Comparison of transient states in step-down power converter (BUCK) in continuous and discontinuous conduction mode

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Abstract. The object of this paper is a step-down (BUCK) power converter working in the continuous conduction mode (CCM) or discontinuous conduction mode (DCM). Two types of transient states in a converter have been analyzed and observed experimentally: slow transients, described by averaged models of a converter and fast transients, in the course of a single switching period. The averaged models of converter working in CCM, presented in various sources are similar, with some differences concerning only the description of parasitic effects. The averaged models for DCM depend on the adopted modeling method. Models obtained by the switch averaging approach are second-order models (containing two reactive elements in equivalent circuit representation). Models obtained by the separation of variables approach are first-order models. The experimental results given in this paper show the first-order type of transients. Another group of experiments concern fast transients in the course of a single switching period. The oscillations of inductor voltage in the part of a switching period are observed for DCM.

Key words: step-down converter, BUCK, transient states, CCM, DCM.

List of symbols

C	–	capacitance,
D	–	diode,
d_A, D_A	–	duty ratio and its steady-state value,
f, f_S	–	frequency, switching frequency,
G	–	load conductance,
H_g, H_d	–	small-signal conductances: input-to-output and control-to-output,
i	–	with subscripts in capital letters – instantaneous values of currents,
I	–	with subscripts in capital letters – quiescent values of currents,
I	–	with subscripts in small letters – small-signal representations of currents in s domain,
K	–	ideal controlled switch (transistor),
L	–	inductance,
M_V, M_I	–	voltage and current static transmittance,
R	–	load resistance,
R	–	with subscripts – parasitic resistances,
T_S	–	switching period,
v	–	with subscripts in capital letters – instantaneous values of voltages,
V	–	with subscripts in capital letters – quiescent values of voltages,
V	–	with subscripts in small letters – small-signal representations of voltages in s domain,
β	–	(with subscripts) – coefficients in small-signal model of converter in DCM,
θ	–	small-signal representations of duty ratio in s domain.

1. Introduction

Pulse-width modulated DC-DC converters may operate in CCM (continuous conduction mode) or DCM (discontinuous conduction mode). A typical converter may be intentionally designed to operate in DCM or, working principally in CCM, may move to DCM in the case of load current drop below some critical levels. Some converters designed for application in PFC (power factor corrections) systems may serve as examples of practical use of DCM [1, 2].

The analysis and experimental observations of transient states in the basic step-down (BUCK) converter is a main purpose of this paper. A special attention is devoted to comparison of transients in two operation modes: CCM and DCM. Transient states in a switching converter may be considered in two separate time scales namely, one may distinguish slow transients (lasting by many switching periods) and fast transients (within single switching period). The analysis of slow transients is based on averaged models of a converter. Averaged models represent relations between values of currents and voltages averaged over single switching period $T_S = 1/f_S$, where f_S is switching frequency of converter.

The averaged models of switching converters may be derived in several ways, for example, by state-space averaging, switch averaging or by separation of variables. The differences in averaged models obtained in different ways for CCM mode are not serious and concern only the way of description of parasitic effects. Averaged models for DCM obtained by different methods differ substantially as is shown in later text.

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In Sec. 2 of the paper, the main features of a converter working in the continuous conduction mode and the discontinuous conduction mode are discussed. In Sec. 3 the averaged models of BUCK converter in CCM and DCM are presented. The exemplary results of calculations and measurements of slow and fast transient states in both conduction modes are presented in Sec. 4. Some concluding remarks are given in Sec. 5.

2. Main features of BUCK converter in CCM and DCM

The circuit under investigations, i.e. power stage of basic BUCK converter is shown in Fig. 1. Symbols i_G, v_G etc. represent instantaneous values of currents and voltages; K, D, L, C – the ideal switches (transistor and diode), ideal inductor and capacitor; R_T, R_D, R_L, R_C – parasitic resistances of transistor, diode, inductor and capacitor and G is load conductance.
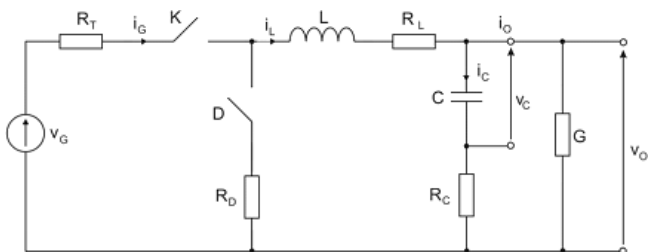

Fig. 1. Power stage of BUCK converter with parasitic resistances

The conditions of working in CCM or DCM can be expressed by a proper relation between converter parameters. It may be shown, for ideal converter [3], that for values of load conductance greater than G_C:

$$G_C = \frac{T_S}{2 \cdot L} \cdot (1 - D_A) \qquad (1)$$

the converter works in CCM. For $G < G_C$ it works in DCM.

