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MODELING OF THE BALL SCREW

The author presents a development of computational model of design of ball screws thread. This model is the basis for computer program, which calculates the geometrical features of the thread for precisely given backlashes and contact angles. The program makes it possible to create a data base of a new generation ball screw of quality competitive to foreign ball screws. The modeling allows one to better select the ball screw and to predict its quality in the early stage of design.

1. Introduction

All the ball thread dimensions (Fig. 1) such as: screw diameter d, lead P, ball diameter d_k, number I_z of loaded turns and the relationship between main radii of the curvature of the screw shaft and the nut, affect rigidity, basic dynamic load rating, mass, efficiency, etc. The screw diameter is calculated on the basis of strength conditions. Good match of these parameters is a confidential know how. For this reason, it is critical to set up a collection of correct acceptable solutions of geometrical parameters of ball screws. Once such a collection is created, i.e. calculated by a computer program, and it makes it possible to selected ball screw dimensions depending on optimisation criteria required by the designer or the customer.

1. Calculating structural thread model

The first step in developing a numerical model is to define decision variables and basic structural parameters. Main decision variables are (see Fig. 1): nominal thread diameter d, lead P, ball diameter d_k, these come from current

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standards. We consider a contact angle α and backlash between the balls Δd_k, as the decision variables. The nominal diameter is the starting value for computation of the calculated diameter $d_{cal} \geq d$. This way, the requirement for backlash between the balls is satisfied. An excessive backlash may lead to drag torque variation exorbitant and thus must be prevented in design [1], [2].

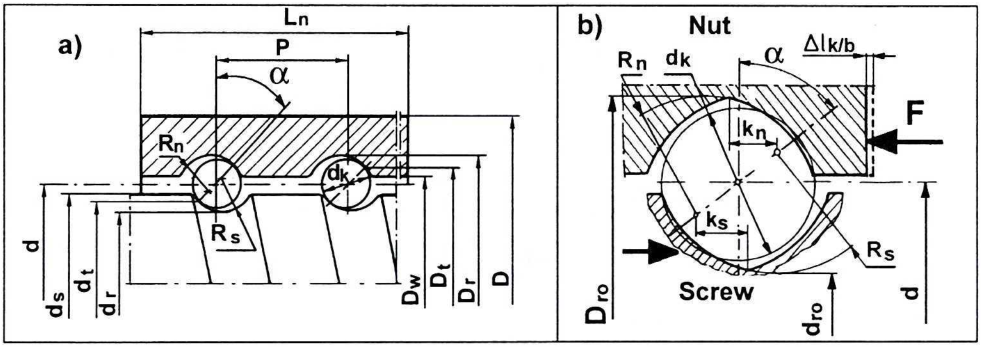

Fig. 1. Basic design thread parameters : a) monoarc thread, b) gothic arc d – nominal diameter, d_k – ball diameter, $R_{s,n}$ – balltrack radius of the screw and nut , respectively, α – contact angle, $k_{s,n}$ – coordinates $R_{s,n}$ radii, P – lead, d_{ro} – screw core diameter calculating, D_{ro} – nut core diameter calculating

Basic conditions and design limitations (besides those on Fig. 1) are:

- $D_w = d + 1.5$, $d_s = d - 1.5$.
- Ratio of balltrack radius to ball diameter called the conformity factor $f_r = 0.51 \div 0.57$; in modern designs we assume $f_r = 0.53 \div 0.55$ when $R_s = R_n = R$.
- Ratio of ball diameter to the lead of screw $d_k / P \leq 0.5$ to ensure a reserve of the material and to avoid an overlay of roots of thread.
- $d_r > 2d_k$; the inequality ensures appropriate proportion between diameter of the screw thread and ball diameter. We calculate the screw diameter from the strength conditions.
- Number of loaded turns of the nut I_z should be in range 2÷6, because when it goes below 2, the sufficient guide length of the screw in nut would not be guarantee. When this value is greater then 6, the load of the extreme coils is so low that the balls practically do not take any load.

