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Fiber-based laser frequency combs

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Abstract. For the last decade a very attractive field of laser physics, namely the optical frequency comb technique, has been intensively developed. Fiber lasers play particular role in that area. The motivation of their development is obtaining broadband comb systems with well-defined and stable mods (comb teeth). This paper presents a basic overview devoted to the fiber-based optical frequency combs.

Key words: laser physics, fiber-based optical frequency combs, fiber lasers, mode-locked fiber lasers.

1. Introduction

The spectrum of a simple passive Fabry-Perot (F-P) optical resonator forms a comb of regularly spread longitudinal resonances (longitudinal modes) separated by a constant magic distance, so called a free spectral range, $\Delta\nu_{FSR} = c/2L$ (c – speed of light, L – length of the resonator). For ring resonators $\Delta\nu_{FSR} = c/L$, because of only one cavity round-trip. When an optical gain medium with some inhomogeneously broadened line is inserted inside a F-P resonator, the multimode laser operation occurs. The special case of laser oscillation, so called a mode-locking operation can appear, when all longitudinal modes are synchronized, i.e. they are precisely separated in frequency and phase. If the gain of the material is broad enough (i.e. from tens to hundreds nanometers), thousands or even millions of excited modes can create a frequency comb. As a result, the spectrum of the output is a sum of many narrow lines. Those lines can be spectrally very narrow, in accordance to the rule: more synchronized modes cause oscillation of a narrower line. In the time domain, the frequency comb corresponds to a train of ultra-short pulses, according to the rule: more synchronized modes – shorter pulses. Trains of ultra-short (sub-picosecond or even femtosecond) pulses can be obtained, depending on the bandwidth of the laser gain medium. The wideband mode-locked lasers with the rigid and stabilized spectral structure are the ideal frequency markers for optical standards. Hence, a huge interest in their investigations. The single-mode, rare-earth doped (ytterbium – 1060 nm, erbium – 1550 nm, thulium – 2000 nm) fiber lasers are particularly attractive media for frequency combs. The effective mode-locking is determined by the dispersion of the laser resonator. Proper dispersion management in the cavity allows to use the full available gain bandwidth of the medium. There are a few well-known techniques of forcing the mode-locked operation of a fiber laser, which is overviewed in this paper. The most popular include nonlinear polarization rotation (NPR), nonlinear optical loop mirrors (NOLM) and mode-locking based on saturable absorbers, like semiconductor saturable absorber mirrors (SESAM), carbon nanotubes (CNT) and recently graphene. The spectrum of available combs can be shifted into new wavelength ranges by nonlinear processes: difference frequency generation (DFG) or second harmonic generation (SHG). The fiber combs have huge potential applications for metrology, telecommunications, medicine, astronomy, micromachining etc. Covering the total optical spectra (from UV over visible to IR) with well controlled frequency comb structure is a great challenge for the future research.

2. Principles of mode-locking

In a mode-locked laser, all longitudinal modes of the laser have a fixed phase relationship between each other [1]. When the phases of the modes are locked, the laser will generate ultra-short optical pulses, as a results of constructive interference between phase-coherent modes in the resonator. If the modes would oscillate randomly (without phase coherence), the output radiation would be multimode, continuous wave (CW). To obtain the mode-locking and pulsed operation, it is required to introduce somehow a mechanism, that will discriminate the CW lasing in the cavity and favor the generation of pulsed light. This can be done by incorporating an active modulator or a passive component (so called saturable absorber) into the laser resonator. The difference between the output radiation from a synchronized and non-synchronized cavity is depicted in Fig. 1.

Intuitively we may guess, that the pulse duration should depend on the number of synchronized modes. Indeed, if we manage to synchronize more modes, we obtain shorter pulses. If we assume that all modes have the same amplitude, the intensity function of the optical field inside the resonator can be expressed as [2]:

$$|E(t)|^2 = \frac{\sin^2\left[(2M+1)\pi\cdot\Delta\nu\cdot t + \dfrac{\phi}{2}\right]}{\sin^2\left[\pi\cdot\Delta\nu\cdot t + \dfrac{\phi}{2}\right]}\cdot E_0^2, \quad (1)$$

where M denotes the number of modes, $\Delta\nu$ is the mode spacing and φ is the locked phase difference between the following

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modes ($\varphi = \varphi_M - \varphi_{M-1}$). Figure 2 shows the calculated field for 3, 6 and 100 synchronized modes. It can be seen, that the amplitude is proportional to the square root of the number of modes and the pulse is consecutively shortened.

The number of modes may be increased in two ways: either by increasing the cavity length and minimizing the mode spacing (more modes fit under the gain curve), or by using an active medium with broad gain bandwidth. The first approach may be very impractical. In the case of bulk solid-state lasers it is not always possible to build a resonator longer than a few meters. Fiber lasers are more flexible and allow to achieve mode-spacing even at the level of kHz with km-long resonators [3], but they require precise dispersion compensation. Typically, very broad gain media are used for ultra-short pulse lasers, like Ti:Sapphire, Yb:KGW, Yb:fiber, Er:fiber, Er/Yb:glass, Cr:LiSaF. Their basic parameters and exemplary results achieved with those media are summarized in Table 1 [4–10].