Figures 2 and 3 present typical approximate waveforms of currents of inductor i_L, transistor i_K (equal to input current i_G) and diode i_D in CCM (Fig. 2) and in DCM (Fig. 3). The switching period T_S in the case of CCM is divided into two subintervals: ON and OFF. In DCM it consists three subintervals: ON, OFF1 and OFF2.

It may be observed, that the differences between maximum and average values of currents are greater in DCM then in CCM. It means that for assumed value of load current, the maximum permissible values of currents in inductor, transistor and diode should be greater for converter working in DCM. This feature is of course the advantage of CCM.

Switching processes of semiconductor devices (transistor and diode) in DCM have in part a "soft" character, because diode current approaches zero before the diode is turned off. After turn-on of transistor in DCM, the transistor current begin to rise gradually in contrast with being hard switched in CCM. As a result the switching processes may be shorter in DCM, with lower level of EMC, therefore higher values of switching frequency may be used.
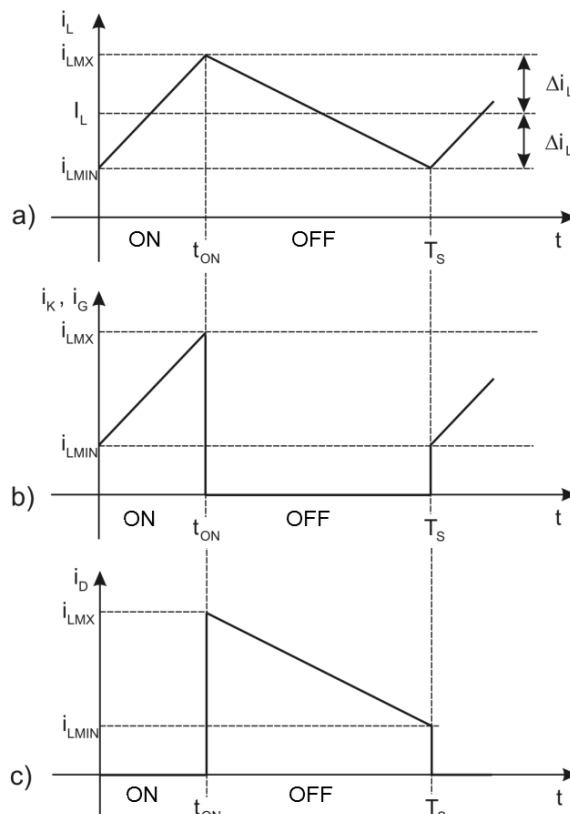

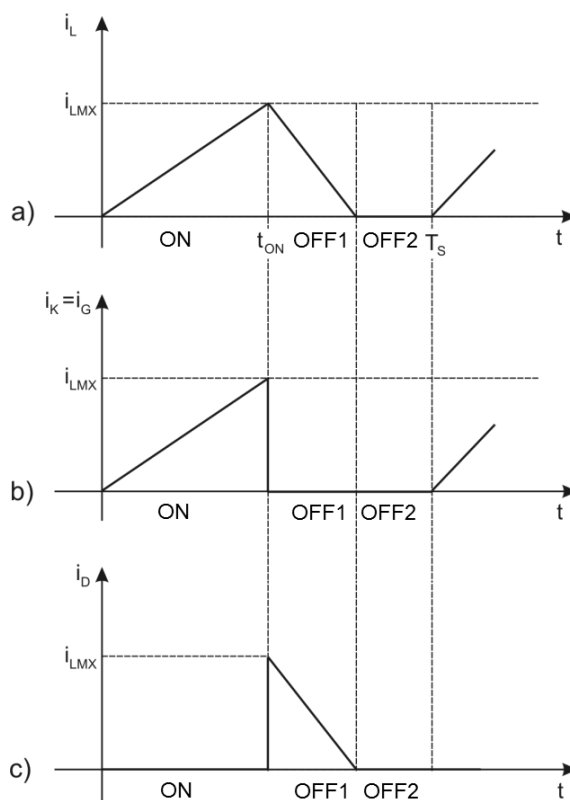Fig. 2. Approximate shape of waveforms of currents in BUCK converter working in CCM