Relationships between contact angle α and another dimensions are described by:

$$k_{s,n} = \frac{(2R_{s,n} - d_k)\sin\alpha}{2}, \tag{1}$$

where $\alpha = \arccos \dfrac{R_s + R_n - 0,5(D_{ro} - d_{ro})}{R_s + R_n}$,

the calculated screw core diameter $d_{ro} = d - 2R_s + \dfrac{2k_s}{\tan\alpha}$,

the calculated nut core diameter $D_{ro} = d + 2R_n - \dfrac{2k_n}{\tan\alpha}$.

For monoarc thread there is: $D_{ro} = D_r$ and $d_{ro} = d_r$. The next calculation will be done for gothic arc thread only, because this type of thread is used in modern designs.

Very often we assum that $R_s = R_n = R$. The contact angle applicable in modern design is $\alpha \cong 45^\circ$. This angle determines the value of load for ball screw and its rigidity. When this angle decreases, the rigidity and load capacity decrease too, and the ball screw is not of competitive quality.

3. Calculation of base operation

The calculation is done under the assumptions that the coil load distribution is regular, machining errors are neglected, and the changes of angle α related to the action of the axial force are neglected too. The nut body and screw shaft have the same Young's Modulus and Poisson's ratio. The calculation is based on the ISO standards [3]. Calculation is done for ball screws assemblies with preload. The preload secures work without clearance and appropriate stiffness. The preload force causes an axial displacement of nuts with respect to the screw equal to $\Delta l_{k/b.w}$ (Fig. 2a). The value of displacement of each of the nuts is equal

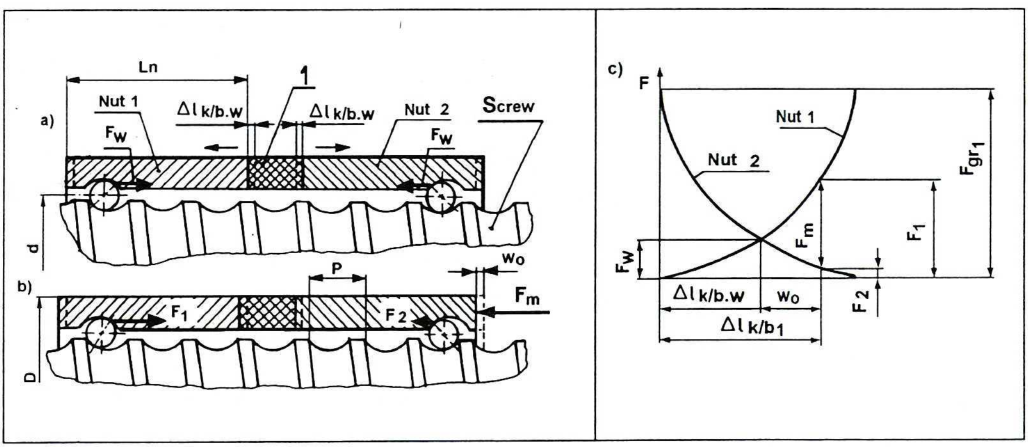

Fig. 2. Axial forces and deflections during: a) preloading, b) loading c) visualisation of the forces and deflections: F_{gr1} – limit load of the Nut 1, $\Delta l_{k/b.w}$ – axial deflection in the ball/balltrack area due to preloading force F_w

to the sum of deflection in screw-ball-nut system after backlash elimination. Preload is most frequently realised by application of a spacer 1 (thick), between the plane of the adhered nuts, or by rotation of both nuts (in both cases the nuts are jostled).

