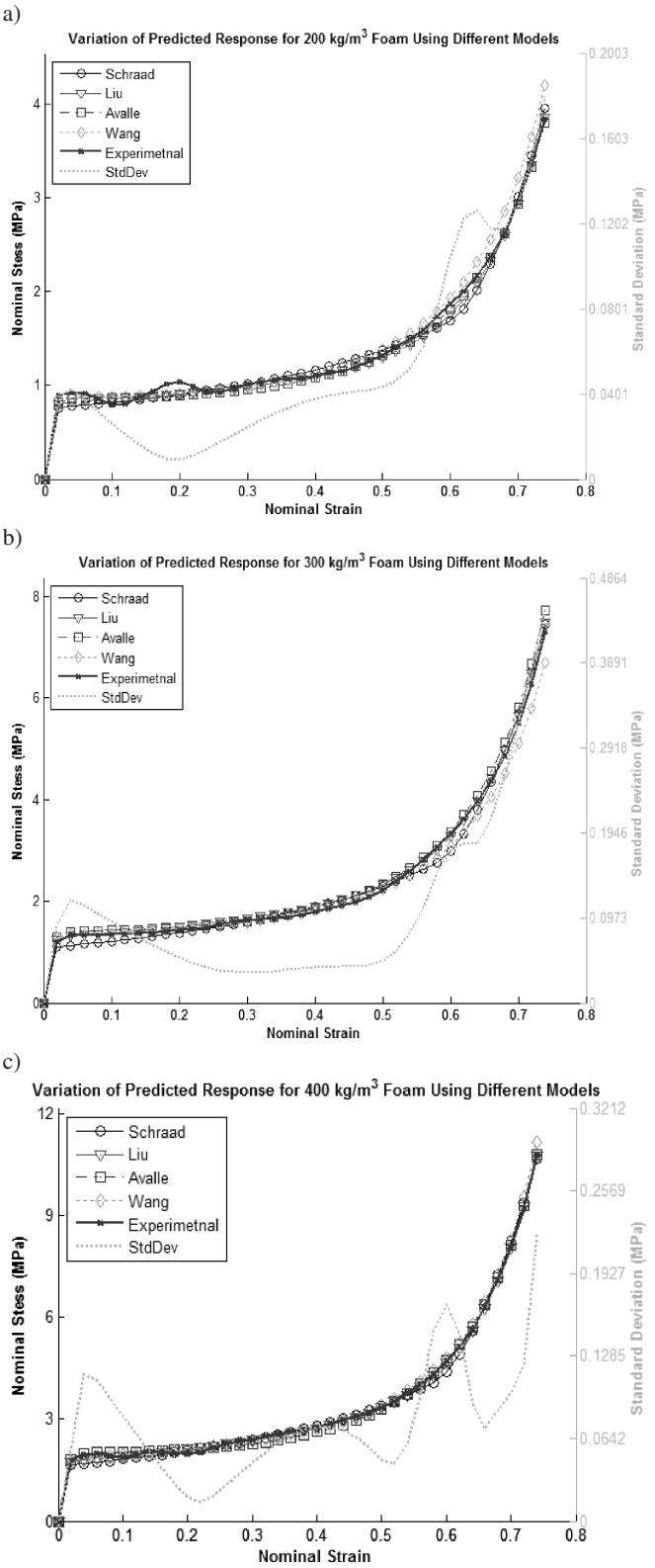


Fig. 5. Predicted foam response through models with manipulated parameters using Type-2 function: a) for 200 kg/m^3, b) for 300 kg/m^3, and c) for 400 kg/m^3 density foam

