THE ARCHIVE OF MECHANICAL ENGINEERING

Vol. L 2003 Number 4

Key words: *vibration isolation, wave effects*

*JERZY MICHALCZYK**), *LESZEK MAJKUT**)

ANALYSIS OF WAVE EFFECTS IN COMPLEX VIBRATION ISOLATION SYSTEMS

The problem of transmitting vibrations with audible frequencies by steel springs, constituting the vibration isolation system was considered in this paper. The analytical relationships allowing determining the value of the transmissibility for the springs resonance frequencies responsible for the transmissibility of high frequency vibrations have been derived and checked by means of FEM method.

Also the occurrence of the increasing stresses in the springs in the areas between the resonances has been shown. The typical system, i.e. the serial system with rubber cushion, has been analyzed, reducing the transmission of high frequency vibrations by the spring. It has been shown that the transmission is reduced not as a result of differences in the wave impedance of the boundary of both media but due to the increased dispersion of energy in the rubber, and the analytical relationships allowing the evaluation of the effectiveness of this method have been derived.

1. Introduction

The vibration isolation systems with the application of the helical steel springs, in addition to a number of advantages, suffer from a disadvantage consisting in the intensive transmission, within the over-resonance range, of the vibrations with frequencies corresponding to the spring natural frequencies [1].

This phenomenon particularly occurs with regard to longitudinal vibrations, or more precisely, rotary-longitudinal ones. For typical springs, with small screw line ascending angles, the factor that couples longitudinal and rotary vibrations can be disregarded [2], which makes it possible to consider

*) *Dept. of Mechanics and Vibracoustics at AGH – University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland; E-mail: michalcz@agh.edu.pl*

further the longitudinal vibrations of the spring as the vibrations of a bar with adequately selected spring and inertia constants.

The near-resonance increase of vibrations of this type causes both the transmission of intense reactions to the structure protected by the spring, as well as large internal stresses in the spring.

However, whereas the range within which the vibration-isolation properties of such systems deteriorate is narrow and concentrates round springs natural frequencies, the range within which the essential additional loads in the spring coils occur is relatively wide [6], filling in the area between the springs natural frequencies.

The basic protection against structure-borne sound in steel springs is their serial association with rubber elements, or applying the additional rubber coating on the springs, so that the springs near-resonance vibrations energy is dissipated owing to the contact friction or material damping in rubber.

The reasonable selection of these protection vehicles requires the possibility of the evaluation of their effectiveness. The methods in use [1], [7], based upon the analysis of the reflection of wave on the boundary of the media with different wave impedances do not emulate the essence of the phenomenon, thereby leading to erroneous results.

The objective of this paper is the formulation of a new analysis of impact of the rubber washer on the processes of transmission of high frequency vibrations, based upon the balance of energy.

With regard to the washer-free springs, for which the literature [3], [8] provides the evaluation of the transmissibility coefficient based upon the linear reology model, the analysis has been performed based upon the material-structural suppression, as this kind of suppression is closer to the vibration-isolation systems nature. The paper also performs the analysis of the influence of the springs stop plates on the course of the phenomenon and shows the existence of the optimal value of a clamp in systems containing a rubber sleeve.

2. The vibration isolation system with the application of the helical springs

Figure 1 shows the vibration isolation unit diagram together with the corresponding model of a bar with continuous distribution of mass and adequately selected constants ρA i EA.

The system has been analyzed, in a number of works, such as [1], [3], [8], where one derived the course of the transmissibility coefficient p, understood as the relation of the amplitude of the force transmitted onto the base to the amplitude of the forcing force P in the frequency function.

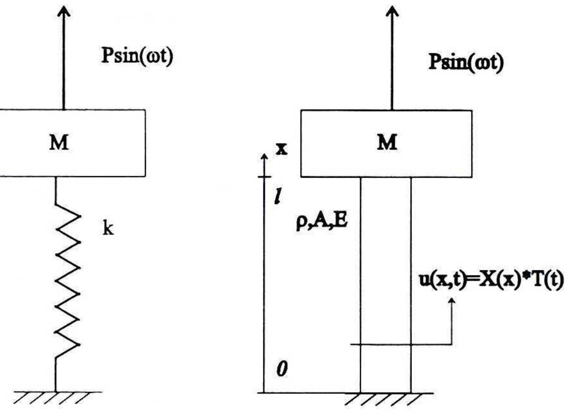

Fig. 1. The vibration isolation system model

The coefficient of the force and displacement transmissibility for the model under analysis is equal to:

$$p = \frac{1}{\left| \cos\left(\frac{\omega}{\sqrt{\frac{k}{m}}}\right) - \frac{M\omega}{\sqrt{km}} \sin\left(\frac{\omega}{\sqrt{\frac{k}{m}}}\right) \right|} \tag{1}$$

where: M – sprung mass, m – spring mass, k – stiffness constant of the spring.

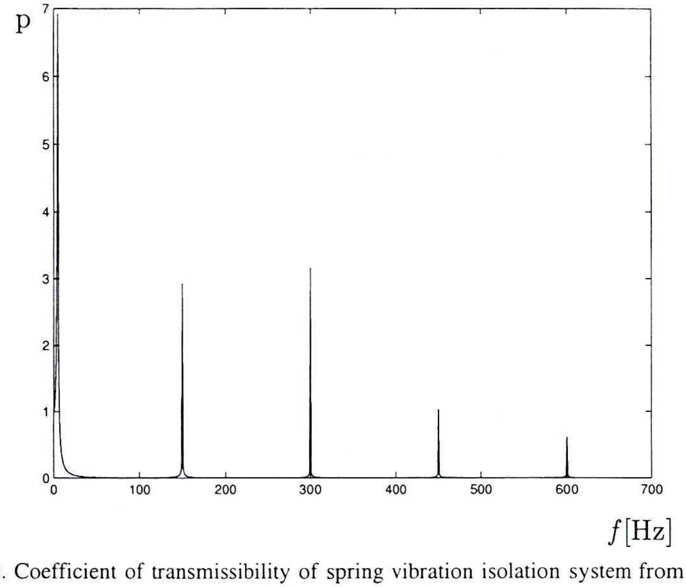

Fig. 2. Coefficient of transmissibility of spring vibration isolation system from Fig. 1, without the suppression

