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EXPERIMENTAL/NUMERICAL PARAMETER IDENTIFICATION FOR VISCO-ELASTIC PROBLEMS

For reasons of reliability, stability, safety and economy, controlling and monitoring the response of structures during the time of use, either permanently or temporally, is of increasing importance. Experimental methods enable in-situ measuring deformations of any kind of structures and enable drawing conclusions over the actual state of the structures. However, to obtain reliable knowledge of the real internal conditions like the strength of materials and the actual stress-state, as well as of their changes over time, caused by ageing, fatigue and environmental influences, always an inverse problem must be solved. That requires special mathematical algorithms. Especially for time-depending material response it might be quite important to know the material parameters at any time and furthermore the internal stress-state also.

Therefore, a method will be presented to solve the inverse problem of parameter identification with reference to linear visco-elastic materials.

1. Introduction

Experimental methods are applied in system identification to analyse the static as well as the dynamic response of complex structures: they are developed as tools to supervise operating systems, machines and installations in order to guarantee a higher degree of safety and to minimise still existing risks. In the future, therefore, methods of experimental mechanics will become very important in strategies of risk-management. The structure must be observed under real operational conditions, recording the actual response. For safety as well as for economical reasons, permanent health monitoring as

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part of a value preservation management is inalienable to obtain information on the effects of ageing, fatigue, environmental influences etc. on the actual state of technical products and structures, to analyse the actual material response as well as the actual stress-strain state and possibly the remaining service-life. Temporary inspection is recommendable also to control the effectiveness of maintenance and retro-fitting measures. In any case, such inspections demand application of non-destructive technologies in order not to impair the usability of the objects under consideration [6].

Modern measuring systems do not consist of the hardware only, but have to include proper software for process control, for transmission, converting and processing the measured input data up to the final output information. The operational capability of the whole measurement system strongly depends on the software. As generally the observed phenomena and measured data, respectively, are not identical with the finally wanted information, mathematical/numerical procedures are necessary for data-processing and evaluation according to the finally wanted information.

2. The inverse problem of parameter identification

Modern measurement techniques, re-worked for routine comfortable practical application in monitoring systems, first of all yield analogue signals, either electrical, optical, acoustical signals of observed and measured phenomena. However, these signals generally do not come up to information necessary for relevant judgements of the actual state of the object under consideration. Processing the observed and recorded analogue signals, digitised data of deformations, related displacements and/or their derivatives will be obtained. However, for reliable assessment of the actual state of structures, information on the internal conditions and parameters are to derive from the measured data. In consequence, it is necessary to develop and to apply proper algorithms for further evaluation of these data. The defective quality of measured data because of errors of the measured data, outliers, lost data, noise-corrupted signals, limited number of measuring points demands advanced methods in statistics and mathematics for further evaluation. The so-called inverse problems arise, most of them are ill-posed as defined by the three *Hadamard* conditions: existence, uniqueness and stability of solutions [1]. Proper numerical procedures are required to obtain unequivocal solutions, which are indispensable to provide the finally wanted results like the stress-state, the internal forces, the internal parameters like material properties, the localisation of damages, the state of stability and safety and further on the service-life to be expected. The mathematical coherences of inverse problems are to find e.g. in [2]. It turns out that for each problem it must be proved which solution meets the physical requirements. The analyst always is responsible to justify the choice of the method and to identify the physical plausibility of the result.

As a result of ageing, fatigue and environmental influences etc. the material characteristic data might change or might have changed already during the time of use and operation. Reliable judgement of the actual state of structures requires the knowledge of time-depending response of materials. For visco-elastic material it is an inalienable necessity to get true information on the characteristic parameters and their dependency on time.

3. Basic relations of visco-elasticity

Considering structures and structural elements respectively, consisting of visco-elastic materials, it will be described in the following, how the internal parameters of such materials like the relaxation bulk-modulus, the relaxation shear-modulus and *Poisson's* ratio can be determined by proper experimental/numerical processes. This is an inverse problem belonging to the class of parameter-identification as the characteristic material parameters as time-functions shall be derived from measured quantities.

Based on *Boltzmann's* principle of superposition [9], the stress-strain relations for visco-elastic material response hold in the *Laplace*-transform [3]

$$\sigma_{ij}(p) = p \cdot 2G(p) \cdot e_{ij}(p) + p \cdot K(p) \cdot e(p) \cdot \delta_{ij} \tag{1}$$

where p denotes the *Laplace*-variable, $K(p)$ the relaxation bulk modulus, $G(p)$ the relaxation shear modulus, δ_{ij} the *Kronecker*-Delta, $e_{ij}(p)$ the strain deviator, $e(p)$ the volume strain.

