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DYNAMIC ANALYSIS OF THE STEERING SYSTEM OF A PASSENGER CAR WITH MCPHERSON SUSPENSION

The objective of this paper is to present a method of dynamic analysis of the steering system of a passenger car with McPherson suspension. The links of the system are modelled as rigid bodies; however, the method enables flexibility of the steering shaft of the car to be taken into account. The geometry of the system is described by using homogenous transformations. Equations of motion are derived on the basis of the Lagrange equations. In the method proposed, the closed loop of links is cut at selected joints and suitable reaction forces are introduced. Dry friction occurring in the steering system is reduced to the prismatic joint between the steering rack and guide. The method can be used in design and optimization of steering systems of passenger cars with McPherson suspension.

1. Introduction

The steering system of a car should enable one to drive over an uneven road without disturbances in the direction. At the same time it should enable the driver to steer and choose the best direction of drive as well as to efficiently manoeuvre in car parks. Thus there is a contradiction in requirements – on the one hand there is a tendency to isolate the driver from the effects of a wheel hitting an uneven surface, and on the other hand it is necessary for the driver to feel the interaction between the road and the

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wheels. In terms of safety, the car's response to steering wheel movements when held to go in a straight line is especially important; it should be necessary to increase the value of the torque applied to the steering wheel in order to change the direction in relation to straight driving. Experimental results show that hysteresis occurs in the courses of those torques (and also other physical parameters of the system) with respect to the angle of the steering wheel. Dry friction occurring in the drive system is considered as a cause of this effect. The friction significantly influences steerability of a vehicle and also its handling. According to experimental measurements and also observations, the friction is especially large in ball-and-socket joints which connect stub axles with suspension arms. It also occurs in joints (rolling bearings) between piston rods of McPherson columns and the vehicle body.

Reliability and efficiency of steering systems is very important for safe driving, and thus it is necessary to carry out full analysis of such systems. In order to examine the dynamics of the steering system of a selected passenger car it is necessary to analyse motion of its links when subject to defined forces, i.e. to solve the dynamic problem when taking into account dry friction in some joints. To this end, a physical model of the system as a multibody system in the form of a closed spatial kinematic chain must be formulated, and then its motion must be described using appropriate differential equations. Conclusions arising from this analysis can be helpful in the design of steering systems and especially in the optimisation process of the design. Results of computer simulations of the model formulated can be used to forecast behaviour of real steering systems. Such results can be reliable if all the essential features of the system are taken into account in the analysis and real data are used for numerical calculations. Considerable difficulties occur in determining parameters of dry friction. Dynamics of steering systems has been analysed in many textbooks on car design, for example [5], [22], but the analysis was usually limited.

There is relatively little Polish literature concerned with complex dynamic analysis of steering systems. These are textbook [11] and monographs [7], [15] as well as in some respects monograph [8] and paper [26]; in the latter the author presents a complex mathematical model of the steering system of a vehicle using calculus of vectors. Papers by Żardecki [32], [33] and Lozia and Zardecki [17] are also worth mentioning. The authors present their own method of modelling dry friction and clearance in steering systems and then analyse how these phenomena influence steerability of the vehicle. Foreign literature is much richer, especially German, one can mention papers by Hiller and his team [4], [9] and also monograph by Kölsch [14] and Wohnhaas [28]. A special attention should be paid to the section of [14] which is concerned with dynamic analysis of steering systems of vehicles with McPherson suspension, in which two phases of friction (kinetic and static) are modelled in the connection between pistons and cylinders of McPherson columns. According to the author, the friction occurring in this connection is especially important for dynamics of a car in motion because in certain conditions the piston in the cylinder can seize (in terms of dry friction the stick friction phase occurs in the connection). As a result, the car becomes unbalanced, and the driver is exposed to a special type of vibration called wobbling, which makes it difficult to steer the car and can cause an accident. A similar problem is also a subject of [19]. Authors of [24] deal with dynamic analysis of steering systems and they describe possibilities of MESA VERDE software in this respect.

A separate problem is kinetic analysis of steering systems, which to some extent can be treated as a preparatory stage for dynamic analysis. Analysis of such systems is especially difficult, since these are spatially complex link mechanisms; papers [12], [13] and parts of [18] should be mentioned among Polish literature. Outside Poland, paper [25] discusses the problems of kinetic analysis of drive systems, whose design involves the steering rack being driven by a pinion from the side of the steering roller. Its authors formulate conclusions useful for designers of such systems. Also [9] and [24] deal with kinetic analysis of steering systems. There are several papers [2], [3], [6], [23] of ADAMS users which are concerned with solving different problems of modelling and investigating steering systems of cars, especially analysis of kinematics.

Interesting results for designer of these systems are presented in [30]. The authors deal with modelling of dry friction in the steering gear (steering roller-pinion) in order to improve its efficiency. The results of experimental measurements of gears analysed are also presented. Authors of [20], [21], [27], [31] are also concerned with modelling and investigating elements of such systems.

2. Physical model

The steering system of a passenger car with McPherson suspension (Fig. 1) consists of the steering rack sliding in the guide of the car body driven by the pinion connected to the lower roller of the steering column. Movement of the steering rack is transferred to stub axles by means of steering rods causing wheel turn. Stub axles are connected with the body by suspension arms and McPherson columns (shock absorbers).

In order to develop a physical model of the system, it is assumed that the body of the car is fixed, and thus 9 links of the system are isolated (Fig. 2):

(1) – steering rack,
(2) – left piston of McPherson column,
(3) – left stub axle with a wheel,
(4) – left steering rod,
(5) – left suspension arm,
(6) – right piston of McPherson column,
(7) – right stub axle with a wheel,
(8) – right steering rod,
(9) – right suspension arm.

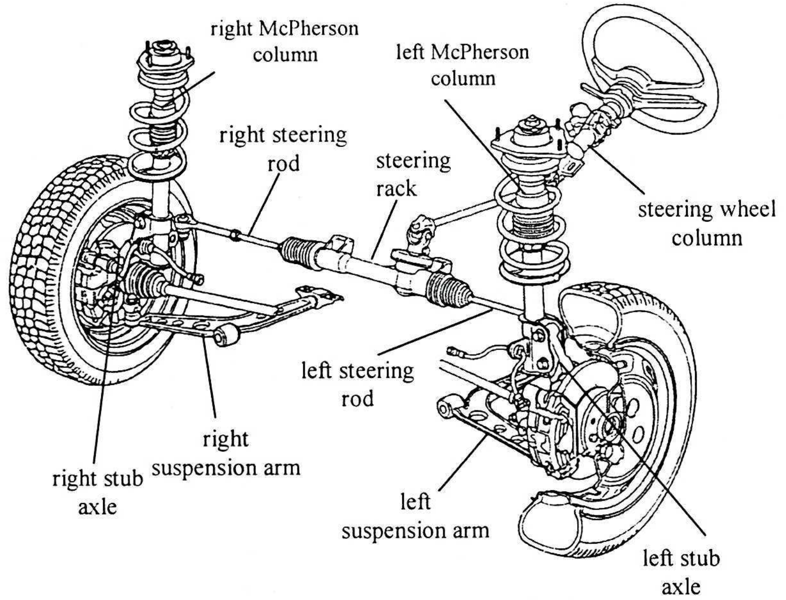


Fig. 1. Steering system of a passenger car with McPherson suspension

It is assumed that the body (1) is connected with a guide placed in the car body by means of a sliding joint P. In the system, there are also two rotary joints R_1 and R_2, respectively, between bodies (5) and (9) and the car body, eight spherical joints $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8$ connecting body (2) with the car body, (1) with (4), (3) with (4), (3) with (5), (6) with the car body, (1) with (8), (7) with (8), (7) with (9), respectively, and two cylindrical joints C_1 and C_2 between (2) and (3), (6) and (7), respectively, (which are right and left pistons and cylinders of McPherson columns fixed to stub axles).

A pinion mounted to the roller of the steering column drives the steering rack. In the model, the assumed stiffness and flexibility of this column is introduced by means of a spring-damping element SDE_1 constructed of a spring element and a parallel damping element. Motion of the elements of the system is caused by a turn of the steering wheel by an angle φ_K.

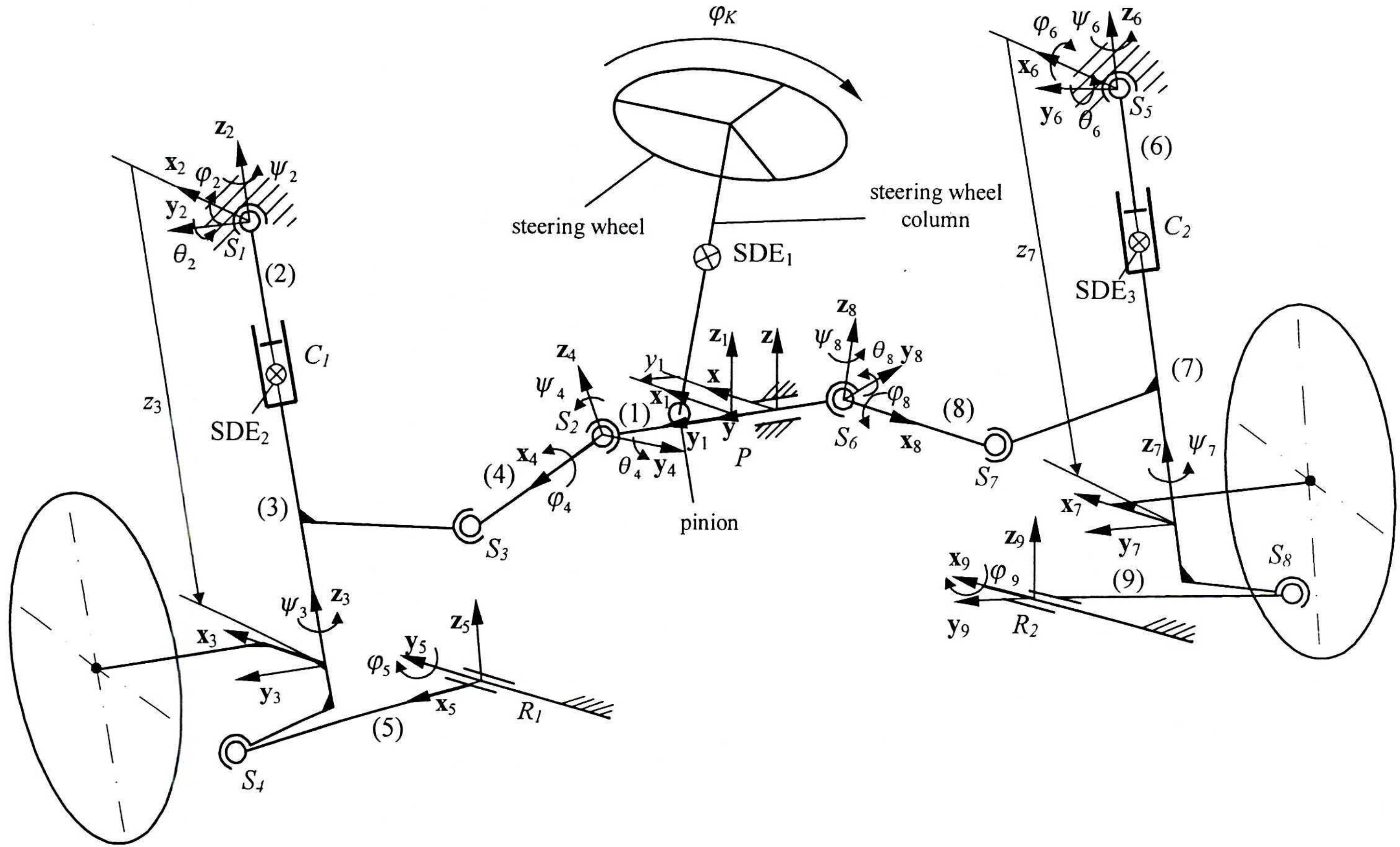


Fig. 2. Physical model of steering system of a passenger car with McPherson suspension