The course of the changes of the coefficient p in the forcing frequency function f [Hz] has been shown in fig. 2 (the operating range of the vibration isolation unit covers the frequencies $\omega > \sqrt{2} \cdot f_1 = 6.75\ s^{-1}$), obtained for data $k = 9 \cdot 10^5$ N/m, $l = 0.2$ m, $M = 1000$ kg, $m = \rho A \cdot l = 10$ kg:

Due to the fact that the suppression has been disregarded, the transmissibility coefficient for each of the resonance frequencies is equal to infinity.

The resonance frequencies may be arrived at approximately, through decomposing the system onto the model with one degree of freedom for the first frequency, and with regard to the value of sprung mass M – onto the model of a bar fastened on both sides for higher frequencies [6]:

$$\omega_1 \cong \sqrt{\frac{k}{M + \frac{1}{3}m}} \tag{2}$$

$$\omega_n \cong (n-1) \cdot \pi \sqrt{\frac{k}{m}} \quad n = 2,3,4,5... \tag{3}$$

The frequency band for which the force transmitted onto the base is amplified is wide only at the proximity of the first free frequency of the system. However, as it has been shown in the work [6], this is not pertinent to internal forces P_w in the springs, whose maximum values are usually found beyond the edge coils and assume large values in the wide neighborhoods of resonance frequencies – Fig. 3.

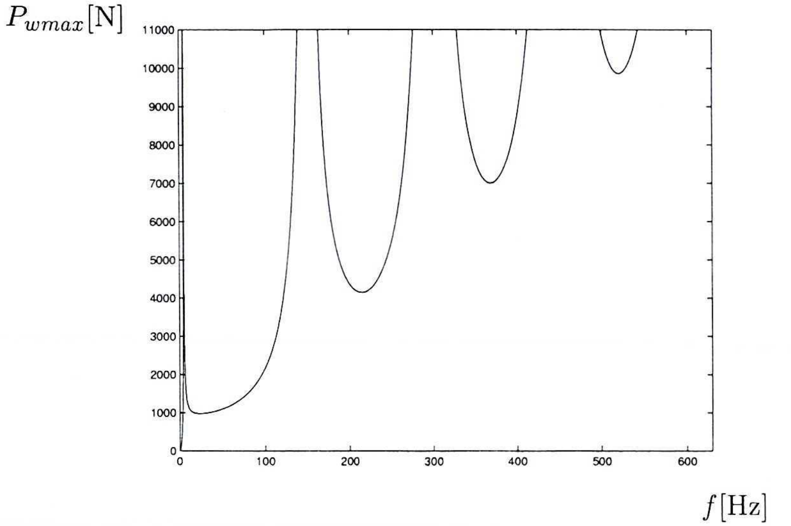

Fig. 3. Maximum internal forces in the spring

These forces lead to the fatigue of the wire in the spring, which causes a danger of the damage to the springs (breaking, fracturing, etc.).

2.1. The analysis of high frequency vibrations transmitted through the vibration isolation systems with linear damping

In order to determine the actual, finite values of transmissibility coefficient p for the springs resonance frequencies, the system as above will be analyzed, with the incorporation of the damping in the bar, acting as a spring in this approach:

$$EA\,\frac{\partial^2 u(x,t)}{\partial x^2} + EA\alpha\cdot\frac{\partial^3 u(x,t)}{\partial x^2\,\partial t} - \rho A\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

In order to solve the equation, the Fourier method of separating the variables is applied, in the following form:

$$u(x,t) = X(x)\cdot e^{i\omega t}$$

After simple transformations, the equation allowing determining the $X(x)$ in the complex numbers domain may be presented as follows:

$$X''(x) + \omega^2\frac{\rho A}{EA\cdot(1+i\omega\alpha)}\cdot X(x) = 0$$

where the roots of characteristic equation:

$$r_{1,2} = \pm\eta\cdot\sqrt{\frac{1}{1+i\omega\alpha}}$$

while:

$$\eta = \omega\cdot\sqrt{\frac{\rho A}{EA}}$$

$$\rho A = \frac{m}{l},\quad EA = k\cdot l$$

therefore, the amplitude of steady vibrations is shown through the following formula:

$$X(x) = C_1 \cdot e^{r \cdot x} + C_2 \cdot e^{-r \cdot x}$$

The boundary conditions have merely been modified through the incorporation in the equation the longitudinal force of the structural damping, thus:

$$X(0) = 0 \qquad EAX'(l) + EA \cdot i \cdot \omega \alpha X'(l) = P - M\omega^2 X(l)$$

After substituting the solution of $X(x)$, together with its derivatives, in the boundary conditions, the constants C_1 and C_2 may be determined, following which the force acting onto the base can be arrived at.

The aforesaid procedure may be easily applied through numerical methods, however, the result in the analytic form cannot be obtained.

The approximate analytic solution of this problem has been presented in [8, volume 3, page 1505] for the first free resonance of springs f_2:

$$p_2 \cong \frac{0.2 \cdot m}{\operatorname{tg}\delta \cdot M} \tag{4}$$

where δ – Loss angle.

