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Industry Expertise Heuristics for Dimensioning Shelf Space of Rack Storage Location in a Distribution Centre with Zone Picking

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Abstract
Appropriate product categorization in distribution centres is important for business success because of the possibility of intuitive product finding by the picker and increased product movement. Both of these factors result in the operational efficiency of the distribution centre. The goal of this paper is to explore a model of shelf space dimensioning of storage location on a rack with vertical and horizontal product categorization in a distribution centre, where the aim is to increase total product movement/profit from all shelves of the rack. This is controlled by a packer who must complete orders by getting the goods from shelves and picking them to the container. In this problem, we develop two heuristics and compare the archived results to the CPLEX solver. The average profit ratios of both heuristics are high and approximately equal to 99%. In 10 cases, optimal solutions have been found by heuristics. The total number of possible solutions to be checked for the largest instance was reduced from $1.33 \cdot 10^{156}$ to $1.19 \cdot 10^{7}$ thanks to the heuristic rules.

Keywords
Distribution centre, Order picking, Decision making/process, Heuristics, Shelf space allocation.

Introduction

According to Dictionary of Business (Law, 2009), a distribution centre (DC) is a facility that receives, temporarily stores, and distributes items in accordance with customer orders as they come in. DCs serve retail stores directly and are customer-focused. Suppliers typically transport products to these distribution centres, which cater to certain retail outlets. These centres are an important component of the order process, and maintaining them can be difficult due to the large number of products they hold. These centres frequently use cutting-edge technology to ensure order system throughput. Warehouse management systems (WMS) and transportation management systems (TMS) are also available in some facilities.

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From the exterior, both warehouses and distribution centres appear to be almost identical, yet their inner workings are drastically different. In summary, warehouses are better suited to those who require long-term storage rather than things that must be delivered promptly. Distribution centres, on the other hand, are built to handle quick item intake and shipping.

Researchers have shown that order-picking activities account for about 65 per cent of a warehouse's overall operating cost (Coyle et al., 2003; Ho and Tseng, 2006; Wu et al., 2017), with travel time accounting for roughly half of all order-picking operations (Ho and Tseng, 2006; Tompkins et al., 2010).

Almost any DC manager says that finding a product to be added to the container with the order on the rack is a pain. The product fulfilment of DCs is growing fast. Nowadays more orders are arriving varied by dimensions and quantities. The distribution centres have modern technology for order processing, warehouse management, and transportation management, among other things.

The DC serves as an important link between suppliers and customers in supply chains. The capability of supply chains is heavily influenced by the performance of DCs. Working with a large number of product items, as well as a large number of orders that comprise diverse combinations of those product items, is one of the issues that distribution centres confront today (Ming-Huang Chiang et al., 2014).

In the warehouse and DC industry, rack space distribution methods are critical. Understanding these strategies can help you improve the allocation of shelf space in distribution centres and later packing processes in day-to-day operations.

An examination of the literature revealed a dearth of data-driven applications in the field of DC design. In the majority of the studies, researchers sought ways to optimize order-picking routes in order to reduce the order-picking trip distance in distribution centres. The problem of dimensioning the shelf space of the storage location on the rack with vertical and horizontal product categorization is examined in this study. This method is used by a packer who has to swiftly locate a product on the rack and add it to the order container in their picking zone.

The following are the research's contributions:

- Identifying parallels between the distribution of shelf space on racks in a distribution centre and the more advanced problem of shelf space allocation on planograms.
- Define the problem in terms of zone-picking at a distribution centre and model it as a mixed-integer linear programming (MILP) problem.
- Developing two heuristics which allow reaching an optimal or near-optimal solution.
- Defining some principles and implementing them in heuristics that allow to significantly reduce the search space.
- Proposing seven steering parameters that control the size of the search space of heuristics and greatly shorten the solution time.

The relevant literature review is presented in the following section. In the next section, a problem definition and mathematical formulation of the model of the problem are given. The heuristics explanation and steering parameters are then described, allowing the search space to be reduced. Next, the results of the computational experiments are reported. Finally, we'll make some conclusions and recommendations.

Literature review

Storage and order picking

One of the most efficient approaches to optimize the supply chain is to implement DC picking enhancements. One selecting approach may fit the processes in the DC more than others that vary according to the size of the DC, the amount and types of products in it, and the overall number of employees.

Stock is transported from factories via DCs to retail outlets in a typical distribution network of a large retailer. Material handling and transportation costs are two significant costs connected with a distribution network. Visser and Visagie (2018) focused on smoothing product movement through the DC over time to reduce DC handling costs and transportation costs between the DC and a retail chain's outlets (Visser and Visagie, 2018).

The design of a storage system from a data-driven perspective is the subject of the research by Tufano et al. (2022). Storage system technology, material handling system, storage allocation strategy, and choosing policy are the four design aspects identified by the authors (Tufano et al., 2022).

The pulling of an item or items in a distribution or fulfilment centre for the purpose of completing a customer's order is known as order picking. It is used to maintain high order average accuracy across all orders exiting in the DC, efficient DC picking necessitates high-tech equipment and labour-intensive processes (Bartholdi and Hackman, 1998).

Pick lines are instructions to order-pickers that inform them where and what to pick, as well as how much and in what quantities. Each pick-line (or, more concisely, pick or line) signifies a location to be visited, and because travel is the most expensive labour expense in a typical DC, the number of pick-lines indicates the amount of manpower needed (Bartholdi and Hackman, 1998).

De Vries et al. (2016) argued that the most prevalent order-picking methods (parallel picking, zone picking, and dynamic zone picking) differ in the degree to which they are variables or dependent, with parallel picking being the most independent, zone picking the most dependent, and dynamic zone picking falling somewhere in the middle. This shows that competitive incentives are best for parallel picking, while cooperation-based incentives are best for zone picking, and both incentives may be useful in a dynamic zone setting (De Vries et al., 2016).

Picking job levels can now exceed processing capacities predicted at the time of DC design, thanks to the rising demand for online shopping. Traditional comprehensive picking is assumed in large online shopping DC in particular. Movement distances become inefficient in approaches where one picker covers the entire area. A combination of total and zone picking might be more efficient (Tanaka et al., 2019). Finding the correct order of DC's picking systems is critical since it can improve picking productivity, order accuracy and eventually end customer happiness.

The process of fulfilling customer orders in total picking is referred to as "batching". Batched customer orders are referred to as "batch orders" in the study of Tanaka et al. (2019). The picker sorts items for each customer while picking in sort-while-picking methods. There is a sorting section separate from the picking area in this situation (Tanaka et al., 2019).

With so many phases of the overall order fulfilment process occurring at the same time, DC leaders need to optimize the DC on a regular basis to improve the layout for more effective picking routes, storage locations for each SKU, and the information transfer from picking to packing, as well as to add new appliances.

Zone-picking

Zone picking is a type of order-picking strategy in which objects are divided into a series of different zones, and pickers are allocated to a zone and educated to pick only within that zone, with no ability to move to other zones (Ho and Tseng, 2006; Ho and Lin, 2017; Bottani et al., 2019). There are two sorts of zone selection strategies: sequential and synchronized (Tompkins et al., 2010). Picking is performed in one zone and one order at a time in sequential zone picking, whereupon the order is handed to the next zone. Alternatively, all items belonging to batched orders are collected concurrently from all zones in synchronized zone picking, and orders are subsequently combined through a sorting mechanism (Bottani et al., 2019).

Zone picking is investigated by the academic society. The issue with zone-picking is that it necessitates all of the labour that goes into balancing an assembly line: a work-content model and task partitioning. An industrial engineer is usually in charge of this (Bartholdi and Hackman, 1998). Zone picking in a warehouse has a significant impact on supply chain productivity as a basic fulfilment operation.

The picking perimeter is divided into several pick zones, with pickers assigned to each zone picking only objects that fall within that zone's boundaries. Lightning pick technologies improve the labour-intensive process' productivity, accuracy, and cost-efficiency. This method is also appropriate for team-based methods such as zone picking (Ho and Tseng, 2006).

Ho et al. (2007) divided a typical logistic warehouse into various geometric zone-picking types for their study. The authors handled each test problem's storage-location assignment problem by splitting the logistic warehouse into various zone types, so they could observe how much better a standard warehouse might be transformed into distinct geometric zone-picking warehouses (Ho et al., 2007).

