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# "NODALISED BEAM" METHOD FOR VIBROINSULATION OF MANUALLY OPERATED TOOLS

In the paper, the authors discuss the possibility to apply the "Nodalised Beam" method for vibroinsulation of manually operated tools. They indicate the difficulties in applying the original method for this purpose. On the bases of the reciprocity principle, the authors propose a method for modifying the system that allows them to avoid the mentioned disadvantages. Equations derived for the modified system that makes it possible to define the position of nodal points. The relations were verified at a test station. Furthermore, a method of tuning the system was proposed.

## 1. Formulating the problem

When applying classical vibroinsulation in many important industrial cases, a contradiction is encountered between the low frequency of free vibration of the system required for high efficiency of vibroinsulation and the simultaneous demand of high static rigidity of the system. Manually operated vibratory tools (vibratory hammers and vibratory compactors, vibratory soil compacting machines, grinding machines etc.) are typical examples here.

Precise and convenient operation of this equipment by the operator requires relatively rigid fastening of the handhold to the vibratory system, which in many cases excludes the use of typical vibroinsulation with the holder. Due to the simplicity of the design, limited weight and dimensions, these tools do not offer suitable conditions for using active vibroinsulation methods [3]. In applications where active methods are more frequently used (e.g. in pneumatically operated tools), a problem is encountered in correct determination of gain factors of integrating elements, similar to the contradictions existing in classical vibroinsulation systems in relation to support elastic constant [5].

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Along with the development of active vibroinsulation methods, the so called "dynamic vibroinsulation" ideas emerged, based on the concept that, with monoharmonic excitation, an effect of "suppressing" undesirable forces and displacements can be obtained by adding vibratory systems in counterphase to the main system [5]. In opposition to Frahm's dynamic eliminators, they do not increase, within the working frequency range, the number of degrees of freedom of the system.

The "Nodalised Beam" method [6] that belongs to this group, was developed in the 1970s in the USA, in connection with the problem of vibroinsulation of the main drive of a helicopter from the load-bearing structure of the body.

In this method, nodal points are sought in the continuous structure that is supported in these points, and subject to harmonic excitation.

The system diagram for a beam subject to harmonic excitation is shown in Fig. 1, where mass $m$ can be freely designed, and the harmonic force can be substituted by kinematic excitation of similar type.

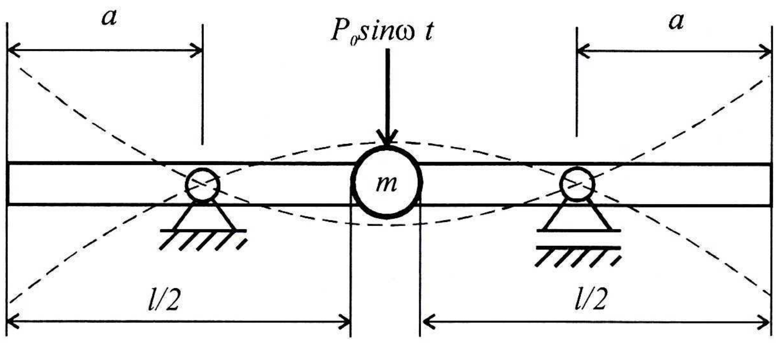


Fig. 1

With beam parameters properly designed for excitation frequency, it is possible to obtain nodal points, while at the same time the static rigidity of the system remains unlimited, which makes this method advantageous over the classical vibroinsulation.

This method, which may be regarded as being related with generally formulated task of structural modification of continuous vibratory systems [7], has shortcomings that make its effective practical application for vibroinsulation in manually advanced machines difficult:

1° In nodal points, only amplitudes of linear displacements cancel each other, but considerably high amplitudes of angular displacements remain, which is adversely sensed by the operator holding, in these points, the machine driving system.

2° Generally, a considerable length of the continuous system is required to obtain appropriate static rigidity of the system, and at the same time to ensure the required beam tuning.

## 2. Modification of the method

In order to avoid the above mentioned difficulties, we will introduce two modifications to the method:

1° Note that according to the vibroacoustic reciprocity principle in formulation stated in paper [1] "vibroacoustic process in the linear system, being the response to harmonics during excitation, initiated in certain point by external factor is constant as the result of swapping the energy application points and observation points".