- **Rigidity calculations**

The rigidity of the system composed of a ball screw and the co-operating machine elements (e.g. machine tool) is defined, in general, by the displacement Δl_w resulting from deflections of screw, nut (or nut assembly), screw bearing, nut mounting and displacement of the ball screw system. Those deflections are generated in the balls and their races in the result of axial loading F_m acting a system. Screw deflection, its bearing system, and the nut mounting depend, however, on the type of the machine in which the ball screw is built-in, not on the ball screw itself. This is why the quality measure of the ball screw system in terms of its rigidity refers only to deflection of the screw and nut $\Delta l_{n/s}$ in the area filled with balls, and to Herzian deflection $\Delta l_{k/b}$ in the area of contact of the ball screw race.

We can obtain the axial rigidity of the nut body and the screw in the space with balls from [3]:

$$j_{n/s} = \frac{2\pi I_z P E tg^2\alpha}{\left(\frac{D^2 + D_t^2}{D^2 - D_t^2} + 1\right)10^3}, \tag{2}$$

where: I_z – number of loaded turns of the nut,

P – lead,

E – Young Module,

$D_t = d + d_k \cdot \cos\alpha$ – diameter of load application on the ball nut.

As both nut bodies act like preloaded rings, the rigidity $j_{n/s.w}$ of the ball nuts system is twice at high as that of a single nut [3]:

$$j_{n/s.w} = 2 j_{n/s}. \tag{3}$$

We can calculate the axial deflection in the ball/balltrack area with thread due to the axial force F on one nut as follows [3]:

$$\Delta l_{k/b} = \left(\frac{F}{k I_z}\right)^{2/3}, \tag{4}$$

where: $k = \dfrac{n_{k1} \sin^{5/2}\alpha \cos^{5/2}\lambda}{C_E^3 C_k^{3/2}}$ – rigidity characteristic of one loaded turn [3],

n_{k1} – number of loaded balls per turn,

λ – lead angle,

C_E – material constant,

C_k – geometry factor.

The rigidity of the ball/balltrack area at which the axial force F acts is [2]:

$$j_{k/b} = \sqrt[3]{F(I_z k)^2} \,. \tag{5}$$

In order to obtain high rigidity in the ball/ balltrack area, the nut systems are preloaded with a force F_w (Fig. 2a). We can obtain this force as [3]:

$$F_w \leq 0.15 C_a \,, \tag{6}$$

where: C_a – basic dynamic axial load rating (see later).

We can calculate the axial deflection of the ball/balltrack area due to the preload force F_w from [3]:

$$\Delta l_{k/b.w} = \left(\frac{F_w}{kI_z} \right)^{2/3} . \tag{7}$$

One assumes that the limited load of nut is [3]

$$F_{gr} \leq 3F_w . \tag{8}$$

We can calculate the displacement for load F_m between the limited load F_{gr1} and preload F_w, as [3]:

$$w_o = \frac{F_m}{3kI_z \left(\Delta l_{k/b.w}\right)^{1/2}} \,, \tag{9}$$

and rigidity of the ball/balltrack area:

$$j_{k/b.w} = \frac{F_m}{w_o} . \tag{10}$$

The resultant rigidity j_u of the preloaded ball nut system in the area of double nuts can be calculated from the following formula [3]:

$$\frac{1}{j_u} = \frac{1}{j_{k/b.w}} + \frac{1}{j_{n/s.w}}, \quad \text{from where} \quad j_u = \frac{j_{k/b.w} \cdot j_{n/s.w}}{j_{k/b.w} + j_{n/s.w}} . \tag{11}$$

