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COMPUTER SIMULATION OF ROTARY ELECTRICAL DISCHARGE MACHINING (REDM)

The paper presents a study of Rotary Electrical Discharge Machining (REDM) process. In REDM, simple shaped cylindrical electrodes are used to generate 3D complex shapes. A theoretical model of machining process is described, which takes into consideration the effect of tool electrode wear on machined surface profile. The software for computer simulation of REDM has been developed. In the paper, the effects of machining parameters on shape error of machined surface are discussed. Results of computer simulation have been confirmed in experimental practice. The simulation model for NC contouring REDM may also be applied to solving numerical control problems and optimization of tool electrode path.

NOMENCLATURE

v – relative tool wear,
MRR – material removal rate,
TWR – tool wear rate,
R – radius of tool,
a – depth of cuta
$y = g(x)$ – path of center of tool,
$y = f(x)$ – initial profile of workpiece,
$y = F(x)$ – final profile of workpiece.

1. Introduction

Today's manufacturing industries are facing challenges from advanced difficult-to-machine materials (tough super alloys, ceramics, and com-

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posites), stringent design requirements (high precision, complex shapes, and high surface quality), and machining costs. Advanced materials play an increasingly important role in modern manufacturing industries, especially, in aircraft, automobile, tool, die and mold making industries. The greatly-improved thermal, chemical, and mechanical properties of the material (such as improved strength, heat resistance, wear resistance, and corrosion resistance), while having yielded enormous economic benefits to manufacturing industries through improved product performance and product design, are making ordinary machining processes unable to machine them economically. The technological improvement of manufacturing attributes can be achieved by high efficiency Rotary Electrical Discharge Machining (REDM) (Electrical Discharge Grinding - EDG or Electrical Discharge Milling), Abrasive Electrodischarge Grinding (AEDG), Rotary Electrochemical Arc/Discharge Machining – RECAM/RCDM or grinding using metallic bond diamond wheels. These machining processes use a rotating tool.

In machining difficult-to-cut materials, effect of tool wear on process performance is very significant.

Tool wear in EDM is characterized by the relative tool-electrode wear, which is generally defined as the ratio of tool wear rate (*TWR*) to material removal rate (*MRR*):

$$\nu = \frac{TWR}{MRR}. \tag{1}$$

Depending upon the operating parameters of REDM, the relative wear may be 0.01-1. Changes in the radius of tool due to wear during machining is expected to reflect in the actual depth of cut and finally in the profile and dimensional accuracy of the machined parts (Fig. 1)

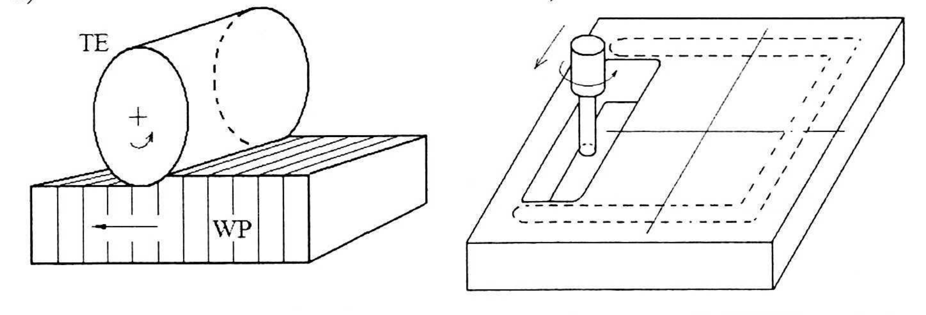

Fig. 1. Example of machining using rotating tool electrode (TE). a) Machining of pocket samples; b) Contouring by REDM

Results of investigation of EDM with rotating electrode reported in [1], [2] show a slope curvilinear profile of bottom surface of the machined groove due to the wear of disk electrode. Controlling the path of electrode can reduce shape error. In the paper [3], preliminary analysis of shape error indicates that one of the main factors leading to shape errors is the wheel wear. More comprehensive study of this problem, based on mathematical modeling and experiments, were reported in [2], where the general differential equation described relationship between tool wear, and initial and final shape of the machined surface was derived. The effect of wheel wear on dimensional accuracy of grinding is known from theory of tolerances. Based on the assumption of constant rate of wear, one can obtain uniform probability density function (PDF) for height of machined parts. At low G-ratio (where $G = \frac{1}{v}$), this assumption is not adequate to real process, and can lead to significant errors in process design [2].

