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Experimental research on the vibration characteristics of canvas and primed canvas of paintings

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Paintings inevitably bear severe mechanical loads during transportation. Understanding the dynamic characteristics of paintings helps to avoid damage during transportation and to effectively slow down their aging. In this contribution, the vibration characteristics of canvas and primed canvas of paintings and their influencing factors are studied experimentally. For this reason, two dummy paintings with canvas in a common orientation and a tilted orientation are investigated, and an experimental setup using an excitation mechanism and a laser Doppler vibrometer is developed. In order to avoid changes of the modal parameters related to humidity or temperature, all experiments were conducted in a climate box. The modal parameters of dummy paintings are identified by means of experimental modal analysis. Also, the difference in modal properties of the two dummy paintings before and after applying the primer are compared. The identified modal parameters are used to reconstruct their eigenmodes. From the identified modal parameters a numerical model is derived, which is then compared to measurements. The comparison shows a good agreement, hence is a hint for the correctness of assuming a modal structure and the quality of the modal parameter identification. Lastly, with the help of the climate box, the influences of humidity and temperature on the eigenfrequencies of dummy paintings are studied.

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1. Introduction

With the increase in loan between museums, paintings are, more likely than before, exposed to shock and vibration. Art conservators have long recognized that transportation may cause irreversible changes in the complex materials of paintings and accelerates their natural aging [1–7]. Researchers and specialized companies have been trying to improve the packaging system to isolate shock or vibration during transportation [8–10]. An experimental investigation of the canvas acceleration during transportation shows that the canvas acceleration can be much higher than the acceleration transferred to the structures through the crate as soon as the excitation spectrum excites its eigenfrequencies [11]. Therefore, studying the dynamic characteristics of paintings is of great significance for art conservators to avoid the excitation of eigenmodes and thereby minimizing damage during transportation.

Part of the research on the vibration characteristics of paintings was done in the last decades. In [6], the vibration behaviour of paintings on a textile carrier was extensively studied. Also, the relation to aging was explicitly examined. Furthermore, some of the eigenfrequencies and eigenmodes of paintings with different layers were experimentally studied, and some nonlinear phenomena in paintings were explained. Thus, the experimental methods and related experimental results in [6] have important significance. In [12], the canvas was defined as a homogeneous orthotropic plate, its material properties were studied, the corresponding finite element models were established, and finally the correctness of the finite element model was verified through experiments. There, the finite element model considered the corner effect, and the analysis results of eigenmodes were of reference significance. However, in order to prevent paintings from being damaged by excessive shock or vibration during transportation, more in-depth research on the vibration characteristics of paintings and their influencing factors is necessary.

Paintings have a complex heterogeneous material structure. For simplification, here they are divided into three layers: canvas, primer, and paint. In this contribution, only the vibration characteristics of canvas alone and canvas with a layer of primer are investigated. The vibration characteristics of more complex paint layers are left for future research. In addition to a dummy painting with canvas oriented in a regular way, i.e., aligned to its weaving direction, a dummy painting with tilted orientation canvas is also studied to identify the influence of the tilted canvas on its eigenmodes, see Fig. 1. Because of the aging painting will deform and slightly distort the canvas. This will affect its vibration characteristics. However, it is difficult to produce such an aged dummy painting, it is approximated with a dummy painting with tilted canvas with the assumption that the distribution of the distortion has a small influence on the vibration characteristics.

For the purpose of studying the vibration characteristics of two dummy paintings, an experimental modal analysis is carried out on them and their modal parameters are identified. All identified parameters are then used for modal reconstruction and compared with experimental results to verify the correctness of modal identification. Moreover, a large number of studies have shown that the material properties of paintings are affected by climate, e.g., [13–15]. Therefore, using a climate box, some experiments are conducted to research the influence of humidity and temperature on the vibration characteristics of dummy paintings, so as to try to establish the mathematical relationship between eigenfrequencies, humidity, and temperature in a small range covering the common climate conditions used to store and transport precious paintings.

The novel contribution of this research lies in the modal parameter identification and modal reconstruction of canvas and primed canvas of paintings, and the reasons for the difference in their modal properties. Moreover, this research establishes a linear relationship between the eigenfrequencies of primed canvas, humidity, and temperature when the climate changes in a small range, which helps to compensate for disturbed eigenfrequencies under changing climate.

This article is divided into several parts. Section 2 contains the description of the two investigated dummy paintings and the experimental setup. Section 3 contains the experimental modal analyses for the two dummy paintings and identification results. In Section 4 the influence of climate on the dummy paintings is researched. Section 5 gives a brief summary.

2. Experimental setup

The two dummy paintings used for this research are shown schematically in Fig. 1. Both dummy paintings are 50 cm wide and 60 cm high. Their canvases are made of linen plain cloth and are prepared by washing, stretching and wetting for several times, i.e., they were decatized. The canvases are then uniformly stretched and stapled to the wooden frame. The difference between the two dummy paintings is that the weaving orientation of one of the canvases is oriented in a regular way, i.e., parallel to the frame, while the other has a 10° tilt angle. Fig. 2 shows the dummy paintings before and after applying the primer and the back of the dummy painting. The primer layer is made of a mixture of chalk and gypsum and a gelatine

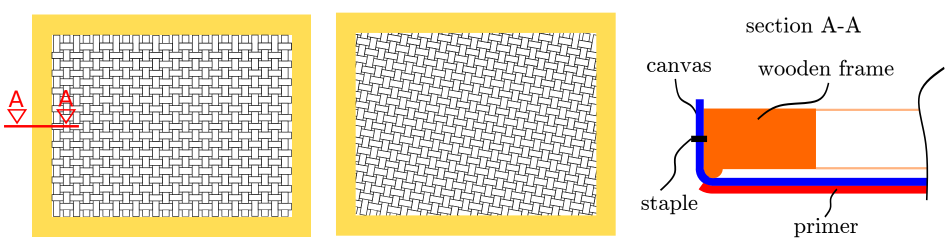

Fig. 1. The dummy paintings with regular orientation and tilted orientation canvases and their section

(a) unprimed canvas (b) primed canvas (c) back of the dummy painting

Fig. 2. The investigated dummy paintings

solution in water. The primer is heated and then applied on the canvas in several layers with a brush.