The pickers can be coordinated in a variety of ways. Zone-picking divides the bays into zones, and each picker is allocated to a specific region: Worker 1 is responsible for picking all goods lying within bays $1, \ldots, b1$; worker 2 is responsible for picking all items lying within bays $b1 + 1, \ldots, b2$ and so on. Managers aim to balance the expected work among the pickers throughout each picking session when building such order-picking systems. The problem with this is that it only balances the work on average across the picking period, which means that everyone will have completed the same total number of picks – even though the line can be drastically out of balance from order to order (Bartholdi and Hackman, 1998).

Discrete zone picking is an example of a procedure that was working but could be improved. For example, a normal order can include things from five different zones. The WMS issued orders per zone instead of sending one person through all five zones to fill that order. Order pickers would pick their allocation and then take the totes to a location where all of the totes for order were wedded and then physically transported to packing in the manual procedure (Trebilcock, 2022).

For several order-picking strategies, Lin and Lu (1999) established the optimum order profiles. The zone-picking method is one of the techniques (Ho and Tseng, 2006). Ho and Lin (2017) proposed a new zone-picking network that gives order routing freedom while addressing the challenges created by the fixed-sequence-route limitation in a sequential zone-picking route.

Whereas routing flexibility allows the proposed zone-picking network to avoid the challenges created by a sequential zone-picking line's fixed-sequence-route restriction, it introduces a new problem: tote dispatching. This issue, as well as another one – order selection – has an impact on the proposed zone-picking network's performance. Ho and Lin (2017) examined both difficulties and presented many solutions varying from simple to complex regulations (Ho and Lin, 2017).

Some scientists investigated zone selecting problem (Yu and de Koster, 2009; De Koster et al., 2012; van Gils et al., 2018; Wu et al., 2020). A typical zone selection problem entails determining the optimal number of zones. Furthermore, there are algorithms for implementing zone-picking strategies optimally (Kuo et al., 2016). Zone selecting, on the other hand, is rarely linked to other design elements; the only exception is research by van Gils et al. (2018), which looked at storage assignment policies, zoning, routing, and batching all at the same time.

The zone selecting technique was studied by De Koster et al. (2012), with the goal of reducing the overall time to complete a batch by estimating the ideal number of zones. Yu and de Koster (2009) used a queuing theory-based approximation model to investigate the influence of zone selection and batching in a pick-and-pass order picking system.

Zone picking has a number of benefits and drawbacks that are often dependent on the size, scope, and managerial style of the operation.

Zone picking enables the creation of zones based on specific criteria, such as fast-moving and slow-moving SKUs or a zone dedicated to high-security SKUs. Another advantage is that each zone can have its own storage strategy, order-picking technology, and order-picking equipment.

There are two drawbacks of zone picking:

- orders are separated, and therefore, they must be re-consolidated before the shipment has occurred;
- labour resources must be allocated throughout all available order-picking zones (van Gils et al., 2017).

To address the first drawback, progressive zoning or synchronized picking are used. Orders are chosen zone by zone in progressive zoning. Synchronized zoning is a policy that allows all order pickers to work in the same order at the same time, each in their own zone. Following the selection process, all orders are aggregated using a sorting mechanism (De Koster et al., 2007).

Van Gils et al. (2017) focused on options for dealing with zone picking's second disadvantage. Space, labour, and technology resources should be deployed across all order-picking zones to fulfil customer orders quickly and efficiently (Gu et al., 2007). In addition, order pickers must be distributed across warehouse zones, which necessitates flexible workforce planning. Order pickers, for example, may be shifted to other pick zones, necessitating worker cross-training. Furthermore, the timing of transferring a cross-trained employee to another zone, as well as the new order-picking zone to which the person should be allocated, should be determined (van Gils et al., 2017).

Intuitive solutions for distribution centres and retail stores

Heuristics can differ in terms of speed and accuracy. Solutions could be applied in a retail environment as well as in distribution centres. While every store and DC scenario is different, the retailer or DC leader might choose to utilize heuristics if speed is critical and result accuracy is not the top priority. Applying heuristics for shelf space planning ensures managers that they frequently get not optimal but satisfactory solutions to each problem.

Ostermeier et al. (2021) proposed an integrative approach to the planning challenges in the retail industry. To allow joint evaluation of all the subproblems involved, they proposed an integrated zone picking and vehicle routing problem with time frames and restricted intermediate storage. A specific heuristic solution strategy, called a general variable neighbourhood search, is used to solve the identified problems (Ostermeier et al., 2021).

The modified class-based heuristic and the association seed-based heuristic are two storage assignment heuristics presented by Ming-Huang Chiang (2014) that aimed to help with effective order choice when using weighted support count. The usefulness of the proposed methodologies is tested using real-world data from a food distribution centre (Ming-Huang Chiang, 2014).

Wu et al. (2017) designed a greedy heuristic-based solution strategy, focusing on the following topics: (1) assessing if merging sequence control can be utilized to reduce idle and order fulfilment periods; (2) constructing a mathematical model and an effective heuristic algorithm.

Evaluation and eventual choice can be influenced by attention only. The visual characteristics of an assortment impact spontaneous attention. The prominence inside the assortment is a critical feature of the assortment that controls attentiveness. Items that are more visually prominent owing to brightness, colour, size, or a number of facings "jump out" of the presentation, attracting automatic attention and increasing the duration of focus, which might possibly influence choices (Kahn, 2017).

Easy-to-process assortments are preferred and assessed to have greater perceived diversity. Complexity must be reduced to allow assortments to be analyzed quickly (Kahn, 2017).

A heuristic approach to automated shelf space allocation was provided by Landa-Silva et al. (2009). This method was developed in partnership with a retailer who was well-versed in the problem and the requirements for a computer approach. Gajjar and Adil (2010) proposed local search heuristics for addressing the retail shelf space allocation problem using a linear profit function, which gives starting arrangements before iteratively improving the profit of candidate solutions via adjustment moves.

The advantage of utilizing heuristics to solve retail and DC space planning problems or make a product choice is that it consistently produces satisfactory results. Heuristics can be useful in problem-solving.

When it comes to problem-solving, the necessity for accuracy of the result or speed of achieving the result determines which strategy to adopt. Heuristics are more commonly used in everyday retail cases. Finding an optimal solution would be a very time-consuming process. Instead, the better option is to use heuristics based on previous experience choosing the ones that worked effectively for such cases.

Methodology

Nomenclature

S total number of shelves

P total number of products

K total number of categories

T total number of tags

i shelf index, $i = 1, \ldots, S$

j product index, $j = 1, \ldots, P$

k category index, $k = 1, \ldots, K$

t tag index, $t = 1, \ldots, T$

r orientation index, $r \in \{0, 1\}$

$$r = \begin{cases} 0, & \text{for front orientation} \\ 1, & \text{for side orientation} \end{cases}$$

Shelf parameters:

s_i^l length of the shelf i

s_i^d depth of the shelf i

s_{ti}^g binary tag t of the shelf i

$$s_{ti}^g = \begin{cases} 1, & \text{if shelf } i \text{ is tagged,} \\ 0, & \text{otherwise} \end{cases}$$

Product parameters:

p_j^w width of the product j

p_j^d depth of the product j

p_j^u unit movement/profit of the product j

p_j^l cluster of the product j

p_{tj}^t tag t of the product j

p_j^k category of the product j

p_j^s supply limit of the product j

p_{jr}^w width or depth of the product j on orientation r

$$p_{jr}^w = \begin{cases} p_{j0}^w, & \text{if } r = 0 \text{ width for front orientation} \\ p_{j1}^w, & \text{if } r = 1 \text{ depth for side orientation} \end{cases}$$

$p_j^{o_2}$ side orientation binary parameter of the product j

$$p_j^{o_2} = \begin{cases} 1, & \text{if side orientation is available for product } j \\ 0, & \text{otherwise} \end{cases}$$

$f_j^{\min}$ minimum number of SKUs of the product j

$f_j^{\max}$ maximum number of SKUs of the product j

$s_j^{\min}$ minimum number of shelves on which the product j can be allocated

$s_j^{\max}$ maximum number of shelves on which the product j can be allocated

Category parameters:

c_k^m minimum category size as a percentage of the shelf length

c_k^t category size tolerance between shelves in the category as a percentage of the shelf length