The goal of this work is to analyze the effect of tool wear on the surface profile and shape error of machined parts. The control problem of machining using rotating tool is also studied.

2. Mathematical modeling

The purpose of this mathematical modeling and computer simulation is to determine the profile of generated surface $y = F(x)$, taking into account the change in tool diameter, which occurs due to the tool wear of rotary tool during machining. The final surface profile depends on the input data, such as setting depth of cut a_0, initial surface profile $y = f(x)$, radius of tool R_0 and a curvilinear path of the center of tool $y = g(x)$ as the result of controlling the motion of tool along x and y – axes (Fig. 2).

In determining the profile of machined surface, the following assumptions are made:

- The inter electrode gap s is constant and included in the effective radius of tool i.e. $R = R(electrode) + s$,
- Actual depth of cut, a, is determined by the position of points A and B, which are the point of tangency of the tool electrode to the generated profile and the intersection point of the tool and the initial profile $y = f(x)$, respectively (Fig. 2),
- Feed rate along axis y is significantly lower in comparison with the feed rate V_f i.e. the value of slope $\frac{dg}{dx} << 1$,
- Changes in the tool shape along the axis of rotation are neglected.

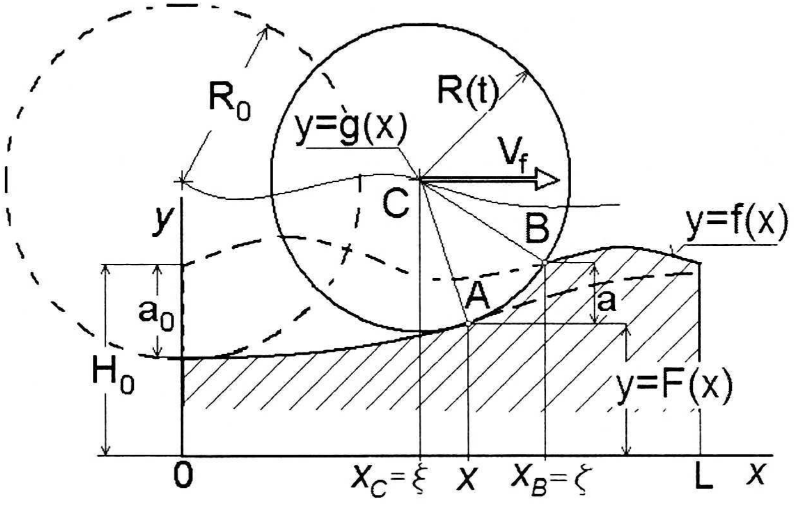

Fig. 2. Scheme of machining with curvilinear path of rotating tool

Let us consider the state of machining process after machining time t, when the position of the tool is described by a coordinate $x_C = \xi$, the feed rate is $V(t) = V_f$ and the effective tool radius due to wear is given by $R(t)$.

The material removal rate MRR at $\frac{dg}{dx} << 1$ is:

$$MRR = a \cdot b \cdot V_f = [(f(\zeta) - F(x)] \cdot b \cdot V_f, \tag{2}$$

where V_f is the feed rate, b is the width of surface machined and ζ is a function of x, $\zeta = \zeta(x)$.

The coordinates of intersection point $B(\zeta, f(\zeta))$ can be calculated from the relation $AC = BC$ (Fig. 2):

$$(x - \xi)^2 + [g(\xi) - F(x)]^2 = (\zeta - \xi)^2 + [g(\xi) - f(\zeta)]^2. \tag{3}$$