The practical calculations according to the aforesaid relation are hindered through the fact that the dissipation of energy in actual systems, determining the amplitude of the near-resonance vibrations, takes place by way of non-linear damping of material-structural type [4], [5], whose mathematical model significantly differs from the linear damping model.

This restriction doesn't apply to other reports in this field, such as [3], where the methodology of determining the transmissibility coefficient for wide band of frequencies, based upon the receptance method and damping model in the complex Young's modulus, has been given.

2.2. The analysis of transmitting high frequency vibrations through the vibration isolation system – the system with material-structural damping and kinematic excitation from the foundation

In order to derive the analytic solution in the form convenient for engineering calculations, the phenomenon of transmitting the vibrations in the spring free resonance will be contemplated only approximately, while the correctness of simplifying assumptions made will be evaluated through the comparison with the accurate solution.

In the first approximation, let us disregard the vibrations of the termination of the spring, fastened to the mass *M*, as these are low in comparison with the resonance vibrations of the spring coils. In this case, the form of the spring vibrations corresponds to its free vibrations in double fixed:

$$X(x) = C\sin\left(\frac{\omega_2}{c}x\right)$$

where:
C – amplitude,
ω_2 – natural frequency of the spring in double fixing, $\omega_2 \cong \pi\sqrt{k/m}$
$c = l \cdot \sqrt{k/m}$,
other denotations as above.

(As it can easily be checked through the replacement, this form fulfil the conditions of double fixed bar).

The full description of vibrations, following the incorporation of their resonance character, i.e. the angle of phase shift $\gamma = -\pi/2$, assumes the following form:

$$u(x,t) = C\sin\left(\frac{\omega_2}{c}x\right)\cdot\sin\left(\omega_2 t - \frac{\pi}{2}\right) = -C\sin\left(\pi\frac{x}{l}\right)\cdot\cos(\omega_2 t)$$

Therefore, the force in the spring, at its bottom ($x = 0$) termination, is equal to:

$$P(0,t) = EA\frac{\partial u(x,t)}{\partial x} = -EA\cdot\frac{\pi}{l}C\cos\left(\frac{\pi\cdot 0}{l}\right)\cdot\cos(\omega_2 t) = -CEA\frac{\pi}{l}\cdot\cos(\omega_2 t)$$

while the external force at the bottom termination:

$$P^*(0,t) = -P(0,t) = CEA\frac{\pi}{l}\cdot\cos(\omega_2 t)$$

The power of the external force $P^*(0,t)$ along the shift of the bottom termination of the spring, arising from the base vibrations $u_0(0,t) = C_0\sin(\omega_2 t)$, is equal to:

$$N(t) = P^*(0,t)\cdot\frac{\partial u_0(0,t)}{\partial t} = C_0\,CEA\frac{\pi}{l}\omega_2\cdot\cos^2(\omega_2 t)$$

while its work for the vibrations period $T_2 = 2\pi/\omega_2$:

$$L_{T_2} = \int_0^{2\pi/\omega_2} N(t)dt = C_0\, CEA\frac{\pi}{l}\omega_2 \int_0^{2\pi/\omega_2} \cos^2(\omega_2 t)dt = \pi^2 C_0 C\frac{EA}{l}$$

This work covers the losses of energy in the course of vibrations ΔE_{T_2}, which losses can be determined e.g. from the model of material damping with the coefficient $\psi = 2\pi\eta$, η-loss factor adjusted in order to take account of also the structural damping [5]:

$$\Delta E_{T_2} = \psi U_{max} = \frac{1}{2}\psi EA\int_0^l \left[\frac{\partial X(x)}{\partial x}\right]^2 dx = \frac{1}{2}\psi EAC^2\frac{\omega_2^2}{c^2}\int_0^l \cos\left(\frac{\omega_2}{c}x\right)dx$$
$$= \frac{\psi}{4}\pi^2 \cdot \frac{EA}{l}C^2$$

Comparing $L_{T_2} = \Delta E_{T_2}$, we obtain the formula allowing determining the amplitude C.

$$C = \frac{4C_0}{\psi}$$

Therefore, the force acting on the mass M, can be arrived at through the relation:

$$P^*(l,t) = -P(l,t) = EA\frac{\pi}{l}\cdot\frac{4C_0}{\psi}\cdot\cos\left(\frac{\pi\cdot l}{l}\right)\cdot\cos(\omega_2 t) = \frac{-4\pi C_0 EA}{\psi l}\cdot\cos(\omega_2 t)$$

while the amplitude of this force is equal to:

$$|P(l,t)|_{max} = \frac{4\pi C_0 EA}{\psi l}$$

Treating the mass M as unconstrained, subjected to harmonic force $P(l,t)$, the amplitude of vibrations C_M of this mass may be arrived at through the following equation:

$$|P(l,t)|_{max} = M\cdot C_M\cdot\omega_2^2$$

from where:

$$C_M = \frac{4\pi C_0 EA}{M\omega_2^2 \psi l}$$

therefore the transmissibility coefficient p_2 for this case is equal to:

$$p_2 = \frac{C_M}{C_0} = \frac{4\pi EA}{M\omega_2^2 \psi l} = \frac{4\pi k}{M\omega_2^2 \psi} = \frac{4\pi k}{M\pi^2 \frac{k}{m} \psi}$$

Finally,

$$p_2 = \frac{4}{\pi \cdot \psi} \cdot \frac{m}{M} \tag{5}$$

The table 1 provides the vibration transmissibility coefficients, determined through the application of numerical methods (FEM), as well as analytically, from the equation (5) for the parameters of the system as above the Fig. 2.

Table 1
Vibration transmissibility coefficients, arrived at by means of approximate and reference (FEM) methods

$\psi = 0.13$		$\psi = 0.065$	
analyt.	FEM	analyt.	FEM
0.098	0.098	0.196	0.196

3. System with rubber cushion

As mentioned in the introduction, the basic protection from the structure - borne sound in the steel springs is their serial association with the rubber elements, or the application of additional rubber coats that attenuate the resonance vibrations of springs owing to the contact friction or structural damping.