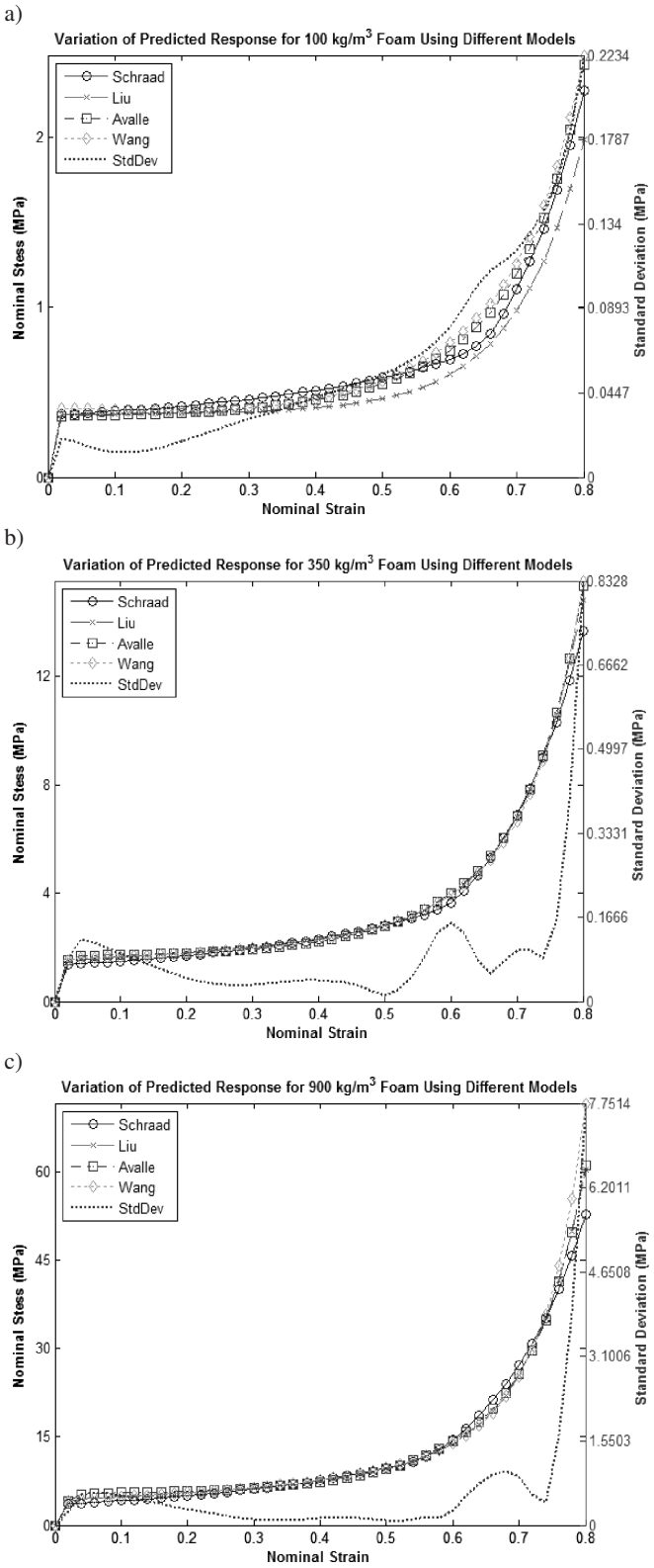


Fig. 6. Predicted foam response through models with manipulated parameters using Type-1 function: a) for 100 kg/m^3, b) for 350 kg/m^3, and c) for 900 kg/m^3 density foam