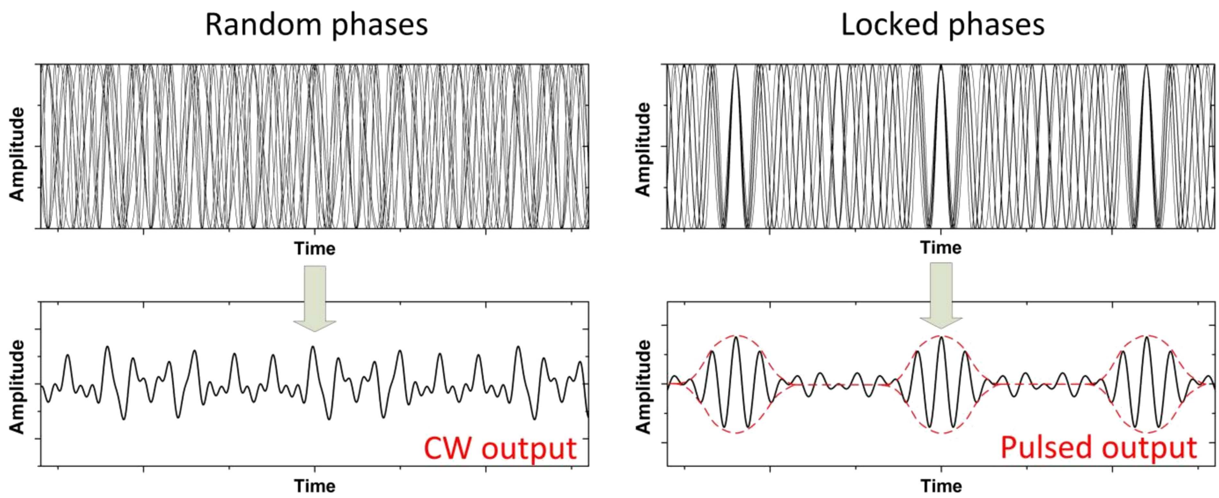

Fig. 1. Difference between a phase-locked cavity and a cavity with random phases

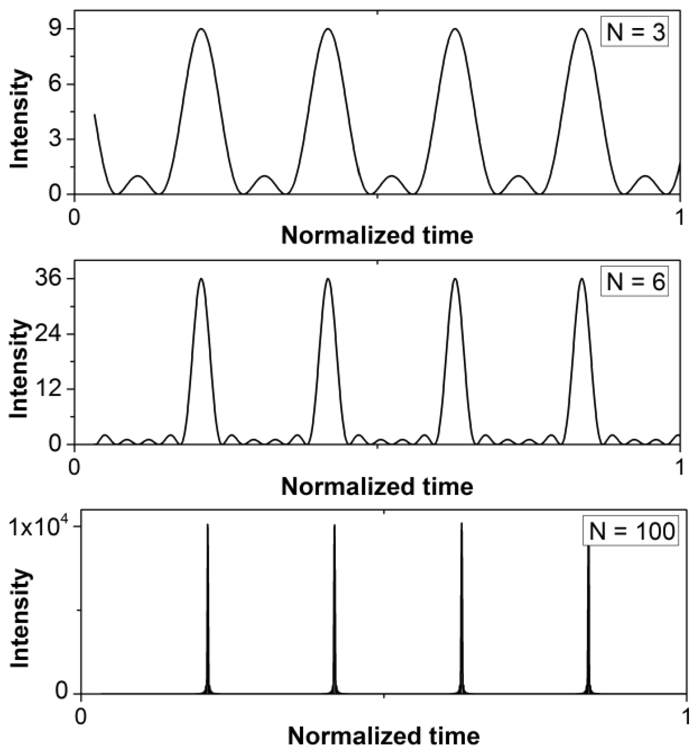

Fig. 2. Comparison between a mode-locked laser output with 3, 6 and 100 synchronized modes

Table 1
State-of-the-art mode-locked oscillators and their parameters

	Ti:Sapphire	Er:fiber	Yb:fiber	Yb:KGW	Er/Yb:glass	Cr:LiSAF
Center wavelength	800 nm	1560 nm	1040 nm	1030 nm	1560 nm	890 nm
Bandwidth	750 nm	∼40 nm	∼40 nm	∼5 nm	∼14.7 nm	24 nm
Achieved pulse	5 fs	55 fs	36 fs	290 fs	170 fs	50 fs
Reference	[5]	[6]	[7]	[8]	[9]	[10]

The pulse temporal duration and spectral width are bound together with a parameter, called time-bandwidth product (TBP). The minimum TBP is obtained for so called transform-limited pulses and is equal to 0.44 and 0.315 for Gaussian and sech^2-shaped pulses, respectively. As an example, lets consider a typical, soliton mode-locked, erbium-doped fiber laser with a common ring cavity, operating at 1550 nm center wavelength with 10 nm bandwidth. Typically, the length of such resonators may be around 5 meters. The fundamental mode spacing of such laser is equal to *c/nL*, which is approx. 40 MHz (assuming 1.5 refractive index for silica fibers at 1550 nm). It means, that with 10 nm bandwidth (1.25 THz) there are around 31000 longitudinal modes, oscillating with synchronized phases. The shortest pulse duration achievable from such laser can be calculated from the TBP. It is common to approximate soliton pulses with sech^2 shape (TBP = 0.315), so the bandwidth-limited pulse can be as short as 252 fs. Whether the pulse is Gaussian or sech^2-shaped depends on the resonator design, especially on the cavity net dispersion.

2.1. Dispersion. The dispersion plays a crucial role in designing ultra-fast lasers and optical systems. Dispersion is a dependence of the phase velocity of a wave on its frequency. In general, in dispersive media some frequencies may propagate faster or slower than others. The frequency-dependent phase of an optical pulse $\varphi(\omega)$ can be represented as a Taylor's expansion [11]:

$$\begin{aligned}\phi(\omega) = \phi_0 + (\omega-\omega_0)\left(\frac{d\phi(\omega)}{d\omega}\right)\\ +\frac{1}{2}(\omega-\omega_0)^2\left(\frac{d^2\phi(\omega)}{d\omega^2}\right)\\ +\frac{1}{6}(\omega-\omega_0)^3\left(\frac{d^3\phi(\omega)}{d\omega^3}\right)+\dots,\end{aligned} \tag{2}$$

where ω_0 is the center frequency. The first term of the expansion, $\varphi_{0,}$ is a constant and denotes the initial phase at ω_0. The second term represents the group delay (GD). The third term is the group delay dispersion (GDD), which is the most important parameter in the case of ultra-fast mode-locked lasers. The subsequent terms are higher order dispersions, e.g. third order dispersion (TOD) and are often practically negligible, but in complex optical systems dealing with very short pulses (at the sub-50 fs level) it is also necessary to take into account the impact of TOD. The GDD is expressed in fs^2 or ps^2 units. Sometimes also the term group velocity dispersion (GVD) is used, which is GDD per unit length (expressed e.g. in ps^2/m). It is also helpful to distinguish the chromatic dispersion used in telecommunications (expressed in ps/nm*km) from the GDD/GVD used by the ultrafast laser community. Both terms refer to the same phenomenon, although the term "positive dispersion" may be understood differently by both communities. In principle, the dispersion may be normal – the group velocity decreases with increasing optical frequency (+ps^2) or anomalous ($-$ps^2), where the group velocity increases with increasing frequency. We can therefore imagine, that dispersion causes temporal broadening or shortening of spectrally broad optical pulses propagating through dispersive media, depending on the dispersion sign [12].

Mode-locked lasers may operate in various regimes, depending on the total cavity dispersion. One may distinguish three types of dispersion regimes: 1) all-anomalous, 2) all-normal, 3) balanced (near-zero) [13]. In the first regime the total GDD of the cavity is anomalous (negative GDD). It is typical for lasers operating in the infrared ($> 1.3\mu$m), e.g. for Er-doped silica fiber lasers. Such lasers emit optical solitons, so their output optical spectrum represents a typical soliton-like shape, which may be fitted with sech^2 curve. The spectrum often contains characteristic peaks – Kelly's sidebands, resulting from the periodical soliton perturbation in the cavity [14]. Typically such lasers can easily generate transform-limited sub-ps pulses, but their energy is strongly limited by the soliton stability area. The all-normal regime is typical for Yb-doped fiber lasers. In the 1 μm wavelength range standard optical fibers have opposite dispersion sign than at 1.55 μm. In this regime, the output spectrum has a characteristic shape with steep edges and a flat, very broad top. The pulses (so called dissipative solitons) are often chirped (not transform-limited) and require recompression. The third regime requires careful dispersion management of the cavity, in order to achieve near-zero total net dispersion. In such lasers (so called stretched-pulse lasers), the circulating pulse is subsequently broadened and shortened by two opposite dispersion fibers. The achieved output spectra are very broad and the pulses often require external recompression. Such regime allows to obtain very short pulses, but the cavity design may be challenging (often the usage of dispersion-compensating fibers, DCFs, is necessary). The comparison between three dispersion regimes and corresponding optical spectra is presented in Fig. 3.