The whole experimental setup shown in Fig. 3 consists of the dummy painting mounted with rubber bands on an aluminium support, as well as an electrodynamical shaker of model PCB TMS-K2007E01, a Polytec PSV-500 scanning laser Dopper vibrometer (LDV) and a climate box measuring 2×2×2.5 m with a humidity controller of model Preservatech Mini One. The dummy painting is rigidly connected to a wooden plate which is used to install hooks and rubber bands, except that a part of the plate is removed to allow the LDV to physically access the canvas. Four hooks are installed on the upper and lower sides of the wooden plate. Then, rubber bands hang the entire experimental object on the aluminium support through the hooks, forming a very soft suspension which can be approximately considered as a free suspension.

Fig. 3. The whole setup to conduct an experimental modal analysis of the dummy painting in a climate box with approximately free suspension

In the experimental modal analysis of this contribution, the input of the system is the exciting force, and the output is the vibration velocity of the canvas. Hence, the so-called mobilities are measured. For the purpose of measuring the vibration velocity, on the back of both canvases 95 reflective stickers are attached, which are spaced 5 cm apart. An approximate 11×9 array composed of all measurement points is sufficient to robustly identify at least the first 8 eigenmodes of the dummy painting. The velocity response at the predefined points on the canvas is then measured with the LDV which can automatically scan all the measurement points on the painting sequentially without moving the LDV.

For this experiment, mainly an impulse excitation is adopted for experimental modal analysis to identify the modal parameters of the dummy painting. It is generated by an electrodynamical shaker and directly acts on the attached wooden plate of the dummy painting from the back. The electrodynamical shaker has a modified tip mechanism that generates a very short hit and avoids double hits. Essentially, the shaker plus tip mechanism composes an automatic impulse hammer. During measurements, a function generator automatically provides the shaker with a short trigger signal which leads to a single hammering motion of the shaker. The contact force between the dummy painting frame and the shaker is measured by a PCB 208C01 ICP for a sensor.

All experiments are carried out in a climate box. The relative humidity adjustment range of the climate box is 35% to 80%, where 55% is used for the experiments if not separately mentioned, and the temperature is kept constant at 22°C. During the experimental modal analysis, the humidity and the temperature in the climate box were kept as stable as possible to reduce fluctuation of the dummy painting modal parameters.

3. Experimental modal analysis

In the following, the experimental modal analysis procedure for the investigated dummy paintings is described. While using the rectangular membrane only for getting a first idea of the modal behaviour, the changes in the modal properties of the dummy painting before and after applying the primer are compared, as well as the influence of the tilted canvas on the modal properties. All the modal experiments are carried out in the climate box with a relative humidity 55%±0.5% and a temperature of 22°C±0.3°C.

3.1. Rectangular membrane

The painting and the membrane are stretched and then fixed along their edge, and their twisting and bending moments are small enough to be considered as negligible as a first approximation. For this reason, the membrane is used as the most simplified mechanical model of the dummy painting in this contribution. Although the simplified model will turn out to show a somewhat different modal behaviour from the investigated dummy paintings, it can still provide some simplified idea of the modal behaviour which can make it easier to understand the reason for some basic behaviour. Moreover, in principle, the general tendencies for the lower order modes derived from the simplified membrane model are still valid, such as the eigenmodes and the distribution of eigenfrequencies. Therefore, although very simplified, the rectangular membrane is used for comparison with the modal properties of the investigated paintings.

Considering a rectangular vibrating membrane with length a and width b, its general solution for obtaining the displacement $u(x, y, t)$ of a point (x, y) of the membrane from rest $(u = 0)$ at time t is according to [16]

$$u(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u_{mn}(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn}(t)\Phi_{mn}(x, y) \tag{1}$$

with

$$F_{mn}(t) = B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t, \tag{2}$$

$$\Phi_{mn}(x, y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \tag{3}$$

$$\lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}, \qquad c^2 = T/\rho, \qquad m = 1, 2, \cdots, \quad n = 1, 2, \cdots. \tag{4}$$

In these equations, T is the tension per unit length caused by stretching the membrane, which is the same in all points and in all directions and does not change during the motion. The homogeneous membrane density is represented by ρ. The different half sine wave numbers m and n in the length and width directions represent the different vibration patterns.

The displacement $u(x, y, t)$ of a rectangular vibrating membrane is composed of all the so-called eigenfunctions u_{mn} which themselves can be decomposed into a time domain function $F_{mn}(t)$ and a spatial domain function $\Phi_{mn}(x, y)$. The functions $F_{mn}(t)$ depend on λ_{mn}, B_{mn} and B^*_{mn}. The geometrical quantities B_{mn} and B^*_{mn} can be obtained from initial conditions and the orthogonality of Fourier components. The numbers λ_{mn} are called the eigenvalues. Therefore, the vibration frequencies of the rectangular membrane are $\lambda_{mn}/2\pi$. Obviously, the eigenfrequencies of the rectangular membrane depend on the tension T, the density ρ and its dimensions a, b. When using Eq. (4) to estimate the eigenfrequencies of the investigated dummy paintings, it is necessary to identify the tension T and the density ρ, which are difficult to precisely measure for real paintings without disassembling and consequently destroying the paintings.