Re-transformation of eq. (1) yields the *Volterra*-integral equation of the 2nd kind

$$\begin{aligned}\sigma_{ij}(t) = 2G(0^+) \cdot e_{ij}(t) - \int_{0^+}^{t} \frac{\partial}{\partial \tau} 2G(t-\tau) \cdot e_{ij}(\tau) d\tau + K(0^+) \cdot e(t) \cdot \delta_{ij} \\ - \int_{0^+}^{t} \frac{\partial}{\partial \tau} K(t-\tau) \cdot e(\tau) \cdot \delta_{ij} d\tau\end{aligned} \tag{2}$$

with the initial values $G(0^+)$ and $K(0^+)$ at time $t = 0^+$.

According to [2] *Volterra's* integral equations of the 2nd kind are well-posed and have a unique and stable solution always in an appropriate setting. Thus the inverse of eq. (1) holds

$$\mathbf{x}(t) = \mathbf{A}(0^+) \cdot \mathbf{y}(t) - \int_{0^+}^{t} \frac{\partial \mathbf{A}(t-\tau)}{\partial \tau} \cdot \mathbf{y}(\tau) \cdot d\tau \tag{3}$$

The output data $\mathbf{y}(t)$ are taken by measurements in time-intervals Δt, which means that the data are not available as functions, unless they are expanded in proper series. Therefore, eq. (3) is transformed into a finite difference⁄summation representation according to [7].

$$\mathbf{x}(t_n) = \frac{1}{2}\left\{\mathbf{A}(0^+) \cdot [\mathbf{y}(t_n) - \mathbf{y}(t_{n-1})] - \sum_{v=1}^{n-1} \mathbf{A}(t_n - t_v) \cdot [\mathbf{y}(t_{v-1}) - \mathbf{y}(t_{v+1})] + \mathbf{A}(t_n) \cdot [\mathbf{y}(t_1) + \mathbf{y}(0^+)]\right\} \tag{4}$$

Regarding this relation, eq. (2) runs

$$\begin{aligned}\sigma_{ij}(t_n) &= G(0^+) \cdot [e_{ij}(t_n) - e_{ij}(t_{n-1})] - \sum_{v=1}^{n-1} G(t_n - t_v) \cdot [e_{ij}(t_{v-1}) - e_{ij}(t_{v+1})] \\ &+ G(t_n) \cdot [e_{ij}(t_1) + e_{ij}(0^+)] + \frac{1}{2}\left\{K(0^+) \cdot [e(t_n) - e(t_{n-1})] - \sum_{v=1}^{n-1} K(t_n - t_v)\right. \\ &\left.\cdot [e(t_{v-1}) - e(t_{v+1})] + K(t_n) \cdot [e(t_1)] + e(0^+)]\right\} \cdot \delta_{ij}.\end{aligned} \tag{5}$$

As to error propagation, it can be anticipated that the selected particular finite-difference/summation representation should be extremely stable. The curves of the strains run steadily and smoothly; with progressive time the increments in time intervals Δt are decreasing. Obviously, the errors in data do not grow in size in subsequent stages of computation because of the generally fading memory of most of the visco-elastic materials.

The equilibrium conditions for a three-dimensional stress-state hold

$$\sigma_{ij,j}(t) + X_i(t) = 0, \qquad i, j \in [1/3] \tag{6}$$

The derivatives of eq. (5) in direction of the axes of a Carthesian co-ordinate system are introduced into the equilibrium conditions eq. (6), yielding three *Volterra*-integral equations of the 2^{nd} kind in discrete formulation, substituting the strain deviator and the volume dilatation by their derivatives $e_{ij,j}$ and $e_{,j}$. In these equations, the bulk modulus $K(t)$ and the shear modulus $G(t)$ are unknown, whereas the elements of the strain deviator and the volume strain and their derivatives, respectively, are obtained from measured data of displacements and/or their gradients respectively, provided these quantities can be measured for all i,j.