Tag parameters:

b_t^n band name of the tag t, $b_t^n = \{H; H^+; V^+\}$

b_{tij}^t product to shelf compatibility tag

$$b_{tij}^t = \left\{ \begin{array}{ll} 1, & \text{if } s_{ti}^t = p_{tj}^t \wedge b_t^n = \{H\} \\ 0, & \text{otherwise} \end{array} \right\},$$

$t = 1, \ldots, T$, for the horizontal shelf level for big products

$$b_{tij}^t = \left\{ \begin{array}{ll} & \min p_{tj}^t; 1) \wedge b_t^n = \{V^+\} \\ 1, & \text{if } p_{tj}^t = 1 \wedge s_{ti}^t = p_{tj}^t \wedge b_t^n = \{H^+\} \\ 0, & \text{if } p_{tj}^t = 1 \wedge s_{ti}^t \neq p_{tj}^t \wedge b_t^n = \{H^+\} \\ 1, & \text{if } p_{tj}^t = 0 \wedge b_t^n = \{H^+\} \end{array} \right\},$$

$t = 1, \ldots, T$, for the horizontal and vertical shelf level for small products

Decision variables:

$$x_{ijr} = \begin{cases} 1, & \text{if product } j \text{ is placed on shelf } i \text{ on orientation } r \\ 0, & \text{otherwise} \end{cases}$$

product placement binary variable, for all $i = 1, \ldots, S$, $j = 1, \ldots, P$, $r \in \{0, 1\}$: $x_{ijr} \in \{0, 1\}$

f_{ijr} the number of SKUs of the product j on the shelf i on orientation r

$$y_j = \begin{cases} 0, & \text{if product } j \text{ is on front orientation} \\ 1, & \text{if product } j \text{ is on side orientation} \end{cases}$$

orientation of the product j, for all $j = 1, \ldots, P$: $y_j \in \{0, 1\}$

Heuristics parameters:

x_{ij} sequence of shelf allocations

f_{ij} sequence of product allocations

Problem definition

In supply chain logistics, zone picking works in a similar way to a regular assembly line. Cartons or other containers are transferred from zone to zone by a manual cart or collaborative mobile robot, where

SKUs are subsequently loaded from each zone. Picking is completed once the order is fulfilled and the items are transported to packaging and shipping regions. Zone picking is a sort of order which entails dividing stock-keeping units (SKUs) into a number of distinct zones, with distribution centre staff teams assigned to pick up orders from within each zone. Each picker is assigned to its own zone. They manage and complete SKUs within that zone for each order.

In terms of meeting client requests and conducting successful decision-making processes, distributors acknowledge the importance of practical modelling approaches. In a distribution centre, distributors must assign rack shelf space to the products that must be placed on rack shelves.

The problem investigated in this research has been formulated in Czerniachowska et al. (2022 a, b). In this research, we modelled the problem as MILP model. So it was possible to solve it using commercial or non-commercial solvers optimally. Another problem formulation which considers vertical product categories and vertical position effect is presented in Czerniachowska and Hernes (2020 a, b), Czerniachowska and Hernes (2021). Moreover, linearization techniques for the transformation non-linear shelf space or storage location problems are presented in Czerniachowska and Lutosławski (2022).

When allocating products on racks very frequently, it is necessary to categorize products vertically, based on one characteristic, and categorize the same products horizontally, based on another characteristic. Such models allow doing this.

The subject of dimensioning shelf space on the rack is addressed in this study by a rack that is spatially separated into vertical groups to help the picker choose the product for the container with the order. Distributors arrange their products on shelves in a variety of ways. They may categorize product packaging into notable horizontal or vertical subgroups based on product types, quantities, or weight. Each shelf in an investigated rack is horizontally marked, and each shelf is given a specific tag. The shelves of the rack are also marked vertically, which implies that a tag is provided to each category and applied to all shelves. These tags identify which products are associated with a specific category region. When the container arrives in the picker's zone, the picker will be able to immediately recognize each product type on the rack and pick it up and place it in the container.

The primary issue can be described this way: a specified number of products must be placed on the shelves of a zone's rack. The products on the shelf are divided into vertical groups. Each vertical category is assigned to a rack by the wholesaler, who places the category with the smallest size on the rack. Distribution centres can stay more organized with such categorization. In order to build relevant racks, distributors often need to contact prospective buyers or assess past data about the movement of merchandise. To maximize total movement/profit, estimate the required shelf space for each category on a rack that defines the number of SKUs for each product.

A range of product brands is commonly found in distribution centres. The basis for how pickers search for the products is product categorizing and tagging (or product hierarchy). It presents a rack that links things based on their characteristics to categorize products, product classes or groups that are used. Each product category is presented in a vertical way. The products and shelves are also labelled horizontally. Each product may have multiple tags at the same time. Several tags may be provided at the same time on each shelf. Tagging improves the efficiency of all pickers in DC. Therefore the sophisticated categorization of intuitive, searchable navigation is important in terms of the picker experience.

As an example of shelf tags, consider the following:

- a shelf is used for counted/measured/heavy/light products;
- a shelf is used for a specific product package (without package, box, can, bottle, or plastic bag);
- a shelf is located at touch/eye/hat level.

As an example, consider the following. We create three alternative tags $b_t^n = \{H; H^+; V^+\}$ in this research. Tags can be used to label shelves and products, and each shelf or single product can have one or more tags assigned:

- H – In the horizontal layout, the shelf is reserved for goods of a specified selling category (cell in countable pieces, cut and cell in metres).
- H^+ – The shelf in the horizontal layout is for goods that must be put in the specific levels (floor/touch/eye/hat level). This means that some products are positioned at the level to make them easier for pickers to be found. Other products that are not area-specific may be placed at these tiers as well.
- V^+ – The shelf in the vertical pattern is assigned to a specific product category. In terms of vertical product categories, all shelves might be organized according to the product weight, type, colour or package.

In the researched scenario, Fig. 1 displays the distinctiveness of the vertical and horizontal bands on a rack. The vertical classification of products into two categories is an example. The lowest shelf is reserved for measured items (such as cables). Countable in pieces, products can be placed on other shelves.

The following levels are stated in the given rack: lowest level for measured items, touch level and eye level for countable items, and the highest shelf is intestinally left without a tag (all countable products can be placed there). Some of the products on a rack are assigned the following tags:

- there are two vertical categories (V^+) on the rack;
- the lowest shelf is for measured products of both categories (H);
- all shelves except the lowest one are for countable products of both categories (H);
- the product D in the first category (V^+) is a countable (H) and placed at eye-level (H^+);
- the product L in the second category (V^+) is a countable (H) and placed at touch level (H^+).

The cornerstone of a strong DC is organizing the products. Various tags $b_t^n = \{H; H^+; V^+\}$ could be added to the products at the same time. Only one form of grouping is defined by the tag. The defined tag could be segmented. Because some products can be grouped together, they should all be placed on the same shelf (e.g. charger and cable). Distributors can better avoid out-of-stock situations by setting aside extra applicable products on shelves. Some products may not be assigned to any shelf level; instead, only a vertical group should be formed. Product classification is essential for a successful business (Fig. 1).

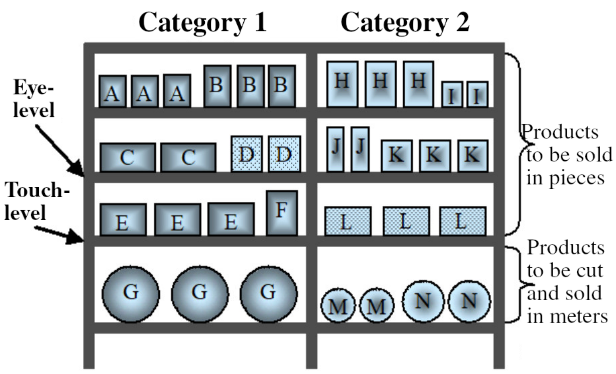

Fig. 1. Rack with vertical and horizontal bands: 2 vertical categories (dark, light colours), 2 horizontal (products in pieces, metres) categories

Several shelves reflect a single product category. Each object might be placed on several shelves. Multiple SKUs are possible for a product. The number of product SKUs is important to manufacturers and distributors. The distributor determines the minimum and maximum number of shelves for each product on the shelf, as well as the minimum and maximum number of SKUs for each product on the shelf, in order to make the commodities visible to the picker. There's no doubt that the distributor's team devotes a significant amount of time, money, and effort to determining the appropriate shelf share for each product. The most popular brands receive the most SKUs. The supply limit of a product determines its maximum availability if it is placed on many shelves. The category size is adjustable by varying the number of products in the defined category.