The tool wear rate TWR, which is defined as the volume of tool material removed per unit time is given by,

$$TWR = -2\pi \cdot b \cdot R(t) \cdot \frac{dR}{dt}. \tag{4}$$

Since

$$\frac{dR}{dt} = \frac{dR}{dx}\frac{dx}{d\xi}\frac{d\xi}{dt} = \frac{dR}{dx}\frac{dx}{d\xi}V_f, \tag{5}$$

and according to the relation from Fig. 2,

$$R^2(t) = [g(\xi) - F(x)]^2 + (x - \xi)^2 \tag{6}$$

the rate of change of the tool radius can be expressed as following:

$$\frac{dR}{dx} = \frac{1}{R}\left([g(\xi) - F(x)]\left(\frac{dg}{d\xi}\frac{d\xi}{dx} - \frac{dF}{dx}\right) + (x - \xi)\left(1 - \frac{d\xi}{dx}\right)\right). \tag{7}$$

By substituting of Equations (2, 5 and 7) into expression for v, Equation (1), and performing transformation, we obtain the equation describing the profile of machined surface:

$$\frac{dF}{dx} + \frac{v}{2\pi}\frac{F}{[g(\xi) - F]} = \frac{v}{2\pi}\frac{f(\zeta)}{[g(\xi) - F]} + \frac{dg}{d\xi}, \tag{8}$$

with the initial condition $F = F(0) = g(0)\text{-}R(0)$.

In general, the value of relative tool wear is depending on the depth of cut, i.e. $v = v[f(\zeta) - F(x)]$. This function can be only determined experimentally.

To complete the systems of Equations (8) and (3), an additional equation is formulated from the condition for tangency of the tool to the profile machined surface at the point A as follows:

$$[g(\xi) - F(x)]\frac{dF}{dx} = x - \xi. \tag{9}$$

The systems of Equations (3, 8 and 9) describe a relation between the profile of tool path $g(x)$, the initial shape $f(x)$ and the final profile $F(x)$ of workpiece surface in a general case of machining using a rotating tool with consideration of tool wear. The derived mathematical model can be used in the following processes: REDM/EDG (EDMilling), AREDM, RE-CAM/RCDM and grinding (in this case from definition of the G-ratio, $v = 1/G$). Based on the presented mathematical model, computer simulation of evolution of workpiece profile can be carried out by using software described in section 3.

Two tasks can be formulated:
(1) Direct problem – The tool path, $g(x)$, and the initial shape of surface, $f(x)$, are known but the resulting shape of the machined surface, $F(x)$ is unknown, and needs to be predicted.
(2) Inverse problem, or control problem – For a required shape of the machined surface, $F(x)$, the tool path, $g(x)$, needs to be determined.

These tasks have been solved numerically using the Finite Difference Method and an iterative procedure.

In many cases in practice, the changes in the radius and the difference between coordinates x and ξ for points A and B, respectively) are usually small as compared to the initial radius (in Eq. (8) the value of $\frac{d\xi}{dx} \cong 1$). The relative wear during some machining operation can be also assumed as constant, i.e. it is not dependent on the actual depth of cut. For these conditions, the system Equations (3, 8 and 9) are reduced to following form:

$$\frac{dF}{dx} + m \cdot F = m \cdot f(x) + \frac{dg}{dx}, \tag{10}$$

where $m = \dfrac{v}{2\pi R_0}$.

Simplified mathematical model, described by above mentioned Eq. (10) can be successfully used for planning REDM operations and solution both mentioned above problems.
Similar model has been derived in the study of Micro EDM using an end tool electrode for the conditions which allowed application of the Uniform Wear Method [4].

2.1. Direct problem with using simplified mathematical model of REDM

The first problem is directly connected with the analysis of accuracy based on the following solution of Equation (10):

$$F(x) = \left[\int_0^x [m \cdot f(x) + g'(x)] \cdot \exp(-m \cdot x) \cdot dx + g(0) \right] \cdot \exp(-m \cdot x) \tag{11}$$

Let us consider elementary case of machining presented in Fig. 3, when the initial surface is flat $f(x) = H_0 = const$ (a height of part) and $g(x) = H_0 - a_0 + R_0 = \text{constant}$.