Fig. 3. Approximate shape of waveforms of currents in BUCK converter working in DCM

3. Averaged models of BUCK converter

Averaged models of switching DC-DC converters are widely used for description of converter dynamics in frequency range much smaller than switching frequency. Such models are necessary in the design process of control circuits of a converter [3–8]. Averaged models represent the relations between currents and voltages in the power stage of a converter averaged over single switching period. In particular, they describe transient states in converter for time intervals containing many switching periods.

There are several approaches to averaged models creation described in the literature, namely state-space averaging [3, 4], switch averaging [3–8] and separation of variables [9, 10]. The models obtained by switch averaging technique are most frequently used. The idea of switch averaging approach is presented in the most clear way in a book [3], Ch. 7.5. It is based on the division of a power stage of a converter into linear, time invariant subcircuit and a pair of switches (for basic converters containing only two switches). The pair of switches is replaced by averaged equivalent subcircuit and connected to first subcircuit. In the separation of variables approach, the main variables are divided into two groups, the first containing the variables having the same average values in each subinterval of switching period and second – the other variables. The average values of the variables of the second group are expressed as a combination of the variables of the first group and duty ratio coefficient [9, 10]. In both approaches mentioned above the first step is derivation of large-signal averaged model and in the second step, small-signal model is found.

The example of averaged models of the non-deal BUCK converter working in CCM derived with separation of variables approach is presented in Fig. 4 [10].
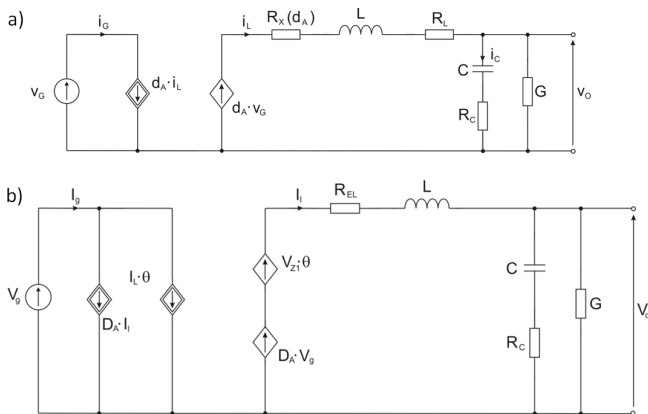

Fig. 4. Averaged models of non-ideal BUCK converter working in CCM obtained by separation of variables approach Ref. 10; a) large signal, b) small signal

Equivalent resistances R_X and R_{EL} are described by following equations:

$$R_X = R_D + d_A \cdot (R_T - R_D), \qquad (2)$$

$$R_{EL} = R_T \cdot D_A + R_D \cdot (1 - D_A) + R_L. \qquad (3)$$

Small-signal transmittances for averaged quantities used usually for description of converter dynamics in frequency domain are defined by following formulas:

$$H_g = \left. \frac{V_o}{V_g} \right|_{\theta=0}, \qquad (4)$$

$$H_d = \left. \frac{V_o}{\theta} \right|_{V_g=0}. \qquad (5)$$

The exemplary form of transmittance H_g of nonideal BUCK converter in CCM, obtained from the model of Fig. 4b has the form:

$$H_g(CCM) =$$
$$\frac{D_A \cdot (s \cdot C \cdot R_C + 1)}{s^2 \cdot L \cdot C_Z + s \cdot (G \cdot L + C_Z \cdot R_{EL} + C \cdot R_C) + R_{EL} \cdot G + 1}, \qquad (6)$$

where

$$C_Z = C \cdot (1 + R_C \cdot G). \qquad (7)$$

The transmittance for ideal converter (without parasitic resistances) is obtained from Eq. (6) by putting $C_Z = C$, $R_{EL} = R_C = 0$. It should be pointing out, that ideal transmittances of BUCK converter in CCM obtained with switch averaging approach and by separation of variables technique are strictly the same. The differences in transmittances of real converters result only from differences in description of parasitic resistances [3, 4, 8, 9]. The transmittance expressed by Eq. (6), for typical values of parameters has a pair of complex poles. As a result, the time-domain response for step change of input voltage has the form of damped oscillation. The equivalent circuit in the form shown in Fig. 4b, is a second order system.