Spring-damping elements SDE_2 and SDE_3, constructed like SDE_1, are placed in the connections of piston with McPherson columns (axes of those elements, determining directions of their actions, coincide with column axes).

The geometry of the system will be described using homogenous transformations [1]. To this end for further considerations it is assumed:

$\mathbf{p}^{(i)}$ – denotes a vector of generalised coordinates defining the position of body (i) with respect to the local coordinate system $\mathbf{x}_p\ \mathbf{y}_p\ \mathbf{z}_p$ assigned to the preceding body p (the one closer to the car body), (1a)

$\mathbf{q}_i$ – denotes a vector of generalised coordinates defining the position of body (i) with respect to the inertial system $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$. (1b)

Coordinate systems are formed by Cartesian unit vectors. The origin of the inertial system $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ (Fig. 2) is placed in the middle between the articulated joints S_2 and S_6 (having assumed that the wheels of the car are directed straight ahead). Versor $\mathbf{z}$ marks the direction of the gravity force.

If p is a number of the preceding body (i) then the following takes place:

$$\mathbf{q}_i = \begin{bmatrix} \mathbf{q}_p \\ \mathbf{q}^{(i)} \end{bmatrix}. \tag{2}$$

In order to describe the relative position of bodies, the following notations are used:

x_i – translation in direction $\mathbf{x}_i$ of the local coordinate system,
y_i – translation in direction $\mathbf{y}_i$ of the local coordinate system,
z_i – translation in direction $\mathbf{z}_i$ of the local coordinate system,
φ_i – rotation about axis $\mathbf{x}_i$ of the local coordinate system,
θ_i – rotation about axi-s $\mathbf{y}_i$ of the local coordinate system,
ψ_i – rotation about axis $\mathbf{z}_i$ of the local coordinate system.

The generalised coordinate vectors defining the position of each link are defined below.

(1) Steering rack

Its position with respect to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system is defined by the vector of one component:

$$\mathbf{q}_1 = \mathbf{q}^{(1)} = [y_1], \tag{3a}$$

and the transformation matrix from the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ system to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system is as follows:

$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3b}$$

It is assumed that the origin of the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ system is placed in the mass centre of the steering rack which lies on a line connecting the centres of articulated joints S_2 and S_6.

(2) Left piston

Its position with respect to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system is defined by the vector:

$$\mathbf{q}_2 = \mathbf{q}^{(2)} = [\psi_2 \quad \theta_2 \quad \varphi_2]^T, \tag{4a}$$

and the transformation matrix from the $\mathbf{x}_2\ \mathbf{y}_2\ \mathbf{z}_2$ system to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system takes the form:

$$\mathbf{B}_2 = \begin{bmatrix} c\psi_2 & -s\psi_2 & 0 & x_2 \\ s\psi_2 & c\psi_2 & 0 & y_2 \\ 0 & 0 & 1 & z_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_2 & 0 & c\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\varphi_2 & -s\varphi_2 & 0 \\ 0 & s\varphi_2 & c\varphi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4b}$$

where: x_2, y_2, z_2 define the position of origin of the $\mathbf{x}_2\ \mathbf{y}_2\ \mathbf{z}_2$ system in the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system.

In general, the following notations are assumed in the case of particular angles: $s\alpha = \sin\alpha$ and $c\alpha = \cos\alpha$.

(3) Left stub axle

Its position with respect to the $\mathbf{x}_2\ \mathbf{y}_2\ \mathbf{z}_2$ system assigned to body (2) and with respect to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system is defined by the following vectors:

$$\mathbf{q}^{(3)} = [z_3 \quad \psi_3]^T, \tag{5a}$$

$$\mathbf{q}_3 = \begin{bmatrix} \mathbf{q}_2^T \\ \mathbf{q}^{(3)T} \end{bmatrix} = [\psi_2 \quad \theta_2 \quad \varphi_2 \quad z_3 \quad \psi_3]^T. \tag{5b}$$

The transformation matrix from the $\mathbf{x}_3\ \mathbf{y}_3\ \mathbf{z}_3$ system to the $\mathbf{x}_2\ \mathbf{y}_2\ \mathbf{z}_2$ and

x y z systems are as follows:

$$\mathbf{A}^{(3)} = \begin{bmatrix} 1 & 0 & 0 & {}^{2}x_3 \\ 0 & 1 & 0 & {}^{2}y_3 \\ 0 & 0 & 1 & {}^{2}z_3+z_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\psi_2 & -s\psi_3 & 0 & 0 \\ s\psi_3 & c\psi_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{5c}$$

$$\mathbf{B}_3 = \mathbf{B}_2\mathbf{A}^{(3)}, \tag{5d}$$

where: ${}^{2}x_3$, ${}^{2}y_3$, ${}^{2}z_3$ define the position of the origin of the $\mathbf{x}_3\ \mathbf{y}_3\ \mathbf{z}_3$ system in the $\mathbf{x}_2\ \mathbf{y}_2\ \mathbf{z}_2$ system.

(4) Left steering rod

Its position with respect to the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ system of body (1) and with respect to the **x y z** system is defined as:

$$\mathbf{q}^{(4)} = [\psi_4 \ \ \theta_4 \ \ \varphi_4]^T, \tag{6a}$$

$$\mathbf{q}_4 = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}^{(4)} \end{bmatrix} = [y_1 \ \ \psi_4 \ \ \theta_4 \ \ \varphi_4]^T. \tag{6b}$$

Transformation matrices from the $\mathbf{x}_4\ \mathbf{y}_4\ \mathbf{z}_4$ system to the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ and **x y z** systems take the following forms:

$$\mathbf{A}^{(4)} = \begin{bmatrix} c\psi_4 & -s\psi_4 & 0 & {}^{1}x_4 \\ s\psi_4 & c\psi_4 & 0 & {}^{1}y_4 \\ 0 & 0 & 1 & {}^{1}z_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\varphi_4 & -s\varphi_4 & 0 \\ 0 & s\varphi_4 & c\varphi_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{6c}$$

$$\mathbf{B}_4 = \mathbf{B}_1\mathbf{A}^{(4)}, \tag{6d}$$

where: ${}^{1}x_4$, ${}^{1}y_4$, ${}^{1}z_4$ define the origin of the $\mathbf{x}_4\ \mathbf{y}_4\ \mathbf{z}_4$ system in the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ system.

(5) Left suspension arm

Its position in the inertial system **x y z** is defined by the one component vector:

$$\mathbf{q}_5 = \mathbf{q}^{(5)} = [\varphi_5], \tag{7a}$$

and the transformation matrix from the $\mathbf{x}_5\,\mathbf{y}_5\,\mathbf{z}_5$ system to the $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ system can be written as:

$$\mathbf{B}_5 = \begin{bmatrix} 1 & 0 & 0 & x_5 \\ 0 & c\varphi_5 & -s\varphi_5 & y_5 \\ 0 & s\varphi_5 & c\varphi_5 & z_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7b}$$

where: x_5, y_5, z_5 define the position of the origin of the $\mathbf{x}_5\,\mathbf{y}_5\,\mathbf{z}_5$ system in the $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ system.

(6) Right piston

Its position in the inertial system $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ is defined by the vector:

$$\mathbf{q}_6 = \mathbf{q}^{(6)} = [\psi_6 \;\; \theta_6 \;\; \varphi_6]^T, \tag{8a}$$

and the transformation matrix from the $\mathbf{x}_5\,\mathbf{y}_5\,\mathbf{z}_5$ system to the $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ system takes the form:

$$\mathbf{B}_6 = \begin{bmatrix} c\psi_6 & -s\psi_6 & 0 & x_6 \\ s\psi_6 & c\psi_6 & 0 & y_6 \\ 0 & 0 & 1 & z_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_6 & 0 & s\theta_6 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_6 & 0 & c\theta_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\varphi_6 & -s\varphi_6 & 0 \\ 0 & s\varphi_6 & c\varphi_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8b}$$

where: x_6, y_6, z_6 define the position of the origin of the $\mathbf{x}_6\,\mathbf{y}_6\,\mathbf{z}_6$ system in the $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ system.

(7) Right stub axle

Its position in the $\mathbf{x}_6\,\mathbf{y}_6\,\mathbf{z}_6$ system of body (6) and in the $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ system is defined by the following vectors:

$$\mathbf{q}^{(7)} = [z_7 \;\; \psi_7]^T, \tag{9a}$$

$$\mathbf{q}_7 = \begin{bmatrix} \mathbf{q}_6 \\ \mathbf{q}^{(7)} \end{bmatrix} = [\psi_6 \;\; \theta_6 \;\; \varphi_6 \;\; z_7 \;\; \psi_7]^T, \tag{9b}$$

and the transformation matrices from the $\mathbf{x}_7\,\mathbf{y}_7\,\mathbf{z}_7$ system to the $\mathbf{x}_6\,\mathbf{y}_6\,\mathbf{z}_6$ and $\mathbf{x}\,\mathbf{y}\,\mathbf{z}$ systems are as follows:

$$\mathbf{A}^{(7)} = \begin{bmatrix} 1 & 0 & 0 & {}^{6}x_7 \\ 0 & 1 & 0 & {}^{6}y_7 \\ 0 & 0 & 1 & {}^{6}z_7 + z_7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\psi_7 & -s\psi_7 & 0 & 0 \\ s\psi_7 & c\psi_7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{9c}$$

$$\mathbf{B}_7 = \mathbf{B}_6 \mathbf{A}^{(7)}, \tag{9d}$$

where: ${}^{6}x_7$, ${}^{6}y_7$, ${}^{6}z_7$ define the position of the origin of the $\mathbf{x}_7\ \mathbf{y}_7\ \mathbf{z}_7$ system in the $\mathbf{x}_6\ \mathbf{y}_6\ \mathbf{z}_6$ system.