4. Solution for two-dimensional stress-states

In the case of a two-dimensional plane stress state, defined by

$$\sigma_{33} = \sigma_{13} = \sigma_{23} = 0; \qquad \varepsilon_{13} = \varepsilon_{23} = 0; \qquad \varepsilon_{33} \neq 0$$

two equilibrium conditions are at disposal.

$$\sigma_{\alpha\beta,\beta}(t) + X_\alpha(t) = 0, \qquad \alpha,\beta \in [1,2] \tag{7}$$

which run with reference to eq. (6):

$$\begin{aligned} 2G(t_n)\cdot \Sigma^*_{\alpha 0} + K(t_n)\cdot \bar{\Sigma}_{\alpha 0} = \sum_{\nu=1}^{n-1} \left\{ 2G(t_n - t_\nu)\cdot \Delta^*_{\alpha\nu} + K(t_n - t_\nu)\cdot \bar{\Delta}_{\alpha\nu} \right\} \\ + 2G(0^+)\cdot \Delta^*_{\alpha n} + K(0^+)\cdot \bar{\Delta}_{\alpha n} + X_\alpha \end{aligned} \tag{8}$$

and with the denotations

$e^*_\alpha(t) = \frac{1}{3}[\kappa\cdot \varepsilon_{\alpha\alpha,\alpha}(t) - \lambda\cdot \varepsilon_{\beta\beta,\alpha}(t)] + \varepsilon_{\alpha\beta,\beta}(t)$, $\alpha \neq \beta$, (not to sum up over α and β), and $\bar{e}_\alpha(t) = \lambda\cdot \varepsilon_{\beta\beta,\alpha}(t)$, (considering Einstein's convention), where κ and λ stand for $\kappa = (2-\nu)\cdot(1-\nu)^{-1}$, $\lambda = (1-2\nu)\cdot(1-\nu)^{-1}$, and with the abbreviations

$\Sigma^*_{\alpha 0} = e^*_\alpha(0^+) + e^*_\alpha(t_1)$, $\Delta^*_{\alpha\nu} = e^*_\alpha(t_{\nu-1}) - e^*_\alpha(t_{\nu+1})$, $\Delta^*_{\alpha n} = e^*_\alpha(t_{n-1}) - e^*_\alpha(t_n)$, $\bar{\Sigma}_{\alpha 0}$, $\bar{\Delta}_{\alpha\nu}$, $\bar{\Delta}_{\alpha n}$ respectively.

Because of the fading memory of visco-elastic material, computing of K and G at any time t_n is to carry out in time-intervals Δt_ν, for each t_n always beginning at $t_{0^+} = 0$, introducing the successive results of the preceding steps.

By means of optical measurement techniques, either the spatial displacements u_1, u_2, (u_3) or the gradients $u_{1,1}$, $u_{2,2}$, $u_{1,2}$, $u_{2,1}$, $(u_{3,1}, u_{3,2})$ of the spatial displacements related to the co-ordinate axes (x_1, x_2) on the surface of the object can be measured, yielding the components of the strain tensor ε_{11}, ε_{22}, $\varepsilon_{12} = \varepsilon_{21}$. The optical field methods do not yield the displacement gradients perpendicular to the object surface in the direction of the x_3-axis by direct measurement. However, considering the condition $\sigma_{33} = 0$, the relation between the component ε_{33} of the strain tensor and ε_{11} and ε_{22} holds

$$\varepsilon_{33} = -\frac{\nu}{1-\nu}\varepsilon_{\alpha\alpha} = -\frac{\nu}{1-\nu}u_{\alpha,\alpha} \tag{9}$$

where ν denotes *Poison's* ratio, which generally depends on time t like G and K, although its change over time is very small [8]. As linear material response can be described by only two material characteristic quantities, *Poisson's* ratio can be substituted by the bulk modulus K and the shear modulus G. It is easy to realise then that, eqs. (8) become non-linear. The unknown material parameters K and G are connected to each other in such a way, that a direct solution might become quite difficult. In the time-space the relation between the three parameters K, G and ν holds at time $t = 0^+$:

$$\nu(0^+) = \frac{3K(0^+) - 2G(0^+)}{6K(0^+) + 2G(0^+)} \tag{10}$$

With an initial estimate ν^{est}, approximate values $K^{est}(0^+)$ and $G^{est}(0^+)$ may be calculated on the basis of the measured data ε_{ij}^{meas} and introduced into eq. (10) yielding an improved value of $\nu(0^+)$.
Then, $K(0^+)$ and $G(0^+)$ are calculated again. This one-step iteration procedure can be repeated in each evaluation step Δt_ν.

After having carried out the calculations for the first time intervals, it may be decided whether the changes of *Poisson's* ratio over time are to be considered further on or whether $\nu(t)$ may be set approximately equal to $\nu(0^+)$.