If the product is distributed in a cartoon box, there are four methods to display it on the shelf: front, side, top, and tilt. But in this research, only two of them are used: rotating it on the front or side. Because the width and depth of bottles are the same, and turning the bottle does not reduce the total width occupied by objects on the shelf, orientation is obviously not applied. All goods are orientated in the front by default. Flipping the product 180 degrees is not considered. As a result, based on the packaging and label visibility displayed on the package, the orientation binary parameter determines whether the product can be laid on its second side orientation.

Only the front exposed SKU row is investigated in this study. The quantity of vertical and depth SKUs is not taken into account. The shelf depth changes because the bottom shelves of a rack are generally deeper, but the product depth and shelf depth are only evaluated for the front SKU row. If the product's depth on the shelf is exceeded in both front and side orientations, but space is still available, the object may be rotated or moved to a deeper shelf.

To overcome the issue, the distributor must first make a decision whether the product should be positioned on the shelf, then define the number of SKUs that should be on each shelf, then decide whether it should be placed on the front or side orientation, and consider a set of constraints classified into the following: shelf constraints, product constraints, orientation constraints, and bands constraints. The purpose of space distribution and product placement on the rack is to increase overall rack movement and profit. Manufacturers gain from having a greater number of SKUs for their brands. They may generate more sales in the distribution centre with additional shelf space on the rack.

For the space distribution problem specification provided, we introduce the notion using the decision variables mentioned below:

- x_{ijr} – if the product j is located on the shelf i on orientation r;
- f_{ijr} – the number of SKUs of the product j on the shelf i on orientation r;
- y_j – orientation of the product j.

Problem formulation

We use the linear version of the problem formulation from Czerniachowska et al. (2022a). The problem can be formulated as follows:

$$\max \sum_{j=1}^{P} \sum_{i=1}^{S} \sum_{r=0}^{1} p_j^u f_{ijr} \tag{1}$$

subject to:

Rack shelf constraints:

- shelf length limit

$$\forall(i) \left[\sum_{j=1}^{P} \sum_{r=0}^{1} p_{jr}^w f_{ijr} \le s_i^w \right] \tag{2}$$

- shelf depth if a product is placed on front orientation

$$\forall(i, j, p_{j1}^w > s_i^d) \; [f_{ij0} = 0] \tag{3}$$

- shelf depth if a product is placed on side orientation

$$\forall(i, j, p_{j0}^w > s_i^d) \; [f_{ij1} = 0] \tag{4}$$

Product constraints:

- minimum and maximum number of products

$$\forall(i, j) \left[f_j^{\min} x_{ijr} \le \sum_{r=0}^{1} f_{ijr} \le f_j^{\max} x_{ijr} \right] \tag{5}$$

- supply limit if the same product is placed on multiple shelves

$$\forall(j) \left[\sum_{i=1}^{S} \sum_{r=0}^{1} f_{ijr} \le p_j^s \right] \tag{6}$$

- product is placed on the shelf

$$\forall(i, j, r) \; [f_{ijr} \le x_{ijr} f_j^{\max}] \tag{7}$$

- if products are grouped into clusters, they are placed on the same shelf

$$\forall(i)\forall(a, b\colon\; p_a^l = p_b^l,\; a, b = 1, \ldots, P) \left[\sum_{r=0}^{1} x_{iar} = \sum_{r=0}^{1} x_{ibr} \right] \tag{8}$$

Multi-shelves constraints:

- minimum and maximum number of shelves

$$\forall(j) \left[s_j^{\min} \le \sum_{i=1}^{S} \sum_{r=0}^{1} x_{ijr} \le s_j^{\max} \right] \tag{9}$$

- if the product is placed on multiple shelves, the next shelf only is available

$$\forall(j)\forall(a, b : |a - b| \ne 1 \wedge a < b, a, b = 1, \ldots, S) \left[\sum_{r=0}^{1} x_{ajr} + \sum_{r=0}^{1} x_{bjr} \le 1 \right] \tag{10}$$

Orientation constraints:

- side orientation is possible for the product

$$\forall(i, j) \; [y_j \le p_j^{o_2}] \tag{11}$$

- only one orientation (front or side) is available for the product

$$\forall(i, j) \left[\sum_{r=0}^{1} x_{ijr} \le 1 \right] \tag{12}$$

Rack bands constraints:

- tags compatibility for the shelves and products

$$\forall(i, j) \left[\prod_{t=1}^{T} b_{tij}^t \ge \sum_{r=0}^{1} x_{ijr} \right] \tag{13}$$

- minimum category size if the category exists on the shelf

$$\forall(i, k) \left[\left(\sum_{j=1, p_j^k = k}^{P} \sum_{r=0}^{1} p_{jr}^w f_{ijr} \ge [s_i^l \cdot c_k^m] \right) \vee \left(\sum_{j=1, p_j^k = k}^{P} \sum_{r=0}^{1} f_{ijr} = 0 \right) \right] \tag{14}$$

- category size tolerance

$$\forall(k) \left[\max_{i=1,\ldots,S} \left(\sum_{j=1, p_j^k = k}^{P} \sum_{r=0}^{1} p_{jr}^w f_{ijr} \right) - \min_{i=1,\ldots,S} \left(\sum_{j=1, p_j^k = k}^{P} \sum_{r=0}^{1} p_{jr}^w f_{ijr} \right) \le \left[\max_{i=1,\ldots,S} (s_i^l) \cdot c_k^t \right] \right] \tag{15}$$

Relationships constraints:

- SKU relationships

$$\forall(i, j, r) \; [f_{ijr} \ge x_{ijr}] \tag{16}$$

- SKU and orientation relationships

$$\forall(i,j)\ \left[f_{ij0} \leq (1-y_j)f_j^{\max}\right] \tag{17}$$

- SKU and orientation relationships

$$\forall(i,j)\ \left[f_{ij1} \leq y_j f_j^{\max}\right] \tag{18}$$

Decision variables:

- the product placed is on the shelf

$$\forall(i,j,r)\ \left[x_{ijr} \in \{0,1\}\right] \tag{19}$$

- the number of SKUs

$$\forall(i,j,r)\ \left[f_{ijr} = \{f_j^{\min} \ldots f_j^{\max}\}\right] \tag{20}$$

- orientation

$$\forall(j)\ \left[y_j \in \{0,1\}\right] \tag{21}$$

Heuristics development

Heuristics are effective methods for distributors to react to decisions or difficulties. Two heuristics are presented and implemented in this study to solve the shelf space distribution problem on the rack with vertical and horizontal product categories. Two algorithms are described below to tackle the modelled problem. Then we'll compare these algorithms in terms of computation.

Heuristics are regarded as tools for decision-making since they enable distributors to arrive at a conclusion quickly and with little information processing. For the distributor, algorithmic thinking can be extremely valuable. It is possible to develop different heuristics solutions based on the common practical rules in the daily process. Analyzing the pros and cons of common practices help to make decisions in more procedural ways. Along these lines, the proposed heuristics solution procedure is depicted.

Let x_{ij} is a sequence of shelf allocations of the rack, consequently, $x_{ij} = \sum_{r=0}^{1} x_{ijr}$ binary value indicates if the product is placed on the shelf of the rack. Let f_{ij} is a sequence of product allocations on the rack, consequently $f_{ij} = \sum_{r=0}^{1} f_{ijr}$, a positive integer value indicates the number of SKUs of the product is placed on the shelf. Variable y_j means to determine the front or side orientation of each product.