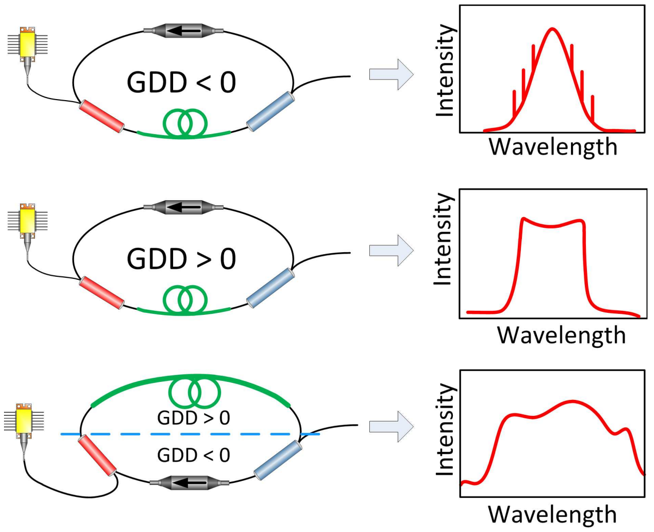

Fig. 3. Comparison between various dispersion regimes in fiber lasers

3. Overview of mode-locking techniques

As mentioned in the previous paragraph, in order to achieve the pulsed operation of a laser, we need to introduce a mode-locking mechanism into the cavity. There are two types of mode-locking: active and passive. The active mode-locking requires the usage of light modulators (e.g. electro-optic or acousto-optic) driven by external signal sources. Such a modulator introduces controlled losses into the cavity, resulting in a pulsed operation. However, typical active mode-locked fiber lasers achieve pulse durations longer than 1 ps. The following part of this section focuses on passive mode-locking techniques.

In a passively mode-locked laser, a nonlinear optical component – a saturable absorber is placed into the cavity in order to modulate the losses. A saturable absorber is an optical component, which possesses 3^{rd} order nonlinear susceptibility. Its transmission (or reflection, depending on the design) is power-dependent in a way, that it absorbs low-intensity light. After illumination with high intensity light, the absorption saturates and the SA becomes transparent. An ideal fast saturable absorber introduces no non-saturable losses, has a very low saturation fluence and the highest possible modulation depth (the contrast between on/off state). A graph illustrating the power-dependent absorption of a saturable absorber with indicated basic parameters is shown in Fig. 4.

In order to estimate the parameters of a saturable absorber, usually a simple two-level model is used [13]:

$$\alpha(I) = \frac{\alpha_0}{1 + \dfrac{I}{I_{sat}}} + \alpha_{NS}, \tag{3}$$

where I is the light intensity, I_{sat} denotes the saturation intensity and α_0 and α_{NS} are modulation depth and non-saturable losses, respectively.

In case of fiber lasers, there are also fiber-based techniques of mode-locking, where a specific cavity design may act as a saturable absorber. Such methods include nonlinear polarization rotation (NPR) and nonlinear optical/amplifying loop mirror (NOLM/NALM).

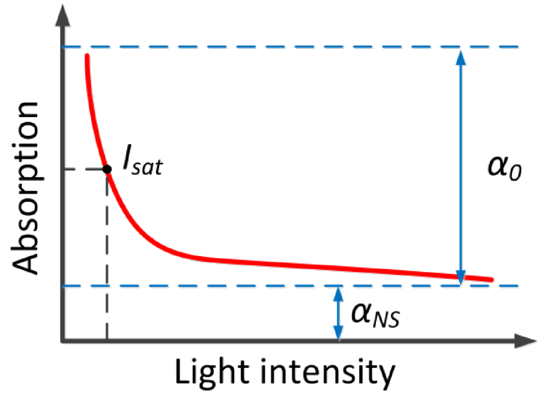

Fig. 4. Power-dependent absorption of a saturable absorber

3.1. NOLM/NALM mode-locking. Efficient mode-locking of fiber lasers may be achieved using interferometric techniques, i.e. by incorporating a Sagnac loop. The most common approach is to develop so called figure eight lasers (F8L) depicted in Fig. 5. Such setup was proposed by M. Fermann et al. in 1990 [15]. The right-hand side of the resonator constitutes a nonlinear amplifying loop mirror (NALM), which consists the active fiber and a polarization controller. The left-hand side of the setup is a directional loop. Both loops are connected together with a 50/50 coupler.

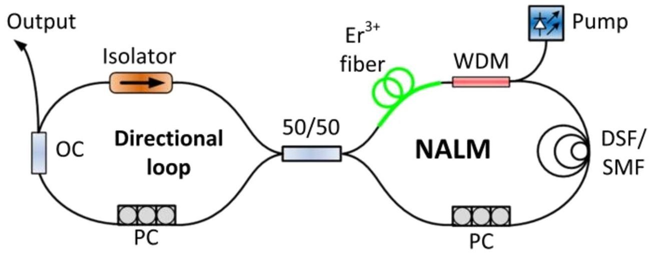

Fig. 5. Typical figure-eight laser. OC – output coupler, PC – polarization controller, DSF – dispersion shifted fiber, SMF – single mode fiber

The pulses counterpropagating in the NALM differ in their amplitude and phase, because of the asymmetrically placed amplifier. The pulses interfere in the 50/50 coupler, but only the highest intensity light is to be transmitted to the directional loop. Thus, a NALM may act as a saturable absorber – its transmission depends on the light intensity. The transmission of a NALM with symmetrical coupler can be expressed as:

$$T = \sin^2\left[(G-1)\gamma \cdot I\frac{L}{4}\right], \tag{4}$$

where G is the gain of the amplifier, γ is the fiber nonlinear coefficient, I denotes the light intensity and L is the fiber length.

In some cases the nonlinear loop mirror may be passive (without an amplifier). Such arrangement is called nonlinear optical loop mirror (NOLM). In the case of NOLM, the gain is equal to 1, thus, the transmission from Eq. (4) is 0. It is therefore needed to use an asymmetrical coupler to achieve nonlinear transmission of the NOLM [16].

3.2. Nonlinear polarization rotation. In a nonlinear polarization rotation mode-locked laser a simple polarizer (or a polarization beamsplitter) acts as a fast saturable absorber. A typical NPR laser setup is shown in Fig. 6. The basic principle of operation of a NPR laser is as follows: the pulse propagating through the resonator experiences different phase shifts of the orthogonal polarization components. The rotation of the polarization is an intensity-dependent process, so the polarization of the pulse peak is rotated differently than the polarization of pulse fringes or a continuous wave component. Thus, a proper adjustment of the polarization controller in the cavity allows to discriminate low intensity light. During each roundtrip only the high-intensity pulse peak is transmitted by the polarizer, just like in the case of NALM/NOLM.