5. Solution for plates-in-bending

The following considerations refer to the shear-elastic plate theory of the 1st order (*Mindlin*-theory) [4]. In contradiction to the classical plate theory according to *Kirchhoff* and the *Bernoulli*-hypothesis, the normal to the "neutral plane" does not remain perpendicular to the neutral plane in the deformed state (Fig. 1). Under the presuppositions i), the material to be homogeneous and isotropic, ii) the central plane of the plate to be the "neutral plane" for deformations due to bending, iii) the deflections to be small compared with the plate thickness, which itself should be small with reference to the other geometric dimensions, iv) σ_{33} to be negligible small compared to the other components and will be set equal to zero, the shear stresses in the planes (x_1, x_3) and (x_2, x_3), their effects on the rotation of the plate normal are taken into account.

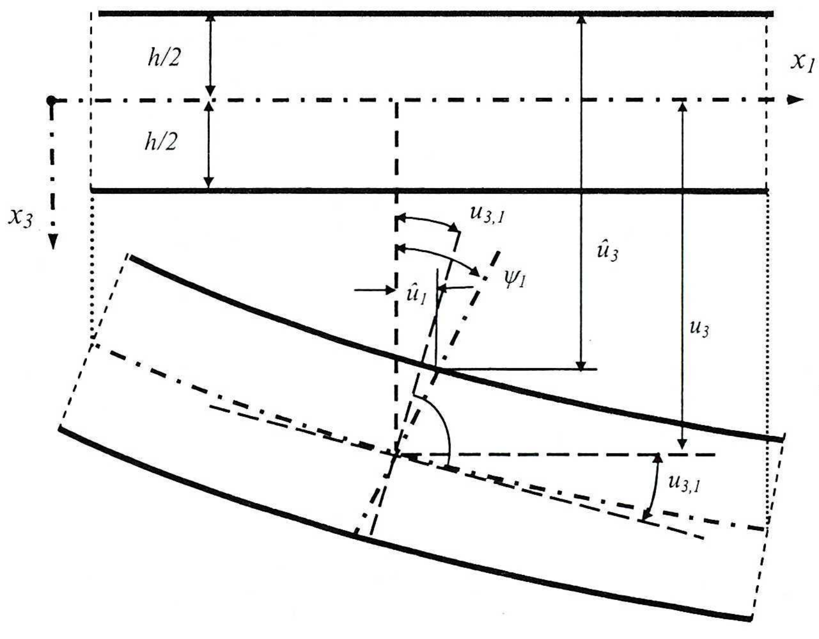

Fig. 1. Kinematics of the shear-elastic plate, section $x_2 = \text{const}$

Let $\hat{u}_\alpha(x_1, x_2)$, $\hat{u}_3(x_1, x_2)$, $\alpha \in [1,2]$ be given by measurements on the plate surface. With $\psi_\alpha(x_1, x_2) = 2/h \cdot \hat{u}_\alpha(x_1, x_2)$, the cinematic relations hold

$$\begin{aligned} u_\alpha(x_1, x_2, x_3) &\approx \psi_\alpha(x_1, x_2) \cdot x_3 = \frac{2}{h} \hat{u}_\alpha(x_1, x_2) \cdot x_3, \\ u_3(x_1, x_2) &\approx \hat{u}_3(x_1, x_2), \end{aligned} \tag{11}$$

subsequently yielding the components of the strain tensor

$$\varepsilon_{\alpha\beta}(x_1, x_2, x_3) = \frac{1}{h} [\hat{u}_{\alpha,\beta}(x_1, x_2) + \hat{u}_{\beta,\alpha}(x_1, x_2)] \cdot x_3. \tag{12}$$

Because $\sigma_{33} \approx 0$ the strain component ε_{33} holds with reference to Einstein's convention

$$\varepsilon_{33} = -\frac{\nu}{1-\nu}\cdot\frac{2}{h}[\hat{u}_{\alpha,\alpha}(x_1, x_2)]\cdot x_3 \tag{13}$$

Considering $u_3(x_1, x_2, x_3) \approx \hat{u}_3(x_1, x_2)$, the shear strain in the planes (x_α, x_3) is given to

$$\varepsilon_{3\alpha}(x_1, x_2, x_3) = \frac{1}{2}\left[\hat{u}_{3\alpha}(x_1, x_2) + \frac{2}{h}\cdot\hat{u}_\alpha(x_1, x_2)\right], \tag{14}$$