Based upon [1], such a typical solution is presented in figure 4, where the steel spring 1 is founded on the steel abutment 2, under which the cushion from flat rubber 3 has been located, while the effect of dry friction has been materialized by means of the rubber sleeve 4 put on the spring with a certain inceptive tension. The fastening screws 5 are insulated with the rubber sleeves 6 allowing the small vibrations of the steel abutment.

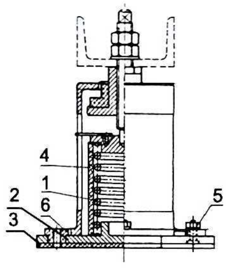

Fig. 4. The vibration isolation system model

The solutions of this kind produce a number of doubts associated with the absence of the methodology of optimized or even proper selection of the system parameters.

The effectiveness of the rubber cushion application is usually accounted for [1], [7] through the fact that the energy of wave motion is weakened (reflected) on the boundary of environments:

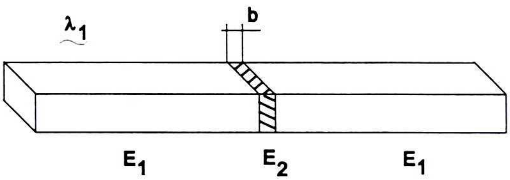

Fig. 5. The wave system model with inclusion

The analysis of the aforesaid model leads to the relationship for the insert effectiveness D [dB]:

$$D = 10 \cdot \lg \frac{I_{incident}}{I_{transition}} = 20 \lg \frac{\pi b E_1}{\lambda_1 E_2}$$

where:

$$\lambda_1 = \frac{c_{steel}}{\omega / 2\pi} = 33.4\ m$$

In analysis case for $b = 1$ cm and rubber 55°Sh effectiveness is on level $D = 36.4$dB.

However, in reality, this is the standing rather than running wave that occurs here. In addition, there is no relationship between p and I – the transmissibility coefficient for e.g. force vibration isolation may reach significant values even where no power is transmitted onto the base, i.e. the base is fully stiff.

These discrepancies show that the aforesaid approach is unsuitable, and therefore the necessity exists of determining the effectiveness of the rubber cushions based upon another model.

In order to explain the doubts produced, the contemplation is proposed of the system shown in figure 6, where the rubber cushion has been described as the discrete element with stiffness constant k_g, and with the material damping factor ψ_g (such a description of the cushion has its reasons in the very high first partial frequency of the cushion, depending on the quotient of the velocity of the longitudinal wave in the rubber to the height of the cushion).

Let us disregard the vibrations of the terminations of the spring as they are negligible in comparison with resonance spring vibrations.

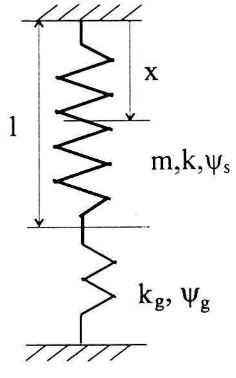

Fig. 6. The model of interaction of the system of steel spring-rubber

The differential equation of the spring vibrations has the following form:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0$$

$$c = \sqrt{\frac{EA}{m/l}} = l\sqrt{\frac{k}{m}}$$

The boundary conditions for the free vibrations can be written as follows:

$$u(0,t) = 0 \qquad EA\frac{\partial u(l,t)}{\partial x} = u(l,t) \cdot k_g$$

The solution, obtained through the Fourier method, is the following series:

$$u(x,t) = \sum_{i}^{\infty} C_i \sin(\eta_i x) \cdot \sin(\omega_i t + \gamma_i)$$

where:

C_i, γ_i – depends on initial conditions

$\omega_i = \eta_i \cdot c$

while η_i are determined through the roots of the equation.

$$\mathrm{tg}\,(h\eta) = -\frac{k}{k_g} \cdot (h\eta)$$

The problem will be further contemplated for the describing series $u(x, t)$, but only with regard to the first $i = 1$, i.e. for:

$$u(x,t) \cong C \sin(\eta_1 x) \cdot \sin(\omega_1 t + \gamma_1)$$

In order to calculate the value of energy losses in each subsystem, the maximum potential energy in a) the spring, and b) the rubber, must be arrived at:

a) the spring:

$$U_{max} = \int_{(s)} \frac{1}{2} EA \frac{\partial u}{\partial x} du = \frac{1}{2} EA \int_0^l [C\eta_1 \cos(\eta_1 x)]^2 dx =$$

$$= \frac{1}{2} EAC^2 \eta_1^2 \left[\frac{x}{2} + \frac{1}{4\eta_1} \sin(2\eta_1 x) \right]_0^l =$$

$$= \frac{klC^2 \eta_1^2}{8} \left[2l + \frac{1}{\eta_1} \sin(2\eta_1 l) \right]$$

b) the rubber:

$$U_{g\max} = \frac{1}{2} k_g C^2 \sin^2(\eta_1 l)$$

The loss of energy for the vibrations period, as a result of material-structural suppression, may be determined from the following formula:

$$\Delta E_s = \psi_s U_{smax} = C^2 \frac{k\psi_s}{8} [2(l\eta_1)^2 + l\eta_1 \sin(2\eta_1 l)]$$

$$\Delta E_g = \psi_g U_{gmax} = \frac{C^2}{2} k_g \psi_g \sin^2(\eta_1 l)$$

therefore the relation of the energy losses is equal to:

$$\Delta = \frac{\Delta E_g}{\Delta E_s} = \frac{4\psi_g k_g}{\psi_s k} \cdot \frac{\sin^2(\eta_1 l)}{[2(l\eta_1)^2 + l\eta_1 \sin(2\eta_1 l)]} \tag{6}$$

Let us now formulate the following hypothesis:

The phenomenon of transmission of vibrations through the steel springs, occurring near the resonance frequencies of these springs is not restricted with dynamic factors (the reflection of waves of the boundary of media), but depends on the system suppression properties, which are increased following the serial insertion of the rubber cushion into the system.

