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Fractional order $PI^{\lambda}D^{\mu}$ controller with optimal parameters using Modified Grey Wolf Optimizer for AVR system

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In this paper, an automatic voltage regulator (AVR) embedded with fractional order PID (FOPID) is employed for the alternator terminal voltage control. A novel meta-heuristic technique, a modified version of grey wolf optimizer (mGWO) is proposed to design and optimize the FOPID AVR system. The parameters of FOPID, namely, proportional gain (K_P), the integral gain (K_I), the derivative gain (K_D), λ and μ have been optimally tuned with the proposed mGWO technique using a novel fitness function. The initial values of the K_P, K_I, and K_D of the FOPID controller are obtained using Ziegler-Nichols (ZN) method, whereas the initial values of λ and μ have been chosen as arbitrary values.The proposed algorithm offers more benefits such as easy implementation, fast convergence characteristics, and excellent computational ability for the optimization of functions with more than three variables. Additionally, the hasty tuning of FOPID controller parameters gives a high-quality result, and the proposed controller also improves the robustness of the system during uncertainties in the parameters. The quality of the simulated result of the proposed controller has been validatedby other state-of-the-art techniques in the literature.

Key words: integer order PID controller, fractional order PID controller, automatic voltage regulator, evolutionary optimization, Grey Wolf Optimizer

1. Introduction

The quantitative analysis helps us to understand any large scale systems. The exact understanding of any system demands accurate mathematics, leading the researchers towards fractional calculus. Hence fractional calculus has much importance for an accurate and complete understanding of physical phenomena

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of any system [1]. Fractional calculus is around 300 years old, and the researches on fractional calculus, fractional modelling and control are on its boom from the past few decades [2]. Fractional calculus has been broadly used for modelling and control purposes in different engineering fields and industrial applications [3, 4]. CRONE (Commande Robust d'Order Non-Entier, meaning Non-integer-order Robust Control) control was the first fractional controller proposed and applied on many systems by Oustaloup [5]. Many toolbars are available for working with fractional calculus and fractional order controller design, such as CRONE, NINTEGER, and FOMCON, among others [6, 7]. The CRONE toolbar is the most powerful tool for simulating fractional-order systems. The CRONE Toolbox inspired the other toolbars, NINTEGER and FOMCON.

The fractional PID controller is one of the different forms of fractional controllers that have been implemented to achieve fine control operation for practical systems [8–13]. It provides two extra tuning knobs, the fractional power of integral control (λ) and differential control (μ) [14]. These extra degrees of freedom assist the researchers in designing an excellent controller for any system. A large number of publications on fractional-order systems and fractional-order control have been published. FOPID controllers have also been used in real-time applications such as servo motor velocity control [15], controlling of DC-motor with elastic shaft [16], Non-minimum phase system [17], Magnetic levitation system [10, 18] and controlling of the automatic voltage regulator (AVR) system [19–21]. Many optimization strategies have been employed in recent years to obtain the most significant value for fractional control variables. Many optimization approaches for designing the best controller are also included in the toolboxes. For determining the best outcome for the FOPID controller, the Nelder's-Mead optimization (NMO) algorithm [21], Interior-point technique, Sequential quadratic programming (SQP), and Active set based approach have been provided in the FOMCON toolbox [7].