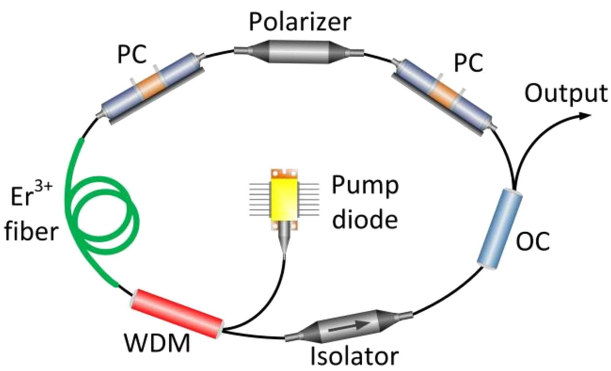

Fig. 6. Typical all-fiber NPR ring laser setup

The nonlinear polarization rotation mechanism currently allows pulses as short as 55 fs from dispersion-managed Er-doped fiber lasers [6]. Up till now, the highest repetition rates from all-fiber NPR lasers in ring resonators were achieved in our group. In 2010, Nikodem et al. demonstrated a fundamentally mode-locked Er-fiber laser with 169 MHz mode spacing and 111 fs pulse duration [17]. One year later Sobon et al. presented a setup based on Er/Yb double-clad fiber capable of generating 10 GHz pulse trains in harmonic mode-locking regime [18].

3.3. Semiconductor saturable absorbers. The SESAM technique of mode-locking of lasers was established almost exactly 20 years ago [19]. Typically, a SESAM contains a semiconductor Bragg reflector and a quantum well absorber, developed by molecular beam epitaxy (MBE) technique [20]. An exemplary SESAM structure [21] is shown in Fig. 7a. We can safely say, that SESAMs are the most common saturable absorbers used nowadays in commercially available solid-state lasers, as well as in laboratory experiments. In case of fiber lasers, this technique needs to compete with well-established and efficient NPR and NALM. A typical laser resonator setup incorporating a SESAM is presented in Fig. 7b.

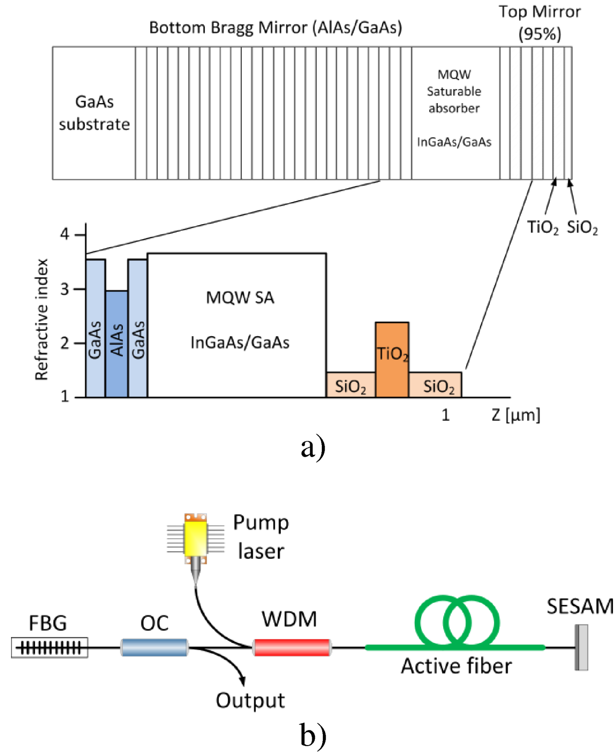

Fig. 7. A typical SESAM structure (a), typical fiber laser mode-locked by SESAM (b)

3.4. Carbon nano-materials. The usage of saturable absorbers based on carbon nano-materials was firstly proposed in 2003, when S. Set et al. demonstrated the noise-suppressing properties of carbon nanotubes (CNTs) [22]. Since then, many various setups were presented, based on Erbium and Ytterbium-doped fibers, as well as bulk gain media [23–25]. CNTs are allotropes of carbon with a cylindrical structure, with typical diameters of 1–3 nm and lengths up to micro or even millimeters. It has been revealed, that single-walled CNTs exhibit strong saturable absorption with high modulation depth and relatively low saturation energy. Typically CNT-based saturable absorbers are prepared as a solution of CNTs in various solvents, for example ethanol [26], or a mixture with polymers (polyvinyl alcohol – PVA [25] or poly-3-hexylthiophene – P3HT [27]). Afterwards, the solution may be deposited on optical substrates or fiber connectors (e.g. by optically driven deposition [26]). Currently, CNT-based saturable absorbers allow to achieve sub-100 fs pulses from Er-doped fiber oscillators [28]. Due to the energy band structure of nanotubes [29], the absorption band strongly depends on their diameter, e.g. nanotubes with ~1 nm diameter are most suitable for 1 μm lasers, while 1.55 μm lasers require 1.3 nm diameter. This fact is considered to be the main drawback of CNT-SA. However, recent work has shown the possibility of using one CNT-SA to mode-lock lasers operating at 1 and 1.55 μm by using the E22 transition [30].

The second carbon material used for mode-locking is graphene. This single-layer of carbon atoms has experienced a great interest of scientists in the past few years, mainly due to its electrical properties [31]. Graphene possesses also unique optical properties, which makes it suitable for mode-locking of fiber lasers. Single layer of graphene absorbs a significant fraction of incident light (2.3%, which scales linearly with the number of layers). Additionally, the absorption saturates at very low intensities and this process is wavelength-independent. The first graphene-based fiber laser was presented only three years ago by Bao et al., in 2009 [32]. Since then various setups have been presented, utilizing erbium [33, 34], and ytterbium [35]. There are several methods of preparing graphene suitable for mode-locking: 1) epitaxial growth by chemical vapor deposition (CVD) on various substrates (mostly Cu and Ni) [36], 2) direct exfoliation of natural graphite by ultrasonification it in various organic solvents e.g. dimethyloformamide (DMF) [37], 3) mechanical exfoliation of graphene from bulk graphite [38–41]. An exemplary setup of an Er-doped fiber laser mode-locked by CVD-graphene saturable absorber is shown in Fig. 8 [35]. The laser was delivering 315 fs soliton pulses at 1562 nm center wavelength with over 11 nm bandwidth.