- Based on the orientation (11)–(12) and cluster (8) restrictions, determine how many sets of shelf allocation sequences there may be for each of the product orientations (front or side).
- If there are too many sets of shelf allocation sequences, analyze the input information and find less proficient sets of sequences with the help of intuitions and product distribution expertise. Exclude shelf allocation sequences that are predicted to produce poor results due to the category size tolerance (15) and minimum category size (14) constraints. The purpose of this stage is to intuitively choose a set of shelf allocation sequences that should yield a favourable result. Later on, they will be processed. "Generate sets of shelf allocation sequences by 2 or 4 products," "don't generate sets of shelf allocation sequences by 1 or 3 or 5 products," etc., should be the output of this stage. In addition, there could be a variety of reasons why distributors choose to assign or not allocate products on shelves or to allocate more products on one shelf than the other.
- Create a set of shelf allocation sequences that enables placing the products on the shelves with consideration to the compatibility tags $b_t^n = \{H; H^+; V^+\}$, i.e. arrange the initially found shelf allocation sequences on each shelf and exclude incorrect product to shelf allocations, using the sets of shelf allocation sequences from the previous stage. After that, eliminate any allocations that do not meet the shelf tag compatibility (13), minimum and maximum number of shelves (9) or next shelf (10) requirements.
- Create a set of product allocation sequences for each shelf sequence, taking into account product allocation (7), minimum and maximum product numbers (5), shelf length (2), shelf depth (3)–(4), and supply limit (6) requirements.
- Remove the sequences that do not satisfy the minimum category size (14) and category size tolerance (15) requirements from the list of product allocation sequences that have been accomplished.
- Because the remainder of the requirements of the relationship in this approach will be satisfied as well, these procedures do not entail checking them.

If, after implementing the steps described previously, there are still too many allocations, return to step 2 and generate less number of sets of shelf allocation sequences. Next, repeat all steps below. Try to solve the problem partly by generating fewer shelf allocation sequences with different product quantities. If the solution quality obtained in the end is not satisfying, return to step 2 and try the method with other product quantities.

After these preparation processes are done, the number of shelf and product sequences can be estimated. This is required in order to estimate the number of sequences that can be generated and checked in the primary solution-finding phase in a reasonable amount of time. The point is that distributors must decide next how they want to solve the defined problem and which method to be used could be appropriate to achieve the goal.

As a result, based on the intuitive retail experience, just a percentage of all conceivable sequences that will produce a satisfying result are generated. In addition, utilizing the minimum and the maximum numbers of product parameters, the future predicted movement/profit of a product allocation sequence might be estimated in advance. As a result, various steering parameters could be altered to reduce the algorithm's execution time. When the quality of the steering parameters is low, it results in a variety of expenses. Therefore the steering parameters should be maintained in order to save time and money.

The following are the steering parameters:

- Parameter 1: The list of numbers of products that are used in creating sequences of shelf allocations.
- Parameter 2: Number of created product allocations to be checked on each shelf.
- Parameter 3: Number of created product allocations to be checked in each category.
- Parameters 4, 5: Minimum and maximum widths for each category between which solution is predicted to be found. The sum of maximum widths for each category is allowed to exceed the shelf width. This allows us to check more allocations and obtain a better solution. The solutions outside the defined category widths are not considered.
- Parameter 6: Input movement/profit for each category to be checked; this means that the product allocations with the profit below this input level are not considered.
- Parameter 7: Total movement/profit for an end solution across all shelves; this means that allocations with a profit below this input level are not taken into account.

Each shelf allocation that meets the stated criteria will be investigated, but steering parameters will reduce the number of generated product allocations, each of which has a different number of SKUs. The above-enumerated steering parameters must be thoroughly examined because, in some cases, small size and lower total movement/profit of one category may result in a significantly better movement/profit of the other category, i.e. the total movement/profit will be greater than if each of the categories occupied roughly equal space. There is a larger field for the experiment with steering parameters if there are more categories on the rack.

Later phases will minimize the number of product allocations according to expertise rules established for each heuristic so that just a small portion of them will be examined. Then, using the produced product allocations, choose the best solution among them.

The developed heuristics execute as follows.

- Heuristics H1 – For each parent shelf allocation for each shelf in each category, create product allocations so that they will be in not descending order of category width, next in not ascending order of category movement/profit. Take only the product sequences inside the category width range defined by parameters 4 and 5. Exclude the product sequences with the category movement/profit below one defined by parameter 6. Take the appropriate number of product allocations on each shelf defined by the steering parameter 2. At this stage, product sequences on each shelf for each category are generated. For each movement/profit, take one allocation with the minimum category widths. Exclude the rest allocations. Combine them with the consequent product sequences of other categories. The resulting product sequences sort in not ascending order of profit ratio; next, sort the parts inside sorted ones in not ascending order of movement/profit. Take the defined by the steering parameter 3 number of product allocations for each category. Take only the product sequences with a total movement/profit higher than defined by parameter 7. Find the solution with the maximum total profit.
- Heuristics H2 – For each parent shelf allocation for each shelf in each category, create product allocations so that they will be in not descending order of category width, next in not ascending order of category movement/profit. Take only the product sequences inside the category width range defined by parameters 4 and 5. Exclude the product sequences with the category movement/profit below one defined by parameter 6. Take the appropriate number of product allocations on each shelf defined by the steering parameter 2. At this stage, product sequences on each shelf for each category are generated. For each category width, take one allocation with the maximum movement/profit. Exclude the rest allocations. Combine them with the consequent product sequences of other categories. The resulting product sequences sort in not ascending order of profit ratio; next, sort the parts inside sorted ones in not ascending order of movement/profit. Take the defined by the steering parameter 3 number of product allocations for each category. Take only the product sequences with a total movement/profit higher than defined by parameter 7. Find the solution with the maximum total profit.

N.B. Parameter 1 is used for the creation of shelf sequences before creating the product sequences. Parameter 1 is not mentioned because heuristics H1 and H2 explain how to handle product sequences. Heuristics H1 and H2 differ from each other by grouping options (underscored values). Thanks to this rule, a large amount of partial solutions are excluded, and the most reasonable one is left. As a result, different steering parameter values may be applied to them, resulting in varied effects. Profit ratio refers to the ratio of total movement/profit of products distributed on a rack divided by occupied space, which excludes open space on each shelf from computations.

Choosing the right strategy is often crucial to finding the best results. Distributors value unique and fresh concepts. In retail, frequently, there is no need to get absolutely the best possible optimal solution. The product shelf space distribution task can be completed using the step-by-step approach outlined above. Because the processes are simple and straightforward, the initial rack space distribution problem may be split down into components.

When problems are simple and straightforward, the solution can be achieved very quickly. A difficult task, on the other hand, necessitates a thorough approach and deep field knowledge in order to identify the appropriate solution. Therefore different techniques are of great interest.

Computational experiment

The optimal solution could not be identified in all problems since not all problems can be considered linear. But in our case, the problem is modelled as a linear one, so an optimal solution is available by the CPLEX solver. In order to apply these heuristics to more complex rack space distribution situations in the future, we explored if the recommended principles inside the heuristics provide good outcomes. Therefore we compare the heuristic solution with the optimal one. The CPLEX solver is used to ensure that heuristics produce the correct result while solving a problem.

Experiments were conducted on 10, 15, ..., 30-product sets, which must be allocated on a 4-shelf rack with shelf lengths 250 cm, 375 cm, 500 cm, 625 cm, and 750 cm. Product sets with different product parameters, such as width, depth, movement/profit, etc., were generated. For 10, 15, 20 product sets, 2 vertical categories on the rack were created. For 25 and 30 product sets, 3 vertical categories on the rack were created.

The experiments were conducted using the computer with the following parameters:

- Processor: AMD Ryzen 5 1600 Six-Core Processor 3.20 GHz
- RAM: 16 GB.
- System type: 64-bit Operation System, x64-based processor
- Windows 10

Heuristics were created in MS SQL Server 2008 R2. Microsoft SQL Server Management Studio: 10.50.4000.0. The optimal solution was found using a commercial solver – IBM ILOG CPLEX Optimization Studio Version: 12.7.1.0.

The is no solution for the 25 product set on 750 cm shelves that could not be found because the minimum category size constraint (14) for category 2 on the lowest shelf is not satisfied even for the maximum possible number of items on the shelf.

The performance of the created heuristics H1 and H2 versus the commercial CPLEX solver solution is shown in Table 1. The profit ratio in the columns denotes the difference between the optimal movement/profit obtained by the CPLEX solver and the movement/profit found using heuristics. For each heuristic in each test instance, such a measure has been calculated.