Note: For lucidity the reference to the co-ordinates (x_1, x_2) will be omitted in the following.
With eqs. (12), (13) and (14) the volume dilatation holds

$$e = \lambda\cdot\frac{2}{h}\hat{u}_{\alpha,\alpha}\cdot x_3 \tag{15}$$

and the components of the deviatoric strain tensor become

$$e_{11} = \frac{2}{3h}(\kappa\cdot\hat{u}_{1,1} - \lambda\cdot\hat{u}_{2,2})\cdot x_3, \quad e_{22} = \frac{2}{3h}(\kappa\cdot\hat{u}_{2,2} - \lambda\cdot\hat{u}_{1,1})\cdot x_3,$$
$$e_{12} = \frac{1}{h}(\hat{u}_{1,2} + \hat{u}_{2,1})\cdot x_3. \tag{16}$$

$$e_{31} = \varepsilon_{31} = \frac{1}{2}\left(\hat{u}_{3,1} + \frac{2}{h}\cdot\hat{u}_1\right), \; e_{32} = \varepsilon_{32} = \frac{1}{2}\left(\hat{u}_{3,2} + \frac{2}{h}\cdot\hat{u}_2\right). \tag{17}$$

The denotations κ and λ in eqs. (21) and (22) stand for

$$\kappa = \frac{2-\nu}{1-\nu}; \quad \lambda = \frac{1-2\nu}{1-\nu}. \tag{18}$$

The eqs. (15) and (16) are inserted into eq. (2), yielding the stress-strain relations for shear-elastic plates consisting of visco-elastic material. The internal forces $m_{\alpha\beta}$ are obtained by integrating the respective stresses over the plate thickness h.

To get the shear-forces q_α, the shear-stresses $\sigma_{3\alpha}$ are to integrate over h. These shear-stresses are independent of x_3, i.e. they are constant over the plate thickness. However, in fact they are distributed parabolic and at $x_3 = \pm h/2$ the boundary conditions require $\sigma_{31} = \sigma_{32} = 0$. This discrepancy shows the character of approximation of *Mindlin's* plate model, which violates the local equilibrium conditions. For correction the deformation energy will be determined, considering parabolic distribution of the shear stresses over h on one hand, on the other constant distribution over a corrected plate thickness h_s [5].

$$I)\ \sigma_{3\alpha}(x_3) = q_\alpha \frac{S(x_3)}{I} = q_\alpha \cdot \frac{6}{h^3}\left[\frac{h^2}{4} - x_3^2\right], \quad W_I = \frac{1}{2G}\int_{-h/2}^{+h/2} \sigma_{3\alpha}^2(x_3)\cdot dx_3 = \frac{1}{2G} q_\alpha^2 \cdot \frac{1}{h}\cdot\frac{5}{6}; \tag{19}$$

$$II)\ \sigma_{3\alpha} = \text{const over } h, \quad W_{II} = \frac{1}{2G}\int_{-h/2}^{+h/2} q_\alpha^2 \frac{1}{k_s\cdot h^2}\cdot dx_3 = \frac{1}{2G} q_\alpha^2 \cdot \frac{1}{k_s \cdot h}.$$

The deformation energy W_I is set equal to W_{II}, then the corrected thickness h_s amounts to 0,833 h. This consideration is true at any time t, therefore the shear forces can be formulated depending on time t as follows:

$$q_\alpha(t) = G(0^+)\cdot h_s\left[\hat{u}_{3,\alpha}(t) + \frac{2}{h}\hat{u}_\alpha(t)\right] - h_s\int_{0^+}^{t}\frac{\partial}{\partial\tau}G(t-\tau)\left[\hat{u}_{3,\alpha}(\tau) + \frac{2}{h}\hat{u}_\alpha(\tau)\right]d\tau \tag{20}$$

The respective derivatives of the internal moments and the eqs. (20) are inserted into the equilibrium conditions, which must be satisfied at any time t

$$m_{\alpha\beta,\beta}(t) - q_\alpha(t) = 0 \tag{21}$$

After some intermediate transformations, considering the discrete solution of Volterra's-integral equations, (see eq. (4)) two equations will be obtained enabling the calculation of the bulk-modulus $K(t_n)$ and the shear-modulus $G(t_n)$ based on measured quantities of the displacements $\hat{u}_1(t)$, $\hat{u}_2(t)$, $\hat{u}_3(t)$ or their derivatives respectively:

$$\begin{aligned} G(t_n)\cdot\Sigma_{\alpha 0}^{*} + K(t_n)\cdot\bar{\Sigma}_{\alpha 0} &= \sum_{v=1}^{n-1}\left[G(t_{n-v})\cdot\Delta_{\alpha v}^{*} + K(t_{n-v})\cdot\bar{\Delta}_{\alpha v}\right] \\ &\quad + G(0^+)\cdot\Delta_{\alpha n}^{*} + K(0^+)\cdot\bar{\Delta}_{\alpha n} \end{aligned} \tag{22}$$