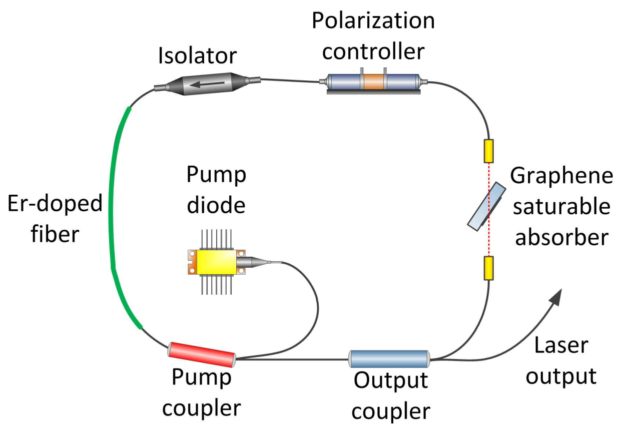

Fig. 8. CVD-graphene mode-locked Er-doped fiber laser

Similar setup allowed to achieve sub-400 fs soliton pulses using saturable absorbers based on graphene oxide and reduced graphene oxide [42].

4. Stabilization of frequency combs

A mode-locked laser emits a train of very short optical pulses, spaced equally by the resonator roundtrip time. Such a temporal pulse train corresponds to a "comb" of Dirac deltas in the frequency domain, which is illustrated in Fig. 9.

Unfortunately, the pulses are not quite identical. The pulse envelope travels with its group velocity v_g, while the carrier propagates with its own phase velocity. In consequence, in each cavity roundtrip the carrier shifts with respect to the envelope, resulting in so called carrier-envelope phase shift $\Delta\phi_{CE}$ This shift corresponds to a frequency offset f_0, indicated in Fig. 9. This means, that the frequency of the m-th mode f_m is not exactly an integer multiple of the repetition rate f_{rep} and needs to be expressed as [43]:

$$f_m = m \cdot f_{rep} + f_o. \quad (5)$$

As a consequence, in order to reveal the absolute frequency of any comb tooth, we need to measure two frequencies: the repetition frequency f_{rep} and offset frequency f_o. The measurement of f_{rep} is relatively easy, since it is commonly in the range from tens to hundreds MHz. However, measuring the offset is very challenging, but necessary to build extremely stable frequency standards. Those frequency may be measured by using interferometric techniques using the mode-locked laser itself – such setup is called self-referencing frequency comb.

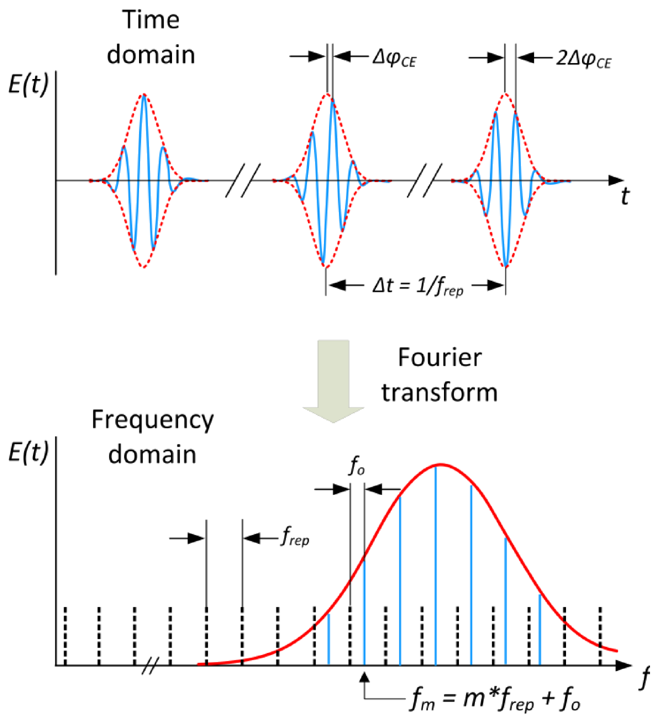

Fig. 9. Time domain and frequency domain of an ultrashort pulse train

4.1. Self-referencing frequency comb. The idea of an universal frequency comb synthesizer with absolutely known frequencies was proposed by T. Hänsch in 1997. Consider a mode-locked fiber laser with an output optical spectrum spanning an octave (which means, that the highest frequency is at least two times higher than the lowest frequency in the spectrum). The lowest frequency from the spectrum, written as f_m, may be simply doubled using a nonlinear second-harmonic crystal. As a result, we obtain a signal with approximately the same frequency as a comb line at the high-frequency side of the spectrum, f_{2m}. A simple heterodyne beat between those two frequencies gives us a difference frequency, which yields the offset f_o:

$$2f_m = f_{2m} = 2(m \cdot f_{rep} + f_o) - (2m \cdot f_{rep} + f_o) = f_o. \quad (6)$$

This method, illustrated in Fig. 10, is known as the *f-2f interferometer* and is widely used to stabilize the frequency combs [4, 43–45]. It seems to be the simplest way to determine absolute optical frequencies. However, the 2-2f interferometer requires a laser source with octave-spanning spectrum. It is extremely difficult to obtain such spectra directly from the oscillator – it is possible only with broad gain media like Ti:Sapphire [5]. In the case of other lasers (like fiber oscillators) octave-spanning spectra need to be generated in a different way. The most common method is the use of optical fibers with high nonlinear coefficient – so called highly nonlinear fibers (HNLFs), or specially designed microstructured fibers (i.e. photonic crystal fibers, PCFs). In such a fiber, due to intense nonlinear effects, mainly self-phase modulation (SPM), the input spectrum can be significantly broadened, covering even more than an octave. The only requirement is to provide pulse energies high enough to induce the nonlinear effects. For this purpose often amplifiers are used, in order to boost the power launched into the HNLF. A schematic of such setup is shown in Fig. 11.

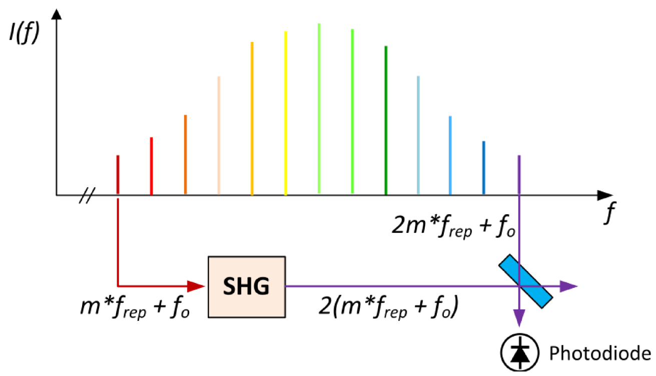

Fig. 10. The idea of a self-referencing frequency comb via f-2f interferometer

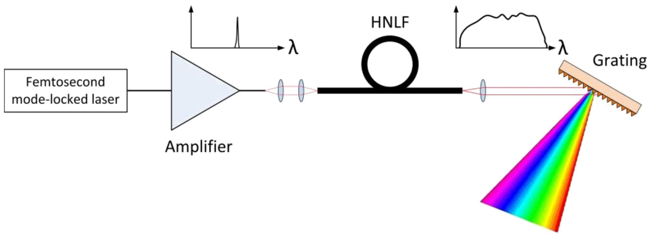

Fig. 11. Supercontinuum generation in HNLF

The used PCFs and HNLFs allow to generate very broad supercontinuum (SC). For example, Washburn et al. demonstrated supercontinuum generation in HNLF, pumped by a preamplified 1.55 μm oscillator, spanning from 1100 to 2400 nm [44]. A typical setup of a self-referenced frequency comb, utilizing a HNLF for generating octave-spanning spectrum is shown in Fig. 12. The supercontinuum is divided into two parts by a dichroic mirror. The infrared part is frequency-doubled in a SHG crystal and then interfered with the visible part of the SC. The beating signal is used to lock the frequency of the mode-locked laser.