The differences in dynamic averaged models of BUCK converter working in DCM obtained with the use of switch averaging and separation of variables approaches are more serious. The structure of averaged models of ideal BUCK converter working in DCM obtained by switch averaging technique, is shown in Fig. 5 [3, 7]. This model contains two reactive elements L and C, therefore it is an object of the second order.
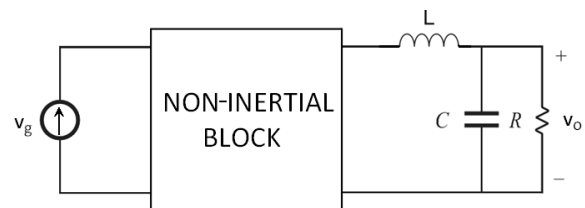

Fig. 5. The general form of averaged model of BUCK converter in DCM, according to Ref. 3, Ch. 7.5

For such a structure of a model, the denominator of small signal transmittances has to be similar to denominator of transmittance H_g (CCM) in Eq. (6) and the location of their poles depends similarly on the values of G, L and C as in the case of Eq. (6). Several modifications of averaged models obtained by switch averaging technique may be found in the literature [5–7, 11–17]. The common feature of these models is a two-pole form of main transmittances.

Averaged models of the BUCK converter in DCM obtained with separation of variables approach are shown in Fig. 6 [10]. These models are of the first order because contain only one inertial element C. Such form of the averaged model results from the fact, that the average value of the voltage on the element L is zero even in transient states [9, 10]. Small signal transmittance H_g (DCM) obtained from model of Fig. 6b is presented by Eq. (8).

$$H_g(DCM) =$$
$$\frac{[M_V \cdot R_G \cdot (2 - M_V) + D_A \cdot R_P] \cdot (s \cdot C \cdot R_C + 1)}{s \cdot (C \cdot R_C \cdot R_{PG} + C_Z \cdot R_Y^2) + R_{PG} + G \cdot R_Y^2}, \quad (8)$$

where

$$M_V = 0.5 \cdot D_A \cdot R_O \cdot G_Z$$
$$\cdot(\sqrt{(D_A + R_P \cdot G)^2 + 4 \cdot G/G_Z} - D_A - R_P \cdot G), \quad (9)$$

$$R_P = R_L + (R_T + R_D)/2, \quad (10)$$

$$R_G = 2 \cdot L/T_S, \quad (11)$$

$$R_Y = R_G \cdot M_V/D_A + R_P, \quad (12)$$

$$G_Z = 1/R_G, \quad (13)$$

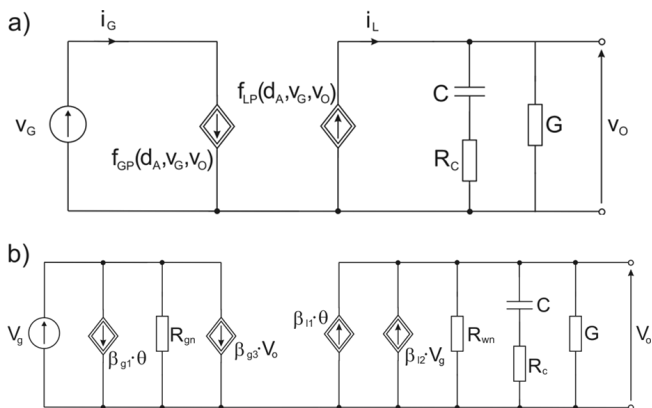$$R_{PG} = D_A \cdot R_P + R_G. \quad (14)$$

Fig. 6. Averaged models of BUCK converter in DCM obtained by separation of variables, according to Refs. 9, 10 a) large signal model, b) small signal model

It is evident that, according to description based on separation of variables approach, there exist essential difference between dynamics of a converter in CCM mode described by equivalent circuit in Fig. 4 and transmittance H_g (CCM) given by Eq. (5), and in DCM described by equivalent circuit of Fig. 6 and transmittance H_g (DCM) given by Eq. (8). The differences in the form of transmittances should correspond to differences in the waveforms obtained in transient states.