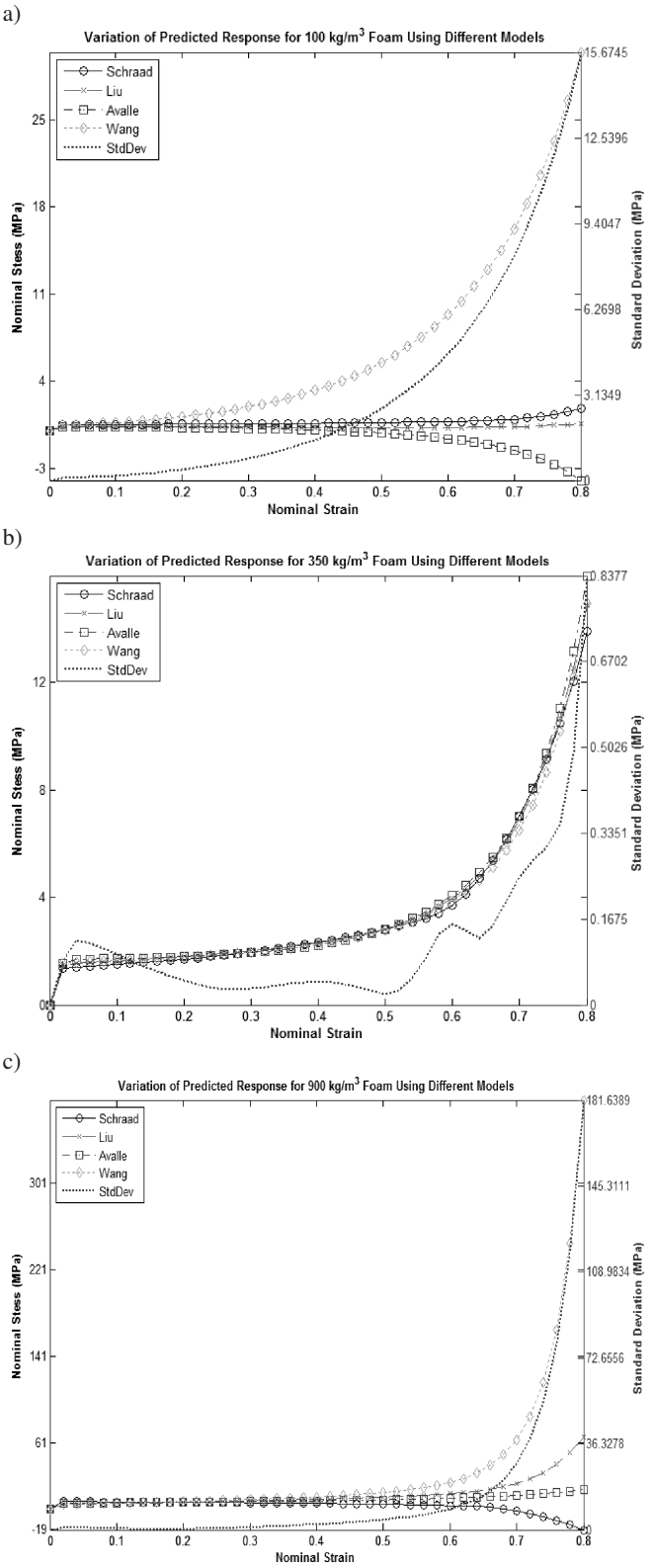


Fig. 7. Predicted foam response through models with manipulated parameters using Type-2 function: a) for 100 kg/m^3, b) for 350 kg/m^3, and c) for 900 kg/m^3 density foam

5.2. Response prediction through models with interpolated/extrapolated parameters. To predict the response of any required foam density, first the parameters of each model are evaluated using relevant parameter-density relationship function (see Table 3). Then these manipulated parameters are plugged-in the respective phenomenological model to extract the mechanical response of the foam with required density. The predicted response for three cases namely extrapolation towards lower end, interpolation and extrapolation towards higher end are considered. Foam densities of 100, 350 and 900 kg/m^3 are selected for this purpose, respectively. The response of these selected densities is shown in Figs. 6 and 7. The standard deviation of the output of four models is used as a virtual check of accuracy of output of the models, as the experimental data of these densities is not available (as is discussed in section 0). It is evident that Type-1 function although not accurate enough to define the parameter-density relationship, in comparison to Type-2 function, is a reasonable choice for response prediction of wider range of densities. The prediction through models with manipulated parameters using Type-2 function is reasonable only for the case of parameter interpolation while it is not suitable for extrapolation cases.

5.3. Design decisions based on predicted response: energy absorption. Energy absorbed by unit volume of a foam material, deformed to a specific strain ε, may be expressed by following relation

$$W = \int_0^{\varepsilon} \sigma(e)de. \tag{10}$$

It means that energy absorbed by the foam may be obtained by evaluating area under the stress-strain curve up to a specific strain. Absorbed energy with respect to the stress (the concept known as energy-absorption diagram [12]) may help to optimally select a foam density to absorb a specific amount of impact energy with minimum stress. Or alternatively, this may help to evaluate energy absorbed at a specified level of stress for a specific foam density. As the calculated absorbed energy is solely dependent on stress-strain response, the erroneous or badly predicted response would affect the designer decision to select some specific foam density for a particular impact application. Moreover, if such a predicted material response is going to be used in some numerical simulation, like that of functionally graded foam under impact or crash loading, it may yield misleading results.