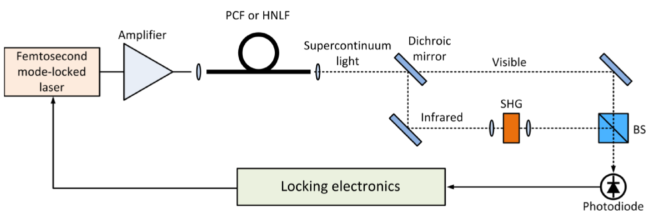

Fig. 12. Self-referenced frequency comb with nonlinear fiber for supercontinuum generation

5. Extending the spectral coverage of frequency combs

Despite the availability of many different gain media (also in combination with nonlinear broadening), the spectral coverage of frequency combs generated directly from oscillators is not broad enough to meet the requirements of some metrological applications. Many effort is taken today to extend the available wavelengths from the extreme ultraviolet to far infrared. Again, nonlinear effects are very helpful in this case. Therefore, we may enter the UV range by second-harmonic generation (SHG) and high-harmonic generation (HHG) in nonlinear crystals or gases. The IR can be reached by the use of $\chi^{(2)}$ effects like difference frequency generation (DFG) or optical parametric oscillation (OPO). Current available spectral coverage by different techniques is summarized on the graph in Fig. 13 [46].

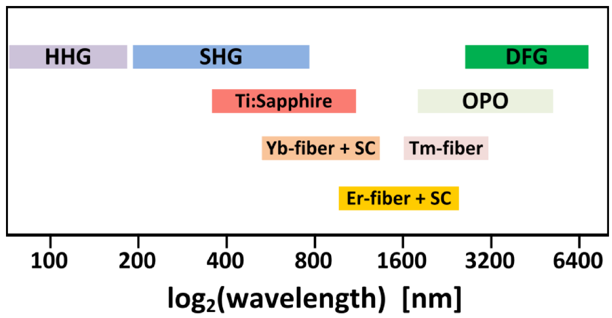

Fig. 13. Spectral coverage of frequency combs

5.1. Conversion into MIR. Two main approaches used to extend the frequency combs into the mid-IR range are: difference frequency generation and optical parametric oscillation. Both are illustrated in Fig. 14.

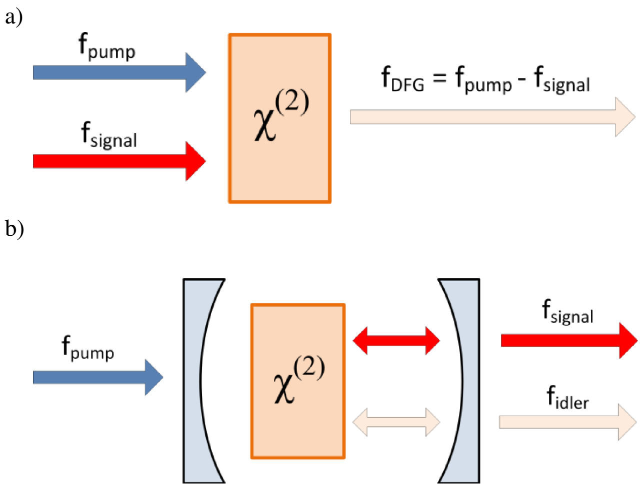

Fig. 14. DFG (a) and OPO (b) schemes

In the DFG process (Fig. 14a) two input beams: pump and signal (with frequencies f_{pump} and f_{signal}, respectively), interact generating a new beam, with the frequency being a difference of the input frequencies. This approach allows an easy access into the mid-infrared region. For example, a

1.55 μm comb may be mixed with a 1.06 μm. As a result of this process, the comb is converted into 3.39 μm [47].

In the OPO (Fig. 14b) the nonlinear crystal is placed in an optical resonator. The optical gain is provided by the parametric amplification process in $\chi^{(2)}$ medium. As a result, two new beams are generated: a signal and idler. OPOs pumped by 1.06 and 1.55 μm frequency combs allow to convert the radiation typically into the 3–4 μm range [48].

5.2. Conversion into EUV. Extending the frequency comb range into the UV region is highly desirable for spectroscopy and fundamental science. Reaching the $<$ 300 nm region requires sophisticated high-harmonic generation schemes. There are two approaches of HHG: by using gas jets and bulk nonlinear crystals (see Fig. 15). The first approach is very common in ultrafast science and allows to generate attosecond EUV pulses. Typically Kr [49] or Xe [50] are used. As an example, an enhancement cavity with Xe gas allowed to generate 23^{rd} harmonic from an Yb-doped frequency comb ($<$ 40 nm) [49]. The second method utilizing bulk crystals does not allow to reach such extreme UV wavelengths, but it has been recently shown that a combination of LBO and BBO crystals can successfully convert a Ti:sapphire frequency comb to 205 nm (4^{th} harmonic) [51].

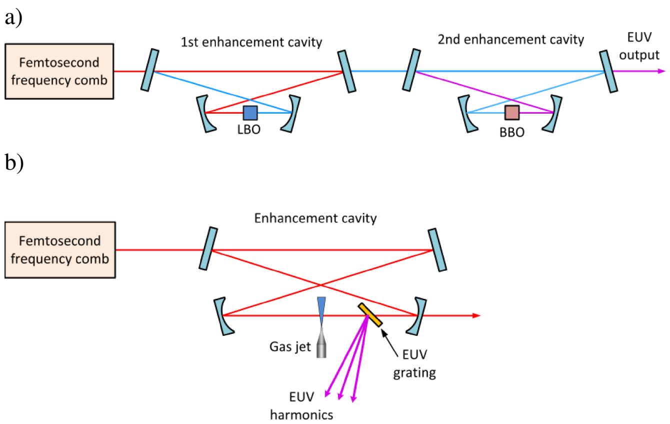

Fig. 15. Typical HHG generation schemes. Setup with nonlinear bulk crystals (a) and with gas jet and enhancement cavity (b)

6. Summary

Recent development in mode-locked fiber lasers has led to a breakthrough in the frequency comb technology. Presently, frequency combs find widespread applications in precise frequency metrology, dimensional metrology, optical clocks, high-resolution spectroscopy, terahertz wave generation, astronomical measurements and many others. Fiber-optic technology strongly contributed to the development of frequency combs in the past decade. Fiber-based optical combs are used widely around the world thanks to their compactness, robustness and cost effectiveness, in comparison to traditional Ti:Sapphire lasers. Current work in the field of frequency combs is mainly focused on extending the spectral coverage from the deep UV up to middle IR. Various nonlinear techniques, like HHG or DFG seem to be effective tools, which enable us to cover the whole spectrum with frequency combs and find new, exciting applications.