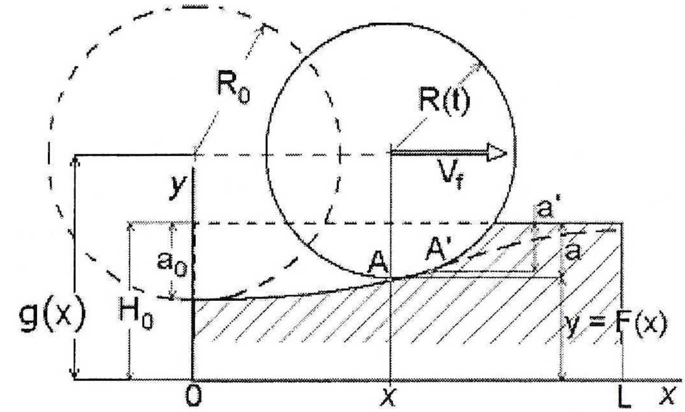

Fig. 3. Scheme of machining of flat surface using rotating tool

Due to the curvature of the profile (Fig. 3), the effective depth of cut a' is determined by a contact point A' ($a' < a_0$). The performed estimation shows that the relative error of the assumed depth of cut (related to $\frac{d\xi}{dx} \cong 1$) is approximately equal to $\varepsilon \cong \frac{\vartheta^2 a_0}{2\pi^2 D_0}$. For example, when: $v = 1$, $a_0 = 1$ mm, $D_0 = 10$ mm, the relative error $\varepsilon = 0.005$ or $\varepsilon\% = 0.5\%$, therefore in practice, the difference between a' and a can be neglected and Eq. (10) and its solution expressed by Eq. (11) can be used. For this case of REDM, Eq. (11) takes the following form:

$$F(x) = H_0 - a_0 \exp(-mx) \tag{12}$$

where $m = \frac{v}{2\pi R_0}$.

While considering accuracy, the profile is conveniently described in the coordinates relative to the required height $H_0 - a_0$, i.e. by the deviation $h = F(x) - (H_0 - a_0)$:

$$h(x) = a_0[1 - \exp(-mx)]. \tag{13}$$

For more universal application of the solution (13), a non-dimensional form of notations is used. The non-dimensional variables are defined as $\bar{h} = h/a_0$, and $\bar{x} = x/L$, where L is the length of workpiece. In non-dimensionalized form, Eq. (13) can be written as following:

$$\bar{h} = 1 - \exp(-A \cdot \bar{x}) \tag{14}$$

where $A = m \cdot L = \dfrac{v \cdot L}{2\pi \cdot R_0}$.

The profiles of machined surface obtained for different values of A are shown in Fig. 4. The dashed line ($A = 1$), presents the profile neglecting changes in *TWR* during machining, it can be seen that the differences between profiles can be significant.

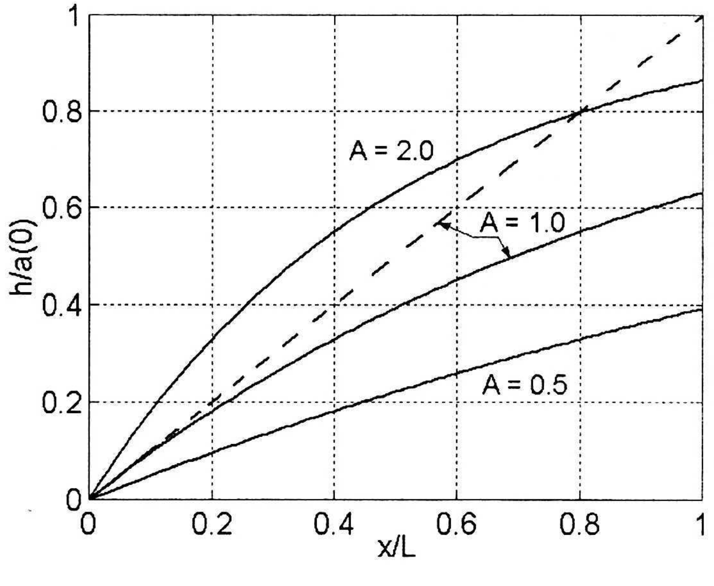

Fig. 4. Profile of machined surface at different values of A

The maximal shape error is (Figure 4):

$$\bar{h}_m = 1 - \exp(-A) \tag{15}$$

or

$$h_m = a_0\left[1 - \exp\left(-\frac{v \cdot L}{\pi \cdot D_0}\right)\right] \tag{16}$$