- **Basic dynamic axial load rating**

When load distribution is optimal (the directions of loads are parallel to the screw shaft and the nut) basic dynamic axial load rating (based on a ball track hardness of 60 HRc) is calculated from [4]:

$$C_a = C_i I_z^{\,0.86} . \tag{12}$$

where: $C_i = C_s \left[1 + \left({C_{ss}}/{C_n} \right)^{10/3} \right]^{-0.3}$ – dynamic axial load rating for the ball screw shaft per single loaded turn. The dynamic axial load rating for the ball screw shaft C_s is [4]:

$$C_s = f_c \left(\cos\alpha\right)^{0.86} n_{k1}^{2/3} d_k^{\,1.8} \tan\alpha \left(\cos\lambda\right)^{1.3} , \tag{13}$$

where:

$$f_c = 9.32 \cdot 10 \left(1 - \frac{\sin\alpha}{3}\right) \frac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1+\gamma)^{\frac{1}{3}}} \left(\frac{1}{1 - \frac{1}{2 f_{rs}}} \right)^{0.41}, \qquad \gamma = \frac{d_k}{d}\cos\alpha,$$

$$C_{ss} / C_n = \left(\frac{1-\gamma}{1+\gamma}\right)^{1,723} \left(\frac{2 - \frac{1}{f_{rn}}}{2 - \frac{1}{f_{rs}}} \right)^{0,41}.$$

Expressions $f_{rs} = \frac{R_s}{d_k}$, $f_{rn} = \frac{R_n}{d_k}$ – define the so called conformity factor. It is assumed that $R_s = R_n = R$, so $f_{rs} = f_{rn} = f_r$.

- **Mass calculations**

The nut dimensions are calculated in an approximate way (we do not take into consideration the dimensions of the nut flange). The nut length is proportional to the number of loaded turns and the lead, and it is increased by the packing ring dimensions:

$$L_n \approx P(I_z + 3). \tag{14}$$

Outer diameter of the ball nut for internal type of ball recirculation can be calculated as follows:

$$D \geq d + 4d_k. \tag{15}$$

Having the dimensions, on can calculate the mass m of the nut.

4. PARKON program description

The computer aided design program PARKON [2] was designed based on the formulas and the model discussed above. The program calculates and prints a table of database where the geometrical features and appropriate structural properties are grouped. The nominal diameter is the starting value for the calculation of diameter $d_{cal} > d$. This way, the requirement for backlash between the balls Δd_k is satisfied. A short part of a printout example is shown below:

PARKON program for design parameters of ball screws

Input data:
Contact angle................................ ....44,9 ≤ Alfa ≤ 45.0 [°]
Assumed total backlash between the balls .DELDK ≅ .10 [mm]
Limiting the relation of the ball diameter to lead dk/P ≤ 0.5
Nominal diameters d [mm]:
20.0 25.0 32.0 40.0 50.0 63.0 80.0 100.0

Lead P [mm]:

5.0 6.0 8.00 10.0 12.0 16.0 20.0 40.0

Number of nut turns Iz:

2 3 4 5 6

Ball diameters dk [mm]:

3.000 3.175 5.556 6.350 9.525 10.000

Conformity factor R/dk = fr:

.52 .53 .54 .55

Ca – basic dynamic axial load rating [N]

k – rigidity characteristic of one turn [N/ μm 3/2]

ju – theoretical rigidity of a double nuts system [N μm]

D – outer diameter of nut [mm]

Ln – length of nut [mm]

m – nut mass [kg]