Providing a constant input voltage has always been a very challenging task in power system applications. The AVR system is used to stabilize the voltage value when suddenly change of load for power supply demand. Conventional Proportional-Integral-Derivative (PID) [21], Proportional-Integral-Derivative-Acceleration (PIDA) [22], Fraction Orders PID (FOPID) [23, 24] and Sugeno Fuzzy Logic (SFL) [25] are some types of controllers that have already been implemented on AVR system to control its terminal voltage. Most of the conventional controllers succeed in solving stability-related issues, but they fail to rectify the issues related to nonlinear loads, variable operating points, and time delay. In such cases, optimization techniques can help by tuning the parameters of conventional PID and FOPID controllers [26]. Artificial Intelligent (AI) based techniques such as neural network and fuzzy logic have been successfully used to optimize the controller parameters, but it requires complex analysis and also goes through a very large convergence time. In recent years, Meta-heuristic optimization techniques are used to tune the controller parameters [27, 28]. These techniques are based on simple concepts and are very easy to implement. Moreover, they do not require information gradient. Evolutionary algorithm, swarm-based algorithms, physics-based algorithms and human behaviour based algorithms are the four major classes of this group. Many optimization algorithms are in practice in these categories, but the most popular algorithms of Evolutionary algorithm, swarm-based algorithm, physics-based algorithm and human behaviour based algorithm are Genetic Algorithm (GA) [29], Particle Swarm Optimization (PSO) [24,30], Gravitational Search Algorithm (GSA) [31] and Teaching Learned Based Optimization (TLBO) [32,33] respectively. As controlling the terminal voltage of AVR system is a concern, Panda, Sahu has used a MOL-based optimized PID controller, and Mohanty (2012), which produces better results than ABC, PSO, and DE [34]. A CS-based PIDA controller has been implemented by Deacha (2012), which produces a better response than TS and GA [35]. Several other optimization techniques have also been presented for optimization of the parameters of conventional PID for AVR systems [6, 16, 36–38]. Similarly, various optimization techniques have also been used to optimize FOPID controller parameters for AVR systems [39, 40].

A modified form of GWO algorithm (mGWO) with a novel fitness function has been presented in this work. The mGWO is implemented to locate the finest controller parameters by minimizing the newly defined fitness function value. Although the basic GWO algorithm proposed in [41] has been widely used for tunning fuzzy control system [42], feature subset selection [43], automatic generation control of interconnected power system [44], evolutionary population dynamics [42,45,46], optimizing the parameters of conventional PID controllers [47,48] and many other applications but fails in providing the best solution in case of function with more than three variables. This inspires the necessary modification in the algorithm to find the optimum FOPID controller for AVR system. In the proposed technique, ZN method is used for defining the approximate region of the prey. The wolves start their movement in searching of the food on the basis of these parameters tuned using ZN method. Using Oustaloup's approximation approach and a frequency band of $\omega \in (10^{-3}–10^{+3})$ rad/sec, the suggested controller's fractional-order terms are approximated into integer ones [49].

The remaining structure of this paper is as follows: Section 2 presents the proposed modified form of the GWO algorithm. Section 3 describes the considered research problem and objective function. In Section 4, all the simulation and mathematical results have been shown, and finally, Section 5 concludes this paper.

2. Proposed modified Grey Wolf Optimizer (mGWO)

Most of the Swarm Intelligence (SI) methods are motivated by a specific species' hunting and searching behaviour in nature. As the Grey wolves are very famous for their hunting behaviour in packs and lies on the top of the food chain.

Hence, the motivated algorithm should also generate the best result than the others. Here, two different forms of GWO algorithm have been presented.

2.1. Basic Grey Wolf Optimizer

The basic GWO algorithm is a recent meta-heuristic algorithm inspired by the Grey wolves of the Canidae family. It was initially proposed [41]. The algorithm is motivated by the behaviour of the social hierarchy of a wolf pack. The dominating behaviour of the wolf claims the social hierarchy of wolves. The social hierarchy of the wolves of GWO establishes four organization levels:

i) The most dominating level is the apex of the hierarchy, and the boss of this level is known as alpha (α). It gives the best population solution.

ii) The following level of the hierarchy (i.e., the second level) is known as beta (β), which aids in the hunting process and provides a second-best answer.

iii) The third best solutionis given by the third level of the hierarchy, which is denoted as delta (δ) wolf. It performs as a subordinate. According to [41], delta represents wolves such as scouts, sentinels, elders, hunters and care-takers.

iv) The rest of the wolves in the hierarchy are denoted as omega (ω) and they act as the camp followers. This is the last level of the hierarchy [50].

Only the movement of the first three higher-ranked wolves (α, β, and δ) mimic the encirclement of prey by wolves. The Grey Wolves perform many steps like Pursuing, encircling, and harassing the prey until it gets tiered and stops moving and at last, they attack the prey. These steps have been modelled mathematically as:

2.1.1. Encircling the prey

The wolves' actions necessitated the use of equations:

$$\vec{D} = \left|\vec{C}.\vec{X}_P(t) - \vec{X}(t)\right|, \tag{1}$$

$$\vec{X}(t+1) = \vec{X}_P(t) - \vec{A}\vec{D}, \tag{2}$$

$$\vec{A} = 2\vec{a}\vec{r}_1 - \vec{a}, \tag{3}$$

$$\vec{C} = 2.\vec{r}_2\,, \tag{4}$$

where $\vec{A}$ and $\vec{C}$ are coefficient vectors, $\vec{X}_P$ is the prey's position vector, $\vec{X}$ is the grey wolf's position vector, t is the current iteration, $\vec{r}_1$ and $\vec{r}_2$ are randomly chosen variables in the range of [0, 1].