4. Calculations and experiments

Slow and fast transients in BUCK working in CCM or DCM have been observed by measurements and calculations. In the laboratory model of a power stage of the BUCK converter, the active switch (K) is transistor IRF 1324 with the driver IRS2186, diode – MBRS34OT3. Parameters of other

elements, measured at frequency 1 kHz, are: $L = 23.5\ \mu H$; $C = 44.7\ \mu F$; $R_T = 4\ m\Omega$; $R_L = 57\ m\Omega$; $R_C = 35\ m\Omega$; $R_D = 141\ m\Omega$. The frequency and duty ratio of switching are: $f_S = 100$ kHz, $D_A = 0.3$. The value of load resistance ensuring the CCM was $R_1 = 5\ \Omega$ and for DCM: $R_2 = 20\ \Omega$. The exemplary waveforms of slow transients within the time range from 0 to 2 ms (200 switching periods) corresponding to excitation by the step change of the input voltage of the form:

$$v_G(t) = V_G + V_m \cdot \mathbf{1}(t) \quad (15)$$

are presented in Fig. 7. A waveform of output voltage in a converter excited by an input voltage step given by Eq. (15) for CCM (load resistance R_1), shown in Fig. 7a, is obtained experimentally. Waveforms in Figs. 7b and 7c have been calculated according to an averaged model obtained by separation of variables – with parasitic resistances included (Fig. 7b) and without parasitic (Fig. 7c). The responses are damped oscillations, typical for a second order system. Good consistency of experiment and calculations with parasitics included are observed. The waveform obtained for ideal converter differs substantially from reality.
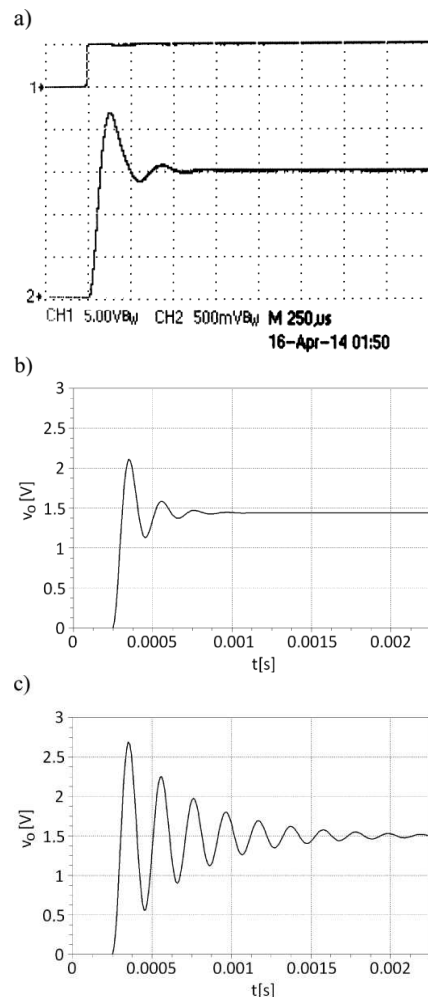

Fig. 7. Exemplary observations of slow transients in CCM – experiment (a) and simulations (b, c)

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Waveforms of output voltage for a converter working in DCM (for load resistance 20 Ω) for the same excitation as in previous example are shown in Fig. 8. Waveform in Fig. 8a is obtained experimentally. Waveform in Fig. 8b is calculated from averaged models obtained by separation of variables. The consistency of measurements and calculations based on averaged model obtained with separation of variables [9, 10] is quite good.
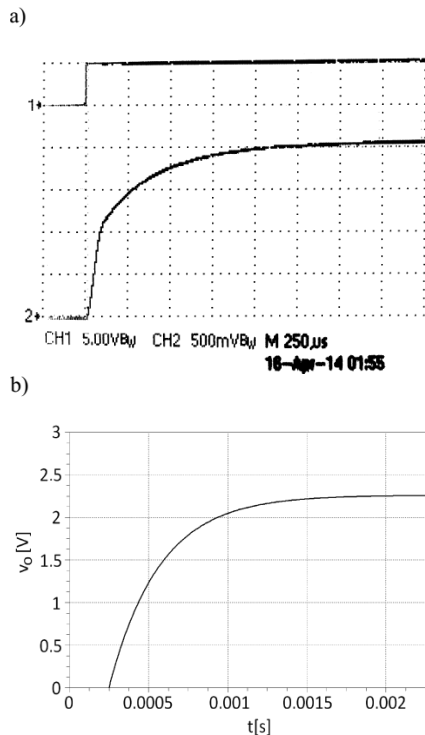
a)

b)