The spatial domain functions $\Phi_{mn}(x, y)$ are called eigenmodes and represent vibration patterns. The eigenmodes depend only on the dimensions of rectangular membrane a and b. Therefore, substituting $a = 50$ cm and $b = 60$ cm into $\Phi_{mn}(x, y)$, the first 8 eigenmodes of the rectangular membrane with the same dimensions as the investigated dummy paintings can be obtained, as shown in Fig. 4. These eigenmodes are used for comparison with the measured eigenmodes of the investigated dummy paintings.

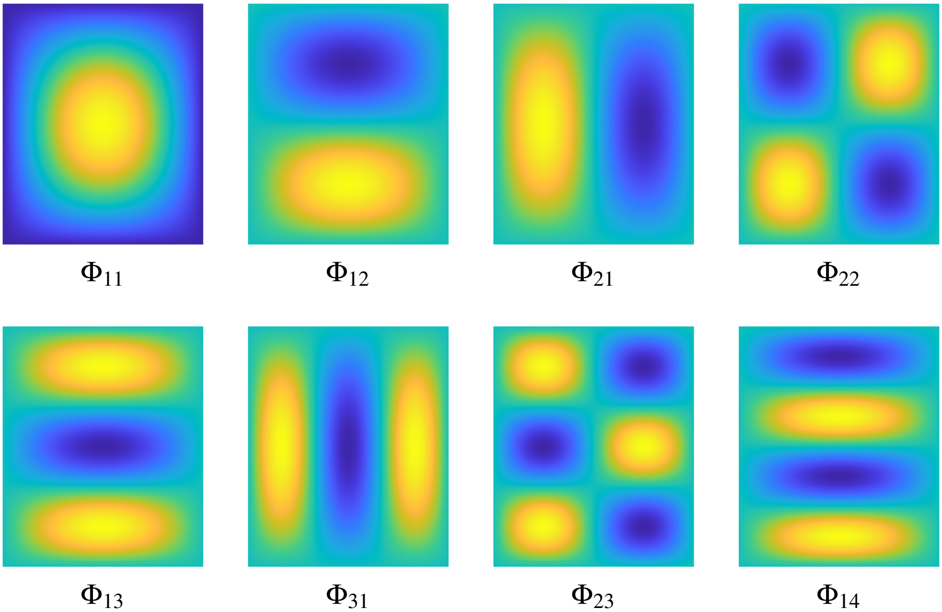

Fig. 4. The first 8 computed eigenmodes of the standard rectangular membrane

3.2. Modal parameter identification

In the performed experiments, the force input to the wooden plate is measured for three different excitation points and the velocity output of the canvas is measured at 95 points. Therefore, 285 mobility Frequency Response Functions (FRFs) can be calculated from the measured data for every canvas. In order to process so much data, the complex mode indicator function (CMIF) is adopted [17]. It is calculated through a singular value decomposition at each considered frequency. Note that when the FRFs correspond to the matrix of mobilities, using the real part of the FRFs is much more discriminating with respect to close modal frequencies [18]. Thus, the singular value decomposition on the real part of the FRFs matrix at every frequency point is performed here, which results in

$$Re(\boldsymbol{Y}(\omega)) = \boldsymbol{U}(\omega)\boldsymbol{\Sigma}(\omega)\boldsymbol{V}(\omega)^{\mathrm{H}}, \tag{5}$$

where ω is the frequency, $\boldsymbol{U}(\omega)$ is the matrix of left singular vectors, $\boldsymbol{V}(\omega)$ is the matrix of right singular vectors, and $\boldsymbol{\Sigma}(\omega)$ is the matrix of singular values. The superscript $^{\mathrm{H}}$ denotes the Hermitian of a matrix. All the singular values at every frequency point form the complex mode indicator function

$$CMIF_{\mathrm{k}}(\omega) = \Sigma_{\mathrm{k}}(\omega), \qquad k = 1, 2, \cdots, K, \tag{6}$$

where K is the total number of references. More references can identify the eigenmodes more comprehensively, and can better distinguish very close eigenmodes.

For the conducted experiments, there are three CMIF curves as $K = 3$. Normally, the curve formed by the first eigenvalues contains most of the modal information. The detected peaks on the CMIF indicate the existence of eigenmodes, and the corresponding frequencies of these peaks give an estimate for the damped eigenfrequencies. In addition, considering that the excitation spectrum is usually concentrated in the low frequency range during paintings transportation [7, 10], only the vibration characteristics of dummy paintings below 100 Hz are analysed here.

The resulting measured CMIF curves for the unprimed canvases with regular and tilted orientation are displayed in Fig. 5. In the low frequency range up to 14 Hz, due to the soft suspension of the dummy painting, the eigenfrequencies related to rigid body modes can be found. Then, the only two obvious peaks at 49 Hz and 97 Hz are caused by the first and second eigenmodes of the wooden frame. Between 14 Hz and 49 Hz, lots of very sharp peaks can be observed that were identified as measurement noise. This is because the canvas has a large damping and comparatively small resonance peaks, which results in small velocity amplitudes. At the same time, due to the high sensitivity of an LDV, a small signal-to-noise ratio results. In the end, a lot of spurious noise forms these sharp peaks. Between 49 Hz and 97 Hz, although there is no effect of spurious noise, the large damping of the canvas also yields no obvious observable peaks. The tilted canvas shows almost the same behaviour in Fig. 5.