2.1.2. Hunting behaviour

The positions of the remaining wolves are renewed and are regarded using the best results α, β, and δ of the algorithm. The following expressions (5), (6), and (7) are used to update the positions of all wolves:

$$\left.\begin{aligned}\vec{D}_{\alpha_i} &= \left|\vec{C}_1.\vec{X}_{\alpha_i}(t) - \vec{X}_i(t)\right| \\ \vec{D}_{\eta_i} &= \left|\vec{C}_2.\vec{X}_{\eta_i}(t) - \vec{X}_i(t)\right| \\ \vec{D}_{\delta_i} &= \left|\vec{C}_3.\vec{X}_{\delta_i}(t) - \vec{X}_i(t)\right|\end{aligned}\right\}, \tag{5}$$

$$\left.\begin{aligned}\vec{X}_{i1} &= \vec{X}_{\alpha_i}(t) - \vec{A}_1.\vec{D}_{\alpha_i} \\ \vec{X}_{i2} &= \vec{X}_{\eta_i}(t) - \vec{A}_2.\vec{D}_{\eta_i} \\ \vec{X}_{i3} &= \vec{X}_{\delta_i}(t) - \vec{A}_3.\vec{D}_{\delta_i}\end{aligned}\right\}, \tag{6}$$

$$\vec{X}_i(t+1) = \frac{\vec{X}_{i1} + \vec{X}_{i2} + \vec{X}_{i3}}{3}, \tag{7}$$

where i and $\vec{X}_i(t+1)$ denotes the number of iterations and the top search agent of i-th iteration, respectively.

2.1.3. Attacking prey

It is critical for the wolves to get closer to the prey in order to attack it. As a result, the distance between the positions of wolves and prey must be narrowed. Only by lowering the value of coefficient vector $\vec{A}$ can this be accomplished. The vector $\vec{A}$ is shrunk by lowering the value of variable a as much as possible:

$$a = 2 - \left(\frac{2}{J_{\max_i}}\right), \tag{8}$$

where a is reduced from 2 to and $J_{\max_i}$ is the maximum value of the objective function in i-th iteration.

2.2. Modified Grey Wolf Optimizer (MGWO)

The proposed approach has been presented for updating the position of wolves. Encircling of the prey by wolves in MGWO requires the following equations.

$$\vec{D} = \left|\vec{C}.\vec{X}_p(t) - \vec{X}(t).U(-2a, 2a)\right|, \tag{9}$$

$$\vec{X}(t+1) = \vec{X}(t).\vec{A}.U(-2a, 2a). \tag{10}$$

Here, the term $U(-2a, 2a)$ gives a uniformly distributed random number in the interval $[-2, 2]$. Where, exploitation occurs if $U(-2a, 2a)$ gives a number closer to 1; otherwise, exploration occurs. Calculation of coefficients 'A' and 'C' is done in the same manner as in (3) and (4), respectively. The positions of the wolves are updated according to the best fitness value computed, as shown below.

$$\vec{X}(t+1) = \begin{cases} \vec{X}(t).\vec{A}.U(-2a, 2a), & \text{fitness } \left\{\vec{X}(t+1)\right\} < \text{fitness } \left\{\vec{X}(t)\right\}, \\ \vec{X}(t) & \text{otherwise.} \end{cases} \tag{11}$$

3. Application of proposed mGWO algorithm for FOPID- AVR system

3.1. Automatic voltage regulator (AVR) system

Power system network is mainly designed to work at a convinced frequency and terminal voltages. Any disturbances raised by a swing in the turbine's output, load variation, high impedance of field winding or deviation in transmission line parameters may lead to instability in the terminal voltage of the system. These instabilities may lead to overall system collapse or, it may cause spoil to any coupled equipment. The two independent control loops are designed to focus on these parameters, namely the load frequency control and AVR. TheAVR system is found cheaper and very effective for controlling the generator's terminal voltage [51,52]. It is a combination of four sub-systems, namely amplifier, exciter, generator, and sensor. A linearized model of the AVR system has been taken here for ease by considering the major time constants, and it has been assumed that the system has no non-linearities such as saturation or other. Figure 1 depicts the schematic block diagram of the AVR system. Each component's (sub-systems) transfer function has been shown in the corresponding block.