Mathematically, by defining vibroacoustic processes taking place in the limited space $V$, caused by volumetric sources $q^{(1)}$ and $q^{(2)}$ as $p^{(1)}$(r) and $p^{(2)}$(r), whereas $F^{(1)}$ and $F^{(2)}$ – forces acting on the system, this principle can be defined as follows:

$$\iint_S \left[ \frac{\partial p^{(1)}(r)}{\partial n} F^{(2)} - \frac{\partial p^{(2)}(r)}{\partial n} F^{(1)} \right] dS = -i\omega\rho \iiint_V \left[ q^{(1)} p^{(2)}(r) - q^{(2)} p^{(1)}(r) \right] dV. \quad (1)$$

where:

$n$ – normal direction toward surface $S$,

$\omega$ – frequency of harmonic coercion,

$\rho$ – medium density.

In particular, in relation to the vibration process excitation by harmonic forces in the stable medium, the following takes place:

$$\frac{v_2}{F_1} = \frac{v_1}{F_2}, \quad (2)$$

where:

$v_2$ – radial velocity in point 2 caused by force $F_1$ applied in point 1,

$v_1$ – velocity in point 1 in direction of force $F_1$ applied in point 1, caused by force $F_2$ applied in point 2 in direction $v_2$.

By using relation (2), it is possible to reverse points of application of excitation and the nodal points in "Nodalised Beam" method, in the way shown on the half model in Fig. 2a, b.

Note that in case (b) the beam does not perform the rotary motion in the section crossing the nodal point, which allows us to place in it, for example, an operator's handhold. Furthermore, mass $m$ concentrated in this point may be of any value and its overall dimensions can be unlimited, which allows the handhold to be freely shaped.

2° In order to overcome the other disadvantage of the method, mentioned in p.1, connected with the necessity to ensure a considerable length of the beam, we will analyze the system shown in Fig. 3.

A concentrated mass $m_s$ was introduced at the free end of the beam, which makes it possible to shorten the system.

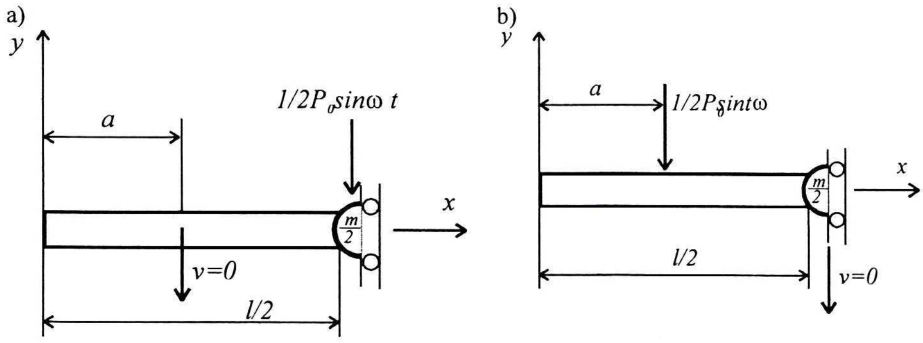


Fig. 2

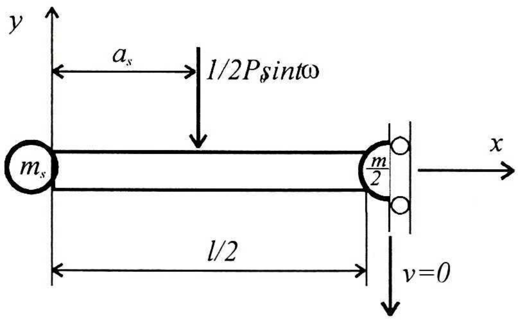


Fig. 3

### 2.1. Mathematical model of the initial system

In the case shown in Fig. 2a, due to condition:

$$v(a,t) = \frac{\partial \ y(a,t)}{\partial \ t} = 0 ,$$

which for harmonic function leads to the condition:

$$v(a,t) = j\omega \ \ y(a,t) = 0 ,$$

we must find such a coordinate $x=a$ so that $y(a,t) = 0$.

In order to find quantity $a$, the beam will be defined by differential equation in the class of generalized functions, which will be resolved by Fourier's method of separation of variables. Due to the fact that the sought solution relates to steady-state processes, the elements describing transient processes were neglected in the solution and only geometric form of forced vibration was analyzed.

The equation describing the model shown in Fig. 2a has the following form:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\left(\rho F+\frac{m}{2}\delta\left(x-\frac{l}{2}\right)\right)\frac{\partial^2 y(x,t)}{\partial t^2}=\frac{P_0}{2}\delta\left(x-\frac{l}{2}\right)\sin\omega t, \tag{3}$$

after separating the variables in form:

$$y(x,t)=X(x)T(t),$$

a system of ordinary differential equations (4), (5) is derived:

$$X^{(4)}(x)-\lambda^4 X(x)=P_1\delta\left(x-\frac{l}{2}\right)+\lambda^4 X\left(\frac{l}{2}\right)\delta\left(x-\frac{l}{2}\right), \tag{4}$$

$$\ddot{T}(t)+\omega^2 T(t)=\sin\omega t, \tag{5}$$

where:

$\lambda^4=\omega^2\dfrac{\rho F}{EI}$,

$\omega$ – frequency of coercion force,

$\mu=\dfrac{m}{\rho F}$,

$P_1=\dfrac{P_0}{2\,EI}$.