(8) Right steering rod

Its position with respect to the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ system assigned to body (1) and the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system is defined by the vectors:

$$\mathbf{q}^{(8)} = [\psi_8\ \ \theta_8\ \ \varphi_8]^T, \tag{10a}$$

$$\mathbf{q}_8 = \begin{bmatrix} \mathbf{q}_7 \\ \mathbf{q}^{(8)} \end{bmatrix} = [y_1\ \ \psi_8\ \ \theta_8\ \ \varphi_8]^T, \tag{10b}$$

and the transformation matrices from the $\mathbf{x}_8\ \mathbf{y}_8\ \mathbf{z}_8$ system to the $\mathbf{x}_7\ \mathbf{y}_7\ \mathbf{z}_7$ and $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ systems can be written in the form:

$$\mathbf{A}^{(8)} = \begin{bmatrix} c\psi_8 & -s\psi_4 & 0 & {}^{1}x_4 \\ s\psi_8 & c\psi_8 & 0 & {}^{1}y_8 \\ 0 & 0 & 1 & {}^{1}z_8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_8 & 0 & s\theta_8 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_8 & 0 & c\theta_8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\varphi_8 & -s\varphi_8 & 0 \\ 0 & s\varphi_8 & c\varphi_8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{10c}$$

$$\mathbf{B}_8 = \mathbf{B}_1 \mathbf{A}^{(8)}, \tag{10d}$$

where: ${}^{1}x_8$, ${}^{1}y_8$, ${}^{1}z_8$ define the position of the origin of the $\mathbf{x}_8\ \mathbf{y}_8\ \mathbf{z}_8$ system in the $\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1$ system.

(9) Right suspension arm

Its position with respect to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system is defined by the one component vector:

$$\mathbf{q}_9 = \mathbf{q}^{(9)} = [\varphi_9], \tag{11a}$$

and the transformation matrix from the $\mathbf{x}_9\ \mathbf{y}_9\ \mathbf{z}_9$ system to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system takes the form:

$$\mathbf{B}_9 = \begin{bmatrix} 1 & 0 & 0 & x_9 \\ 0 & c\varphi_9 & -s\varphi_9 & y_9 \\ 0 & s\varphi_9 & c\varphi_9 & z_9 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{11b}$$

where: x_9, y_9, z_9 defines the position of the origin of the $\mathbf{x}_9\ \mathbf{y}_9\ \mathbf{z}_9$ system in the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system.

3. Mathematical model

The equations of motion are derived from the Lagrange equations.

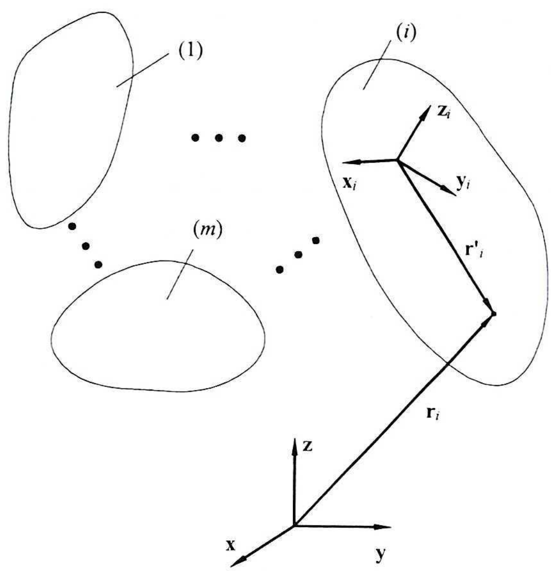


Fig. 3. Multibody system

Using the relations presented in section 2 and the methodology presented in [1], it is assumed that the motion of body (i) of the multibody system (where $i = 1,2,...,m$), shown in Fig. 3, is described in the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system by the vector of n_i generalised coordinates, which can be written in the following form:

$$\mathbf{q}_i = [q_{i1} \; \dots q_{ik} \dots \; q_{in_i}]^T, \tag{12}$$

and the coordinates of an arbitrary point in body (i) are transformed to this system according to the formula:

$$\mathbf{r}_i = \mathbf{B}_i(\mathbf{q}_i)\mathbf{r}'_i, \tag{13}$$

where: $\mathbf{B}_i$ – is the transformation matrix from the $\mathbf{x}_i \; \mathbf{y}_i \; \mathbf{z}_i$ system to the $\mathbf{x} \; \mathbf{y} \; \mathbf{z}$ system,

$\mathbf{r}'_i = [x'_i \; y'_i \; z'_i \; 1]^T$ – is a coordinate vector of the point in the $\mathbf{x}_i \, \mathbf{y}_i \, \mathbf{z}_i$ system assigned to body (i),

$\mathbf{r}_i = [x_i \; y_i \; z_i \; 1]^T$ – is a coordinate vector with respect to the $\mathbf{x} \; \mathbf{y} \; \mathbf{z}$ system.

Kinetic and potential energies of body (i) can be presented as follows [10]:

$$T_i = tr\{\dot{\mathbf{B}}_i \mathbf{H}_i \dot{\mathbf{B}}_i^T\}, \tag{14a}$$

$$V_i = m_i g \boldsymbol{\theta}_3^T \mathbf{B}_i \mathbf{r}'_{Ci}, \tag{14b}$$

where: $tr\{ \}$ – is the trace of a matrix,

$\dot{\mathbf{B}}_i$ – is the matrix derivative with respect to time of matrix $\mathbf{B}_i$,

$\mathbf{H}_i = \int_m \mathbf{r}'_i \, \mathbf{r}'^T_i \, dm_i$ – is the mass matrix of body (i),

m_i – is the mass of body (i),

g – is the acceleration of gravity,

$\boldsymbol{\theta}_3^T = [0 \;\; 0 \;\; 1 \;\; 0]$,

$\mathbf{r}'_{Ci}$ – is the coordinate vector of the mass centre of body (i) with respect to the $\mathbf{x}_i \; \mathbf{y}_i \; \mathbf{z}_i$ system.

The Lagrange operators, after some transformations, take the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_{i,k}} - \frac{\partial T}{\partial q_{i,k}} = \sum_{j=1}^{n_i} a_{k,j}^{(i)} \ddot{q}_{i,j} + h_k^{(i)}, \text{ for } i = 1, \dots, m \text{ and } k = 1, \dots, n_i, \tag{15a}$$

$$\frac{\partial V}{\partial q_{i,k}} = V_k^{(i)} = m_i g \boldsymbol{\theta}_3^T \mathbf{B}_{i,k} \mathbf{r}'_{Ci}, \tag{15b}$$

where: T, V – are the kinetic and potential energies of the system,

$$a_{k,j}^{(i)} = tr\{\mathbf{B}_{i,k} \mathbf{H}_i \mathbf{B}_{i,j}^T\},$$

$$h_k^{(i)} = \sum_{j=1}^{n_i} \sum_{l=1}^{n_i} tr\{\mathbf{B}_{i,k} \mathbf{H}_i \mathbf{B}_{i,j,l}^T\} \dot{q}_{i,j} \dot{q}_{i,l},$$

$$\mathbf{B}_{i,j} = \frac{\partial \mathbf{B}_i}{\partial q_{i,j}},$$

$$\mathbf{B}_{i,j,l} = \frac{\partial^2 \mathbf{B}_i}{\partial q_{i,j} \partial q_{i,l}}.$$

If force $\mathbf{P}$ defined with respect to the $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$ system by the components of the vector:

$$\mathbf{P} = [P_x \ \ P_y \ \ P_z \ \ 0]^T \tag{16}$$

acts at body (i) at the point with the following coordinates:

$$\mathbf{r}'_{i,P} = [x'_{i,P} \ \ y'_{i,P} \ \ z'_{i,P} \ \ 1]^T \tag{17}$$

in the local coordinate system $\mathbf{x}_i\ \mathbf{y}_i\ \mathbf{z}_i$, then the generalised forces arising can be expressed as:

$$Q_k^{(i)}(\mathbf{P}) = [\mathbf{B}_{i,k} \mathbf{r}'_{i,P}]^T \mathbf{P} \ \text{ for } k=1,...,n_i. \tag{18}$$

Potential energy of spring deformation and the function of energy dissipation of the spring-damping element SDE_1 of the steering wheel column can be expressed respectively as follows:

$$V^{(1)} = \frac{1}{2} c_1 [i_K \varphi_K - y_1]^2, \tag{19a}$$

$$D^{(1)} = \frac{1}{2} b_1 [i_K \dot{\varphi}_K - \dot{y}_1]^2, \tag{19b}$$

where: c_1, b_1 – are stiffness and damping coefficients respectively,
i_K – is the ratio of the steering system.

This ratio can be written in the form:

$$i_K = \frac{1}{2} d_K, \tag{20}$$

where: d_K – is the pitch diameter of the pinion.

Potential energy of deformation and the function of energy dissipation of SDE_1 and SDE_2 can be presented in the forms:

$$V^{(2)} = \frac{1}{2} c_2 z_3^2, \tag{21a}$$

$$D^{(2)} = \frac{1}{2} b_2 \dot{z}_3^2, \tag{21b}$$

and

$$V^{(3)} = \frac{1}{2} c_3 z_7^2, \tag{22a}$$

$$D^{(3)} = \frac{1}{2} b_3 \dot{z}_7^2, \tag{22b}$$

where: c_2, c_3, b_2, b_3 – are stiffness and damping coefficients of respective spring-damping elements.