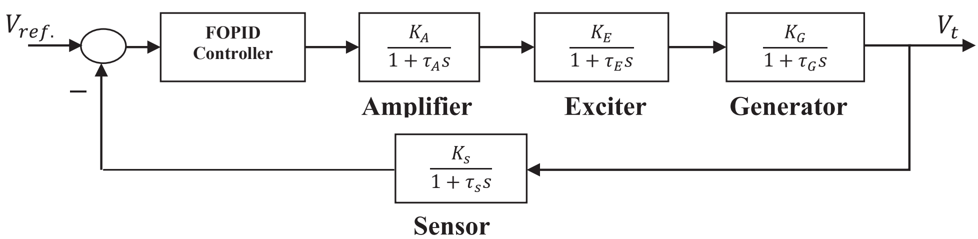


Figure 1: Block diagram of an AVR system

An effective objective or fitness function will be needed to optimize the values of K_P, K_I, K_D, λ and μ for getting the most suitable controller. The objective function considered for this work is defined in terms of rise-time (RT), settling-time

(ST), peak-overshoot (PO), Gain-Margin (G.M), Phase-Margin (P.M), integral time-weighted absolute error (ITAE) and integral time-weighted square error (ITSE). These considerable values of each parameter has been multiplied by weight factors w_i and then combined together as shown as:

$$J = \frac{(\text{ITAE} + \text{ITSE}) * w_1 + (\text{RT} + \text{ST} + \text{PO}) * w_2}{(\text{G.M} + \text{P.M.}) * w_3}, \tag{12}$$

where w_1, w_2 and w_3 are the weighting factors. Selection of w_i is tricky and designer has to try multiple times for getting suitable values of weighting factors.

The Idea of defining the fitness function in terms of RT, ST, PO, GM and PM is to make the proper balance between time-domain and frequency-domain characteristics of the system during each iteration of optimization. The weighting factors are chosen according to their percentage of contribution for the desired result. The ITAE and ITSE are defined as follows:

$$\text{ITAE} = \int_0^T T\,|e(t)|\,\mathrm{d}t, \tag{13}$$

$$\text{ITSE} = \int_0^T T e^2(t)\,\mathrm{d}t, \tag{14}$$

where $e(t)$ is the error, t is the time period, and T is total simulation time.

The error $e(t)$ at time t is calculated as:

$$e(t) = 1 - \text{step}(G_C)\big|_t, \tag{15}$$

where G_C is the closed-loop transfer function of the system with controller.

3.2. A summary of fractional calculus

Around 300 years back in 1695 the two scientists Leibniz and L'Hˆopital initially offer the fractional derivative in terms of thehalf-order derivative. This was the first time when fractional calculus comes into the picture. They gave a general representation for differentiation and integration both as ${}_{\alpha}D_t^r$, where α and t are limits of the operation. Different definitions integro-differential operator and most significant properties of this non-integer order calculus are discussed below.

3.2.1. The definition of integro-differential operator

The integro-differential operator's continuous domain definition is as follows:

$$
{}_{\alpha}D_t^r = \begin{cases} d^r/dt^r & R(r) > 0, \\ 1 & R(r) = 0, \\ \displaystyle\int_{\alpha}^{t} (\mathrm{d}t)^r & R(r) < 0. \end{cases} \tag{16}
$$

where r is the integration or differentiation order, r might be either a real or a complex number. Twodifferent definitions of differ-integral for fractional-order systems are present in the literature. One is given Grunwald-Letnikov (GL), and other is given by Riemann-Liouville (RL) [54, 56].

Before optimizing the FOPID settings, the fractional terms of the controller must be approximated into an integer order transfer function. For the approximation of fractional order terms into integer order, there are numerous approximation methods accessible in the literature. For this aim, the well-known Oustaloup's approximation algorithm is used.