SPR – efficiency of the ball screw

d	dcal	P	Iz	dk	R	fr	ks,n	Ca	k	ju	D	Ln	m	SPR
20	20.00	5.0	2	3.000	1.56	0.52	0.04	6782	41	525	32.00	25.0	0.08	0.94
						....	....	....						
20	20.00	12.0	6	6.350	3.49	0.55	0.22	24350	15	904	45.40	108.0	1.01	0.99
						....	....	....						
63	63.01	5.0	2	3.000	1.65	0.55	0.11	8469	110	862	75.01	25.0	0.21	0.84
63	63.66	5.0	2	3.175	1.65	0.52	0.04	13084	147	1109	76.36	25.0	0.24	0.85
						....	....	....						
63	64.61	10.0	6	6.350	3.49	0.55	0.22	60188	72	3212	90.01	90.0	2.01	0.96
63	63.00	12.0	2	3.000	1.56	0.52	0.04	12027	150	1348	75.00	60.0	0.52	0.93
						....	....	....						
63	63.46	16.0	2	10.000	5.50	0.55	0.35	40337	51	1136	103.46	80.0	3.01	0.98
63	63.78	16.0	3	3.000	1.56	0.52	0.04	17137	152	2099	75.78	96.0	0.87	0.94
63	63.35	20.0	6	10.000	5.50	0.55	0.35	103546	51	3267	103.35	180.0	6.75	0.99
						....	....	....						
80	80.20	5.0	2	3.000	1.56	0.52	0.04	13318	194	1140	92.20	25.0	0.27	0.81
						....	....	....						
100	101.66	20.0	6	10.000	5.30	0.53	0.21	165406	105	5856	141.66	180.0	9.98	0.98
100	101.66	20.0	6	10.000	5.40	0.54	0.28	148134	96	5349	141.66	180.0	9.98	0.98
100	101.66	20.0	6	10.000	5.50	0.55	0.35	136204	90	5016	141.66	180.0	9.97	0.98

5. Simulation tests

The purpose of testing of the accepted calculated model is verification of the target functions. Figure 3 shows an example that illustrates how the conformity factor and the lead affect the rigidity of ball screws. The analysis was done for turns number $I_z = 3$.

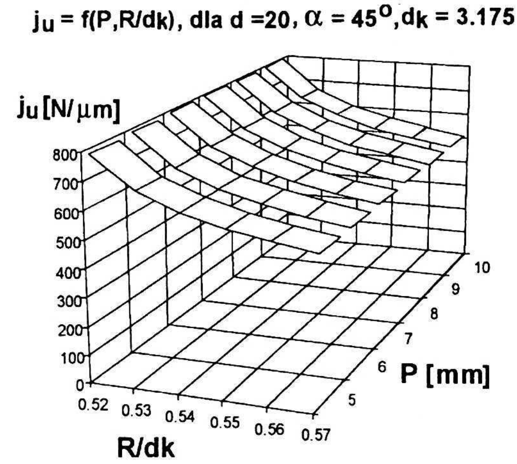

Fig. 3. Rigidity j_u for some leads and conformity factors

An appropriate choice of the values of conformity factor is the main problem when we design thread of a ball screw. The problem consists in maximisation of the load for ball screw when the dimensions of nut remain minimal (dimensions of nuts are proportional to balls diameter and the lead). When we want to increase the load, we should ensure maximum balltrack area. It means that one has to take the conformity factor near 0.5, while the ISO standard recommends this coefficient equals to 0.53÷0.55 like in ball-bearing.

Figures 4÷6 is show results of the simulation research when $f_r = 0.55$ for differences of nut diameters ($d = 20 \div 100$) and nut turns numbers ($I_z = 2 \div 6$).

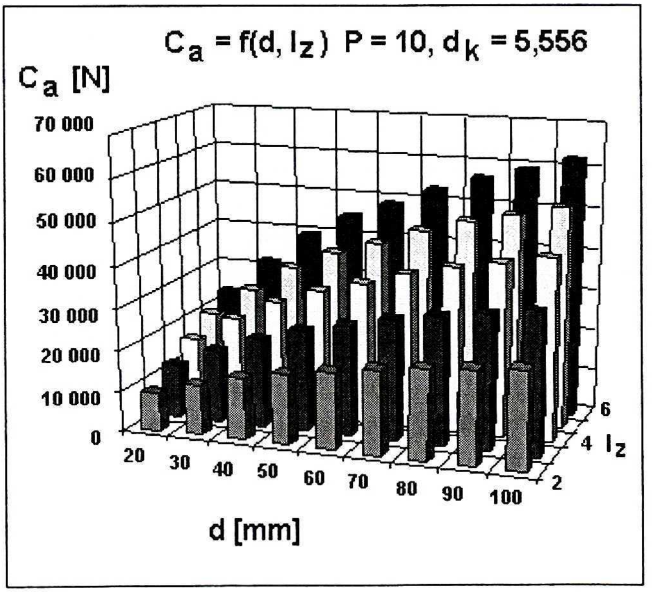

Fig. 4. Basic dynamic axial load rating C_a as function of screw diameter d and nut turns number I_z