It could be observed that heuristics H1 and H2 found the optimal solution for 10 from 24 test instances. For the set of 30 products, heuristics H1 found an optimal solution for the 250 cm shelves, but heuristics H2 found an optimal solution for the other 750 cm shelves. This proves that both heuristics are useful to be implemented as that deal differently with different test instances. Both heuristics found the solution, the quality of which is above 97%. The average profit ratio of both heuristics is more than 99%. This demonstrates the validity of the suggested heuristics and the propriety of the steering parameters, which significantly minimize the number of solutions to be supplied without compromising the outcome. Only profitable solutions are developed and evaluated in the heuristics' subsequent steps.

H1 was slightly better than heuristics H2, having an average profit ratio of 99.21% compared to 99.12% of H2. For 10 and 15 product sets, both heuristics found optimal solutions for 8 from 10 test instances. For the instances with 2 categories (10, 15, 20 products), both heuristics achieved the same profit ratio. The differences started on larger test instances where products were divided into 3 categories.

The CPLEX solution time ranged from 1 to 17 seconds. The fastest solutions were found by both heuristics in 0.01 minutes. Heuristics H1 was a bit slower, with an average time of 1.36 minutes compared to

Table 1
Performance of the developed heuristics

Prod.	Shelf width	Profit ratio of H1	Profit ratio of H2	Time of H1 [min]	Time of H2 [min]
10	250	100.00%	100.00%	0.01	0.01
	375	100.00%	100.00%	0.03	0.01
	500	100.00%	100.00%	0.03	0.02
	625	100.00%	100.00%	0.03	0.02
	750	99.52%	99.52%	0.04	0.02
15	250	100.00%	100.00%	0.12	0.07
	375	100.00%	100.00%	0.10	0.10
	500	99.12%	99.12%	1.62	1.54
	625	100.00%	100.00%	0.04	0.04
	750	100.00%	100.00%	0.20	0.19
20	250	98.94%	98.94%	0.46	0.43
	375	98.31%	98.31%	1.81	1.72
	500	99.02%	99.02%	2.96	2.38
	625	99.43%	99.43%	2.59	0.93
	750	97.27%	97.27%	1.63	1.57
25	250	98.79%	97.85%	0.18	0.14
	375	98.22%	98.16%	1.02	0.54
	500	99.51%	99.32%	0.46	1.07
	625	97.07%	97.40%	3.59	2.31
	750				
30	250	100.00%	99.67%	0.40	0.14
	375	100.00%	100.00%	1.03	0.48
	500	99.17%	99.17%	0.34	0.15
	625	97.28%	97.28%	5.55	0.50
	750	99.45%	100.00%	8.31	1.67
Min		97.07%	97.27%	0.01	0.01
Avg		99.21%	99.19%	1.36	0.67
Max		100.00%	100.00%	8.31	2.38

Table 2
Percentage of created product allocations to be checked on each shelf (steering parameter 2) of the developed heuristics H1

Prod.	Shelf width	Checked alloc. on shelf 1	Checked alloc. on shelf 2	Checked alloc. on shelf 3	Checked alloc. on shelf 4
10	250	100.00%	100.00%	100.00%	100.00%
	375	100.00%	100.00%	100.00%	100.00%
	500	100.00%	100.00%	100.00%	100.00%
	625	100.00%	100.00%	100.00%	100.00%
	750	100.00%	100.00%	100.00%	100.00%
15	250	100.00%	100.00%	100.00%	100.00%
	375	100.00%	100.00%	100.00%	100.00%
	500	100.00%	100.00%	100.00%	100.00%
	625	100.00%	100.00%	100.00%	100.00%
	750	100.00%	100.00%	100.00%	100.00%
20	250	100.00%	9.65%	25.16%	33.04%
	375	100.00%	6.22%	7.95%	2.99%
	500	100.00%	2.00%	3.71%	0.70%
	625	100.00%	0.07%	3.08%	0.11%
	750	100.00%	0.02%	2.89%	0.06%
25	250	100.00%	2.51%	11.27%	11.53%
	375	80.00%	0.41%	4.20%	3.28%
	500	100.00%	6.56%	1.92%	1.75%
	625	42.92%	3.02%	0.99%	0.11%
	750				
30	250	100.00%	20.58%	37.34%	20.08%
	375	64.94%	3.63%	14.22%	4.81%
	500	100.00%	0.61%	4.19%	1.04%
	625	35.46%	0.16%	1.66%	0.33%
	750	28.17%	0.05%	0.16%	0.13%
Min		28.17%	0.02%	0.16%	0.06%
Avg		89.65%	43.98%	46.61%	45.00%
Max		100.00%	100.00%	100.00%	100.00%

the 0.67 minutes for heuristics H2. The biggest instance was solved in 8.31 minutes by heuristics H1, but heuristics H1 used only 1.67 minutes for the same instance. This proves the usage of different grouping options implemented in heuristics.

Tables 2–3 and Tables 4–5 present the steering parameters of the developed heuristics H1 and H2 correspondingly. The percentage in the tables refers to the number of solutions received after processing the solution set with the reduction settings, not to the comparison of checked solutions to all potential solutions. So, initially, the reduced number of solutions were created on shelf sequences on the defined range of product numbers (steering parameter 1), for the range of category widths (steering parameters 4, 5), which are higher than movement/profit for each category (steering parameter 6), and for which the overall movement/profit is higher than one defined by steering parameter 7. The number of solutions to compare is obtained at this stage. Following that, the reduced number of solutions is evaluated for each shelf (steering parameter 2) and each category (steering parameter 3). Then, in the columns "checked allocations on

Table 3
Percentage of created product allocations to be checked in each category (steering parameter 3) of the developed heuristics H1

Prod.	Shelf width	Checked alloc. for category 1	Checked alloc. for category 2	Checked alloc. for category 3
10	250	100.00%	100.00%	
	375	100.00%	100.00%	
	500	100.00%	100.00%	
	625	100.00%	100.00%	
	750	100.00%	100.00%	
15	250	100.00%	100.00%	
	375	100.00%	100.00%	
	500	100.00%	100.00%	
	625	100.00%	100.00%	
	750	100.00%	100.00%	
20	250	100.00%	100.00%	
	375	100.00%	100.00%	
	500	100.00%	100.00%	
	625	100.00%	100.00%	
	750	100.00%	100.00%	
25	250	22.11%	81.14%	0.11%
	375	4.83%	35.34%	0.04%
	500	0.98%	0.47%	1.67%
	625	0.04%	3.46%	0.39%
	750			
30	250	3.35%	6.19%	45.05%
	375	0.72%	0.85%	6.43%
	500	8.77%	64.43%	0.19%
	625	0.65%	0.54%	0.14%
	750	9.73%	6.39%	35.84%
Min		0.04%	0.47%	0.04%
Avg		64.63%	70.78%	9.99%
Max		100.00%	100.00%	45.05%

Table 4
Percentage of created product allocations to be checked on each shelf (steering parameter 2) of the developed heuristics H2

Prod.	Shelf width	Checked alloc. on shelf 1	Checked alloc. on shelf 2	Checked alloc. on shelf 3	Checked alloc. on shelf 4
10	250	100.00%	100.00%	100.00%	100.00%
	375	100.00%	100.00%	100.00%	100.00%
	500	100.00%	100.00%	100.00%	100.00%
	625	100.00%	100.00%	100.00%	100.00%
	750	100.00%	100.00%	100.00%	100.00%
15	250	100.00%	100.00%	100.00%	100.00%
	375	100.00%	100.00%	100.00%	100.00%
	500	100.00%	100.00%	100.00%	100.00%
	625	100.00%	100.00%	100.00%	100.00%
	750	100.00%	100.00%	100.00%	100.00%
20	250	100.00%	9.65%	25.16%	33.04%
	375	100.00%	6.22%	7.95%	2.99%
	500	100.00%	2.00%	3.71%	0.70%
	625	100.00%	0.07%	3.08%	0.11%
	750	100.00%	0.02%	2.89%	0.06%
25	250	100.00%	2.51%	11.27%	11.53%
	375	80.00%	0.41%	4.20%	3.28%
	500	100.00%	6.56%	1.92%	1.75%
	625	42.92%	3.02%	0.99%	0.11%
	750				
30	250	100.00%	20.58%	37.34%	20.08%
	375	64.94%	3.63%	14.22%	4.81%
	500	100.00%	0.61%	4.19%	1.04%
	625	35.46%	0.16%	1.66%	0.33%
	750	100.00%	1.31%	0.78%	0.89%
Min		35.46%	0.02%	0.78%	0.06%
Avg		92.64%	44.03%	46.64%	45.03%
Max		100.00%	100.00%	100.00%	100.00%

a shelf" and "checked allocations for category", the ratios of taken solutions to total reduced solutions are determined. Therefore, 100% in the table signifies that reducing the solution numbers using steering parameters 1, 4–7 is sufficient, and all solutions that could be generated according to the steering parameters were checked.