During machining of parts loaded together in a pocket and cut in one pass of the rotary tool, the achieved height of part in the machined set is a random variable (Figure 1a). Based on solution (13), statistical characteristics of distribution of height of the machined parts have been obtained [2]:

Probability Density Function (PDF)

$$y = \frac{1}{a_0 A \cdot [1 - h(x)/a_0]} \tag{17}$$

Cumulative Distribution Function (CDF)

$$Y = -\frac{1}{A} \ln\left[1 - \frac{h(x)}{a_0}\right] \qquad 0 \le \frac{h}{a_0} \le \bar{h}_m \tag{18}$$

Mean value

$$E(H) = a_0\left[1 - \frac{1}{A}(1 - \exp(-A))\right] \tag{19}$$

Variance

$$\sigma^2 = \frac{1 - \exp(-A)}{A}\left[1 - (1 - \exp(-A))\,\frac{2 + A}{2A}\right] a_0^2 \tag{20}$$

Process Capability Index (PCI)

$$C_p = \frac{T}{6 \cdot \sigma} = \frac{A\sqrt{2}}{\sqrt{2A - (1 - \exp(-A)) \cdot (2 + A)}} \tag{21}$$

2.2. Inverse problem solved by using simplified mathematical model of REDM

The solution of the inverse problem is given by:

$$y = g(x) = g(0) + F(x) - F(0) + \int_0^x m\,[F(x) - f(x)]\,dx \tag{22}$$

For example, the tool wear can be compensated by moving tool or workpiece along y-axis, in the result of which a flat machined surface can be obtained [1], [2]. For a case when the initial surface is flat $f(x) = H_0$ and the initial depth of cut is equal to a_0, the required flat shape is $F(x) = H_0 - a_0$ (Fig. 2). Therefore, according to solution (22), the linear path of tool is needed, which is described as:

$$y = g(x) = H_0 + R_0 - a_0 \cdot m \cdot x. \quad (23)$$

For machining with a constant feed rate, V_f, along x-axis, the compensation of the tool wear needs a relative motion of the tool/workpiece with a constant feed rate along y-axis, V_y, given by,

$$V_y = - a_0 \cdot m \cdot V_f = - \frac{v \cdot a_0}{2 \cdot \pi \cdot R_0} V_f. \quad (24)$$

Therefore, the conclusion about linear path of tool electrode for compensation of wear presented in papers [1], [2] is confirmed by the theoretical results (23 and 24).

3. Computer simulations

The software developed in this study supports process design for the NC – REDM. The software is easy to use, as it contains menu driven application allowing evaluating the effect of various parameters on process performance.

Programming of kinematics of the tool electrode can be achieved by supplying the program with the tool path in x direction described. The geometry of the surface at the beginning of machining can be defined in three ways:

- by supplying the software with the surface equation, $y=f(x)$;
- by supplying the software with the coordinates of the surface (for example, a surface just obtained after a NC – REDM process cycle can be used as a starting surface for the next NC – REDM operation).

The important feature of the software is the capability to define the relative tool wear as a function of depth of cut. The results of simulation can be presented in two ways:

- tables of coordinates of points of the workpiece surface
- 2D graphs.

Fig. 5 shows an example of a graph of the machined profile after REDM of a surface with waviness, approximated by function $y = a/(1 + b\sin(2\pi x/\lambda)$. The tool path was given by $g(x) = H_0 - a_0 + R_0 =$ constant (Fig. 3).