Based upon this hypothesis, as well as the approximately inversely proportional dependence of amplitude of vibrations in the resonance on the damping, the formula has been devised, determining the transmissibility coefficient for the system with the rubber cushion p_{s+g}, where the value of this coefficient for the system without the cushion p_s is known:

$$\frac{p_{s+g}}{p_s} \cong \frac{\Delta E_s}{\Delta E_s + \Delta E_g} = \frac{1}{1 + \Delta} \quad \text{or} \quad p_{s+g} \cong \frac{p_s}{1 + \Delta} \tag{7}$$

where the coefficient p_s, as referring to the first resonance of the spring, may be arrived at from the formula (5) i.e.:

$$p_s = \frac{4}{\pi \cdot \psi} \cdot \frac{m}{M}$$

while Δ is determined through the relation (6).

The aforesaid relations allow the approximate evaluation of the impact of the rubber cushion on the vibrations transmissibility coefficient in the first natural frequency of the spring.

A simple form of the relation between p_s, p_{s+g} and Δ, may produce doubts whether this relation actually reflects the essence of the relationships between these quantities.

In order to verify the proposed method of determining the transmissibility coefficient p_{s+g} for the first resonance frequency of the spring, its value has been calculated based upon formulas (5, 7, 6) for various ratios of k_g / k. The results obtained for the vibration isolation system previously analyzed have been presented in Fig. 7, where the results obtained through the application of the FEM method are additionally given for reference and comparison.

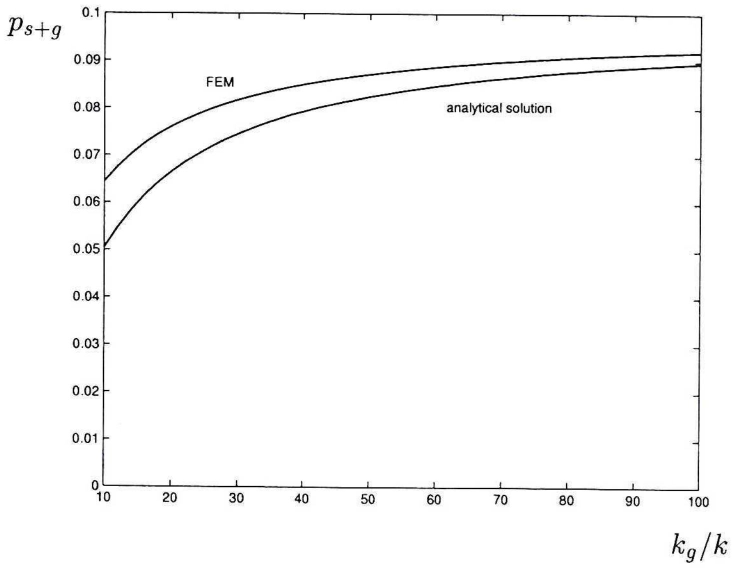

Fig. 7. The transmissibility coefficient p_{s+g} determined for ψ_s = 0.13 and ψ_g = 0.3

As can be seen from the diagrams, the accuracy of the relationship (7) is significantly determined by the ratio of k_g / k. The relative error p_{s+g} within the range $10 \leq k_g / k \geq 100$ is found between 21.4% and 2.6%. For typical values of k_g / k, in the order of 70, the method error is in the order of 3.8%.

3.1. Optimization of the rubber cushion parameters

In order to determine the influence of k_g on the transmissibility in the main resonance p_1 and the first resonance of the spring p_2, the force vibration isolation system of the mass M, supported on the spring and the rubber cushion with different values of modulus of elasticity, has been subject to analysis.

In numerical analysis, the spring has been adopted with the following constants: $l = 0.2$ m, $EA = 180 \cdot 10^3$N, $\rho A = 50$ kg/m while $M = 1000$ kg, $\Psi_s = 0$.

Two limiting cases of rubber cushions have been adopted for the analysis, notably: ring cushion with parameters $k_g = 14 \cdot 10^6$N/m and $b_g = 0.684 \cdot 10^6/\omega$Ns/m and typical circular cushion with the parameters: $k_g = 71 \cdot 10^6$N/m and $b_g = 3.44 \cdot 10^6/\omega$Ns/m.

The optimal value of rubber parameters has been sought by analyzing the model for 11 different rubber parameters, starting from the "ring" ending on the "solid". The results of the analysis have been tabulated in table 2 (lines 2–12). The values: f_1, p_1 describe the vibrations of mass M on the elastic bearing, while f_2, p_2 – the lowest free resonance of the spring.