For the smallest instances for 10 and 15 product sets all solutions received after reduction by steering parameters 1, 4–7 on shelf and category were checked. Steering parameters were not applied. For the set of 20 products, additionally, steering parameter 2, which decreases the number of solutions to be checked on each shelf, was applied. The steering parameter 3 was still not needed.

For larger instances for 25 and 30 product sets which were divided into 3 categories, the additional steering parameters 2 and 3 were required. As it could be observed, even 0.02% of the reduced numbers of solutions on shelves for both heuristics were enough to

Table 5
Percentage of created product allocations to be checked in each category (steering parameter 3) of the developed heuristics H2

Prod.	Shelf width	Checked alloc. for cat. 1	Checked alloc. for cat. 2	Checked alloc. for cat. 3
10	250	100.00%	100.00%	
	375	100.00%	100.00%	
	500	100.00%	100.00%	
	625	100.00%	100.00%	
	750	100.00%	100.00%	
15	250	100.00%	100.00%	
	375	100.00%	100.00%	
	500	100.00%	100.00%	
	625	100.00%	100.00%	
	750	100.00%	100.00%	
20	250	100.00%	100.00%	
	375	100.00%	100.00%	
	500	100.00%	100.00%	
	625	100.00%	100.00%	
	750	100.00%	100.00%	
25	250	100.00%	100.00%	0.54%
	375	100.00%	100.00%	0.40%
	500	71.94%	77.52%	58.82%
	625	77.52%	100.00%	100.00%
	750			
30	250	100.00%	100.00%	55.56%
	375	100.00%	100.00%	90.09%
	500	100.00%	100.00%	55.87%
	625	100.00%	100.00%	100.00%
	750	100.00%	100.00%	100.00%
Min		71.94%	77.52%	0.40%
Avg		97.89%	99.06%	62.36%
Max		100.00%	100.00%	100.00%

check to get the result. A similar situation was with the reduced numbers of solutions for categories where even 0.04% for heuristics H1 and 0.40% for heuristics H2 were enough.

The reduction rule of usage steering parameters 2 and 3 is the following. If the final solution can be obtained quickly enough, we do not reduce the number of solutions with the steering parameters 2 and 3 in order to obtain a higher-quality result.

The percentage of the number of checked allocations on various shelves was 0.02% to 100.00% for both heuristics. The percentage of numbers of checked allocations in various categories was 0.04% (for heuristics H1) and 0.40% (for heuristics H2) to 100.00%. Because the same input parameters were utilized for both heuristics, the numbers of steering parameters in Table 2 and Table 3 were very similar for both heuristics. The idea was to compare the heuristics themselves in this way because a larger number of solution numbers may result in a better final solution, but it would take longer. We attempted to utilize the same beginning parameters for both heuristics where it was possible. The differences occurred when the steering parameters, which were good for one heuristics caused a very long computation time or the inexistence of any final solution after the execution of the other heuristics.

The generated heuristics' movement/profit steering parameters are shown in Table 6. They were the same for both heuristics. The input movement/profit (the last column) below which the single solutions for a category were not checked is calculated on the basis of the sum of average profits of all shelves of all categories in all reduced partial solutions received after applying steering parameters 1, 6, 7. Two types of the lowest profit to be checked were estimated: the input profit for each category considering all shelves in the category and the total profit considering all shelves and all categories. So, at this point, we can estimate the category's average profit; we could go somewhat lower for one category and significantly higher for another more profitable category. There's no reason to come up with solutions that have a movement/profit that's much lower than the average one across all categories. In both heuristics, this idea is employed.

The input profit ratios greater than 100% signify that only the possible solutions which had greater movement/profits than the average ones were investigated. The value of 100% signifies that the input value for a category or the final result equals the average consequent value. If there were too few partial solutions to be checked, we decreased the input movement/profits and took values which were slightly below the average value. When the instance is large enough, it is advised to increase the input profit in order to decrease the number of solutions. So the proposed method of including/excluding much/less profitable solutions is very valuable.

The profit input ratio was, on average, 110% and varied from 88% for less profitable partial solutions up to 130% for more profitable ones. The lowest profit input ratio was for category 2 and counted at 48%. The highest one was for category 3 and counted 158%. The average values of profit input ratios for 1, 2, 3 categories were 123%, 89%, 96%, consequently.

Table 6
Profit steering parameters of the developed heuristics H1 and H2

Prod.	Shelf width	Profit input ratio for cat. 1	Profit input ratio for cat. 2	Profit input ratio for cat. 3	Profit input ratio
10	250	138%	69%		100%
	375	96%	125%		100%
	500	84%	141%		100%
	625	82%	118%		100%
	750	72%	103%		100%
15	250	130%	91%		102%
	375	139%	108%		122%
	500	137%	95%		120%
	625	144%	98%		123%
	750	146%	93%		123%
20	250	151%	55%		124%
	375	152%	55%		122%
	500	141%	66%		117%
	625	141%	56%		112%
	750	133%	101%		122%
25	250	87%	48%	158%	101%
	375	94%	97%	112%	102%
	500	111%	80%	117%	104%
	625	94%	90%	116%	88%
	750				
30	250	109%	93%	70%	103%
	375	138%	74%	69%	104%
	500	147%	90%	75%	115%
	625	143%	78%	54%	105%
	750	154%	119%	92%	130%
Min		72%	48%	54%	88%
Avg		123%	89%	96%	110%
Max		154%	141%	158%	130%

The total number of shelf allocations in a general case is $(r+1)^{PS} = 3^{PS}$. The product might be allocated in one of three ways, as shown by number 3: (1) if it is not placed on the shelf; (2) if it is placed on the shelf in the front orientation; or (3) if it is placed on the shelf in the side orientation. The total number of product allocations for each number of products in a general case can be calculated as $\prod_{j=1}^{P} (f_j^{\max} - f_j^{\min} + 1)^S$.

Table 7 and Table 8 display the number of created allocations and solutions for heuristics H1 and H2 consequently. The reduced numbers of checked by heuristics H1 and H2 allocations result in the number of solutions (allocations that meet all constraints) in the last column that were found. It could be observed that even the reduced number of allocations allows getting approximately up to 2 million (for heuristics H1)