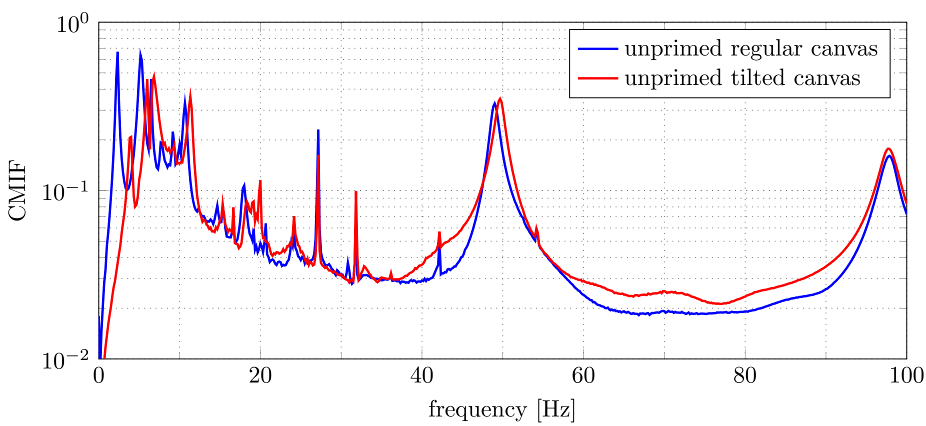

Fig. 5. Measured complex mode indicator function for the unprimed canvases

The resulting measured CMIF curves for the primed canvases with regular and tilted orientation are displayed in Fig. 6. Obviously, after applying the primer, the damping becomes much smaller. Therefore, there are much more pronounced peaks with much larger absolute values. Hence, the signal-to-noise ratio is higher, and noise becomes almost invisible thus the eigenfrequencies are much easier to excite and to identify. Similarly, in the low frequency range up to 14 Hz, due to the soft suspension of the dummy painting, the eigenfrequencies related to rigid body modes can also be found. The first eigenfrequencies of the primed canvases with regular and tilted orientation are 22.33 Hz and 29.5 Hz, respectively. They are relatively far away from the rigid body modes, which means that the suspension is soft enough and that the influence of the rigid body modes can be neglected. Two small residues of the rubber bands suspension can be observed by small peaks at 29.8 Hz for regular canvas and 35 Hz for tilted canvas. They are very small compared to the peaks of the structural modes of the primed canvas. Furthermore, they are always between the first and the second eigenmodes, hence, even though they appear, they do not influence the structural modes of the primed canvas. Nevertheless, they are still identified during parameter identification and modal reconstruction later in this section.

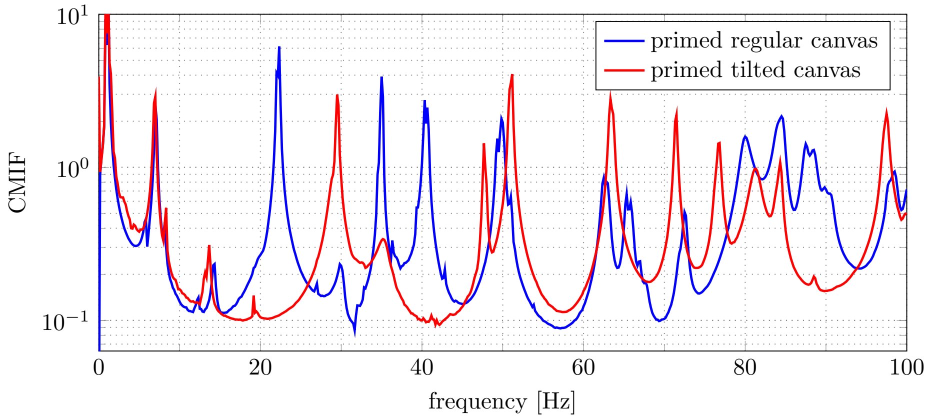

Fig. 6. Measured complex mode indicator function for the primed canvases

The identification results of the first eight eigenfrequencies of the two primed canvases are listed in Table 1, as well as the eigenfrequencies of the corresponding rectangular membranes. Because the values for the tension and density are unknown, their ratio, i.e., the constant c^2 is chosen such that the first eigenfrequency of the membrane matches the first identified eigenfrequency of the primed canvas. After comparison, it can be found that the eigenfrequencies of the primed canvas with tilted orientation are higher than those of the primed canvas with regular orientation. This suggests that the primed canvas with tilted orientation either has a bigger tension or a smaller density. The former may be explained by variations in the tension during the stretching process, the later may be explained by variations in the thickness of the primer due to variations in the application process. However, both were carefully carried out by experienced conservators and thus were carried out with high precision. The differences in the dynamic behaviour, i.e., the eigenfrequencies, must thus be assumed to be within the general variance. On the other hand, it can also be found that the eigenfrequencies of the two primed canvases are higher than what the corresponding rectangular membrane model predicts. This is caused by the small bending stiffness in the primed canvas. Although

Table 1. The first eight eigenfrequencies of the primed canvases and the corresponding rectangular membranes

mode r	1	2	3	4	5	6	7	8
regular canvas [Hz]	22.3	34.5	40.3	49.3	50.3	62.3	65.3	67.0
membrane[1] [Hz]	22.3	32.8	36.6	44	45.5	52.6	54.1	58.0
tilted canvas [Hz]	28.6	46.6	49.3	61.9	69.7	74.7	82.0	86.7
membrane[2] [Hz]	28.6	42.7	47.6	57.2	59.2	68.4	70.3	75.4

[1]rectangular membrane with the same first eigenfrequency as the primed canvas with regular orientation

[2]rectangular membrane with the same first eigenfrequency as the primed canvas with tilted orientation

the rectangular membrane model cannot provide accurate eigenfrequencies, the distribution tendency of the eigenfrequencies of the canvas remains similar to that of the rectangular membrane.