Table 2

Coefficients of transmissibility for different rubber parameters

k_g	b_g	p_1	f_1	p_2	f_2
$3.865 \cdot 10^6$	$188000/\omega$	108.86	4.29	0.15	124.20
$14 \cdot 10^6$	$684000/\omega$	339.02	4.62	0.37	141.21
$19.7 \cdot 10^6$	$959600/\omega$	470.15	4.66	0.50	143.65
$25.4 \cdot 10^6$	$1235200/\omega$	601.25	4.68	0.63	145.05
$31.1 \cdot 10^6$	$1510800/\omega$	732.34	4.70	0.77	145.95
$36.8 \cdot 10^6$	$1786400/\omega$	863.43	4.71	0.90	146.59
$42.5 \cdot 10^6$	$2062000/\omega$	994.52	4.72	1.03	147.05
$48.2 \cdot 10^6$	$2337600/\omega$	1125.6	4.72	1.16	147.41
$53.9 \cdot 10^6$	$2613200/\omega$	1256.7	4.73	1.30	147.70
$59.6 \cdot 10^6$	$2888800/\omega$	1387.8	4.73	1.43	147.93
$65.3 \cdot 10^6$	$3164400/\omega$	1518.8	4.73	1.56	148.12
$71 \cdot 10^6$	$3440000/\omega$	1649.9	4.74	1.69	148.28
∞	∞	∞	4.77	∞	150.15

The value $k_g = 3.865 \cdot 10^6$ N/m, included in the first line of tab. 2, has been chosen so that the value of the first free resonance frequency of the original system (without rubber) is lowered following the introduction of the cushion by not more than 10 %. The value of b_g has been selected proportionally to k_g [5]. In spite of the fact that the application of rubber with such parameters leads to significant lowering of coefficients p, these will be disregarded in the course of other contemplations due to the fact that in order to obtain such values of b_g and k_g, the special rubber vibration isolation unit should be used rather than the flat cushion adopted in this report.

Except for the vibration isolation unit, in the range of changes the rubber cushion parameters, the best values have been obtained for the limiting value of the variability interval (hence the name "the best" rather than optimal). For further analysis, a ring rubber cushion has been used, with the parameters $k_g = 14 \cdot 10^6$ N/m and $b_g = 0.684 \cdot 10^6 / \omega$ Ns/m.

4. Selection of steel abutment mass

If, as it is often the case, the resistance plate with mass comparable with the spring mass is found between the spring and rubber, this plate should be incorporated in the model, e.g. in a way as shown in Fig. 8.

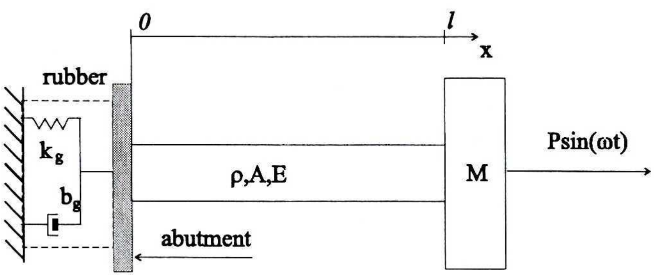

Fig. 8. The vibration isolation system model

In the case of the analysis of the system shown in Fig. 8, the mass of the plate (steel abutment) has been included in the boundary conditions, as the lumped mass:

$$EA\frac{\partial u(0,t)}{\partial x} = k_g \cdot u(0,\mathrm{t}) + b_g \cdot \frac{\partial u(0,t)}{\partial t} + m\frac{\partial^2 u(0,t)}{\partial t^2} \tag{8}$$

$$EA\frac{\partial u(l,t)}{\partial x} = -M\frac{\partial^2 u(l,t)}{\partial t^2} + P \cdot \sin(\omega t)$$

where: m – mass of the abutment,

M – sprung mass.

Table 3 shows the results of the analysis of the system shown in the figure 8 for different values of m – the mass of the abutment. The damping in the spring is disregarded, while the parameters of the rubber are the same as previously.

Table 3

Transmissibility coefficients for different masses

m	p_1	f_1	p_2	f_2	p_3	f_3	p_4	f_4	p_5	f_5	p_6	f_6
0	339.02	4.62	0.3714	141.21	–	–	0.0974	282.74	0.0463	425.25	0.0283	568.86
2	338.99	4.62	0.3365	140.20	0.0669	273.15	0.0262	388.38	0.0229	499.08	0.0275	628.80
5	338.95	4.62	0.2870	138.28	0.0438	245.53	0.0471	332.29	0.0719	464.25	0.0844	609.18
10	338.83	4.62	0.2155	133.65	0.0552	201.46	0.1338	311.39	0.1714	456.04	0.1857	604.21
15	338.74	4.62	0.1642	126.88	0.0971	178.67	0.2323	306.65	0.2730	453.83	0.2878	602.73
20	338.64	4.62	0.1345	118.75	0.1625	167.65	0.3331	304.68	0.3750	452.80	0.3901	602.03
25	338.55	4.62	0.1200	110.70	0.2436	162.08	0.4347	303.62	0.4773	452.22	0.4925	601.61
50	338.07	4.62	0.1191	83.37	0.7225	154.34	0.9453	301.71	0.9891	451.10	1.0046	600.81
100	337.11	4.62	0.1717	60.29	1.7356	151.92	1.9692	300.86	2.0135	450.57	2.0291	600.42

The additional resonance f_3, p_3 has appeared in the spectrum in conjunction with the partial frequency of the rubber-abutment configuration, following which the consecutive resonances have been renumbered, similarly as the corresponding transmissibility coefficients.

As can be seen from the results presented, the introduction of the mass m, in the order of 15–20 kg reduces in this example the coefficients p_2, p_3, corresponding to the lower frequencies of free vibrations of the abutment, down to 0.44% of the value p_2 for $m = 0$. Further increase of the mass leads to the increase of p_3.

5. Spring in the rubber sleeve – viscous damping

The damping in the system may be increased through putting the rubber sleeve on the spring [1] – figure 4.

The dry friction between the spring and rubber, occurring in reality, is replaced, based upon the condition of friction forces works equality, with the viscous friction in the calculation model. The friction force per unit of length is equal to the product of the difference of the spring and rubber velocity in the specific cross-section and the damping coefficient α.