with

$$\Sigma^*_{\alpha 0} = U^*_\alpha(0^+) + U^*_\alpha(t_1), \quad \Delta^*_{\alpha\nu} = U^*_\alpha(t_{\nu-1}) - U^*_\alpha(t_{\nu+1}), \quad \Delta^*_{\alpha n} = U^*_\alpha(t_{n-1}) - U^*_\alpha(t_n),$$

and $\bar{\Sigma}_{\alpha 0}$, $\bar{\Delta}_{\alpha\nu}$, $\bar{\Delta}_{\alpha n}$ respectively, where U^*_α, $\bar{U}_\alpha$ stand for

$$U^*_\alpha(t) = \frac{1}{3}\kappa \cdot \hat{u}_{\alpha,\alpha\alpha}(t) + \frac{1}{2}\hat{u}_{\alpha,\beta\beta}(t) + \frac{1}{6}\cdot\frac{1+\nu}{1-\nu}\hat{u}_{\beta,\beta\alpha}(t) - \frac{3h_s}{h^3}\left[\hat{u}_{3,\alpha}(t) + \frac{2}{h}\hat{u}_\alpha(t)\right],$$

$$\alpha \neq \beta, \text{ (not to sum up over } \alpha \text{ and } \beta) \tag{23}$$

$$\bar{U}_\alpha = \frac{1}{2}\lambda \cdot \hat{u}_{\beta,\beta\alpha} \text{ (acc. to Einstein's convention)} \tag{24}$$

Neglecting the rotation of the plate-normal in the case of small h, a *modified* Mindlin-theory may be considered, where $\psi_\alpha = u_{3,\alpha}$; $\alpha \in [1,2]$. Then the components of the strain tensor hold

$$\varepsilon_{\alpha\beta} = u_{3,\alpha\beta} \cdot x_3, \quad \varepsilon_{33} = -\frac{\nu}{1-\nu}u_{3,\alpha\alpha} \cdot x_3, \quad \varepsilon_{3\alpha} = u_{3,\alpha} \cdot x_3 \tag{25}$$

and the terms $U^*_\alpha(t)$, $\bar{U}_\alpha(t)$; $\alpha \in [1,2]$ in eqs. (14) stand for

$$U^*_\alpha(t) = \frac{1}{3}\kappa \cdot u_{3,\alpha\beta\beta}(t) - \frac{12h_s}{h^3} \cdot u_{3,\alpha}(t), \quad \bar{U}_\alpha(t) = \frac{1}{2}\lambda \cdot u_{3,\alpha\beta\beta}(t) \tag{26}$$

6. Example of application

As an example of application of the above described method, a clamped plate has been inspected (Fig. 2). The surface of deflection has been determined by means of electronic-speckle-pattern interferometry at different time-intervals Δt. After processing the saw-tooth images (Fig. 3), the thus obtained deflections u_3 in the nodal points of an evaluation grid have been subjected to a process of smoothing and adjustment, yielding the approximation-function of the deflection surface $f = u_3 \cdot (x_1, x_2)$ (Fig. 4). The respective derivatives of the approximation function are the input data to execute the calculations according to eq. (22), yielding the material characteristic parameters $K(t)$, $G(t)$ and in addition $E(t)$ as shown in Fig. 5. The results are in good coincidence with those obtained in the material testing.