3.2.2. Description of Fractional Order PID Controller:

The FOPID controller is a generalized version of the ordinary PID controller, denoted by the letters PI D. It increases the system's stability and robustness. Furthermore, the fractional PID controller provides better dynamical system control and is less susceptible to changes in control system parameters. The FOPID controller's standard transfer function is as follows:

$$
C_{\text{FOPID}}(s) = K_P + \frac{K_I}{s^{\lambda}} + Ds^{\mu} quad(\lambda,\ \mu > 0), \tag{17}
$$

where for λ and μ are the fractional power of integral and differential terms respectively.

FOPID controller can realize all classical controllers from a variety of values of λ, and μ, i.e., $\lambda = 1$, $\mu = 1$ delivers classical PID controller, $\lambda = 1$, $\mu = 0$ delivers PI controller and $\lambda = 0$, $\mu = 1$ delivers PD controller. This can also be realized in a two-dimensional plan as given in Figure 2. The darken portion with blue colour will represent the fractional controller.

It has been earlier mentioned that the optimization of control variables (K_P, K_I, K_D, λ, and μ) have been done by GWO algorithm. A brief introduction of this algorithm has been given in the next section.

3.3. Implementation of the algorithm

Implementation process of the algorithm for this works is shown with the flowchart in Figure 3.