Having assumed kinematic drive input of the steering wheel, i.e. assuming that the following functions are known:

$$\varphi_K = \varphi_K(t), \tag{23a}$$

$$\dot{\varphi}_K = \dot{\varphi}_K(t), \tag{23b}$$

the vector of generalised coordinates of the whole system takes the following form:

$$\begin{aligned}\mathbf{q} &= [q_1\ q_2\ q_3\ q_4\ q_5\ q_6\ q_7\ q_8\ q_9\ q_{10}\ q_{11}\ q_{12}\ q_{13}\ q_{14}\ q_{15}\ q_{16}\ q_{17}\ q_{18}\ q_{19}]^T = \\ &= [y_1 | \psi_2\ \theta_2\ \varphi_2 | z_3\ \psi_3 | \psi_4\ \theta_4\ \varphi_4 | \varphi_5 | \psi_6\ \theta_6\ \varphi_6 | z_7\ \psi_7 | \psi_8\ \theta_8\ \varphi_8 | \varphi_9]^T.\end{aligned} \tag{24}$$

Bearing in mind relations (15a), (15b), (19), (20) and (21), we can write the equations of motion of the system considered as follows:

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{F} + \mathbf{Q}, \tag{25}$$

where: $\mathbf{M} = (m_{ij})_{i,j=1...19}$ is a symmetrical matrix and its non-zero elements are:

$$\boldsymbol{m}_{11} = \boldsymbol{a}_{11}^{(1)} + \boldsymbol{a}_{11}^{(4)} + \boldsymbol{a}_{11}^{(8)},$$

$$\boldsymbol{m}_{1,j+6} = \boldsymbol{a}_{1,j}^{(4)} \quad j = 1,2,3,$$

$$\boldsymbol{m}_{1,j+15} = \boldsymbol{a}_{1,j}^{(8)} \quad j = 1,2,3,$$

$$\boldsymbol{m}_{i+1,j+1} = \boldsymbol{a}_{i,j}^{(2)} + \boldsymbol{a}_{i,j}^{(3)} \quad i = 1,2,3 \quad j = i,...,3,$$

$$\boldsymbol{m}_{i+1,j+3} = \boldsymbol{a}_{i,j}^{(3)} \quad i = 1,2,3 \quad j = 1,2,$$

$$\boldsymbol{m}_{i+4,j+4} = \boldsymbol{a}_{i+3,j+3}^{(3)} \quad i = 1,2 \quad j = i,...,2,$$

$$\boldsymbol{m}_{i+6,j+6} = \boldsymbol{a}_{i+1,j+1}^{(4)} \quad i = 1,2,3 \quad j = i,...,3,$$

$$\boldsymbol{m}_{10,10} = \boldsymbol{a}_{1,1}^{(5)},$$

$$\boldsymbol{m}_{i+10,j+10} = \boldsymbol{a}_{i,j}^{(6)} + \boldsymbol{a}_{i,j}^{(7)} \quad i = 1,2,3 \quad j = i,...,3,$$

$$\boldsymbol{m}_{i+10,j+13} = \boldsymbol{a}_{i,j}^{(7)} \quad i = 1,2,3 \quad j = 1,2,$$

$$\boldsymbol{m}_{i+13,j+13} = \boldsymbol{a}_{i+3,j+3}^{(7)} \quad i = 1,2 \quad j = i,...,2,$$

$$\boldsymbol{m}_{i+15,j+15} = \boldsymbol{a}_{i+1,j+1}^{(8)} \quad i = 1,2,3 \quad j = i,...,3,$$

$$\boldsymbol{m}_{19,19} = \boldsymbol{a}_{1,1}^{(9)},$$

$\mathbf{F} = (f_i)_{i=1,...,19}$ is a vector with the following components:

$$f_1 = c_K[i_K\varphi_K - y_1] + b_K[i_K\dot{\varphi}_K - \dot{y}_1] - h_1^{(1)} - h_1^{(4)} - h_1^{(8)} - V_1^{(1)} - V_1^{(4)} - V_1^{(8)},$$

$$f_{i+1} = -h_i^{(2)} - V_i^{(2)} \quad i = 1,2,3,$$

$$f_5 = -h_4^{(3)} - V_4^{(3)} - c_{23}z_3 - b_{23}\dot{z}_3,$$

$$f_6 = -h_5^{(3)} - V_5^{(3)},$$

$$f_{i+6} = -h_{i+1}^{(4)} - V_{i+1}^{(4)} \quad i = 1,2,3,$$

$$f_{10} = -h_1^{(5)} - V_1^{(5)},$$

$$f_{i+10} = -h_i^{(6)} - V_i^{(6)} \quad i = 1,2,3,$$

$$f_{14} = -h_4^{(7)} - V_4^{(7)} - c_{67}z_7 - b_{67}\dot{z}_7,$$

$$f_{15} = -h_5^{(7)} - V_5^{(7)},$$

$$f_{i+15} = -h_{i+1}^{(8)} - V_{i+1}^{(8)} \quad i = 1,2,3,$$

$$f_{19} = -h_1^{(9)} - V_1^{(9)}.$$

It is also necessary to define the generalised forces which are components of vector **Q** in equation (25). The way in which the generalised coordinates are chosen requires that the internal forces acting at points A_L, B_L, A_R, B_R (Fig. 4) are treated as external forces, and they can be defined with respect to the **x y z** system as:

$$\mathbf{R}_{AL} = [R_{ALx} \quad R_{ALy} \quad R_{ALz}]^T, \tag{26a}$$

$$\mathbf{R}_{BL} = [R_{BLx} \quad R_{BLy} \quad R_{BLz}]^T, \tag{26b}$$

$$\mathbf{R}_{AR} = [R_{ARx} \quad R_{ARy} \quad R_{ARz}]^T, \tag{26c}$$

$$\mathbf{R}_{BR} = [R_{BRx} \quad R_{BRy} \quad R_{BRz}]^T. \tag{26d}$$

According to Fig. 3 it has to be taken into account that these forces act as follows:

$\mathbf{R}_{AL}$ – on link (3),
$-\mathbf{R}_{AL}$ – on link (4),
$\mathbf{R}_{BL}$ – on link (3),
$-\mathbf{R}_{BL}$ – on link (5),
$\mathbf{R}_{AR}$ – on link (7),
$-\mathbf{R}_{AR}$ – on link (8),
$\mathbf{R}_{BR}$ – on link (7),
$-\mathbf{R}_{BR}$ – on link (9).

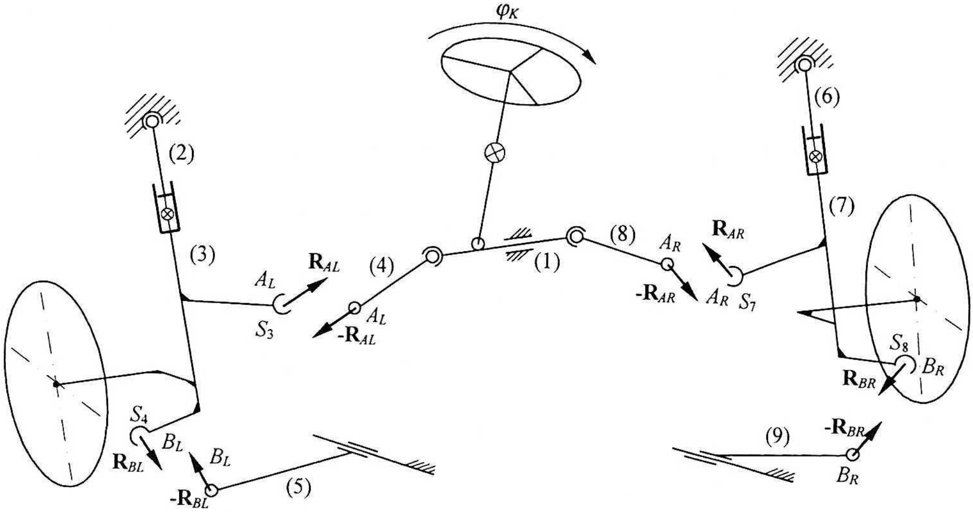


Fig. 4. Reaction forces in spherical joints

If it is assumed that:

$\left.\begin{matrix}\mathbf{r}'^{(3)}_{AL}\\ \mathbf{r}'^{(4)}_{AL}\end{matrix}\right\}$ define coordinates of point A_L with respect to $\mathbf{x}_3\ \mathbf{y}_3\ \mathbf{z}_3$ and $\mathbf{x}_4\ \mathbf{y}_4\ \mathbf{z}_4$ systems respectively,

$\left.\begin{matrix}\mathbf{r}'^{(3)}_{BL}\\ \mathbf{r}'^{(5)}_{BL}\end{matrix}\right\}$ define coordinates of point B_L with respect to $\mathbf{x}_3\ \mathbf{y}_3\ \mathbf{z}_3$ and $\mathbf{x}_5\ \mathbf{y}_5\ \mathbf{z}_5$ systems respectively,

$\left.\begin{matrix}\mathbf{r}'^{(7)}_{AR}\\ \mathbf{r}'^{(8)}_{AR}\end{matrix}\right\}$ define coordinates of point A_R with respect to $\mathbf{x}_7\ \mathbf{y}_7\ \mathbf{z}_7$ and $\mathbf{x}_8\ \mathbf{y}_8\ \mathbf{z}_8$ systems respectively,

$\left.\begin{matrix}\mathbf{r}'^{(7)}_{BR}\\ \mathbf{r}'^{(9)}_{BR}\end{matrix}\right\}$ define coordinates of point B_R with respect to $\mathbf{x}_7\ \mathbf{y}_7\ \mathbf{z}_7$ and $\mathbf{x}_9\ \mathbf{y}_9\ \mathbf{z}_9$ systems respectively,

then the generalised forces arising from reaction forces in spherical joints according to (18) can be presented in the forms:

$$Q_j^{(3)}(\mathbf{R}_{AL}) = (\boldsymbol{\theta}\mathbf{B}_{3,j}\,\mathbf{r}'^{(3)}_{AL})^T\mathbf{R}_{AL} \quad \text{for } j = 1,2,3,4,5, \tag{27a}$$

$$Q_j^{(4)}(-\mathbf{R}_{AL}) = -(\boldsymbol{\theta}\mathbf{B}_{4,j}\,\mathbf{r}'^{(4)}_{AL})^T\mathbf{R}_{AL} \quad \text{for } j = 1,2,3,4, \tag{27b}$$

$$Q_j^{(3)}(\mathbf{R}_{BL}) = (\boldsymbol{\theta}\mathbf{B}_{3,j}\,\mathbf{r}'^{(3)}_{BL})^T\mathbf{R}_{BL} \quad \text{for } j = 1,2,3,4,5, \tag{27c}$$

$$Q_j^{(5)}(-\mathbf{R}_{BL}) = -(\boldsymbol{\theta}\mathbf{B}_{5,j}\,\mathbf{r}'^{(5)}_{BL})^T\mathbf{R}_{BL} \quad \text{for } j = 1, \tag{27d}$$

and

$$Q_j^{(7)}(\mathbf{R}_{AR}) = (\boldsymbol{\theta}\mathbf{B}_{7,j}\,\mathbf{r}'^{(7)}_{AR})^T\,\mathbf{R}_{AR} \quad \text{for } j = 1,2,3,4,5, \tag{28a}$$