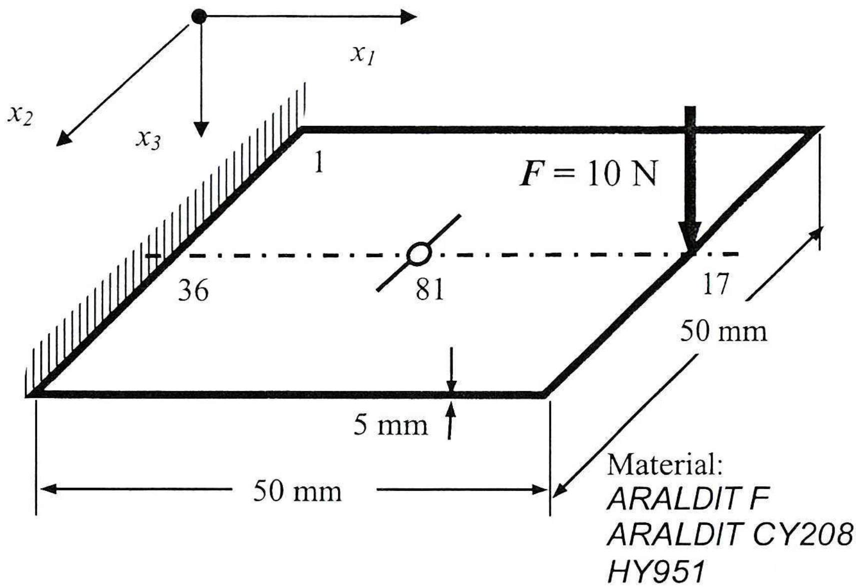

Fig. 2. Example of a clamped plate, dimensions and materials

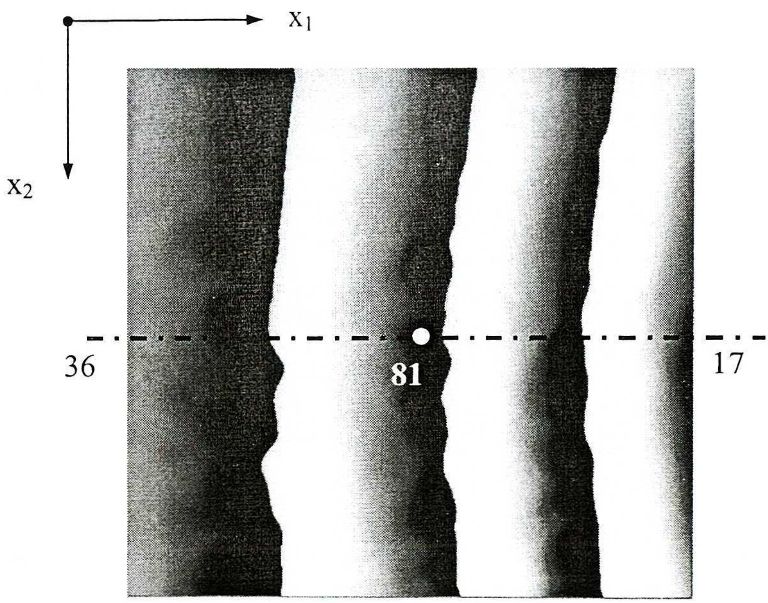

Fig. 3. Saw-tooth image, recorded 10 sec. after loading

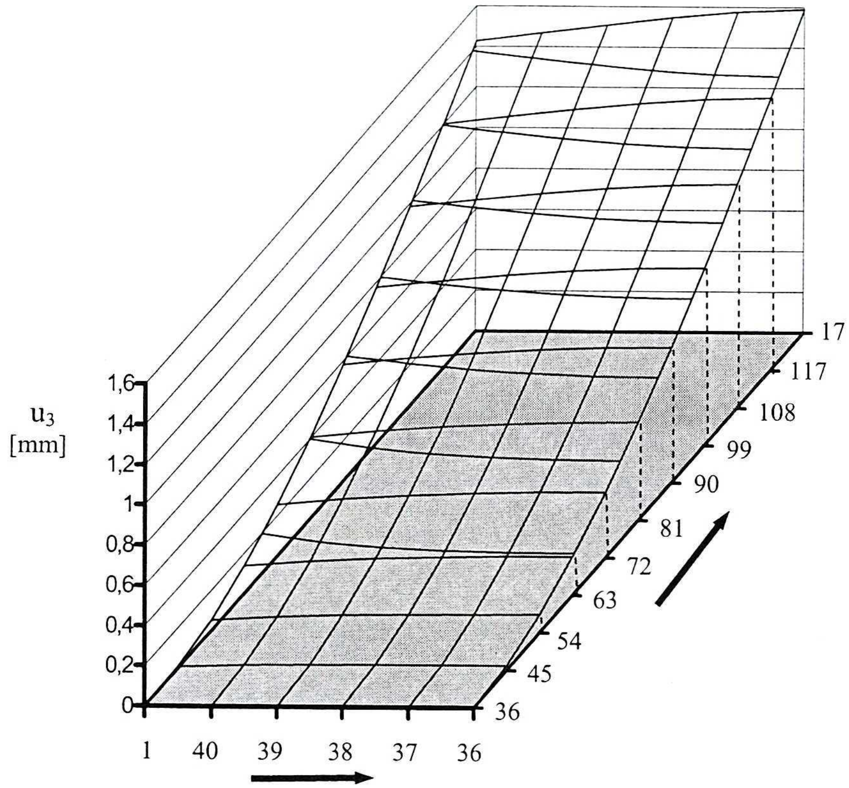

Fig. 4. Deflection-surface $u_3(x_1, x_2)$, 10 sec. after loading

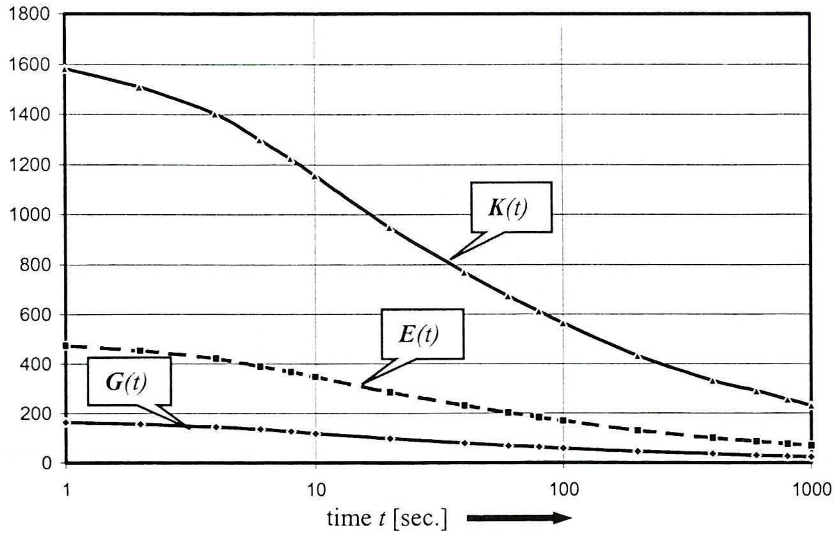

Fig. 5. Material parameters $K(t)$, $G(t)$, $E(t)$

In practical application, notwithstanding the fact that depending on the geometry of the object parts of the surface can not be observed, it is not necessary to carry out the analysis for the entire object surface. A sector of the surface only can be selected, considering as many neighbouring nodal points of the evaluation grid as necessary to determine an approximation function of at least grade three.

7. Conclusion

Monitoring and supervising structures and structural elements, respectively, consisting of polymer or composite material becomes increasingly important as the strength of the material defines the bearing capacity and safety of the structures. Based on the theory of linear visco-elasticity, algorithms have been derived enabling the determination of time-depending material characteristic parameters from in-situ measured deformations of the object/structure under consideration. Optical measurement techniques yield the necessary data for evaluation. As an example of application a plate-in-bending has been chosen, the results of the analysis are reported.

The presented method is based on the assumption, that the measurements start at the time of initial loading of the structure. However, in reality the more interesting case must be considered that the state of structure and the material response are to be determined more