According to the remark made above, general solutions of the above equations were discarded in the solution, in the order to focus on the particular solution, essential for the steady state.

The following function is a solution of equation (3):

$$\begin{aligned}X(x)&=P\cosh\lambda x+Q\sinh\lambda x+R\cos\lambda x+S\sin\lambda x+\\&+\frac{P_1}{2\lambda^3}\left[\sinh\lambda\left(x-\frac{l}{2}\right)-\sin\lambda\left(x-\frac{l}{2}\right)\right]H\left(x-\frac{l}{2}\right)+\\&+\frac{\lambda^4\mu}{4\lambda^3}X\left(\frac{l}{2}\right)\left[\sinh\lambda\left(x-\frac{l}{2}\right)-\sin\lambda\left(x-\frac{l}{2}\right)\right]H\left(x-\frac{l}{2}\right),\end{aligned} \tag{6}$$

constants $P$, $Q$, $R$, $S$, are determined based on boundary conditions in the following form:

$$X''(0)=0,\qquad X'\left(\frac{l}{2}\right)=0,$$

$$X'''(0)=0,\qquad X'''\left(\frac{l}{2}\right)=0.$$

Taking into account the relations for derivatives of solution (6), in equations for boundary conditions, a system of equations is derived, which in matrix notation has the following form:

$$\begin{bmatrix} \sinh\lambda\frac{l}{2}-\sin\lambda\frac{l}{2} & \cosh\lambda\frac{l}{2}+\cos\lambda\frac{l}{2} & 0 \\ \sinh\lambda\frac{l}{2}+\sin\lambda\frac{l}{2} & \cosh\lambda\frac{l}{2}-\cos\lambda\frac{l}{2} & \lambda\frac{\mu}{2} \\ \cosh\lambda\frac{l}{2}+\cos\lambda\frac{l}{2} & \sinh\lambda\frac{l}{2}+\sin\lambda\frac{l}{2} & -1 \end{bmatrix} \begin{bmatrix} P \\ Q \\ X\left(\frac{l}{2}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{P_1}{\lambda^3} \\ 0 \end{bmatrix}.$$

From this system of equations, constants P and Q are determined and put into equation (4). Then, a section with a coordinate $x = a$ in which $v(a,t) = \frac{\partial\ y(a,t)}{\partial\ t} = 0$ is searched.

For: $b$ = 0.02m, $h$ = 0.02m, $E$ = 2.1*10^11 Pa $\rho$ = 7860, $l$ = 1.8m, $m$ = 1.5kg, $P_0$ = 100N we get constants $P$ = 1.8*10^-3, $Q$ = -1.5*10^-3, $a$ = 0.6m.
Therefore, supporting the beam in the section with a coordinate $x = a$ will results in nullifying coefficient $p$ of force transfer to the base.

### 2.2. Mathematical model of the modified system

Differential equation of beam transverse vibration shown in Fig. 2b, with the assumptions as specified above, after separating the variables, leads to equation:

$$X^{(4)}(x) - \lambda^4 X(x) = P_1\delta(x-a) + \lambda^4 X\left(\frac{l}{2}\right)\ \delta\left(x-\frac{l}{2}\right),$$

definitions as above.
The solution in the class of generalized functions has the following form:

$$\begin{aligned} X(x) = {} & P\cosh\lambda x + Q\sinh\lambda x + R\cos\lambda x + S\sin\lambda x + \\ & + \frac{P_1}{2\ \lambda^3}\left[\sinh\lambda(x-a) - \sin\lambda(x-a)\right]H(x-a) + \\ & + \frac{\lambda^4\mu}{4\ \lambda^3} X\left(\frac{l}{2}\right)\left[\sinh\lambda\left(x-\frac{l}{2}\right) - \sin\lambda\left(x-\frac{l}{2}\right)\right]H\left(x-\frac{l}{2}\right), \end{aligned} \tag{7}$$

constants $P$, $Q$, $R$, $S$, are determined, similarly as in the previous example from the boundary conditions associated with the problem presented in Fig. 2b:

$$X''(0) = 0, \qquad X'\left(\frac{l}{2}\right) = 0,$$

$$X'''(0) = 0, \qquad X'''\left(\frac{l}{2}\right) = 0.$$

The above equations are given in matrix notation by moving coercion part to the right hand side of the equation:

$$\begin{bmatrix} \sinh\lambda\frac{l}{2}-\sin\lambda\frac{l}{2} & \cosh\lambda\frac{l}{2}+\cos\lambda\frac{l}{2} & 0 \\ \sinh\lambda\frac{l}{2}+\sin\lambda\frac{l}{2} & \cosh\lambda\frac{l}{2}-\cos\lambda\frac{l}{2} & \lambda\frac{\mu}{2} \\ \cosh\lambda\frac{l}{2}+\cos\lambda\frac{l}{2} & \sinh\lambda\frac{l}{2}+\sin\lambda\frac{l}{2} & -1 \end{bmatrix} \begin{bmatrix} P \\ Q \\ X\left(\frac{l}{2}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{P_1}{2\ \lambda^3}\left[\sinh\lambda\left(\frac{l}{2}-a\right)+\sin\lambda\left(\frac{l}{2}-a\right)\right] \\ -\frac{P_1}{2\ \lambda^3}\left[\cosh\lambda\left(\frac{l}{2}-a\right)+\cos\lambda\left(\frac{l}{2}-a\right)\right] \\ -\frac{P_1}{2\ \lambda^3}\left[\sinh\lambda\left(\frac{l}{2}-a\right)-\sin\lambda\left(\frac{l}{2}-a\right)\right] \end{bmatrix}.$$

With the data as in the previous example, with the determined coordinate $a$, the following was derived:

$$P = 1.1*10^{\wedge}\text{-}3,\ Q = -0.8*10^{\wedge}\text{-}3, \qquad \text{and} \qquad X\left(\frac{l}{2}\right) = 0.$$

The above results confirm the correctness of the conclusions drawn from Liamszew – Fahy theorem.

### 2.3. The model of the modified system with additional mass

The system shown in Fig. 3 was proposed. Where it was necessary to decrease the overall dimensions, and particularly the length of vibroinsulating beam, vibration differential equation that describes the structure with concentrated masses as well as exciting force can be given as:

$$EI\frac{\partial^4 y(x,t)}{\partial\ x^4} + \left(\rho F + m_s\delta(x-0) + \frac{m}{2}\delta\left(x-\frac{l}{2}\right)\right)\frac{\partial^2 y(x,t)}{\partial\ t^2} = \frac{P_0}{2}\delta(x-a_s)\sin\omega t\,,$$

after separating the variables:

$$y(x,t) = X(x)T(t)\,,$$

we will get:

$$X^{(4)}(x) - \lambda^4 X(x) = \lambda^4\mu_s X(0)\delta(x-0) + P_1\delta(x-a_s) + \lambda^4\frac{\mu}{2}X\left(\frac{l}{2}\right)\delta\left(x-\frac{l}{2}\right), \tag{8}$$

$$\ddot{T}(t) + \omega^2 T(t) = \sin\omega\ \ t\,, \tag{9}$$

where: $\lambda^4 = \omega^2\frac{\rho F}{EI}$, $\omega$ – frequency of exciting force, $\mu_s = \frac{m_s}{\rho F}$, $\mu = \frac{m}{\rho F}$,

$P_1 = \frac{P_0}{2\ EI}$, similarly as in the systems discussed before. Similarity as it was

done before, one neglected in this example, general solutions of homogeneous equations connected with the system of equations (8), (9).
The following function is a solution to equation (8):

$$\begin{aligned} X(x) = {} & P\cosh\lambda x + Q\sinh\lambda x + R\cos\lambda x + S\sin\lambda x + \\ & + \frac{\lambda^4\mu_s}{4\lambda^3} X(0)\left[\sinh\lambda(x-0) - \sin\lambda(x-0)\right] H(x-0) + \\ & + \frac{P_1}{2\lambda^3}\left[\sinh\lambda(x-a_s) - \sin\lambda(x-a_s)\right] H(x-a_s) + \\ & + \frac{\lambda^4\mu}{4\lambda^3} X\left(\frac{l}{2}\right)\left[\sinh\lambda\left(x-\frac{l}{2}\right) - \sin\lambda\left(x-\frac{l}{2}\right)\right] H\left(x-\frac{l}{2}\right), \end{aligned} \tag{10}$$

derivatives of function (10) are calculated in a similar way as in the previous examples.
The problem shown in Fig. 3 is connected with the boundary conditions:

$$X''(0) = 0, \qquad X'\left(\frac{l}{2}\right) = 0,$$

$$X'''(0) = 0, \qquad X'''\left(\frac{l}{2}\right) = 0.$$

The above equations are given below in matrix notation, where $a_s$ is a parameter:

$$\begin{bmatrix} \sinh\lambda\frac{l}{2} - \sin\lambda\frac{l}{2} & \cosh\lambda\frac{l}{2} + \cos\lambda\frac{l}{2} & \lambda\frac{\mu_s}{2}\left[\cosh\lambda\frac{l}{2} - \cos\lambda\frac{l}{2}\right] & 0 \\ \sinh\lambda\frac{l}{2} + \sin\lambda\frac{l}{2} & \cosh\lambda\frac{l}{2} - \cos\lambda\frac{l}{2} & \lambda\frac{\mu_s}{2}\left[\cosh\lambda\frac{l}{2} + \cos\lambda\frac{l}{2}\right] & \lambda\frac{\mu}{2} \\ 2 & 0 & -1 & 0 \\ \cosh\lambda\frac{l}{2} + \cos\lambda\frac{l}{2} & \sinh\lambda\frac{l}{2} + \sin\lambda\frac{l}{2} & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} P \\ Q \\ X(0) \\ X\left(\frac{l}{2}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{P_1}{2\lambda^3}\left[\cosh\lambda\left(\frac{l}{2}-a_s\right) - \cos\lambda\left(\frac{l}{2}-a_s\right)\right] \\ -\frac{P_1}{2\lambda^3}\left[\cosh\lambda\left(\frac{l}{2}-a_s\right) + \cos\lambda\left(\frac{l}{2}-a_s\right)\right] \\ 0 \\ -\frac{P_1}{2\lambda^3}\left[\sinh\lambda\left(\frac{l}{2}-a_s\right) - \sin\lambda\left(\frac{l}{2}-a_s\right)\right] \end{bmatrix}.$$

After determining constants *P*, *Q*, *X(0)* from the above equation $X\left(\frac{l}{2}\right)$, they are substituted into equation (10). One can find $a_s$ from the condition that the amplitude of beam deflection in the section with a coordinate $l/2$ be equal to zero $X\left(\frac{l}{2}\right)=0$. With the data as in the previous examples, we get:

$P = 0.74*10\text{^-}3$, $Q = -0.61*10\text{^-}3$, $X(0) = 1.5*10\text{^-}3$, $X\left(\frac{l}{2}\right)=0$, $a_s = 0.369$m.

The above results show the possibility of reducing the length of the vibroinsulation system when applying additional masses $m_s$. For the above data we get: $P = 0.47*10\text{-}3$ $Q = -0.47*10\text{-}3$ $X(0) = 0.95*10\text{-}3$ $X(l/2) = 0$, $a_s = 0.24$m, $l = 1.4$m for $m_s = 1.5$kg which results in shortening the system by 22%. Despite that fact, the system is 1.4m long, which means that is not a suitable solution for handholds of vibratory tools. Further shortening of the beam length was achieved by replacing it with the system shown in Fig. 7, described below.

## 3. Experimental tests

Experimental verification was carried out on the device represented by the diagram shown in Fig. 2b, but equipped in its central part with a shaped handhold – Fig. 4. In order to reduce the length of the continuous system, a helical spring with the parameters as below was used in place of a metal bar $\bar{m} = 3.129$ kg/m, $EI = 2.087$ Nm$^2$, $m = 1.5$ kg, $m_s$=0 kg and for such parameters with excitation frequency $\omega = 159$ rad/s the following was obtained in the analytical way: $l = 0.32$m, $a = 0.08$m.

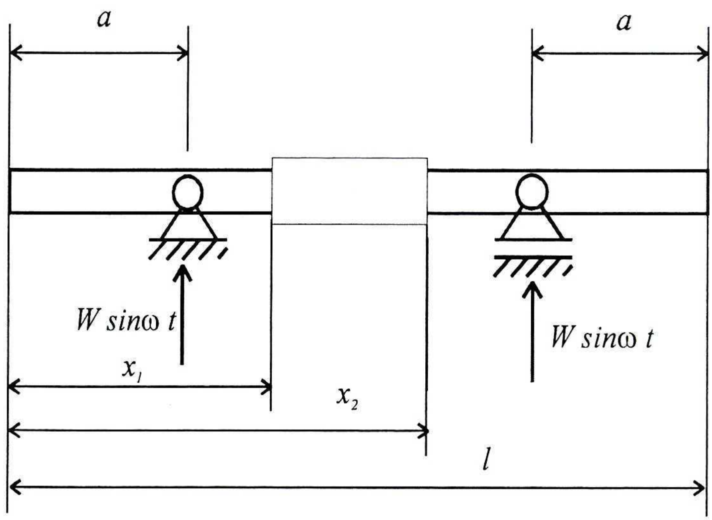


Fig. 4

Since, by assumption, rigidity in the section ($x_1$, $x_2$) is much higher than in the other part of the beam, this part of the beam can be regarded as rigid, nondeformable body. This allows us to simplify the beam model, as shown in Fig. 5.