Fig. 8. Exemplary observations of slow transients in DCM – experiment (a) and simulation (b)

In the second group of experiments the waveforms of selected voltages in the power stage of BUCK converter with constant input voltage in short segment of time (several switching periods) have been observed and calculated. In Fig. 9 the observed waveforms of voltage over inductor coil are presented for load resistance R_1 (CCM) in Fig. 9a and for load resistance R_2 (DCM) in Fig. 9b. According to idealized description, in subinterval OFF2 of switching in DCM, the inductor current and voltage should be zero. In Fig. 9b the oscillations of inductor voltage of frequency about 0.5 MHz are visible in subinterval OFF2. Such oscillations, not predicted by an idealized model of a converter are the result of resonance of inductance L and parasitic output capacitance of a transistor. In other subintervals or in CCM the resonant subcircuit is effectively damped by small resistances of a conducting diode or transistor. In subinterval OFF2, diode and transistor are turned OFF and the oscillations are weakly damped, only by load resistance. Results of SPICE simulations of fast transients also reveals the existence of oscillations of inductor voltage in OFF2 subinterval.
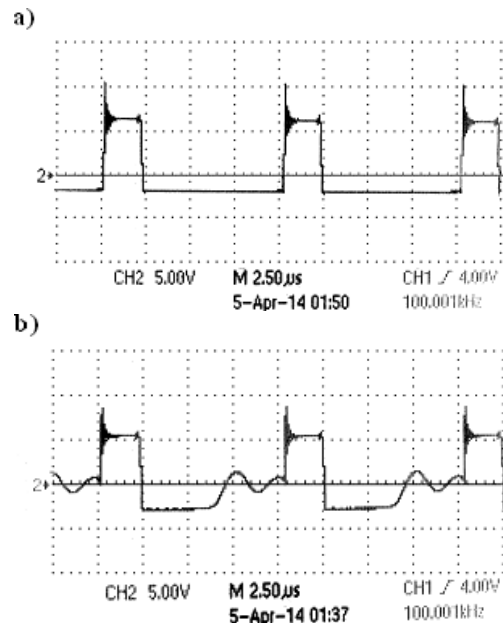
a)

b)

Fig. 9. Experimental observation of fast transients of inductor voltage in CCM (a) and DCM (b)

5. Conclusions

Transient states in BUCK converters for various time scales are discussed in this paper, for two conducting modes: CCM and DCM. The main attention is paid to slow transients for relatively large time intervals, containing many periods of switching. The behavior of converters for such time intervals is usually described by averaged models. Averaged models are typically obtained by switch averaging technique [3–8]. Another possibility proposed recently [9, 10] is to use the separation of variables approach. Both methods of averaged models derivation give nearly the same results for BUCK converter working in the continuous conduction mode (CCM) and the resulting models are of the second order. Results of applying these methods to the discontinuous conduction mode (DCM) differ substantially. An averaged model of two-switch BUCK converter in DCM obtained directly on the base of switch averaging approach (Fig. 5) is of the second order, whereas a model based on separation of variables is of the first order. Calculations of slow transients obtained for step change of input voltage for CCM based on two types of models give damped oscillations of output voltage that is confirmed by experiments. In the case of DCM, the response of the output voltage calculated from model based on separation of variables is non-oscillatory. The output voltage waveforms obtained experimentally are also non-oscillatory. Transient states calculated on the base of second order model resulting from switch averaging approach may be oscillatory or non-oscillatory depending on the detailed combination of circuit parameters.

Another group of experiments, described in the paper, concerns the fast transients in a converter in CCM and DCM. An interesting result is the observation of oscillations of the voltage over inductor coil in one of subintervals of a switching

period in DCM. The proposed explanation of such behavior have been qualitatively confirmed by SPICE simulations.

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