After separating the variables in the form:

$$u(x,t) = X(x)\ T(t)$$

for the spring and

$$v(x,t) = Y(x)\,T(t)$$

for the rubber, the following system of differential equations is obtained:

$$\begin{cases} EA_1 X''(x) + \omega^2 \rho A_1 X(x) + \alpha\,(i\omega X(x) - i\omega Y(x)) = 0 \\ EA_2 Y''(x) + \omega^2 \rho_2 A_2 Y(x) + \alpha\,(i\omega Y(x) - i\omega X(x)) = 0 \end{cases}$$

where:

α – overall coefficient of viscous suppression of the boundary of environments,

ω – frequency of vibrations,

EA_1 – longitudinal stiffness of the spring,

EA_2 – longitudinal stiffness of the rubber; here $EA_2 = 1/15 \cdot EA_1$.

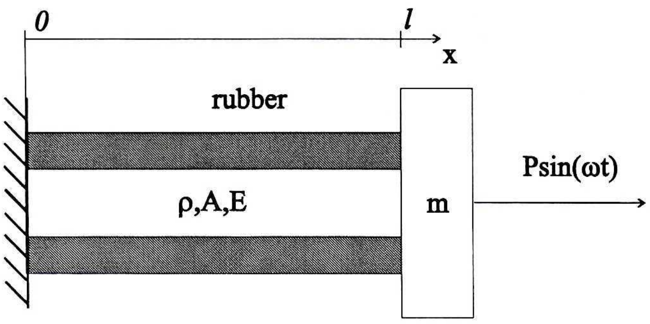

Fig. 9. The vibration isolation system model

It has been assumed that the friction on the boundary between the rubber and spring constitutes the prime source of energy dissipation, which makes it possible to disregard the material suppression in both environments.

Determining from the first equation of the aforesaid system:

$$i\omega\alpha Y(x) = EA_1 X''(x) + (\omega^2 \rho A_1 + i\omega\alpha)\,X(x)$$

differentiating twice

$$i\omega\alpha Y''(x) = EA_1 X^{(4)}(x) + (\omega^2 \rho A_1 + i\omega\alpha)\,X''(x)$$

and inserting into the second equation and arranging the terms, the fourth order equation is obtained:

$$X^{(4)}(x) + \left(\frac{\omega^2 \rho A_1 + i\omega\alpha}{EA_1} + \frac{\omega^2 \rho_2 A_2 + i\omega\alpha}{EA_2}\right) X''(x) + \tag{9}$$

$$+ \frac{(\omega^2 \rho A_1 + i\omega\alpha)\cdot(\omega^2 \rho_2 A_2 + i\omega\alpha) + \omega^2\alpha^2}{EA_1 \cdot EA_2} X = 0 \tag{10}$$

In order to simplify the notation, the following replacements have been used:

$$A = \frac{\omega^2 \rho A_1 + i\omega\alpha}{EA_1} + \frac{\omega^2 \rho_2 A_2 + i\omega\alpha}{EA_2}$$

$$B = \frac{(\omega^2 \rho A_1 + i\omega\alpha)\cdot(\omega^2 \rho_2 A_2 + i\omega\alpha) + \omega^2\alpha^2}{EA_1 \cdot EA_2}$$

The solution to the equation:

$$X^{(4)}(x) + AX''(x) + BX = 0$$

has been sought in the following form:

$$X(x) = C \exp^{\theta \cdot x}$$

therefore, the characteristic equation has the following form:

$$\theta^4 + A\theta^2 + B = 0$$

which roots are:

$$\theta_1^2 = \frac{-A + \sqrt{A^2 - 4B}}{2} \qquad \theta_1^2 = \frac{-A - \sqrt{A^2 - 4B}}{2}$$

The general solution of the equation (11) is presented by means of the function:

$$X(x) = C_1 \exp^{\theta_1 \cdot x} + C_2 \exp^{-\theta_1 \cdot x} + C_3 \exp^{\theta_2 \cdot x} + C_4 \exp^{-\theta_2 \cdot x} \tag{12}$$

while the equation of the amplitude of vibrations in the rubber:

$$Y(x)=\frac{EA_1}{i\omega\alpha}\cdot\left[\theta_1^2 C_1 \exp^{\theta_1\cdot x}+\theta_1^2 C_2 \exp^{-\theta_1\cdot x}+\theta_2^2 C_3 \exp^{\theta_2\cdot x}+\theta_2^2 C_4 \exp^{-\theta_2\cdot x}\right]$$
$$+\frac{\omega^2\rho A_1+i\omega\alpha}{i\omega\alpha}\cdot\left[C_1 \exp^{\theta_1\cdot x}+C_2 \exp^{-\theta_1\cdot x}+C_3 \exp^{\theta_2\cdot x}+C_4 \exp^{-\theta_2\cdot x}\right] \tag{13}$$

The boundary conditions associated with the model shown in figure 9 have the following form:

$$\begin{cases} EA_1 X''(l)+EA_2 Y''(l)-m\omega^2 X(l)=P \\ X(l)=Y(l) \\ X(0)=0 \\ Y(0)=0 \end{cases} \tag{14}$$

Inserting the functions (12) and (13), including their derivatives, into the system of equations (14) we may determine the constants of integration C_1 up to C_4, and the equations of amplitude of forced vibrations may be arrived at.

Figure 10 summarizes the results obtained for different values of clamping force of the sleeve on the rubber, thereby, various values of α.