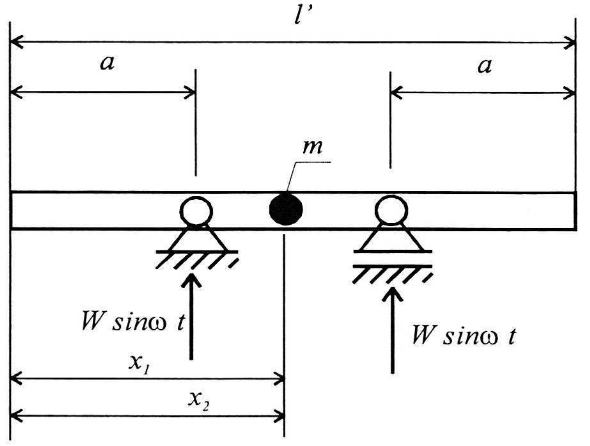


Fig. 5

In the above model, with the beam length $l' = l - (x_2 - x_1)$ and other data as above, the handhold mass $m$ = 1.5 kg was adopted as a concentrated mass located at the point with a coordinate $x = x_1 = l'/2$. This model is defined by equation:

$$X^{(4)}(x) - \lambda^4 X(x) = \lambda^4 X\left(\frac{l'}{2}\right)\delta\left(x - \frac{l'}{2}\right). \tag{11}$$

The following function is a solution to equation (11):

$$X(x) = P\cosh\lambda x + Q\sinh\lambda x + R\cos\lambda x + S\sin\lambda x +$$
$$+\frac{\lambda^4\mu}{4\ \lambda^3}\ X\left(\frac{l'}{2}\right)\left[\sinh\lambda\left(x - \frac{l'}{2}\right) - \sin\lambda\left(x - \frac{l'}{2}\right)\right]\ H\left(x - \frac{l'}{2}\right). \tag{12}$$

After determining the constants as in the previous examples, a function of beam forced vibration amplitudes was derived with the parameters as above. Fig. 6 also indicates lack of vibration in section ($x_2$ - $x_1$).

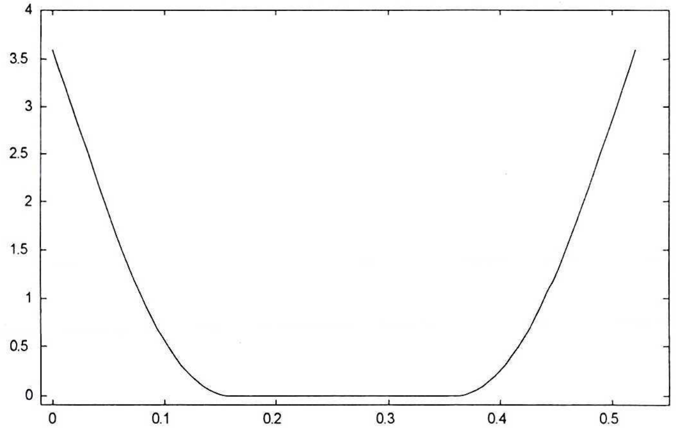


Fig. 6

### 3.1. Experimental tests

The tests were performed on the stand shown in Fig. 7.