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REFERENCES

[1] J.Ye, S.T. Cundiff, *Femtosecond Optical Frequency Comb: Principle, Operation, and Applications*, Springer, Boston, 2005.

[2] J.-C. Diels, and W. Rudolph, *Ultrashort Laser Pulse Phenomena: Fundamentals, echniques, and Applications on a Femtosecond Timescale*, Academic Press, San Diego, 1996.

[3] J. Chen, D. Jia, C. Wang, J. Wang, Z. Wang, T. Yang, "High energy passively mode-locked Erbium-doped fiber laser at tens of kHz repetition rate", *Proc. SPIE* 8307, 83071Z (2011).

[4] S.A. Diddams, "The evolving optical frequency comb", *J. Opt. Soc. Am.* B 27 (11), B51–B62 (2010).

[5] R. Ell, U. Morgner, F. X. Kaertner, J. G. Fujimoto, E. P. Ippen, V. Scheuer, G. Angelow, T. Tschudi, M. J. Lederer, A. Boiko, and B. Luther-Davies, "Generation of 5-fs pulses and octave-spanning spectra directly from a Ti:sapphire laser", *Opt. Lett.* 26 (6), 373–375 (2001).

[6] D. Deng, L. Zhan, Z. Gu, Y. Gu, and Y. Xia, "55-fs pulse generation without wave-breaking from an all-fiber Erbium-doped ring laser", *Opt. Express* 17 (6), 4284–4288 (2009).

[7] F. Ilday, J. Buckley, L. Kuznetsova, and F. Wise, "Generation of 36-femtosecond pulses from a ytterbium fiber laser", *Opt. Express* 11 (26), 3550–3554 (2003).

[8] S. Pekarek, T. Südmeyer, S. Lecomte, S. Kundermann, J.M. Dudley, and U. Keller, "Self-referenceable frequency comb from a gigahertz diode-pumped solid-state laser", *Opt. Express* 19 (17), 16491–16497 (2011).

[9] M.C. Stumpf, S. Pekarek, A.E.H. Oehler, T. Südmeyer, J.M. Dudley, and U. Keller, "Self-referencable frequency comb from a 170-fs, 1.5-μm solid-state laser oscillator", *Appl. Phys.* B 99 (3), 401–408 (2010).

[10] R. Holzwarth, M. Zimmermann, Th. Udem, T.W. Hänsch, P. Russbüldt, K. Gäbel, R. Poprawe, J.C. Knight, W.J. Wadsworth, and P.St.J. Russell, "White-light frequency comb generation with a diode-pumped Cr:LiSAF laser", *Opt. Lett.* 26 (17), 1376–1378 (2001).