Figure 8 compares the energy absorbed by the foams of tested densities; 200, 300 and 400 kg/m^3, calculated using experimental data and using predicted response through parameter manipulation. The stress levels selected for these densities are 1.25, 2, and 3.5 MPa, respectively. It is evident from the figure that prediction through models with manipulated parameters using both types of functions is very close to that of experimental response. Figure 9 shows the predicted absorbed energy for 450, 500 and 600 kg/m^3 densities at 4, 5, and 6 MPa respectively. It is evident that the difference between the predictions of four models with manipulated parameter using Type-1 function is negligible. While the spread of predicted absorbed energies is increasing with Type-2 function, as the foam density, for which parameters are to be extrapolated, is advancing away from the domain of densities with known response.

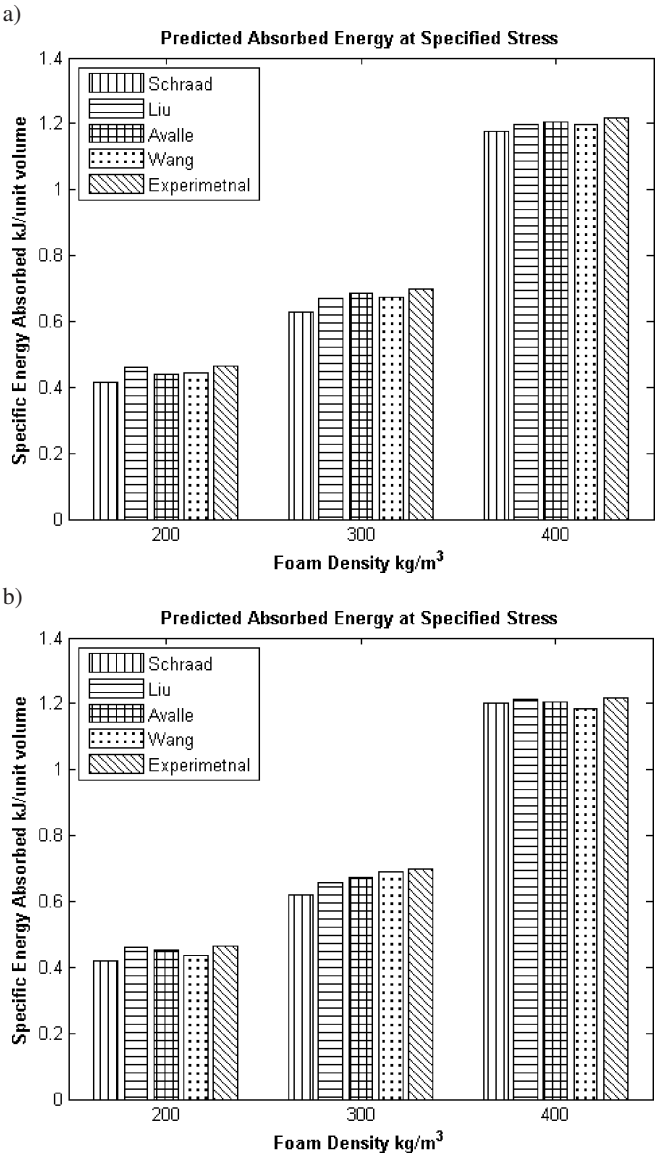
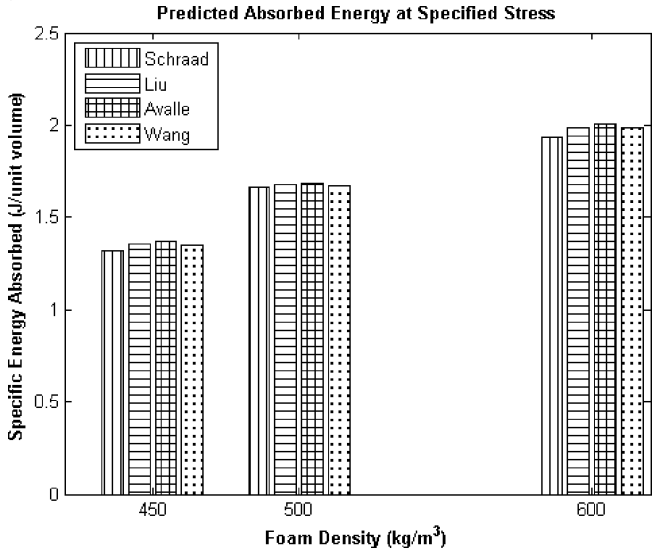