When calculating the CMIF, the left singular vectors, associated with the peaks in the CMIF, lead to approximate eigenmodes. Accordingly, the first eight eigenmodes of the primed canvases are identified, as shown in Figs. 7 and 8. In general, the eigenmodes of the primed canvases are similar to those of rectangular membrane. But many eigenmodes are distorted, such as the modes at 40.3 Hz and 62.3 Hz for the primed canvas with regular orientation, and the modes at 46.6 Hz and 69.7 Hz for the primed canvas with tilted orientation. These distorted eigenmodes are probably caused by uneven distribution of tension and density.

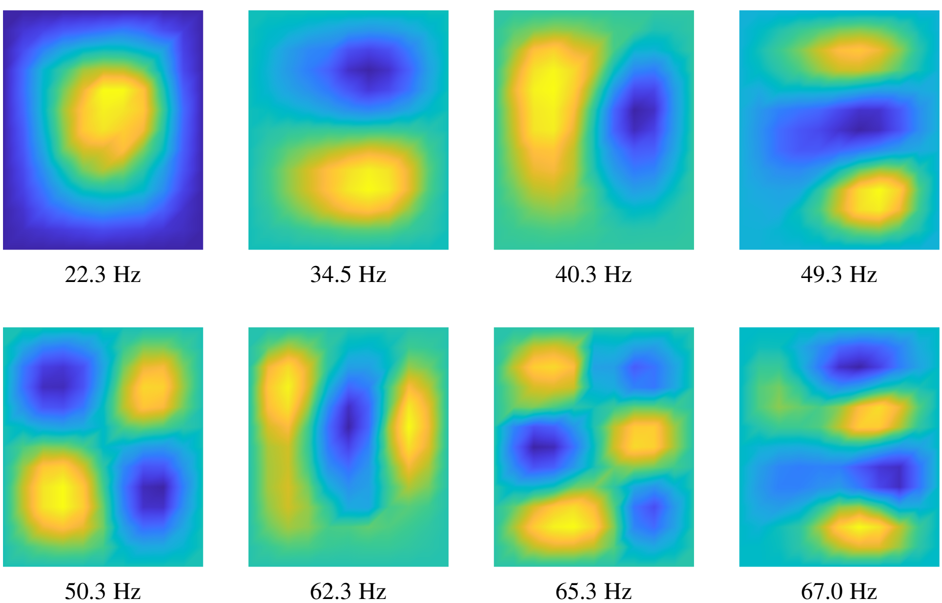

Fig. 7. The measured first eight eigenmodes of the primed canvas with regular orientation

In particular, compared with the rectangular membrane, the fourth and fifth eigenmodes of the primed canvas with regular orientation have been reversed. This may be because their eigenfrequencies are quite close, and slightly unevenly distributed tension or density has changed their order. In addition, for the primed canvas with tilted orientation, starting from the seventh eigenmode, the eigenmodes become severely distorted, which is probably caused by the tilted canvas.

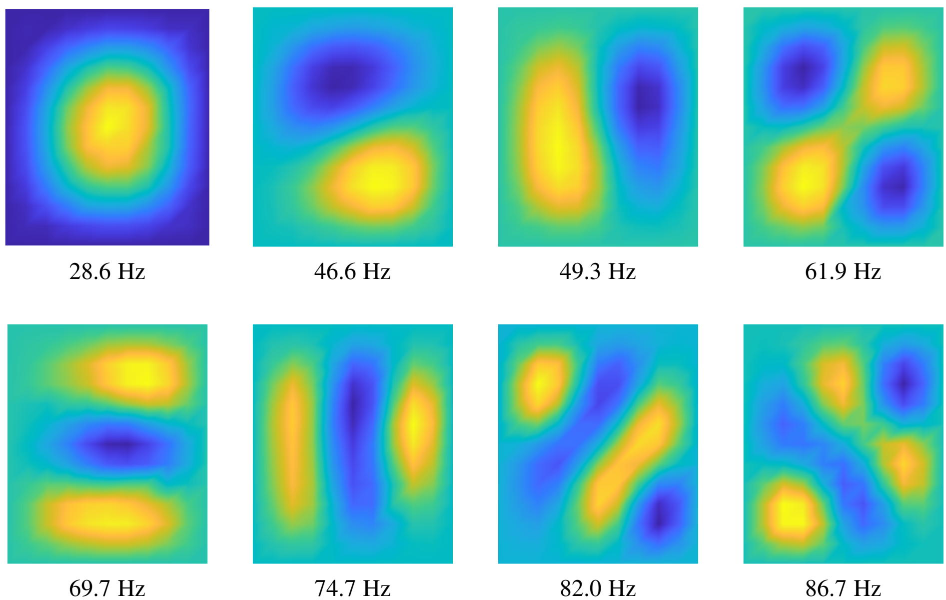

Fig. 8. The measured first eight eigenmodes of the primed canvas with tilted orientation

After identifying the eigenfrequencies and eigenmodes, the enhanced frequency response function (eFRF) is adopted to identify other modal parameters and reconstruct FRFs [18]. The eFRF is developed based on the concept of physical to modal coordinate transformation and is used to manipulate FRFs so as to enhance a particular eigenmode. Normally, the left singular vectors $\boldsymbol{u}_{\mathrm{r}}(\omega_{\mathrm{r}})$ and the right singular vectors $\boldsymbol{v}_{\mathrm{r}}(\omega_{\mathrm{r}})$, associated with the peaks in the CMIF, can be used as an estimate of the modal filter

$$\overline{Y}_{\mathrm{r}}(\omega) = \boldsymbol{u}_{\mathrm{r}}^{\mathrm{T}} \boldsymbol{Y}(\omega) \boldsymbol{v}_{\mathrm{r}}, \tag{7}$$

which accomplishes the eFRF. The advantage of using eFRFs for each mode is that, with a sufficient spatial resolution of the measurements, a curve for each mode is created without any influence of adjacent modes. Therefore, any single degree of freedom parameter identification technique can be used to identify the modal parameters for each mode separately through eFRF [17].