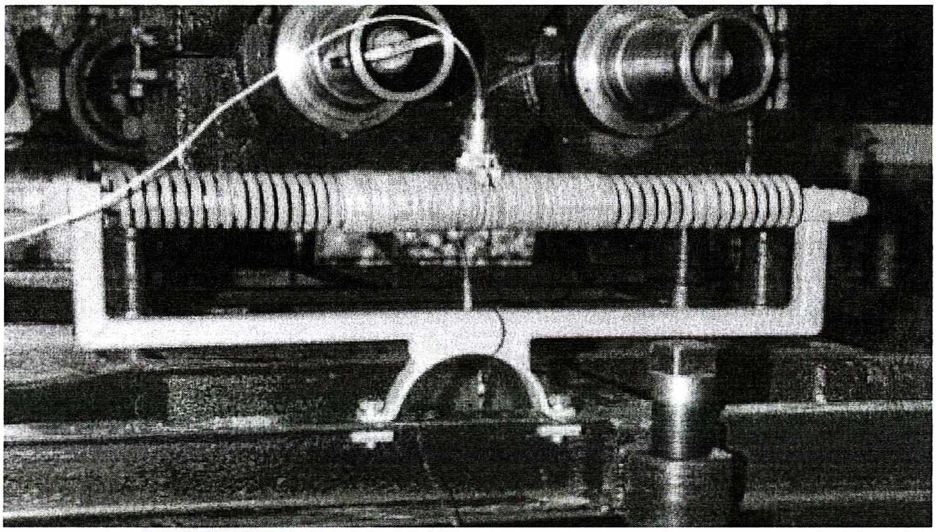

Fig. 7

The vibroinsulation system was installed at the vibratory station with adjustable frequency and amplitude of vibrations. Recording was done by using two-channel dynamic response analyzer HP 3560A.

During tests, one recorded the coercion frequency $f$ [Hz], amplitude of handhold acceleration $a_r$ [mg] and station acceleration $a_s$ [mg]. The obtained results are summarized in Tab.1, providing additionally the value of transfer coefficient $p = \frac{a_r}{a_s}$.

Table 1.

| $f$ [Hz] | $a_r$ [mg] | $a_s$ [mg] | $p = {a_r}/{a_s}$ |
|---|---|---|---|
| 9.25 | 100.76 | 53.38 | 1.89 |
| 13.75 | 970.68 | 89.07 | 10.90 |
| 18.25 | 172.56 | 78.91 | 2.19 |
| 23.00 | 87.32 | 329.60 | 0.26 |
| 24.50 | 27.20 | 318.50 | 0.0085 |
| 26.00 | 68.00 | 399.00 | 0.17 |
| 32.00 | 211.50 | 645.50 | 0.33 |

The presented experimental test results show that the system reaches the transfer coefficient $p$=0.085 at 24.5 Hz close to 25 Hz determined in the analytical way, which proves the justness of theoretical considerations.

It should be noted that the classical vibroinsulation system with the same static rigidity and mass allows for obtaining the value $p \geq 0.33$ at $f$=25 Hz.

## 4. Tuning the system

The system of such a construction ensures effective vibroinsulation for one selected coercion frequency. In the used analytical model, an assumption was made that one had a comprehensive knowledge of beam properties and beam geometry, and the damping influence etc. were neglected. so in this connection vibration amplitude within the range of ($x_1$, $x_2$) of the real system is different from zero. For constructional reasons, changing the position of the support on the beam (spring) is not possible. Therefore, we will consider a certain possibility to tune the system to the coercion frequency. This modification is connected with pretensioning of the spring in section $(a, l'-a)$, i.e. between the supports.

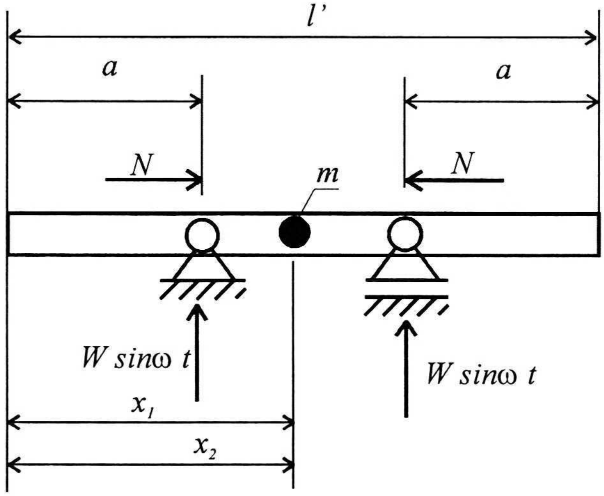


Fig. 8

In such a case (Fig. 8), the analyzed system was divided into three subsystems:

- in the range of ($0$, $a$) the beam was defined with equation:

$$X_1^{(4)}(x) - \lambda^4 X_1(x) = 0,$$

- in the range of $(a,\ l'-a)$ with equation:

$$X_2^{(4)}(x) + \beta \ \ X_2''(x) - \lambda^4 X_2(x) = 0, \qquad (13)$$

where:

$$\beta = \frac{N}{E\ I},$$

N – longitudinal force.