or less long ago after production or construction or initial loading, and no information of the structural history is available. At present, the author is working out a method, based on applying a test-load, to get at least approximate information on the actual state of structural stability and on the history of the material parameters.

It must be mentioned that the algorithms refer to iso-thermal conditions. In further investigations, the influence of temperature will be analysed.

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Eksperymentalna i numeryczna identyfikacja parametrów w zagadnieniach lepko-sprężystych

Streszczenie

Względy niezawodności, stabilności, bezpieczeństwa i ekonomii powodują, że rośnie znaczenie kontrolowania i monitorowania, permanentnie lub w sposób wyrywkowy, czasowych charakterystyk odpowiedzi struktury. Metody eksperymentalne pozwalają mierzyć deformacje w miejscu, gdzie one powstają, w strukturach dowolnego rodzaju, umożliwiają także wyciąganie wniosków co do rzeczywistego stanu badanych struktur. Niemniej, by uzyskać wiarygodną informację o realnych warunkach wewnętrznych, takich jak wytrzymałość materiałów, rzeczywisty stan naprężeń, a także o zmianach tych wielkości w czasie na skutek procesów starzenia, zmęczenia i wpływów środowiska, trzeba zawsze rozwiązać zagadnienie odwrotne. To zadanie wymaga specjalnych algorytmów matematycznych. W szczególności, dla wyznaczenia czasowej funkcji odpowiedzi materiału może okazać się bardzo ważna znajomość parametrów materiału w dowolnej chwili czasu, a co więcej, także stanu naprężeń wewnętrznych.

W pracy będzie więc przedstawiona metoda rozwiązanie zagadnienia odwrotnego identyfikacji parametrów w odniesieniu do liniowych materiałów lepko-sprężystych.