$$Q_j^{(8)}(-\mathbf{R}_{AR}) = -(\boldsymbol{\theta}\mathbf{B}_{8,j}\,\mathbf{r}'^{(8)}_{AR})^T\,\mathbf{R}_{AR} \quad \text{for } j = 1,2,3,4, \tag{28b}$$

$$Q_j^{(7)}(\mathbf{R}_{BR}) = (\boldsymbol{\theta}\mathbf{B}_{7,j}\,\mathbf{r}'^{(7)}_{BR})^T\,\mathbf{R}_{BR} \quad \text{for } j = 1,2,3,4,5, \tag{28c}$$

$$Q_j^{(9)}(-\mathbf{R}_{BR}) = -(\boldsymbol{\theta}\mathbf{B}_{9,j}\,\mathbf{r}'^{(9)}_{BR})^T\,\mathbf{R}_{BR} \quad \text{for } j = 1, \tag{28d}$$

where: $\boldsymbol{\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Finally the following can be written:

$$\mathbf{Q} = \mathbf{D}\mathbf{R}, \tag{29}$$

where:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{D}_{11} & 0 & \mathbf{D}_{13} & 0 \\ \mathbf{D}_{21} & \mathbf{D}_{22} & 0 & 0 \\ \mathbf{D}_{31} & \mathbf{D}_{32} & 0 & 0 \\ \mathbf{D}_{41} & 0 & 0 & 0 \\ 0 & \mathbf{D}_{52} & 0 & 0 \\ 0 & 0 & \mathbf{D}_{63} & \mathbf{D}_{64} \\ 0 & 0 & \mathbf{D}_{73} & \mathbf{D}_{74} \\ 0 & 0 & \mathbf{D}_{83} & 0 \\ 0 & 0 & 0 & \mathbf{D}_{94} \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} \mathbf{R}_{AL} \\ \mathbf{R}_{AR} \\ \mathbf{R}_{BL} \\ \mathbf{R}_{BR} \end{bmatrix},$$

$$\mathbf{D}_{11} = Q_1^{(4)}(-\mathbf{R}_{AL}) = -\boldsymbol{\theta}\mathbf{B}_{4,1}\,\mathbf{r}'^{(4)}_{AL},$$

$$\mathbf{D}_{13} = Q_1^{(8)}(-\mathbf{R}_{AR}) = -\boldsymbol{\theta}\mathbf{B}_{8,1}\,\mathbf{r}'^{(8)}_{AR},$$

$$\mathbf{D}_{21} = \begin{bmatrix} D_{21,1} \\ D_{21,2} \\ D_{21,3} \end{bmatrix} = \begin{bmatrix} Q_1^{(3)}(\mathbf{R}_{AL}) \\ Q_2^{(3)}(\mathbf{R}_{AL}) \\ Q_3^{(3)}(\mathbf{R}_{AL}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{3,1}\,\mathbf{r}'^{(3)}_{AL} \\ \boldsymbol{\theta}\mathbf{B}_{3,2}\,\mathbf{r}'^{(3)}_{AL} \\ \boldsymbol{\theta}\mathbf{B}_{3,3}\,\mathbf{r}'^{(3)}_{AL} \end{bmatrix},$$

$$\mathbf{D}_{22} = \begin{bmatrix} D_{22,1} \\ D_{22,2} \\ D_{22,3} \end{bmatrix} = \begin{bmatrix} Q_1^{(3)}(\mathbf{R}_{BL}) \\ Q_2^{(3)}(\mathbf{R}_{BL}) \\ Q_3^{(3)}(\mathbf{R}_{BL}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{3,1}\,\mathbf{r}'^{(3)}_{BL} \\ \boldsymbol{\theta}\mathbf{B}_{3,2}\,\mathbf{r}'^{(3)}_{BL} \\ \boldsymbol{\theta}\mathbf{B}_{3,3}\,\mathbf{r}'^{(3)}_{BL} \end{bmatrix},$$

$$\mathbf{D}_{31} = \begin{bmatrix} D_{31,1} \\ D_{31,2} \end{bmatrix} = \begin{bmatrix} Q_4^{(3)}(\mathbf{R}_{AL}) \\ Q_5^{(3)}(\mathbf{R}_{AL}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{3,4}\mathbf{r}'^{(3)}_{AL} \\ \boldsymbol{\theta}\mathbf{B}_{3,5}\mathbf{r}'^{(3)}_{AL} \end{bmatrix},$$

$$\mathbf{D}_{32} = \begin{bmatrix} D_{32,1} \\ D_{32,2} \end{bmatrix} = \begin{bmatrix} Q_4^{(3)}(\mathbf{R}_{BL}) \\ Q_5^{(3)}(\mathbf{R}_{BL}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{3,4}\mathbf{r}'^{(3)}_{BL} \\ \boldsymbol{\theta}\mathbf{B}_{3,5}\mathbf{r}'^{(3)}_{BL} \end{bmatrix},$$

$$\mathbf{D}_{41} = \begin{bmatrix} D_{41,1} \\ D_{41,2} \\ D_{41,3} \end{bmatrix} = \begin{bmatrix} Q_2^{(4)}(-\mathbf{R}_{AL}) \\ Q_3^{(4)}(-\mathbf{R}_{AL}) \\ Q_4^{(4)}(-\mathbf{R}_{AL}) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\theta}\mathbf{B}_{4,2}\mathbf{r}'^{(4)}_{AL} \\ -\boldsymbol{\theta}\mathbf{B}_{4,3}\mathbf{r}'^{(4)}_{AL} \\ -\boldsymbol{\theta}\mathbf{B}_{4,4}\mathbf{r}'^{(4)}_{AL} \end{bmatrix},$$

$$\mathbf{D}_{52} = Q_1^{(5)}(-\mathbf{R}_{BL}) = -\boldsymbol{\theta}\mathbf{B}_{5,1}\mathbf{r}'^{(5)}_{BL},$$

$$\mathbf{D}_{63} = \begin{bmatrix} D_{63,1} \\ D_{63,2} \\ D_{63,3} \end{bmatrix} = \begin{bmatrix} Q_1^{(7)}(\mathbf{R}_{AR}) \\ Q_2^{(7)}(\mathbf{R}_{AR}) \\ Q_3^{(7)}(\mathbf{R}_{AR}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{7,1}\mathbf{r}'^{(7)}_{AR} \\ \boldsymbol{\theta}\mathbf{B}_{7,2}\mathbf{r}'^{(7)}_{AR} \\ \boldsymbol{\theta}\mathbf{B}_{7,3}\mathbf{r}'^{(7)}_{AR} \end{bmatrix},$$

$$\mathbf{D}_{64} = \begin{bmatrix} D_{64,1} \\ D_{64,2} \\ D_{64,3} \end{bmatrix} = \begin{bmatrix} Q_1^{(7)}(\mathbf{R}_{BR}) \\ Q_2^{(7)}(\mathbf{R}_{BR}) \\ Q_3^{(7)}(\mathbf{R}_{BR}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{7,1}\mathbf{r}'^{(7)}_{BR} \\ \boldsymbol{\theta}\mathbf{B}_{7,2}\mathbf{r}'^{(7)}_{BR} \\ \boldsymbol{\theta}\mathbf{B}_{7,3}\mathbf{r}'^{(7)}_{BR} \end{bmatrix},$$

$$\mathbf{D}_{73} = \begin{bmatrix} D_{73,1} \\ D_{73,2} \end{bmatrix} = \begin{bmatrix} Q_4^{(7)}(\mathbf{R}_{AR}) \\ Q_5^{(7)}(\mathbf{R}_{AR}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}B_{7,4}\mathbf{r}'^{(7)}_{AR} \\ \boldsymbol{\theta}\mathbf{B}_{7,5}\mathbf{r}'^{(7)}_{AR} \end{bmatrix},$$

$$\mathbf{D}_{74} = \begin{bmatrix} D_{74,1} \\ D_{74,2} \end{bmatrix} = \begin{bmatrix} Q_4^{(7)}(\mathbf{R}_{BR}) \\ Q_5^{(7)}(\mathbf{R}_{BR}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}\mathbf{B}_{7,4}\mathbf{r}'^{(7)}_{BR} \\ \boldsymbol{\theta}\mathbf{B}_{7,5}\mathbf{r}'^{(7)}_{BR} \end{bmatrix},$$

$$\mathbf{D}_{83} = \begin{bmatrix} D_{83,1} \\ D_{83,2} \\ D_{83,3} \end{bmatrix} = \begin{bmatrix} Q_2^{(8)}(-\mathbf{R}_{AR}) \\ Q_3^{(8)}(-\mathbf{R}_{AR}) \\ Q_4^{(8)}(-\mathbf{R}_{AR}) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\theta}\mathbf{B}_{8,2}\mathbf{r}'^{(8)}_{AR} \\ -\boldsymbol{\theta}\mathbf{B}_{8,3}\mathbf{r}'^{(8)}_{AR} \\ -\boldsymbol{\theta}\mathbf{B}_{8,4}\mathbf{r}'^{(8)}_{AR} \end{bmatrix},$$

$$\mathbf{D}_{94} = Q_1^{(9)}(-\mathbf{R}_{BR}) = -\boldsymbol{\theta}\mathbf{B}_{9,1}\mathbf{r}'^{(9)}_{BR}.$$

Having presented the generalised forces in form (29) we can write the equations of motion as follows:

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{D}\mathbf{R} = \mathbf{F}. \tag{30}$$