With the identified modal parameters, the eFRFs can be reconstructed as a single degree of freedom system

$$\overline{Y}_{\mathrm{r_{rec}}}(\omega) = \frac{{}_{\mathrm{r}}A(\omega)}{\omega_{\mathrm{r}}^2 - \omega^2 + 2i\omega_{\mathrm{r}}\omega\zeta_{\mathrm{r}}}, \tag{8}$$

where ${}_{\mathrm{r}}A(\omega)$ are the modal constants and ζ_{r} are the modal damping coefficients. In order to reconstruct the matrix of mobilities, all the reconstructed eFRFs are transformed back into the physical domain and summed up

$$\boldsymbol{Y}_{\mathrm{rec}}(\omega) = \sum_{\mathrm{r}=1}^{\mathrm{R}} \boldsymbol{u}_{\mathrm{r}} \overline{Y}_{\mathrm{r_{rec}}}(\omega)\, \boldsymbol{v}_{\mathrm{r}}^{\mathrm{T}}, \tag{9}$$

where R is the number of identified modes in the considered frequency range. For the primed canvases, the first eight eigenmodes have been reconstructed and the mode caused by the rubber bands between the first and second eigenmodes is also taken into account. The accuracy of the modal parameters identification can be evaluated by directly comparing the measured mobilities and the reconstructed mobilities.

Both the reconstruction of magnitude response and phase response have achieved good agreement. Fig. 9 shows the comparison between the reconstructed mobility and the measured mobility in three different points. Fig. 10 only takes one point as an example to illustrate the reconstruction result of the phase response. Below 20 Hz, because the peaks corresponding to the rigid body modes of the painting are neglected, the reconstruction results and measurement results show a large difference. But this does not affect the reconstruction of the true structural eigenmodes. Above 20 Hz, all three reconstructed mobilities approximate their measured counterparts very well and hence, the parameter identification can be rated as successful.

4. Climate influence

During the measurement for the experimental modal analysis, a sensitivity of the modal parameters with respect to fluctuations of humidity and temperature was observed. It was for this reason that all measurements were conducted in the climate box with highly stable climate. However, even small humidity and temperature fluctuations of only a few degrees Celsius or a few percent humidity can cause shifts of eigenfrequencies of several hertz. It is interesting and important to study the relationship between climate change and eigenfrequency change. If such a relationship would be known, measurements under changing climate could be compared and related. Measurements that need to be carried out over a longer period of time, for example measurements with a sine sweep for all 95 points that take hours to complete, and where small changes in climate are unavoidable, could then be climate-compensated.

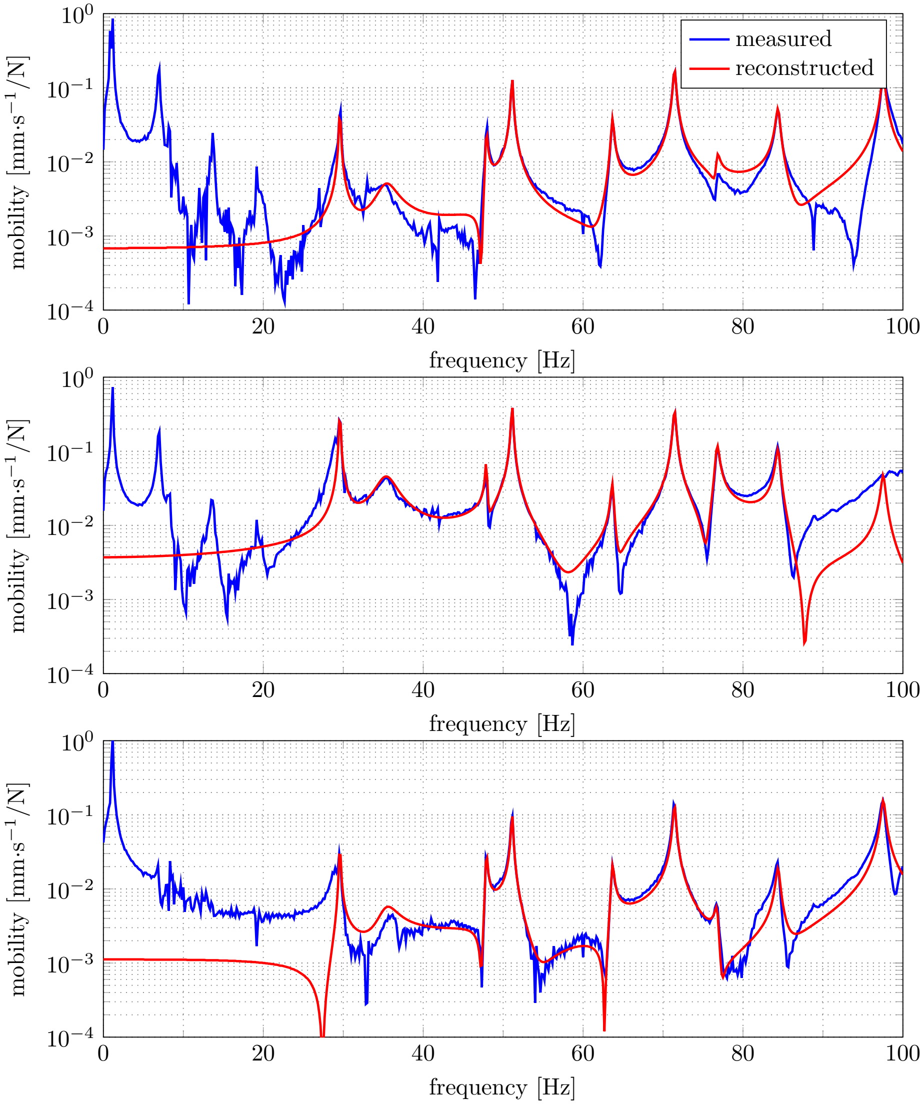

Fig. 9. The measured mobilities of three different points in comparison to the related reconstructed mobilities from the experimental modal analysis (top: upper right of the dummy painting, middle: centre of the dummy painting, bottom: lower left of the dummy painting)