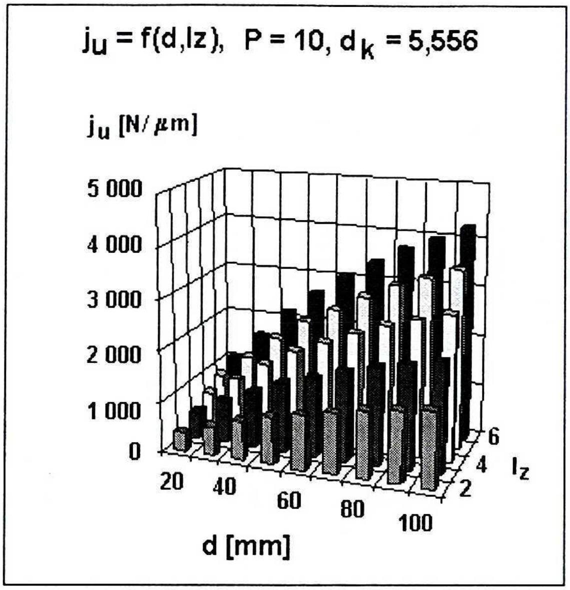

Fig. 5. Rigidity j_u as function of screw diameter d and nut turns number I_z

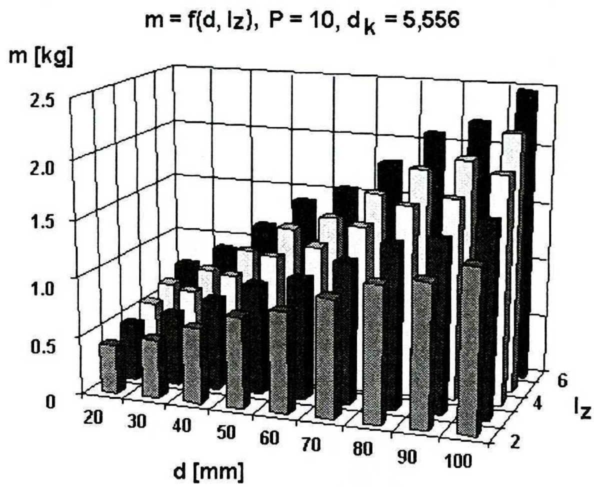

Fig. 6. Screw nut mass as function of diameter d and nut turns number I_z

After deciding on nominal diameter d from strength condition (it depends on mounting method, speed and loading [2]), we can selected a ball screw with other parameters as required (e.g. ball diameter or lead). For comparison, we consider two nominal diameters (d=50, and d=63) for two ball diameters (d_k=3.173, and d_k=9.525). Graphical presentation of the results for dynamic axial load rating are show in Fig. 7 and 8, and the results for nut mass are shown in Fig. 9 and 10.