Fig. 8. Comparison of absorbed energy calculated using experimental data and using response predicted through parameter manipulation. Parameter manipulation using (a) Type-1 function (b) Type-2 function

Here it may be deduced again that predicted absorbed energy, using a response predicted through the Type-1 function is more reasonable and reliable than that of the Type-2 function. Hence for the Aluminum foam being investigated in current study, dependence of model parameters on density of the foam through a power law function should be considered as a suitable choice for making any design decision.

a)



b)

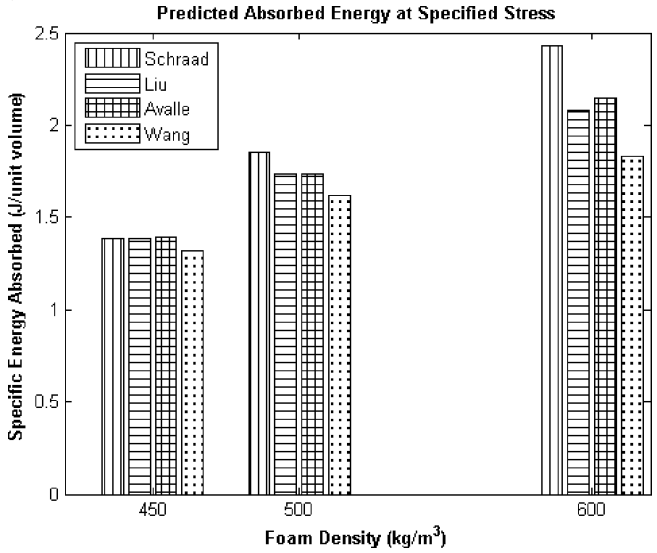


Fig. 9. Comparison of absorbed energy calculated using response predicted through models with manipulated parameters. Parameter manipulation using (a) Type-1 function (b) Type-2 function

6. Conclusions

Prediction of a mechanical response of the wide range of densities of a cellular material using available experimental data of very few densities is explored in this study. Best fitting-parameters of four selected phenomenological models, to fit the available experimental response of three Aluminum foam densities, are first evaluated. The relationship between the best-fitting parameters and density of the foam is established by using two types of functions, namely Type-1 and Type-2 functions. Type-1 function is based on a power law relationship between each parameter and foam density ρ, while the Type-2 function assumes each parameter as a linear combination of ρ^n and ρ, where n is any real number. It is concluded that

- Density dependent law used for parameter-density relationship plays a key role in accurately predicting the response of a cellular material, through parameter manipulation of a phenomenological model.
- The selection of a *reasonable* density dependent law for the parameters of a phenomenological model may help to extract the response of a wide range of densities while utilizing available experimental data of limited densities.
- The Type-1 function, based on power law, is not capturing the parameter-density relationship well for available experimental data. However models with manipulated parameters using this function are capable of predicting the response of a wide range of densities through both parameter interpolation and extrapolation. While Type-2 function, based on liner combination of law, capturing the parameter-density relationship well for available experimental data, is found suitable for response prediction through parameter interpolation case only.
- Similarly, energy absorption calculated using a response predicted through the Type-1 function is accurate for both interpolation and extrapolation cases in contrast to the Type-2 function, where it is predicted well only in a case of parameter interpolation.
- Models with manipulated parameters using the Type-1 function have been found suitable to predict the response and energy absorption of foams having densities ranging from 100 to 900 kg/m^3. This range is 4 times wider than the density domain where experimental response of the foam is available. While the response, using the Type-2 function, is well predicted only in the range of densities of 200 to 400 kg/m^3.
- For the specific case of Aluminum foam being investigated in current study, dependence of selected model parameters on density of the foam through power law is found as a suitable choice for making any design decision i.e. predicting stress-strain or energy absorption response for a foam density which is not available and/or not tested in the laboratory yet.

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