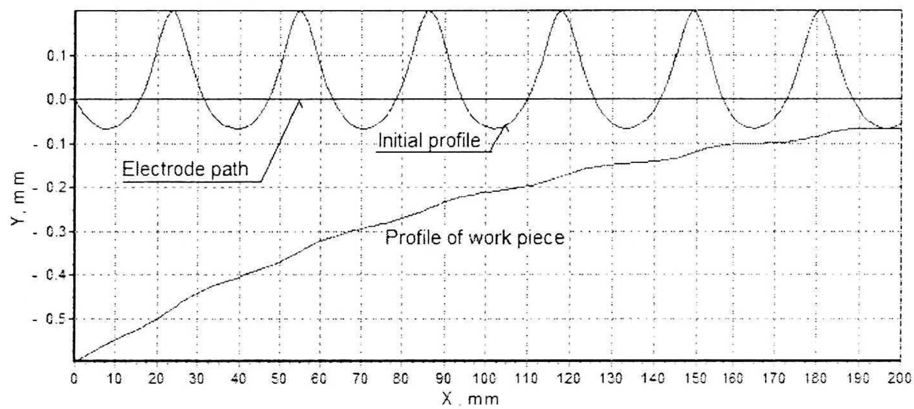

Fig. 5. The profiles from software screen ($R_0 = 10$ mm, $a_o = 0.6$ mm, $v = 0.3$)

Fig. 6 presents the results of simulation of a REDM process with compensation of tool wear by relative motion of electrode/workpiece along *y* axis with constant feed rate according to Eq. (24). An effect of periodical changes in the depth on final shape can be seen on machined surface.

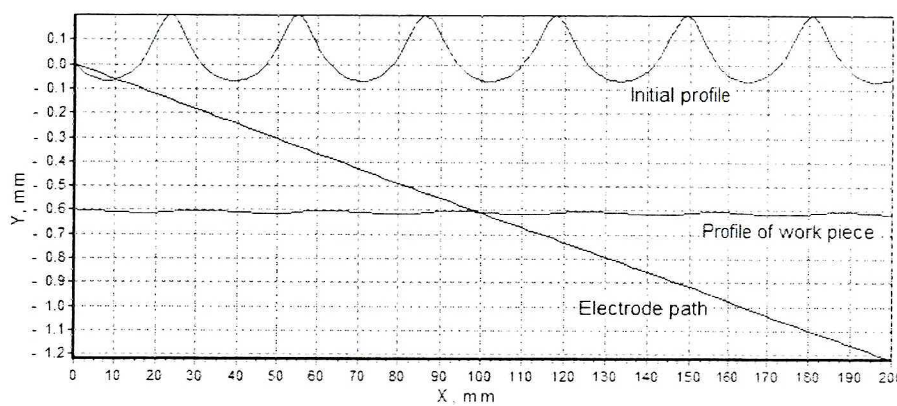

Fig. 6. Surface profile and tool path at REDM with compensation of tool wear ($R_0 = 10$ mm, $a_o = 0.6$ mm, $v = 0.3$)

The effect of vibration of tool electrode on the final profile at machining an initially flat workpiece is presented in Fig. 7. As it is shown in this figure, the vibration of tool is copied on the workpiece surface independent on the tool wear.

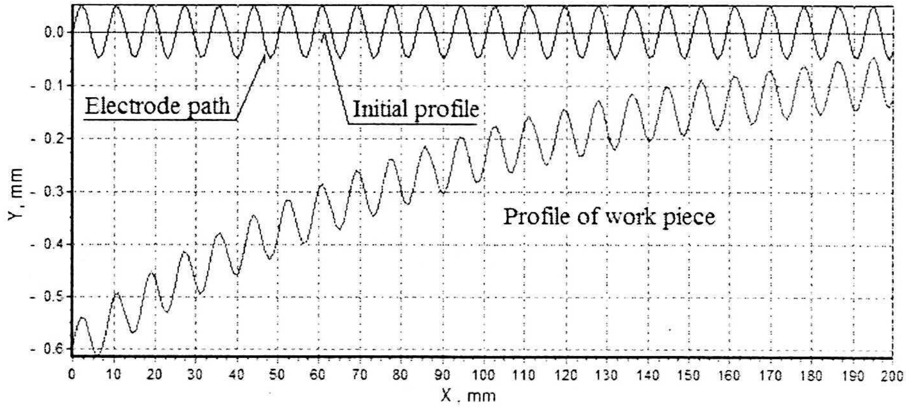

Fig. 7. The effect of vibration of tool on final profile ($R_0 = 10$ mm, $a_o = 0.6$ mm, $v = 0.3$)

4. Experimental verification

The theoretical model and simulation results were verified in Rotary Electrical Discharge Machining (REDM) on M35K Mitsubishi, EDM – NC machine tool. The range for workpiece movement was 0–160 mm for x-axis. The material removed thickness, i.e. the profile of machined surface was measured for a single passage of the tool electrode over a flat surface. The experiments have been performed on tool steel (HRC 56) using a copper electrode.