The number of unknowns is $19+12=31$, while there are only 19 equations. Thus, additional 12 constraint equations have to be formulated and they are as follows:

$$\boldsymbol{\theta}\mathbf{r}_{AL}^{(3)} = \boldsymbol{\theta}\mathbf{r}_{AL}^{(4)}, \tag{31a}$$

$$\boldsymbol{\theta}\mathbf{r}_{BL}^{(3)} = \boldsymbol{\theta}\mathbf{r}_{BL}^{(5)}, \tag{31b}$$

$$\boldsymbol{\theta}\mathbf{r}_{AR}^{(7)} = \boldsymbol{\theta}\mathbf{r}_{AR}^{(8)}, \tag{31a}$$

$$\boldsymbol{\theta}\mathbf{r}_{BR}^{(7)} = \boldsymbol{\theta}\mathbf{r}_{BR}^{(9)}, \tag{31b}$$

while:

$$\mathbf{r}_{AL}^{(3)} = \mathbf{B}_3\mathbf{r}'^{(3)}_{AL}, \tag{32a}$$

$$\mathbf{r}_{AL}^{(4)} = \mathbf{B}_4\mathbf{r}'^{(4)}_{AL}, \tag{32b}$$

$$\mathbf{r}_{BL}^{(3)} = \mathbf{B}_3\mathbf{r}'^{(3)}_{BL}, \tag{32c}$$

$$\mathbf{r}_{BL}^{(5)} = \mathbf{B}_5\mathbf{r}'^{(5)}_{BL}, \tag{32d}$$

and

$$\mathbf{r}_{AR}^{(7)} = \mathbf{B}_7\mathbf{r}'^{(7)}_{AR}, \tag{33a}$$

$$\mathbf{r}_{AR}^{(8)} = \mathbf{B}_8\mathbf{r}'^{(8)}_{AR}, \tag{33b}$$

$$\mathbf{r}_{BR}^{(7)} = \mathbf{B}_7\mathbf{r}'^{(7)}_{BR}, \tag{33c}$$

$$\mathbf{r}_{BR}^{(9)} = \mathbf{B}_9\mathbf{r}'^{(9)}_{BR}. \tag{33d}$$

Having differentiated equations (31) with respect to time one can write the following:

$$-\mathbf{D}^T\ddot{\mathbf{q}} = \mathbf{W}, \tag{34}$$

where: $\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \mathbf{W}_3 \\ \mathbf{W}_4 \end{bmatrix}$,

$$\mathbf{W}_1 = \boldsymbol{\theta}\left[\sum_{i=1}^{n_3}\sum_{j=1}^{n_3}(\mathbf{B}_{3,i,j}\mathbf{r}'^{(3)}_{AL})\dot{q}_{3,i}\,\dot{q}_{3,j} - \sum_{i=1}^{n_4}\sum_{j=1}^{n_4}(\mathbf{B}_{4,i,j}\mathbf{r}'^{(4)}_{AL})\dot{q}_{4,i}\,\dot{q}_{4,j}\right],$$

$$\mathbf{W}_2 = \boldsymbol{\theta}\left[\sum_{i=1}^{n_3}\sum_{j=1}^{n_3}(\mathbf{B}_{3,i,j}\mathbf{r}'^{(3)}_{BL})\dot{q}_{3,i}\dot{q}_{3,j} - \sum_{i=1}^{n_5}\sum_{j=1}^{n_5}(\mathbf{B}_{5,i,j}\mathbf{r}'^{(5)}_{BL})\dot{q}_{5,i}\dot{q}_{5,j}\right],$$

$$\mathbf{W}_3 = \boldsymbol{\theta}\left[\sum_{i=1}^{n_7}\sum_{j=1}^{n_7}(\mathbf{B}_{7,i,j}\mathbf{r}'^{(7)}_{AR})\dot{q}_{7,i}\dot{q}_{7,j} - \sum_{i=1}^{n_8}\sum_{j=1}^{n_8}(\mathbf{B}_{8,i,j}\mathbf{r}'^{(8)}_{AR})\dot{q}_{8,i}\dot{q}_{8,j}\right],$$

$$\mathbf{W}_4 = \boldsymbol{\theta}\left[\sum_{i=1}^{n_7}\sum_{j=1}^{n_7}(\mathbf{B}_{7,i,j}\mathbf{r}'^{(7)}_{BR})\dot{q}_{7,i}\dot{q}_{7,j} - \sum_{i=1}^{n_9}\sum_{j=1}^{n_9}(\mathbf{B}_{9,i,j}\mathbf{r}'^{(9)}_{BR})\dot{q}_{9,i}\dot{q}_{9,j}\right].$$

Finally, the equations of motion (30) and constraint equations (34) can be presented together:

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{D}\mathbf{R} = \mathbf{F}, \tag{35a}$$

$$-\mathbf{D}^T\ddot{\mathbf{q}} = \mathbf{W}. \tag{35b}$$

Numerical integration of equations (35) enables us to calculate vectors of displacements $\mathbf{q}$, generalised velocities $\dot{\mathbf{q}}$ and vector of reaction forces $\mathbf{R}$.

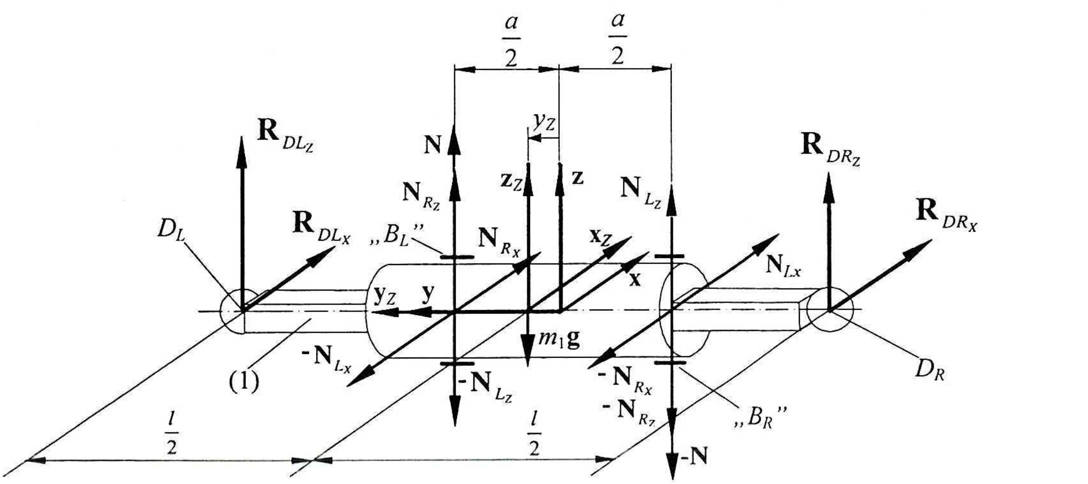


Fig. 5. Reaction forces in sliding joint of steering rack

These equations have to be modified when dry friction in the systems is taken into account [8], [29]. In the problem considered, it is assumed that friction is reduced to the sliding joint P which connects the steering rack

– body (4) with the guide in the car body. Two phases of dry friction can be considered: kinetic friction when the steering rack moves and static friction when this motion is "blocked". The way of considering the friction depends on the model of the joint assumed. The model assumed for these considerations is presented in Fig. 5. It is assumed that the steering rack with length l is connected with the guide by means of two slide bearings „B_L" and „B_R" with zero width (the distance between the bearings is a).

Having projected reaction forces $\mathbf{R}_{DL}$ and $\mathbf{R}_{DR}$ ($\mathbf{R}_{DL_x}$, $\mathbf{R}_{DR_x}$, $\mathbf{R}_{DL_z}$, $\mathbf{R}_{DR_z}$ are the respective projections on axes **x** and **z** of the **x y z** system) and the force of gravity $\boldsymbol{m}_1\mathbf{g}$, (Fig. 5): one can calculate the moments of normal reaction forces acting from the bearings on the steering rack as follows:

$$N_{L_x}a = R_{DL_x}\left(\frac{l}{2} + y_1\right), \tag{36a}$$

$$N_{R_x}a = R_{DR_x}\left(\frac{l}{2} + y_1\right), \tag{36b}$$

$$N_{L_z}a = R_{DL_z}\left(\frac{l}{2} + y_1\right), \tag{36c}$$

$$N_{R_z}a = R_{DR_z}\left(\frac{l}{2} + y_1\right), \tag{36d}$$

$$Na = m_1 g y_1, \tag{36e}$$

where: R_{DL_x}, R_{DR_x}, R_{DL_z}, R_{DR_z} – are components of forces $\mathbf{R}_{DL}$ and $\mathbf{R}_{DR}$ respectively.
Thus normal reaction forces N_{L_x}, N_{R_x}, N_{L_z} and N_{R_z} can be easily calculated.

Components of normal reaction forces at the left bearing are defined as follows:

$$\sum F_{ix}^{(L)} = N_{R_x} - N_{L_x} - \frac{1}{2}(R_{DL_x} + R_{DR_x}), \tag{37a}$$

$$\sum F_{iz}^{(L)} = N_{R_z} - N_{L_z} + N - \frac{1}{2}(R_{DL_z} + R_{DR_z} - m_1 g). \tag{37b}$$

The resultant force at this bearing is then calculated according to the formula:

$$F_L = \sqrt{(\sum F_{i_x}^{(L)})^2 + (\sum F_{i_z}^{(L)})^2}. \tag{38}$$

The force of kinetic friction is defined as:

$$T_L = \mu F_L, \tag{39}$$

where: μ – is the coefficient of kinetic friction.

Similarly, the components of normal reaction forces at the right bearing are defined as:

$$\sum F_{i_x}^{(R)} = N_{L_x} - N_{R_x} - \frac{1}{2}(R_{DL_x} + R_{DR_x}), \tag{40a}$$

$$\sum F_{i_z}^{(R)} = N_{L_z} - N_{R_z} - N - \frac{1}{2}(R_{DL_z} + R_{DR_z} - m_1 g), \tag{40b}$$

and the resultant force is calculated from the formula:

$$F_R = \sqrt{(\sum F_{i_x}^{(R)})^2 + (\sum F_{i_z}^{(R)})^2}. \tag{41}$$

The force of kinetic friction can be calculated as follows:

$$T_R = \mu F_R. \tag{42}$$

In addition, a sliding stone connected with the guide acts on the steering rack by means of a compression spring and this results in additional friction force T_0 dependent on compression of the spring.