In order to investigate the relationship between changing eigenfrequencies due to changing humidity and temperature, the eigenfrequencies are identified from the mobility of only one point using an automatic hammer excitation every 5 minutes for 14 hours, resulting in 168 measurements. The desired humidity of the climate box is set to 55%, but the temperature is left uncontrolled and is thus changing with the indoor lab environment. Then, the triplets consisting of humidity,

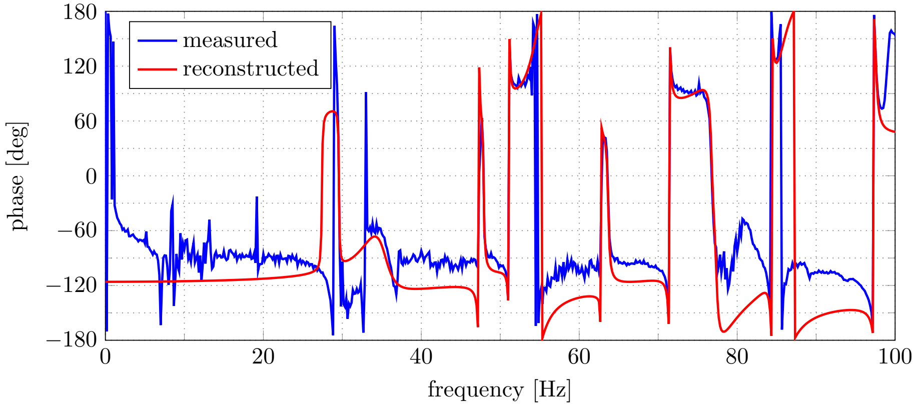

Fig. 10. Comparison of reconstruction results for the phase response corresponding to the bottom plot in Fig. 9

temperature and eigenfrequency are interpolated to gain a relationship of climate versus eigenfrequency.

The experiment is conducted on the primed canvas. One of the measurement results for primed canvas with tilted orientation is shown in Fig. 11. Obviously, it can be seen from the measurement that the eigenfrequencies of the primed canvas are affected by environmental humidity and temperature. Preliminary measurements have shown that the sensitivity with respect to climate variations of the unprimed canvas is negligible compared to the sensitivity for primed canvas. This indicates that it is the primer layer that will cause the vibration characteristics of the painting to be affected by humidity and temperature.

The packaging box of painting is generally designed to keep humidity and temperature as stable as possible, i.e., the humidity and temperature in the packaging

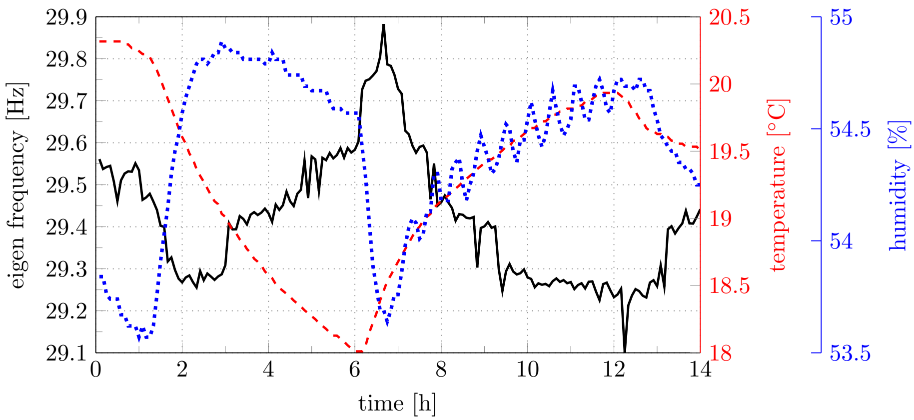

Fig. 11. One measurement of the relationship between the first eigenfrequency (solid line), humidity (dotted line) and temperature (dashed line) for the primed canvas with tilted orientation

box only fluctuate in a small range. As shown in Fig. 12, the results of multiple measurements also show that within a small range, the eigenfrequency approximately depends linearly on humidity and temperature. Therefore, in a small range of humidity and temperature, it is feasible to establish a linear relationship between eigenfrequency, humidity and temperature. The following objective function is used to fit the first 8 eigenfrequencies of the primed canvas with tilted orientation

$$f_r(H,T) = p_0 + p_{\mathrm{H}}H + p_{\mathrm{T}}T, \tag{10}$$

where f_r is the r-th eigenfrequency, H is relative humidity, T is temperature, and p_0, p_{H}, p_{T} are fitting parameters of the two dimensional linear function. At the same time, in order to evaluate the accuracy of linear fitting, the coefficient of determination is adopted

$$R^2 = 1 - \sum_i (y_i - f_i)^2 / \sum_i (y_i - \overline{y})^2, \tag{11}$$

where y_i is the measured value, f_i is the fitted value and $\overline{y}$ is the mean of the measured data. The R^2 value is a statistic quantity that gives some information about the quality of fit of a model, based on the proportion of total variation of outcomes explained by the model [19]. The closer R^2 is to 1, the better the fitting results.