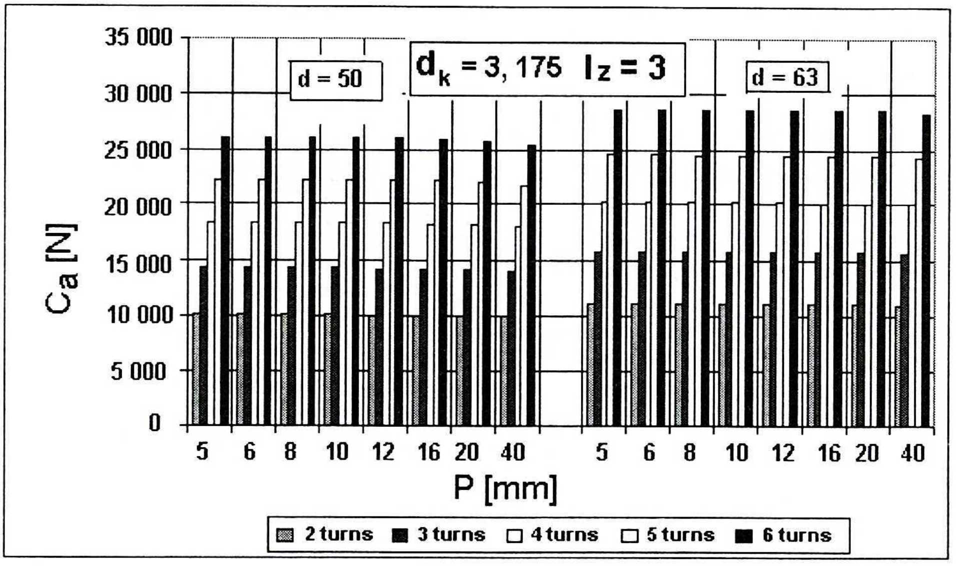

Fig. 7. Relationship between dynamic axial load rating C_a, lead P for nut turns number $I_z = 2 \div 6$, conformity factor $f_r = 0.53$ and ball diameters $d_k = 3.175$

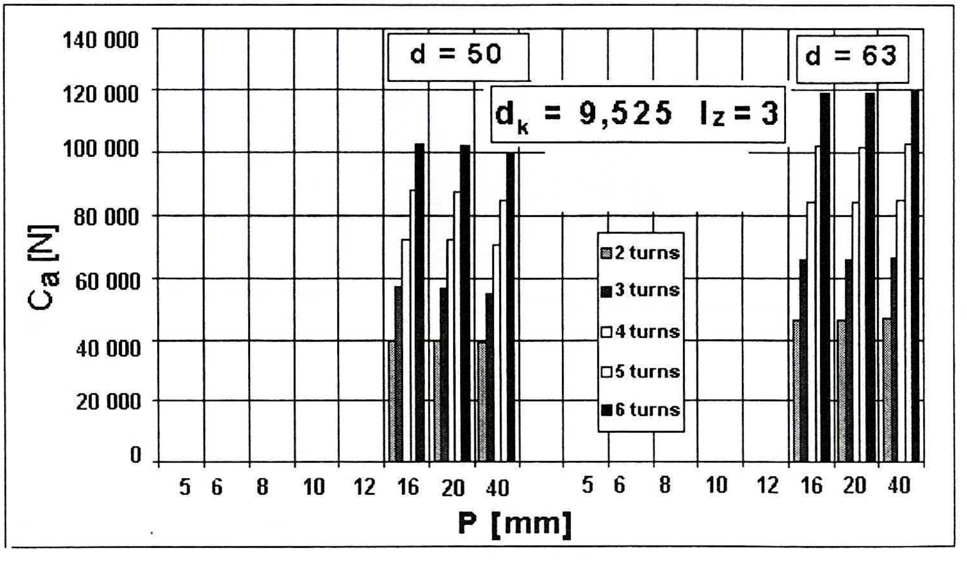

Fig. 8. Relationship between dynamic axial load rating C_a, lead P for nut turns number $I_z = 2 \div 6$, conformity factor $f_r = 0.53$ and ball diameters $d_k = 9.525$