Reaction forces $\mathbf{R}_{DL}$ and $\mathbf{R}_{DR}$ acting at points D_L and D_R (Fig. 6) can be calculated using the Newton equations:

$$\mathbf{R}_{DL} = \boldsymbol{m}_4 \boldsymbol{\theta} \ddot{\mathbf{r}}_C^{(4)} + \mathbf{R}_{AL}, \tag{43a}$$

$$\mathbf{R}_{DR} = \boldsymbol{m}_8 \boldsymbol{\theta} \ddot{\mathbf{r}}_C^{(8)} + \mathbf{R}_{AR}. \tag{43b}$$

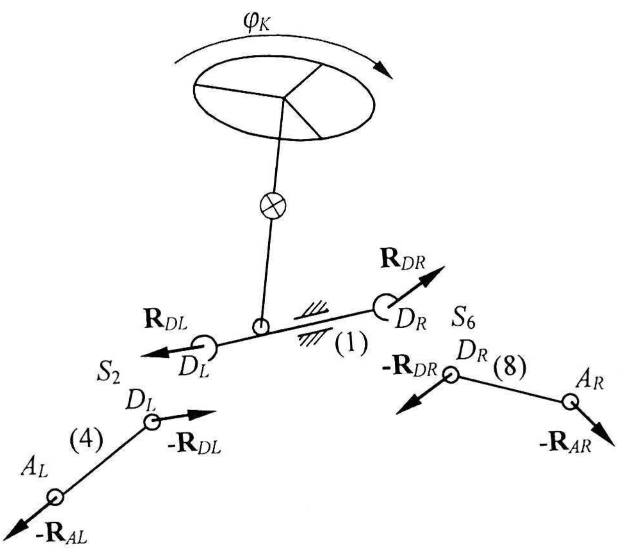


Fig. 6. Reaction forces at chosen spherical joints

Vectors of accelerations of mass centres of links (4) and (8) can be calculated according to the formulae:

$$\ddot{\mathbf{r}}_C^{(4)} = \left[\sum_{i=1}^{n_4} (\mathbf{B}_{4,i}\ddot{q}_{4,i} + \sum_{i=1}^{n_4} \sum_{j=1}^{n_4} \mathbf{B}_{4,i,j}\dot{q}_{4,i}\,\dot{q}_{4,j} \right] \mathbf{r}'^{(4)}_C, \tag{44a}$$

$$\ddot{\mathbf{r}}_C^{(8)} = \left[\sum_{i=1}^{n_8} (\mathbf{B}_{8,i}\ddot{q}_{8,i} + \sum_{i=1}^{n_8} \sum_{j=1}^{n_8} \mathbf{B}_{8,i,j}\dot{q}_{8,i}\,\dot{q}_{8,j} \right] \mathbf{r}'^{(8)}_C, \tag{44b}$$

where: $\mathbf{r}'^{(4)}_C$ – is the vector of coordinates of the mass centre of link (4) in the $\mathbf{x}_4\ \mathbf{y}_4\ \mathbf{z}_4$ system,

$\mathbf{r}'^{(8)}_C$ – is the vector of coordinates of the mass centre of link (8) in the $\mathbf{x}_8\ \mathbf{y}_8\ \mathbf{z}_8$ system.

Two phases of dry friction in joint P are signalled by means of a friction index i_f:

$$i_f = \begin{cases} 0, & \text{for static friction,} \\ 1, & \text{for kinetic friction.} \end{cases} \tag{45}$$

1. $i_f = 0$ In this case the steering rack does not move in relation to the guide in the car body and the force of static friction is not known. Lack of motion in the joint is signalled by the constraint equations:

$$\ddot{y}_1 = 0. \tag{46}$$

In order to calculate the force of static friction T, it becomes a component of the reaction force vector which now takes the form:

$$\mathbf{R}_\mathrm{T} = \begin{bmatrix} \mathbf{R} \\ T \end{bmatrix}, \tag{47}$$

and matrix $\mathbf{D}$ is also modified as follows:

$$\mathbf{D}_\mathrm{T} = [\mathbf{D} \;\; \mathbf{D}'], \tag{48}$$

where: $\mathbf{D}' = [\underset{1}{1} \;\; \underset{2}{0} \; \underset{...}{...} \; \underset{19}{0}]^T.$

The equations of motion and constraint equations in this case take the form:

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{D}_\mathrm{T}\mathbf{R}_\mathrm{T} = \mathbf{F}, \tag{49a}$$

$$-\mathbf{D}_\mathrm{T}^T\ddot{\mathbf{q}} = \mathbf{W}_\mathrm{T}, \tag{49b}$$

where: $\mathbf{W}_\mathrm{T} = \begin{bmatrix} \mathbf{W} \\ \mathbf{0} \end{bmatrix}.$

Transition from the static to the kinetic friction phase ($i_f = 1$) occurs when the following condition is fulfilled:

$$|T| > \mu_s (F^{(L)} + F^{(R)}), \tag{50}$$

where: T – is the force of static friction,
μ_s – is the coefficient of static friction in bearing „B_L" and „B_R".

2. $i_f = 1$ In this case the steering rack moves along the guide in the car body and kinetic friction occurs in joint P. Then the first component of vector $\mathbf{F}$ has to be changed by the component:

$$T = -\operatorname{sgn} \dot{y}_1 (T_L + T_R + T_0). \tag{51}$$

Transition from the kinetic friction phase to the static friction phase ($i_f = 0$) occurs when the following condition is fulfilled:

$$\operatorname{sgn} \dot{y}_1 (t - h) \neq \operatorname{sgn} \dot{y}_1 (t) \tag{52}$$

that is, when the velocity of the steering rack $\dot{y}_1$ changes sign at a given integration step (h – integration step, t – time).

A computer program using Object Pascal (Delphi 6.0) has been developed on the basis of the algorithm formulated.

Initial conditions are calculated assuming that:

$$\dot{\mathbf{q}} = \ddot{\mathbf{q}} = \mathbf{0}, \tag{53a}$$

$$\psi_3 = \varphi_4 = \psi_7 = \varphi_8 = 0, \tag{53b}$$

which means that generalised velocities and accelerations as well as rotation angles of stub axles about pistons and turn angles of steering rods about their axes $\mathbf{x}_4$ and $\mathbf{x}_8$ respectively equal zero. The following system of nonlinear algebraic equations:

$$\mathbf{D}_T \mathbf{R}_T - \mathbf{F} = \mathbf{0}, \tag{54a}$$

$$\mathbf{S}(\mathbf{q}) = \mathbf{0}, \tag{54b}$$

where: $[S_1 \ S_2 \ S_3]^T = \boldsymbol{\theta}[\mathbf{r}_{AL}^{(3)} - \mathbf{r}_{AL}^{(4)}]$,

$[S_4 \ S_5 \ S_6]^T = \boldsymbol{\theta}[\mathbf{r}_{BL}^{(3)} - \mathbf{r}_{BL}^{(5)}]$,

$[S_7 \ S_8 \ S_9]^T = \boldsymbol{\theta}[r_{AR}^{(7)} - \mathbf{r}_{AR}^{(8)}]$,

$[S_{10} \ S_{11} \ S_{12}]^T = \boldsymbol{\theta}[\mathbf{r}_{BR}^{(7)} - \mathbf{r}_{BR}^{(8)}]$,

$S_{13} = y_1$,

has been solved without equations referring to generalised coordinates ψ_3, φ_4, ψ_7, φ_8 by using Newton's iterative method.

In the equations of motion presented external forces acting on the wheels can be taken into account. It is assumed that the wheels do not turn about the stub axles and thus it can be assumed that the following forces act directly on bodies (3) and (7):

$$\mathbf{P}_L = [P_{L_X} \ P_{L_Y} \ P_{L_Z} \ 0]^T, \tag{55a}$$

$$\mathbf{P}_R = [P_{R_X}\ \ P_{R_Y}\ \ P_{R_Z}\ \ 0]^T, \tag{55b}$$

at points with coordinates defined by vectors:

$\mathbf{r}'_{3,P_L}$ – in the $\mathbf{x}_3\ \mathbf{y}_3\ \mathbf{z}_3$ system,

$\mathbf{r}'_{7,P_R}$ – in the $\mathbf{x}_7\ \mathbf{y}_7\ \mathbf{z}_7$ system.

This means that vectors $\mathbf{Q}(\mathbf{P}_L)$ and $\mathbf{Q}(\mathbf{P}_R)$ have to added to the vector $\mathbf{F}$ in equations (25). The components of these vectors can be calculated according to formula (18), and the following is obtained:

$$Q_j^{(3)}(\mathbf{P}_L) = [\mathbf{B}_{3,j}\mathbf{r}'_{3,P_L}]^T\,\mathbf{P}_L \quad \text{for} \quad j = 1,2,3,4,5, \tag{56a}$$

$$Q_j^{(7)}(\mathbf{P}_R) = [\mathbf{B}_{7,j}\mathbf{r}'_{7,P_R}]^T\,\mathbf{P}_R \quad \text{for} \quad j = 1,2,3,4,5. \tag{56b}$$

Thus, vectors $\mathbf{Q}(\mathbf{P}_L)$ and $\mathbf{Q}(\mathbf{P}_R)$ take the forms:

$$\mathbf{Q}(\mathbf{P}_L) = [\underset{1}{0}\ \ \underset{2}{[\mathbf{B}_{3,1}\mathbf{r}'_{3,P_L}]^T\,\mathbf{P}_L}\ \ \underset{\dots}{\dots}\ \ \underset{7}{[\mathbf{B}_{3,5}\mathbf{r}'_{3,P_L}]^T\,\mathbf{P}_L}\ \ \underset{8}{0}\ \ \underset{\dots}{\dots}\ \ \underset{19}{0}]^T, \tag{57a}$$

$$\mathbf{Q}(\mathbf{P}_R) = [\underset{1}{0}\ \ \underset{\dots}{\dots}\ \ \underset{10}{0}\ \underset{11}{[\mathbf{B}_{7,1}\mathbf{r}'_{7,P_R}]^T\,\mathbf{P}_R}\ \ \underset{\dots}{\dots}\ \ \underset{15}{[\mathbf{B}_{7,5}\mathbf{r}'_{7,P_R}]^T\,\mathbf{P}_R}\ \ \underset{16}{0}\ \ \underset{\dots}{\dots}\ \ \underset{19}{0}]^T. \tag{57b}$$

4. Computer simulations

Data for calculations have been assumed on the basis of technical documentation of a car with McPherson suspension made available by the manufacturer. In order to calculate mass parameters of links, their geometry has been imaged using AutoDesk Inventor software. Simulations were carried out on the assumption that the car is placed on a jack, and its wheels hang down freely and are not loaded by any forces acting from the surface.

Dry friction in sliding joint P is considered in numerical simulations on the assumption that kinetic and static coefficients of friction are the same: $\mu = \mu_s = 0.2$. Coefficients of stiffness and damping of SDE_1 are as follows [16]: $c_1 = 200\,\dfrac{Nm}{rad}$ and $b_1 = 0.1\,\dfrac{Nms}{rad}$. Coefficients of stiffness and damping for SDE_2 and SDE_3 are assumed as $c_2 = c_3 = 10^5\,\dfrac{N}{m}$, $b_2 = b_3 = 0$.

Results of numerical simulations presented are obtained for the drive input in the steering wheel shown in Fig. 7.