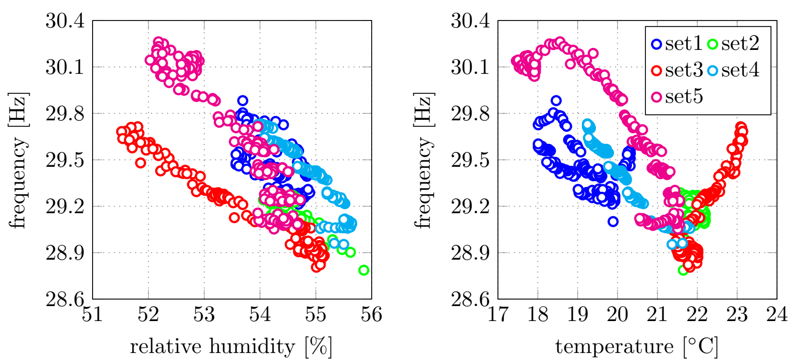

Fig. 12. Five sets of measurement results between eigenfrequency, humidity, and temperature

The results of linear fitting using a least square fitting are listed in Table 2. The lowest R^2 in the fitting results is 0.825 and the highest can reach 0.961, which indicates that the linear fitting obtains a good result. Therefore, these fitting parameters can be used for eigenfrequency compensation.

The purpose of eigenfrequency compensation is that, when changes in eigenfrequencies are caused by inevitable changing climate, the disturbed eigenfrequency can be compensated to the corresponding eigenfrequency under a reference climate. For example, when taking a measurement that lasts for several hours, the humidity and temperature in the climate box inevitably fluctuate, which causes the measured

Table 2. Linear fitting parameters of the relationship between the first 8 eigenfrequencies of the painting, humidity, and temperature.

mode r	1	2	3	4	5	6	7	8
p_0	45.41	72.37	75.86	63.40	71.00	76.40	84.10	97.10
p_{H}	-0.241	-0.378	-0.371	-0.468	-0.520	-0.509	-0.643	-0.668
p_{T}	-0.144	-0.204	-0.243	-0.311	-0.281	-0.350	-0.415	-0.368
R^2	0.892	0.851	0.961	0.863	0.892	0.955	0.825	0.892

eigenfrequencies of the canvas to fluctuate. The fluctuating eigenfrequencies will increase the difficulty of interpreting the results. However, a reference climate can be selected, and the theoretical value of eigenfrequency f_{ref} under this climate condition can be obtained by Eq. (10). The theoretical value of the eigenfrequency under a changing climate f_r is also calculated by Eq. (10). Then, the difference between the reference eigenfrequency f_{ref} and the theoretical eigenfrequency f_r is compensated by the measured eigenfrequency f_m. Finally, the compensated eigenfrequency $\hat{f}_m$ can be obtained, namely

$$\hat{f}_m = f_m + f_{\mathrm{ref}} - f_r . \tag{12}$$

The quality of the compensation depends on the quality of the fitting. The better the fitting result, the smaller the deviation of the compensation result. For example, the measurement results in Fig. 12 are compensated to the eigenfrequencies when the relative humidity is 55% and the temperature is 22°C, which is the climate condition in the experimental modal analysis. The compensation results of the eigenfrequencies are shown in Fig. 13. The maximum deviation of the compensation results is 0.265 Hz, which is much smaller than the differences seen in Fig. 12. Although there are still deviations caused by the measurement procedure, these are much smaller than the fluctuations caused by changing climate.

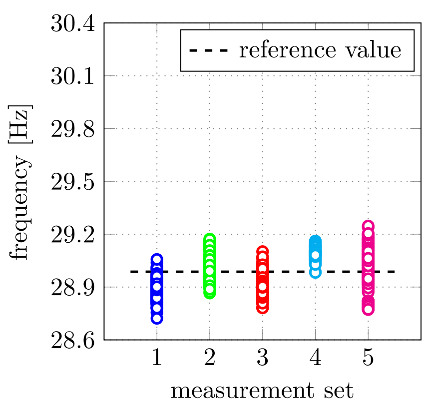

Fig. 13. Compensation values of five sets of measurement results under a relative humidity of 55% and a temperature of 22°C

5. Conclusion

The goal of this paper was to study the vibration characteristics of canvas and primed canvas for paintings and their influencing factors. For this reason, an experimental setup has been designed with which the vibration of the canvas can be measured and an experimental modal analysis has been performed for two dummy paintings with regular and tilted orientation canvas. The experimental modal analysis results show that the eigenmodes of unprimed canvas are difficult to excite due to the large damping, which makes it difficult to identify the modal parameters. However, after the primer is applied, the modal characteristics become much more pronounced and easier to excite and identify. As the most simplified mechanical model of the canvas, the rectangular membrane can be used as a reference to provide general tendencies for low-order modes, such as eigenmodes and distribution of eigenfrequencies. However, the non-negligible bending stiffness of primed canvas will result in higher eigenfrequencies, and the uneven distribution of tension and density in the canvas eventually distort the eigenmodes as compared to the simple membrane. In addition, the canvas with tilted orientation exhibits further distorted the eigenmodes. The mobilities reconstructed by experimental modal analysis show very good agreement with the measured mobilities, which verifies the correctness of the modal parameter identification.

Moreover, the experiment also shows that the eigenfrequencies of the primed canvas are sensitive to humidity and temperature. The dependency of eigenfrequencies from humidity and temperature can be linearly fitted in a small range, which makes it possible to compensate for changes in eigenfrequencies under changing climate.

Nevertheless, further work has to be put into the vibration characteristics of paintings with paint layers and the prediction of vibration during transportation.

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