The following function is a solution to equation (13):

$$X_2(x) = P_2 \cosh \kappa_1 x + Q_2 \sinh \kappa_1 x + R_2 \cos \kappa_2 x + S \sin \kappa_2 x\,, \qquad (14)$$

where:

$$\kappa_1 = \sqrt{\frac{-\beta + \sqrt{\beta^2 + 4\lambda^4}}{2}},$$

$$\kappa_2 = \sqrt{\frac{\beta + \sqrt{\beta^2 + 4\lambda^4}}{2}},$$

- in the range of $(\ l' - a,\ l')$ with equation identical as in the first range:

$$X_3^{(4)}(x) - \lambda^4 X_3(x) = 0\,,$$

in all ranges $\lambda$ is defined identically as:

$$\lambda^4 = \omega^2 \frac{\rho F}{EI},$$

$\omega$ – frequency of kinematic coercion (support vibration), each of the equations defines vibration in the local system.
To set 14 integration constants, 12 resulting from the solution in each range and 2 reactions in the supports $R_a$, $R_b$, there must be 14 equations defining boundary and continuity conditions (to bond the solutions in the range limits.

Boundary conditions are as follows:

$$X_1''(0) = 0\,, \qquad X_3''(l' - (l' - a)) = 0\,,$$

$$X'''(0) = 0\,, \qquad X_3'''(l' - (l' - a)) = 0\,,$$

"bonding" conditions in the section with coeff. $a$:

$$X_1(a) = W\,, \qquad X_2(0) = W\,,$$

$$X_1'(a) = X_2'(0), \qquad X_1''(a) = X_2''(0),$$

$$X_2'''(0) - X_1'''(a) = \frac{R_a}{EI},$$

in the section with coeff. $l' - a$:

$$X_2\big((l' - a) - a\big) = W\,, \qquad X_3(0) = W\,,$$

$$X_2'\big((l' - a) - a\big) = X_3'(0), \qquad X_2''\big((l' - a) - a\big) = X_3''(0),$$

$$X_3'''(0) - X_2'''\big((l' - a) - a\big) = \frac{R_b}{EI}.$$

By proceeding in a standard way, one derived the influence of longitudinal force $N$ on the frequency of kinematic coercion. For this frequency, beam vibration amplitude at the point with a coordinate $l'/2$ was equal to zero i.e. $X_2\left(\frac{l'}{2}\right)=0$. This relation is shown in Fig. 9.

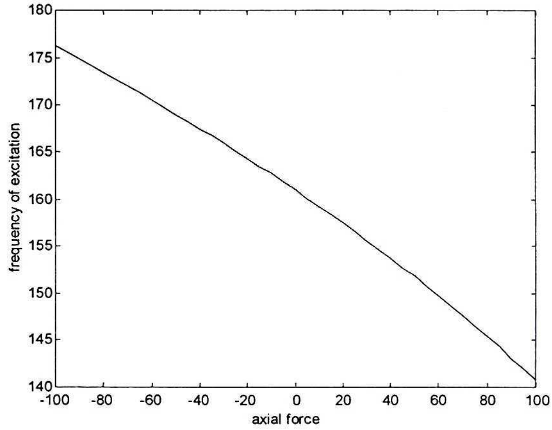


Fig. 9

Force $N$ = 100N changes the length of the beam by less than *1* mm, this is why it was not taken into account in the model. Critical force due to buckling equals $P_{kr}=804\text{N}$, thus by adopting the force within the range up to 100 $N$ brings no risk of loosing stability.

## 5. Conclusions

The paper indicates the possibility of using the reciprocity principle when investigating dynamic vibroinsulation systems of Nodalised Beam type in manually operated tools.

The described vibroinsulation system acts alike to antiresonant vibroinsulation class, except that within the working frequency range it does not introduce additional free vibration frequency into the system. It has tremendous advantage of designing in the desired way, the machine operator's handhold both its shape and weight. When compared to classical vibroinsulation, the analyzed system is characterized by high static rigidity and small mass, which is important for vibroinsulation of manually operated machines. When using classical vibroinsulation, with the same weight of the handhold (1.5 kg), static deflection of the system would be approx. 20 mm which makes precise operation of the tool impossible, while in the presented solution the deflection is only approx. 1.5 mm.

Moreover, the presented vibroinsulation system can be tuned, which is an important feature, for example in the case of machines manufactured in serial production, because particular pieces of serial production have slightly different working frequencies.

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**Adaptacja metody „Nodalised Beam" do wibroizolacji narzędzi ręcznie prowadzonych**

Streszczenie

W pracy rozpatrzono możliwość zastosowania metody „Nodalised Beam" do wibroizolacji narzędzi ręcznie prowadzonych. Wskazano na trudności w zastosowaniu w tym celu metody oryginalnej oraz zaproponowano, oparty na zasadzie wzajemności, sposób modyfikacji układu pozwalający na ominięcie wspomnianych niedogodności. Dla układu zmodyfikowanego wyprowadzono równania pozwalające na określenie położenia punktów węzłowych. Zależności zweryfikowano na stanowisku badawczym. Zaproponowano ponadto metodę dostrajania układu.