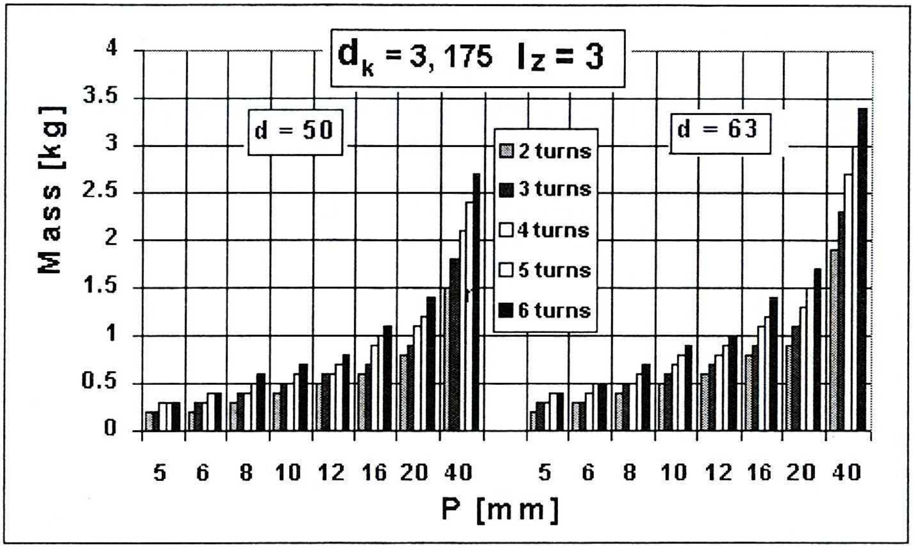

Fig. 9. Relationship between nut mass, lead P, for nut turns number I_z= 2÷6, conformity factor f_r=0,53 and ball diameters d_k = 3,175

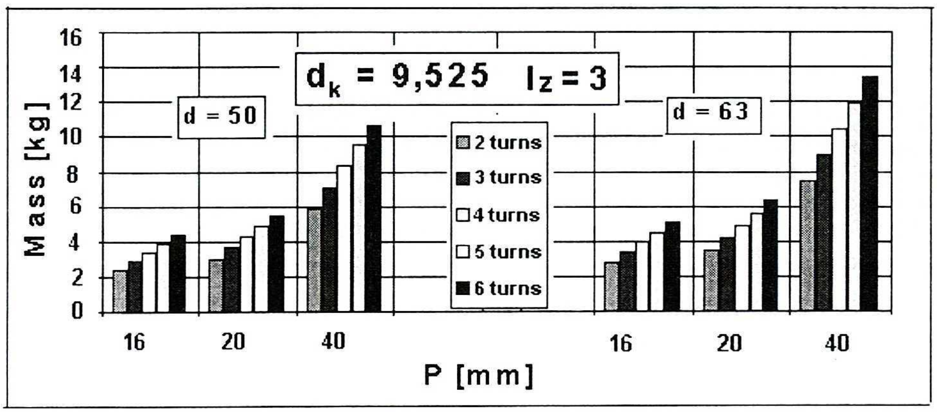

Fig. 10. Relationship between nut mass and lead P, for nut turns number I_z=2÷6, conformity factor f_r = 0,53 and ball diameters d_k = 9,525

6. Conclusions

The numerical model makes it possible to develop a computer program for calculating ball thread dimensions of ball screws. The results of the simulation facilitate selection of the design parameters. When the designer or customer requirements related to the ball screw are known, it is possible to generate a database by means of the PARKON program. From this database we can chose an appropriate ball screw with operating and geometrical parameters exactly as required. The database is significantly larger than factory catalogues issued in Poland, and the program can be used for preparing catalogues as well as factory offers and adverts. The program may be used for both design offices and individual customers.

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Modelowanie przekładni śrubowych kulkowych

S t r e s z c z e n i e

W pracy przedstawiono tworzenie obliczeniowego modelu do projektowania gwintu przekładni śrubowych kulkowych. Model ten jest bazą do programu komputerowego który oblicza właściwości geometryczne gwintu dla założonego luzu osiowego i kąta działania kulek. Program ten umożliwia tworzenie basy danych przekładni nowej generacji o jakości konkurencyjnej do przekładni zagranicznych. Modelowanie ułatwia selekcję i przewidywanie jakości przekładni we wczesnej fazie projektowania.