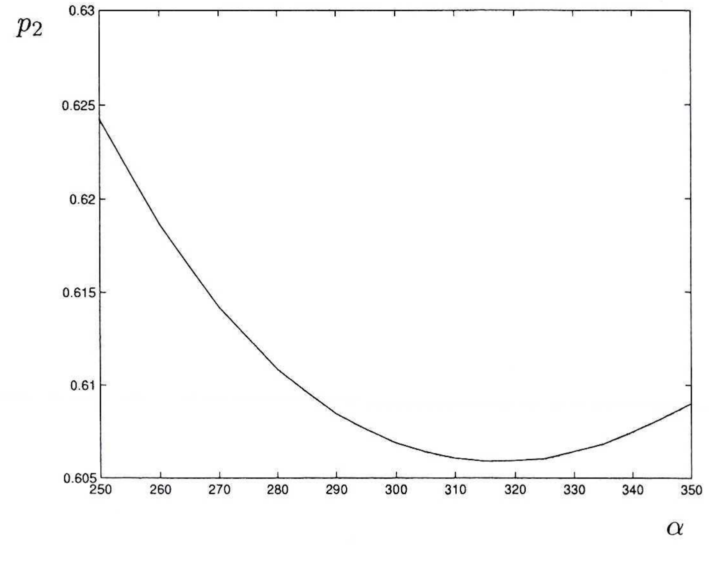

Fig. 10. Transmissibility coefficient p_2 in the function of coefficient α

A-s it can be seen, the α_{opt} i.e. $\alpha(p_2 = min)$ exists in this case, which means the existence of optimal clamping force of the sleeve on the spring.

In order to determinie this point, the method presented above may be directly applied.

For the optimal point, also the coefficients of transfer for the remaining resonances have been calculated.

The amplitude of force *R*, transmitted onto the base in the contemplated problem of vibration isolation (Fig. 9), can be arrived at from the following formula:

$$R = EA_1 X'(0) + EA_2 Y'(0)$$

Table 4 summarizes the coefficients *p* for the first five maximum values (resonances), together with the corresponding frequencies [Hz]. The suppression in the spring and in the rubber (p_1 = *infty*) has been disregarded in this analysis.

Table 4

Maximal coefficients of amplitude amplification, shown in figure 9

p_1	∞	f_1	4.92
p_2	0.606	f_2	149.90
p_3	0.187	f_3	299.88
p_4	0.111	f_4	449.91
p_5	0.080	f_5	599.93

As it can be seen, the damping on the boundary of the rubber-spring environments, does not restrict the resonance vibrations of the first form (vibrations of mass *M*), which is concurrent to the phenomenological analysis of the system. With regard to the internal resonances of the springs, the results obtained are worse than the results of the very structural damping in the spring, which shows the purposefulness of modifying the system from figure 9 so that the interaction method of both elements is changed, as shown e.g. in [1, p. 249].

6. Summary and conclusions

1°. The report shows that the "wave" method of evaluation of the effectiveness of the rubber cushion inserted into the spring vibration isolation system to reduce the high frequency vibrations transmission is unreasonable.

2°. The new method of evaluating the effectiveness, based upon the balance of energy in the resonance, has been formulated, and the relationship obtained has been checked using the MES method.
3°. With respect to the springs without rubber cushions, the formulas allowing arriving at the value of transmissibility coefficient p_2 in the first free resonance of the spring, with the incorporation of the nonlinear model of material and structural damping, have been derived.
4°. The analysis of impact of the resistance plate mass of the spring on the phenomenon of transmitting the vibrations at the proximity of the system first few areas of resonance has been performed.
5°. The mathematical model of interaction between the rubber sleeve and the spring in the spring-sleeve systems has been built, and the existence of the optimum clamping force has been shown from the condition against the minimum of p_2.

The paper was prepared under the research project KBN no 5T12A00622

Manuscript received by Editorial Board, May 21, 2003;
final version, January 19, 2004

REFERENCES

[1] Goliński J. A.: Wibroizolacja maszyn i urządzeń. WNT Warszawa, 1979.
[2] Kožešnik J.: Dynamika maszyn. WNT Warszawa, 1963.
[3] Mead D.: Passive Vibration Control. Wiley & Sons, N. York, 2000.
[4] Michalczyk J.: Dynamika maszyn górniczych cz. I. Wyd. AGH Kraków, 1990.
[5] Michalczyk J.: Maszyny Wibracyjne – obliczenia dynamiczne, drgania, hałas. WNT Warszawa, 1995.
[6] Michalczyk J., Cieplok G.: Wysokoefektywne układy wibroizolacji i redukcji drgań. Collegium Columbinum Kraków, 1999.
[7] Puzyna Cz.: Zwalczanie hałasu w przemyśle. WNT Warszawa, 1979.
[8] Rywin E.: Encyclopedia of Vibration; Vol. 3, chapter Vibration Isolation Teory, pages 1487÷1506. Academic Press, San Diego/Tokio, 2002.
[9] Solecki R., Szymkiewicz J.: Układy prętowe i powierzchniowe. Obliczenia dynamiczne. Warszawa, Arkady, 1964.

Analiza przenoszenia drgań materiałowych w układach wibroizolacji

S t r e s z c z e n i e

W pracy rozpatrzono zagadnienie przenoszenia drgań o częstotliwościach słyszalnych przez sprężyny stalowe stanowiące układ wibroizolacji. Wyprowadzono i zweryfikowano za pomocą

MES zależności analityczne pozwalające na określenie wartości współczynnika przenoszenia dla częstości rezonansowych sprężyn, które odpowiadają za przewodzenia drgań wysokoczęstotliwościowych.

Wskazano na występowanie narastających naprężeń w sprężynach również w obszarach pomiędzy rezonansami. Przeanalizowano typowy układ redukujący przenoszenie drgań wysokoczęstotliwościowych przez sprężynę tzn układ szeregowy z podkładką gumową. Wykazano, że redukcja przenoszenia nie zachodzi na skutek różnic w impedancji falowej obu środowisk lecz na skutek zwiększonego rozproszenia energii w gumie i wyprowadzono zależności analityczne pozwalające na ocenę skuteczności tego sposobu.