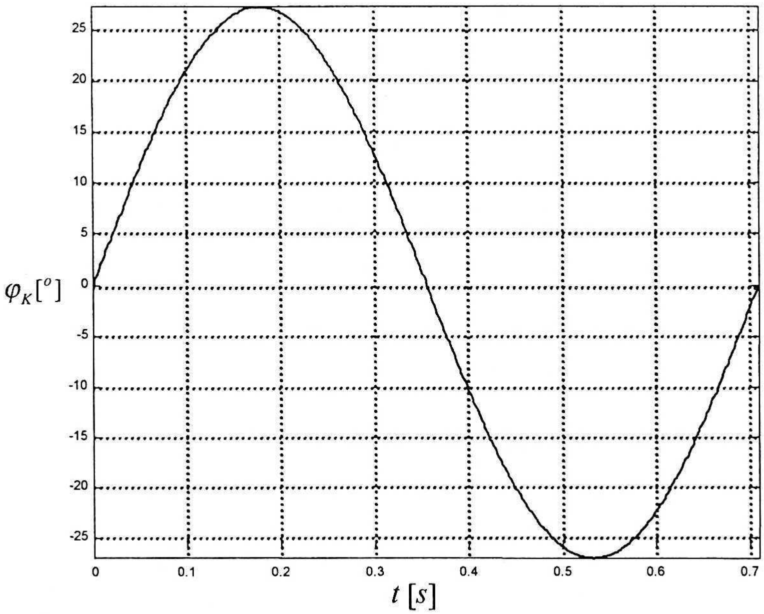


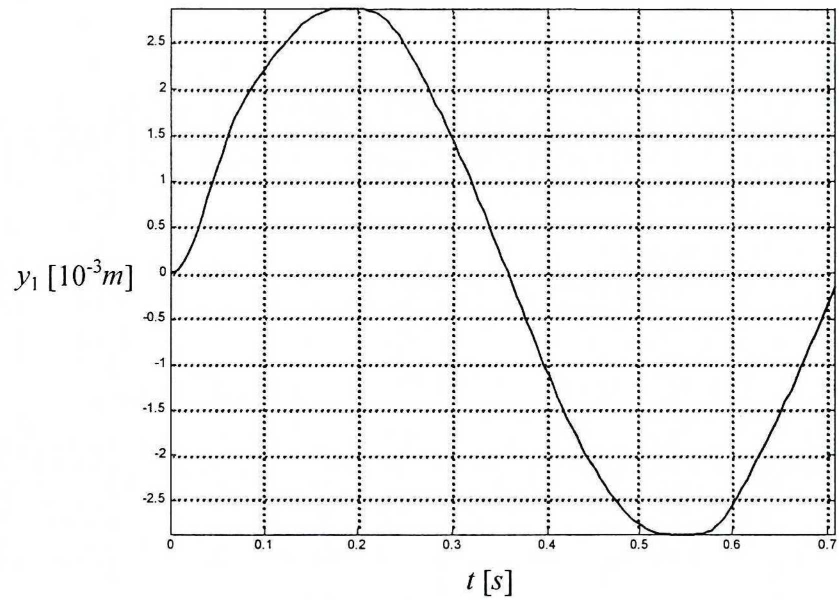
Fig. 7. Change of rotation angle of steering wheel defined as function of time:
$\varphi_K = 0.15\pi \sin 2.8\pi t$

Fig. 8a presents displacement y_1 of the steering rack in time, while Fig. 8b presents its velocity $\dot{y}_1$. In terms of dry friction, static friction phases of short duration can be observed (at the beginning or at the end of motion and also when changing direction). Occurrence of these phases is marked by bold lines in the course of the friction index i_f.

The course of rotation angle ψ_3 of the left stub axle is shown in Fig. 9. This angle, after some simplification, can be treated as the steer angle of the left wheel.

The course of rotation angle φ_5 of the left suspension arm is shown in Fig. 10. Relatively large vibrations of the suspension arm result from omitting the damping in shock absorbers in simulations.

a)



b)

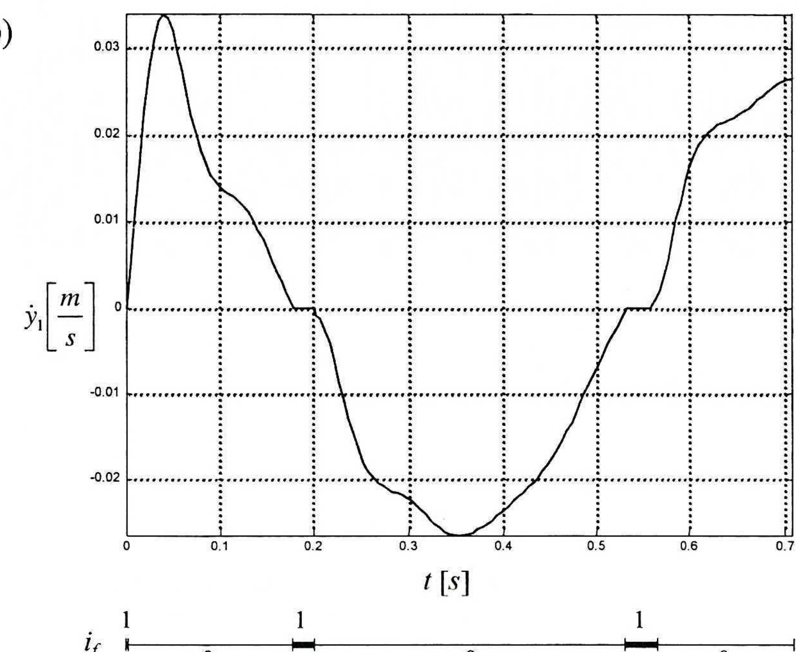


Fig. 8. Motion of steering rack: a) displacement y_1 b) velocity $\dot{y}_1$

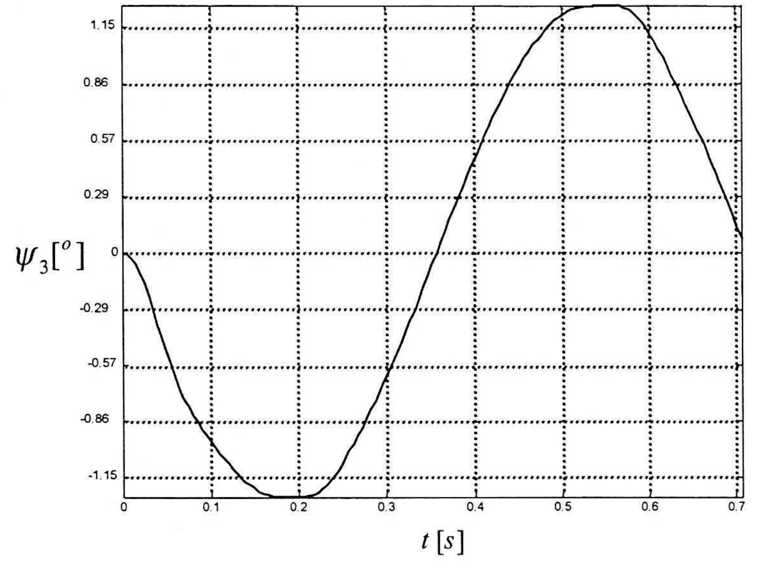


Fig. 9. Course of rotation angle ψ_3 of left stub axle

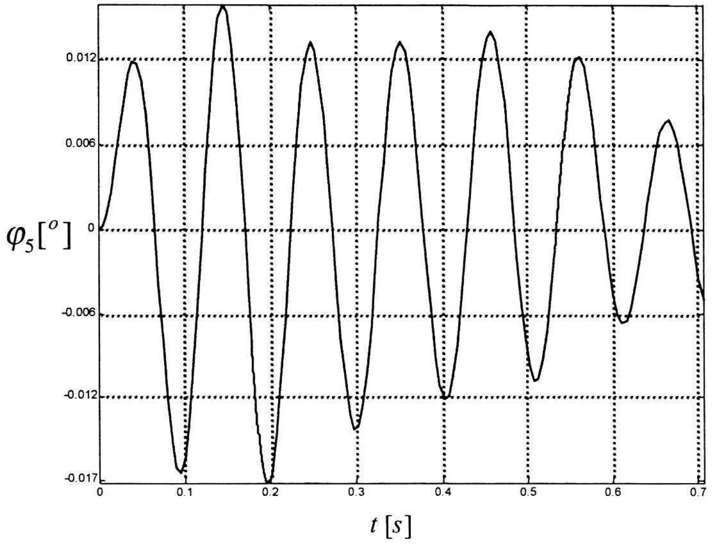


Fig. 10. Course of rotation angle φ_5 of left suspension arm

5. Conclusions

The method of dynamic analysis presented will be of interest to designers of passenger cars, it can be useful especially in the process of computer aided design and optimisation of steering systems.

Results of numerical calculations can be reliable when real data are used for analysis. In order to calculate mass parameters of links AutoDesk Inventor can be used. Special difficulties occur when the parameters of dry friction (which is not fully researched but inevitably occur in joints of the system) are to be defined.

Since they are aware that only a complex model of the whole car would be useful, the authors are working now on physical and mathematical models of a car in motion, where the model of the steering system presented will be included.

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Analiza dynamiczna układu kierowniczego samochodu osobowego z zawieszeniem typu McPherson

Streszczenie

W pracy przedstawiono pewną metodę analizy dynamicznej (w zakresie tzw. prostego zadania dynamiki) układu wieloczłonowego będącego modelem fizycznym układu kierowniczego wybranego samochodu osobowego z zawieszeniem typu McPherson. Analizowany układ stanowiący przestrzenny mechanizm dźwigniowy zbudowany jest z dziesięciu członów modelujących: kierownicę, zębatkę, drążki kierownicze, zwrotnice wraz z kołami, tłoczyska kolumn typu McPherson oraz wahacze. W przyjętym modelu człony traktowane są jako ciała sztywne, można natomiast uwzględnić podatność skrętną kolumny kierownicy. Geometrię układu opisano przy wykorzystaniu tzw. macierzy jednorodnych. Równania ruchu układu wyprowadzono, bazując na formalizmie równań Lagrange'a. Równania te uzupełniono odpowiednimi równaniami więzów geometrycznych. Przyjęty model daje możliwość uwzględnienia tarcia suchego występującego w połączeniach członów układów kierowniczych samochodów, przy czym dla potrzeb przeprowadzonej analizy tarcie to zredukowano do przesuwnego połączenia między zębatką a obudową. Dane do obliczeń przyjęto na podstawie analizy dokumentacji technicznej dotyczącej wybranej marki samochodu osobowego z zawieszeniem typu McPherson, udostępnionej przez jego producenta. W celu wyznaczenia potrzebnych parametrów masowych poszczególnych członów odwzorowano ich rzeczywistą geometrię, korzystając z pakietu oprogramowania Autodesk Inventor. Proponowana metoda analizy dynamicznej układów kierowniczych może zainteresować projektantów mechanizmów samochodowych, a w szczególności może być użyteczna w procesie wspomagania projektowania układów kierowniczych.