[11] D.N. Fittinghoff, B.C. Walker, J. A. Squier, C.S. Toth, C. Rose-Petruck, and C.P.J. Barty, "Dispersion considerations in ultrafast CPA systems", *IEEE J. Sel. Top. Quant*. 4 (2), 430–440 (1998).
[12] I. Walmsley, L. Waxer, and C. Dorrer, "The role of dispersion in ultrafast optics", *Rev. Sci. Instrum.* 72 (1), 1–29 (2001).
[13] S. M. Kelly, "Characteristic sideband instability of periodically amplified average soliton", *Electron. Lett.* 28 (8), 806–807 (1992).
[14] H.A. Haus, "Mode-locking of lasers", *IEEE J. Sel. Top. Quant*. 6 (6), 1173–1185 (2000).
[15] M.E. Fermann, F. Haberl, M. Hofer, and H. Hochreiter, "Nonlinear amplifying loop mirror", *Opt. Lett.* 15 (13), 752–754 (1990).
[16] I.N. Duling, "Compact sources of ultrashort pulses", Cambridge University Press, London, 1995.
[17] M. Nikodem and K. Abramski, "169 MHz repetition frequency all-fibre passively mode-locked erbium-doped fibre laser", *Opt. Commun*. 283 (1), 109–112 (2010).
[18] G. Sobon, K. Krzempek, P. Kaczmarek, K.M. Abramski, and M. Nikodem, "10 GHz passive harmonic mode-locking in er-yb double-clad fiber laser", *Opt. Commun*. 284 (18), 4203–4206 (2011).
[19] U. Keller, D.A.B. Miller, G.D. Boyd, T.H. Chiu, J.F. Ferguson, and M.T. Asom, "Solid-state low-loss intracavity saturable absorber for Nd:YLF lasers: an antiresonant semiconductor Fabry–Perot saturable absorber", *Opt. Lett*. 17 (7), 505–507 (1992).
[20] U. Keller, K.J. Weingarten, F.X. Kaertner, D. Kopf, B. Braun, I.D. Jung, R. Fluck, C. Hoenninger, N. Matuschek, and J. Aus der Au, "Semiconductor saturable absorber mirrors (SESAM's) for femtosecond to nanosecond pulse generation in solid-state lasers", *IEEE J. Sel. Top. Quant.* 2 (3), 435–453 (1996).
[21] A. Jasik, J. Muszalski, J. Gaca, M. Wojcik, and K. Pierscinski, "Ultrashort pulses supported by SESAM absorber", *Bull. Pol. Ac.: Tech.* 58 (4), 477–483 (2010).
[22] S.Y. Set, H. Yaguchi, M. Jablonski, Y. Tanaka, Y. Sakakibara, A.G. Rozhin, M. Tokumoto, H. Kataura, Y. Achiba, and K. Kikuchi, "A noise suppressing saturable absorber at 1550 nm based on carbon nanotube technology", *Optical Fiber Communication Conf.* FL2, CD-ROM (2003).
[23] J.W. Nicholson and D.J. DiGiovanni, "High-repetition-frequency low-noise fiber ring lasers mode-locked with carbon nanotubes", *IEEE Phot. Tech. L.* 20 (24), 2123–2125 (2008).
[24] W.B. Cho, J.H. Yim, S.Y. Choi, S. Lee, A. Schmidt, G. Steinmeyer, U. Griebner, V. Petrov, D.-I. Yeom, K. Kim, and F. Rotermund, "Boosting the nonlinear optical response of carbon nanotube saturable absorbers for broadband mode-locking of bulk lasers", *Adv. Funct. Mater*. 20 (12), 1937–1943 (2010).
[25] Z. Sun, T. Hasan, F. Wang, A.G. Rozhin, I.H. White, and A.C. Ferrari, "Ultrafast stretched-pulse fiber laser mode-locked by carbon nanotubes", *Nano Res.* 3, 404–411 (2010).
[26] J.W. Nicholson, R.S. Windeler, and D.J. DiGiovanni, "Optically driven deposition of single-walled carbon-nanotube saturable absorbers on optical fiber end-faces", *Opt. Express* 15 (15), 9176–9183 (2007).
[27] F. Shohda, T. Shirato, M. Nakazawa, J. Mata, and J. Tsukamoto, "147 fs, 51 MHz soliton fiber laser at 1.56 μm with a fiber-connector-type SWNT/P3HT saturable absorber", *Opt. Express* 16 (25), 20943–20948 (2008).
[28] D. Popa, Z. Sun, T. Hasan, F. Torrisi, F. Wang, and A. Ferrari, "Sub-100fs pulse generation from a fiber oscillator mode-locked by nanotubes", *CLEO:2011 – Laser Applications to Photonic Applications, OSA Technical Digest (CD)* CMK6, CD-ROM (2011).
[29] H. Kataura, Y. Kumazawa, Y. Maniwa, I. Umezu, S. Suzuki, Y. Ohtsuka, and Y. Achiba, "Optical properties of single-wall carbon nanotubes", *Synthetic Met.* 103, 2555 (1999).
[30] J.C. Travers, J. Morgenweg, E.D. Obraztsova, A.I. Chernov, E.J.R. Kelleher, and S.V. Popov, "Using the E22 transition of carbon nanotubes for fiber laser mode-locking", *Laser Phys. Lett*. 2 (2), 144–149 (2010).
[31] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, Y. Zhang, S.V. Dubonos, I.V. Grigorieva, A.A. Firsov, "Electric field effect in atomically thin carbon films", *Science* 306 (5696), 666–669 (2004).
[32] Q.L. Bao, H. Zhang, Y. Wang, Z.H. Ni, Z.X. Shen, K.P. Loh, and D.Y. Tang, "Atomic layer graphene as saturable absorber for ultrafast pulsed laser", *Adv. Funct. Mater*. 19 (19), 3077–3083 (2009).
[33] H. Zhang, D.Y. Tang, L.M. Zhao, Q.L. Bao, and K.P. Loh, "Large energy mode locking of an erbium-doped fiber laser with atomic layer graphene", *Opt. Express* 17 (20), 17630–17635 (2009).
[34] D. Popa, Z. Sun, F. Torrisi, T. Hasan, F. Wang, and A.C. Ferrari, "Sub 200 fs pulse generation from a graphene mode-locked fiber laser", *Appl. Phys. Lett*. 97 (20), 203106 (2010).
[35] L.M. Zhao, D.Y. Tang, H. Zhang, X. Wu, Q. Bao, and K.P. Loh, "Dissipative soliton operation of an ytterbium-doped fiber laser mode locked with atomic multilayer graphene", *Opt. Lett.* 35 (21), 3622 (2010).
[36] G. Sobon, J. Sotor, I. Pasternak, K. Grodecki, P. Paletko, W. Strupinski, Z. Jankiewicz, and K.M. Abramski, "Er-doped fiber laser mode-locked by CVD-graphene saturable absorber", *J. Lightwave Technol.* 30 (17), 2770–2775 (2012).
[37] A. Martinez, K. Fuse, B. Xu, and S. Yamashita, "Optical deposition of graphene and carbon nanotubes in a fiber ferrule for passive mode-locked lasing", *Opt. Express* 18 (22), 23054–23061 (2010).
[38] J. Sotor, G. Sobon, K. Krzempek, and K.M. Abramski, "Fundamental and harmonic mode-locking in erbium-doped fiber laser based on graphene saturable absorber", *Opt. Commun.* 285 (13–14), 3174–3178 (2012).
[39] G. Sobon, J. Sotor, and K.M. Abramski, "All-polarization maintaining femtosecond Er-doped fiber laser mode-locked by graphene saturable absorber", *Laser Physics Letters* 9 (8), 581–586 (2012).
[40] J. Sotor, G. Sobon, and K.M. Abramski, "Scalar soliton generation in all-polarization maintaining, graphene mode-locked fiber laser", *Optics Letters* 37 (11), 2166–2168 (2012).
[41] G. Sobon, J. Sotor, and K.M. Abramski, "Passive harmonic mode-locking in Er-doped fiber laser based on graphene saturable absorber with repetition rates scalable to 2.22 GHz", *Applied Physics Letters* 100 (16), 161109 (2012).
[42] G. Sobon, J. Sotor, J. Jagiello, R. Kozinski, M. Zdrojek, M. Holdynski, P. Paletko, J. Boguslawski, L. Lipinska, and K.M. Abramski, "Graphene oxide vs. reduced graphene oxide as saturable absorbers for Er-doped passively mode-locked fiber laser", *Opt. Express* 20 (17), 19463–19473 (2012).
[43] Th. Udem, R. Holzwarth, and T.W. Haensch, "Optical frequency metrology", *Nature* 416, 233–237 (2002).
[44] B.R. Washburn, S.A. Diddams, N.R. Newbury, J.W. Nicholson, M.F. Yan, and C.G. Jrgensen, "Phase-locked, erbium-fiber-

laser-based frequency comb in the near infrared", *Opt. Lett.* 29 (3), 250–252 (2004).

[45] D.J. Jones, S.A. Diddams, J.K. Ranka, A. Stentz, R.S. Windeler, J.L. Hall, and S.T. Cundiff, "Carrier-envelope phase control of femtosecond mode-locked lasers and direct optical frequency synthesis", *Science* 288 (5466), 635–639 (2010).

[46] A. Schliesser, N. Picqué, and T.W. Hänsch, "Mid-infrared frequency combs", *Nat. Photonics* 6, 440–449 (2012).

[47] P. Maddaloni, P. Malara, G. Gagliardi, and P. De Natale, "Mid-infrared fibre-based optical comb", *New J. Phys*. 8, 262 (2006).

[48] N. Leindecker, A. Marandi, R.L. Byer, and K.L. Vodopyanov, "Broadband degenerate OPO for mid-infrared frequency comb generation", *Opt. Express* 19 (7), 6296–6302 (2011).

[49] D.Z. Kandula, C. Gohle, T.J. Pinkert, W. Ubachs, and K.S.E. Eikema, "Extreme ultraviolet frequency comb metrology", *Phys. Rev. Lett.* 105 (6), 063001 (2010).

[50] A. Cingöz, D.C. Yost, T.K. Allison, A.Ruehl, M.E. Fermann, I. Hartl, and J. Ye, "Direct frequency comb spectroscopy in the extreme ultraviolet", *Nature* 482, 68–71 (2012).

[51] E. Peters, S.A. Diddams, P. Fendel, S. Reinhardt, T.W. Hänsch, and Th. Udem, "A deep-UV optical frequency comb at 205 nm", *Opt. Express* 17 (11), 9183–9190 (2009).