The experimental results are compared with the simulation results in Fig. 8 and Fig. 9. The distribution of measured coordinates of the points of machined profile shows a good agreement between theoretical and experimental results.

Further experiments were carried out at the following setting operation parameters:

Pulse voltage of 40V,

Pulse current I: 55 and 120A,

Pulse on time t_p = pulse off time t_0: $t_p = t_0 = 60$ and 120 μs,

Depth of cut a_o: 1.2 and 1.5mm,

Feed rate V_f: 0.15, 1.0, 2.0, and 4.0 mm/min,

Rotation speed n: 200 rpm.

The measured values of the relative tool wear are presented in the table in (Fig. 9).

A non-dimensional coordinate system $X = \frac{v}{2\pi \cdot R_0} x$ and $\overline{Y} = \frac{y}{a_0}$ was used for verification with different combinations of setting parametrs. In this system coordinates, the final shape was described as follows:

$$\overline{Y} = -\exp(-X) \tag{25}$$

The results are presented in Fig. 9.

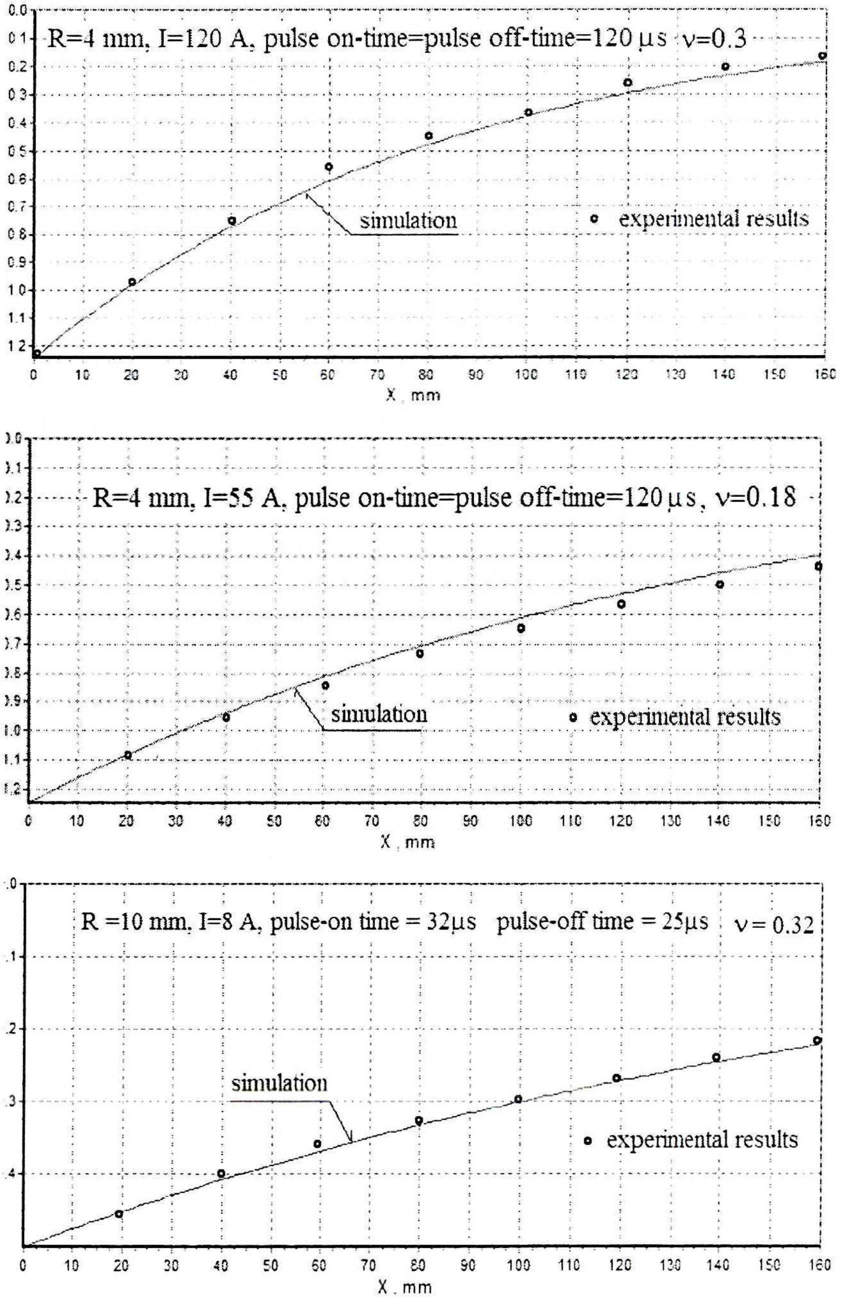

Fig. 8. Comparison of theoretical and experimental results for REDM

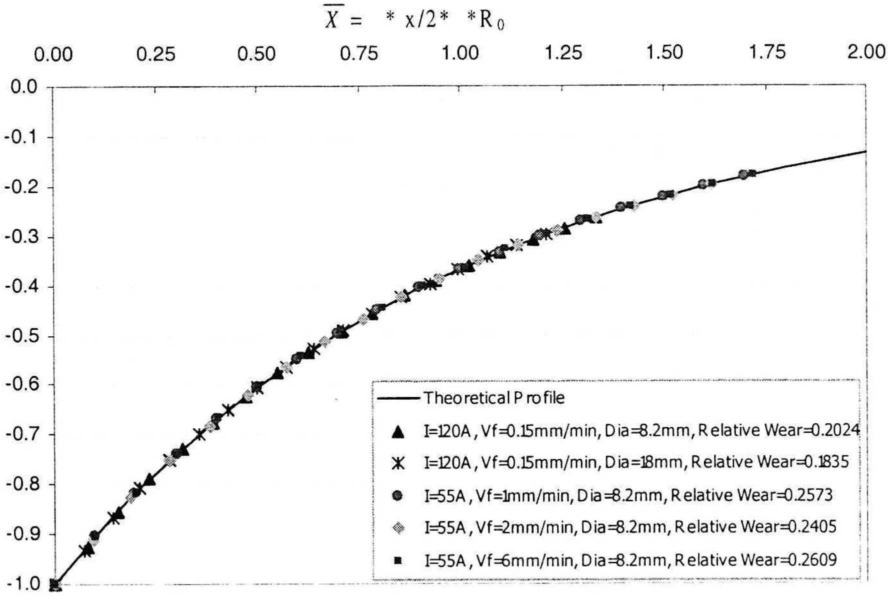

Fig. 9. Comparison of theoretical and experimental results in non-dimensional system coordinates. Setting parameters: $t_p = t_0 = 120$ μs, n = 200 rpm

Experimental verifications show high accuracy of the developed mathematical model and the computer simulation. An overall average of 6 % deviation was found between simulation and experimental results for all cases of experiments.

5. Summary

This paper presents theoretical modeling, simulation and experimental verification of a Rotary Electrical Discharge Machining (REDM). The developed software is useful for REDM process analysis, surface prediction and optimization. The study showes a good agreement between theoretical and experimental results. We can draw the conclusion that this software can be useful for analysis of the REDM process, parameter optimization and surface prediction. This study indicates that computer simulation of REDM has a significant potential for the use in industry applications.

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Symulacja komputerowa obróbki elektroerozyjnej z wirującą elektrodą

S t r e s z c z e n i e

W pracy przedstawiono wyniki badań obróbki elektroerozyjnej z wirującą elektrodą (REDM). W obróbce REDM do wytworzenia złożonych kształtów (3D) używana jest elektroda cylindryczna i sterowanie numeryczne ruchami względnymi. W artykule opracowano model matematyczny procesu kształtowania uwzględniający wpływ zużycia elektrody roboczej na profil obrabianej powierzchni. Omówiono również wpływ parametrów obróbki na błąd kształtu powierzchni obrabianej. Wyniki symulacji komputerowej zostały zweryfikowane doświadczalnie.