Table 7
Number of generated allocations steering by heuristics H1

Prod.	Shelf width	Number of generated alloc. to be checked	Number of generated alloc. after grouping option to be checked	Number of solutions
10	250	$3.86 \cdot 10^4$	$3.86 \cdot 10^4$	8
	375	$2.81 \cdot 10^5$	$2.81 \cdot 10^5$	4
	500	$2.86 \cdot 10^4$	$2.86 \cdot 10^4$	30
	625	$1.68 \cdot 10^5$	$1.68 \cdot 10^5$	3 025
	750	$1.75 \cdot 10^4$	$1.75 \cdot 10^4$	15 652
15	250	$4.14 \cdot 10^5$	$4.14 \cdot 10^5$	22 535
	375	$5.63 \cdot 10^4$	$5.63 \cdot 10^4$	464
	500	$5.42 \cdot 10^5$	$5.42 \cdot 10^5$	21 495
	625	$3.24 \cdot 10^4$	$3.24 \cdot 10^4$	179
	750	$6.54 \cdot 10^4$	$6.54 \cdot 10^4$	9 013
20	250	$5.87 \cdot 10^4$	$5.87 \cdot 10^4$	11 150
	375	$1.12 \cdot 10^5$	$1.12 \cdot 10^5$	54 482
	500	$2.81 \cdot 10^5$	$2.81 \cdot 10^5$	239 593
	625	$7.13 \cdot 10^5$	$7.13 \cdot 10^5$	685 948
	750	$4.73 \cdot 10^4$	$4.73 \cdot 10^4$	47 275
25	250	$4.87 \cdot 10^{11}$	$3.92 \cdot 10^8$	636
	375	$1.40 \cdot 10^{13}$	$2.95 \cdot 10^8$	233 672
	500	$1.29 \cdot 10^{12}$	$1.14 \cdot 10^8$	97 961
	625	$1.83 \cdot 10^{13}$	$7.42 \cdot 10^7$	990 995
	750			
30	250	$2.67 \cdot 10^{10}$	$4.65 \cdot 10^8$	75 813
	375	$6.39 \cdot 10^{12}$	$1.15 \cdot 10^9$	217 205
	500	$2.28 \cdot 10^{11}$	$3.13 \cdot 10^8$	28 015
	625	$4.01 \cdot 10^{14}$	$8.58 \cdot 10^9$	1 598 242
	750	$8.98 \cdot 10^9$	$7.46 \cdot 10^7$	2 436 176
Min		$1.75 \cdot 10^4$	$1.75 \cdot 10^4$	4
Avg		$1.84 \cdot 10^{13}$	$4.78 \cdot 10^8$	282 899
Max		$4.01 \cdot 10^{14}$	$8.58 \cdot 10^9$	2 436 176

Table 8
Number of generated allocations steering by heuristics H2

Prod.	Shelf width	Number of generated alloc. to be checked	Number of generated alloc. after grouping option to be checked	Number of solutions
10	250	$1.01 \cdot 10^5$	$2.03 \cdot 10^3$	1
	375	$1.90 \cdot 10^6$	$1.77 \cdot 10^4$	2
	500	$5.46 \cdot 10^4$	$1.37 \cdot 10^3$	3
	625	$3.80 \cdot 10^5$	$6.21 \cdot 10^3$	184
	750	$2.19 \cdot 10^4$	$3.42 \cdot 10^2$	342
15	250	$4.41 \cdot 10^7$	$5.94 \cdot 10^4$	4 295
	375	$1.27 \cdot 10^7$	$2.53 \cdot 10^4$	307
	500	$3.39 \cdot 10^9$	$1.15 \cdot 10^5$	7 629
	625	$1.29 \cdot 10^6$	$1.21 \cdot 10^4$	48
	750	$6.66 \cdot 10^6$	$1.89 \cdot 10^4$	1 801
20	250	$1.20 \cdot 10^6$	$6.55 \cdot 10^3$	1 524
	375	$4.55 \cdot 10^6$	$1.18 \cdot 10^4$	3 322
	500	$2.40 \cdot 10^7$	$8.80 \cdot 10^3$	5 788
	625	$1.00 \cdot 10^8$	$4.00 \cdot 10^3$	2 893
	750	$2.11 \cdot 10^5$	$1.68 \cdot 10^3$	1 677
25	250	$4.87 \cdot 10^{11}$	$1.19 \cdot 10^7$	108
	375	$1.40 \cdot 10^{13}$	$7.01 \cdot 10^6$	10 880
	500	$1.29 \cdot 10^{12}$	$3.05 \cdot 10^6$	286 037
	625	$1.83 \cdot 10^{13}$	$9.61 \cdot 10^5$	632 132
	750			
30	250	$2.67 \cdot 10^{10}$	$1.79 \cdot 10^6$	9 885
	375	$6.39 \cdot 10^{12}$	$1.28 \cdot 10^6$	68 065
	500	$2.28 \cdot 10^{11}$	$4.81 \cdot 10^5$	6 538
	625	$4.01 \cdot 10^{14}$	$4.61 \cdot 10^5$	59 913
	750	$1.28 \cdot 10^{13}$	$9.18 \cdot 10^5$	168 238
Min		$2.19 \cdot 10^4$	$3.42 \cdot 10^2$	1
Avg		$1.90 \cdot 10^{13}$	$1.17 \cdot 10^6$	52 984
Max		$4.01 \cdot 10^{14}$	$1.19 \cdot 10^7$	632 132

and approximately up to 600 thousand (for heuristics H2) of solutions that were checked during the experiment. The number of generated allocations to be checked before and after the grouping option implemented in heuristics shows that it is possible to decrease the numbers of allocations to be checked, for example, from $4.01 \cdot 10^{14}$ to $8.58 \cdot 10^9$ for 30 products set on 625 cm width.

Table 9 displays the numbers of all possible shelf and product allocations in general cases calculated with regard to the formulas given above. For example, the total number of shelf allocations on all shelf widths for the largest instance was $5.80 \cdot 10^{57}$. Similarly, the total number of product allocations on all shelf widths for the largest instance was $1.33 \cdot 10^{156}$. Obviously, there is impossible to generate and check such an amount of allocations. Therefore heuristics are needed. Table 7 and Table 8 report a significantly less number of allocations processed by the heuristics.

Table 9
Numbers of all possible shelf and product allocations in the general case

Products	Number of shelf allocations	Number of product allocations
10	$1.22 \cdot 10^{19}$	$1.10 \cdot 10^{52}$
15	$4.24 \cdot 10^{28}$	$1.15 \cdot 10^{78}$
20	$1.48 \cdot 10^{38}$	$1.21 \cdot 10^{104}$
25	$5.15 \cdot 10^{47}$	$1.27 \cdot 10^{130}$
30	$5.80 \cdot 10^{57}$	$1.33 \cdot 10^{156}$

We demonstrate how shelf space distribution planning may be improved in real DCs using this case study. It could be beneficial for examining certain scenarios in the physical realm (amount of products, categories, shelves, and rack widths).

Conclusion

When opposed to a warehouse, a DC holds things for a shorter amount of time. As a result, the flow velocity via a DC is significantly higher than that through a warehouse. A customer-centric DC serves as a link between a supplier and its clients. While a warehouse's job is to properly store things, a distribution centre's job is to efficiently satisfy client needs.

A distribution facility, not a warehouse, typically ships retail and warehouse orders. A warehouse, on the other hand, does not typically service external consumers, but a distribution centre does. A distribution centre's operations are far more complicated than those of a warehouse. As a result, order fulfilment facilities are outfitted with modern equipment.

With the advancement of AI technology, researchers have been widely applying machine learning to product categorization for various problems. Therefore in this research, the model with vertical and horizontal tags was presented. The product categorization issues are also related to the e-commerce sites.

In this research, we developed two heuristics to solve the proposed problem of dimensioning the shelf space of storage location on the rack with vertical categories on different problem instances. The results obtained by heuristics were compared to the optimal solution obtained by the commercial CPLEX solver. The average profit ratio of heuristics H1 was 99.21%; in consequence, the average profit ratio of heuristics H2 was 99.19%. Ten instances were solved even optimally by both heuristics.

The methods implemented in heuristics allow for a significantly reduced number of allocations to be checked in order to find the final solution. For example, for the largest instance, the such value was reduced from $1.33 \cdot 10^{156}$ to $1.19 \cdot 10^{7}$. The average solution time of heuristics was fast enough and equal to 1.36 minutes for heuristics H1 and 0.67 minutes for heuristics H2.

Information-processing techniques known as heuristics can be helpful in many situations but can also result in mistakes if used incorrectly. Heuristics is a general-purpose principle that is simply a guess about some process. In this research, we show how industry expertise rules could be transformed into heuristics for usage in a distribution centre. They also could improve the business as well as the distributor and manufacturer's position in the market.

One of the key benefits of using heuristics in retail and DC space planning is the following. Heuristics enable quick decision-making. They can consolidate and simplify a lot of information, reducing the amount of time it takes to make a decision.

Cognitive biases have an impact on people. While people's sentiments can be beneficial in other fields, they can also have a negative impact on people's decisions if they block people from seeing the whole situation with the problem. It is known that the human brain uses shortcuts to help people comprehend information more quickly, typically by drawing on their own past feelings and experiences. Therefore, in this case, the good idea is to use heuristics to solve the problem.

In conclusion, it could be highlighted that heuristics have the advantage of making decision-making relatively simple, but there is also a potential drawback: the solution obtained by using heuristics is not always the optimal one.

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