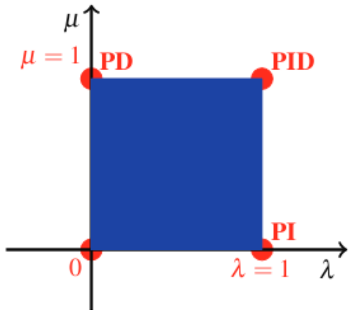


Figure 2: The plane of FOPID controller

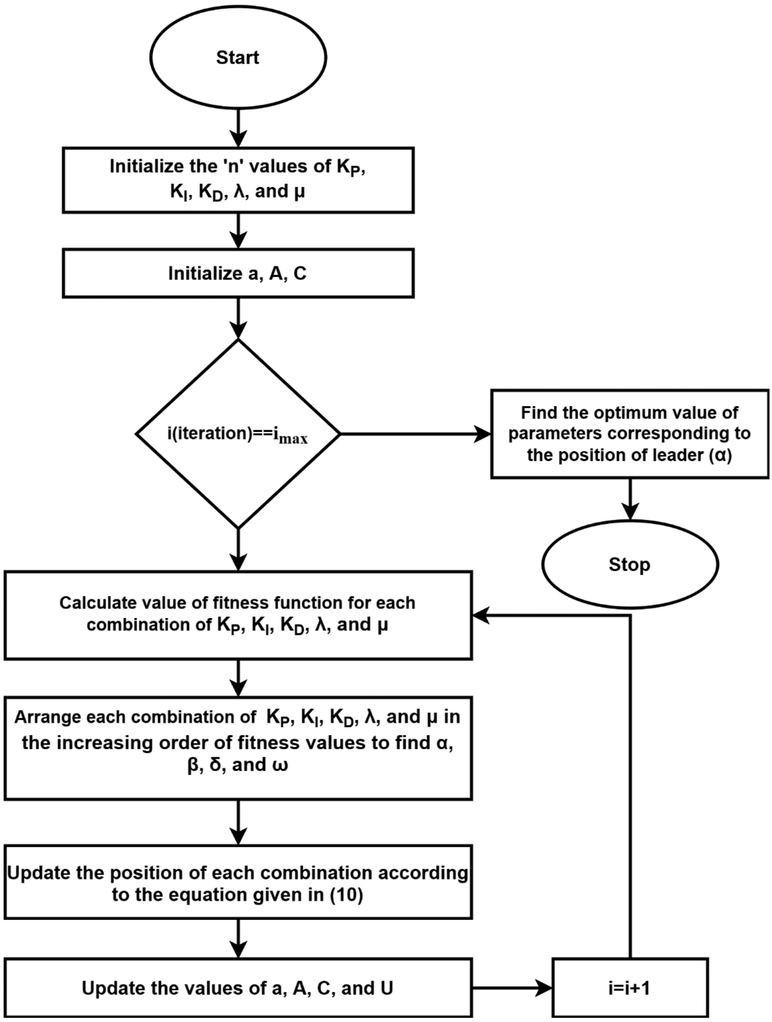


Figure 3: Flowchart of proposed algorithm

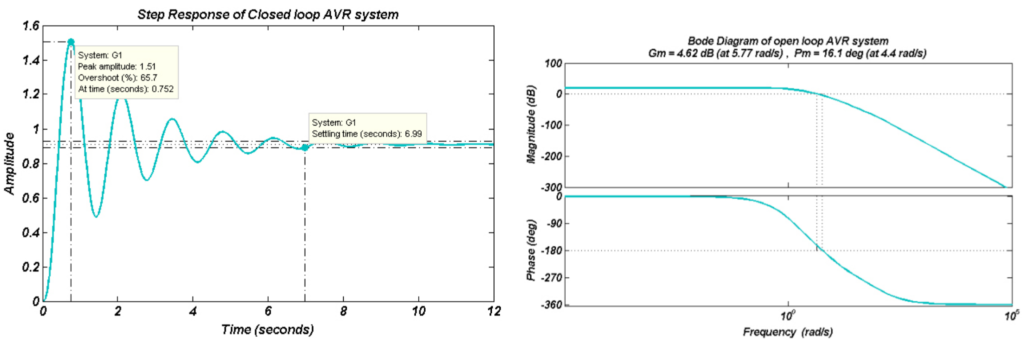
4. Simulation results

4.1. AVR System Analysis without controller

$K_A = 10$, $\tau_A = 0.1$, $K_E = 1$, $\tau_E = 0.4$, $K_G = 1$, $\tau_G = 1$, $K_s = 1$, $\tau_s = 0.01$, are the parameter values used in this study from its typical ranges i.e. $K_A = [10, 400]$, $\tau_A = [0.02, 0.1]$, $K_E = [1, ,400]$, $\tau_E = [0.4, 1.0]$, $K_G = [0.7, 1.0]$, $\tau_G = [1.0, 2.0]$, $K_s = [1.0, 2.0]$, $\tau_s = [0.001, 0.06]$. As a result, the AVR system's closed-loop transfer function without a controller can be written as [21]:

$$G_{\text{AVR}} = \frac{V_t(s)}{V_{\text{ref}}(s)} = \frac{0.1s + 10}{0.0004s^4 + 0.045s^3 + 0.555s^2 + 1.51s + 11}. \tag{18}$$

We can easily calculate the poles and zeros of the closed-loop transfer function of the given AVR system. It has two real poles located at $s_1 = -99.9712$ And $s_2 = -12.4892$ and two complex poles at $s_{3,4} = -0.5198 \pm 4.6642i$ and one zero at $z = -100$. The step response of the AVR system shown in Figure 4(a), shows the oscillating behavior which proves the under-damped nature of the system. Moreover, thephase plot in Figure 4(b) shows very small gain margin (GM) and phase margin (PM). Therefore, we need a controller which can improve the overall performance of the AVR system.



(a) (b)

Figure 4: (a) Closed-loop step response, (b) frequency plot of open-loop AVR system without controller

4.2. Design of MGWO-FOPID controller for AVR system

In this section, An enhanced FOPID controller is designed to improve the AVR system's performance in both the time and frequency domains.Optimization of controller parameter are done using proposed mGWO algorithm and the newly defined fitness function. Numerical values of weights in the fitness function taken for this work are 0.5, 15 and 10 respectively.The performance of the optimized FOPID controller in both domains has been shown in the next subsection.

4.2.1. The performance of MGWO-FOPID controller

The performance of the MGWO-FOPID controller is examined in this section. Figure 5 depicts the performance of all twelve wolves in the time domain. Figure 6 depicts the step response of three of the best wolves, namely alpha, beta, and delta. Table 1 shows the values of the parameters (i.e. K_P, K_I, K_D, λ and μ), as well as the time-domain and frequency-domain characteristics of each wolf. As we can see in Figure 6(b) that the leader of the hierarchy i.e. alpha gives the most excellent controller for the AVR system. The AVR system with this controller gets settled in 0.0653 sec and also has zero percent of overshoot in the system.

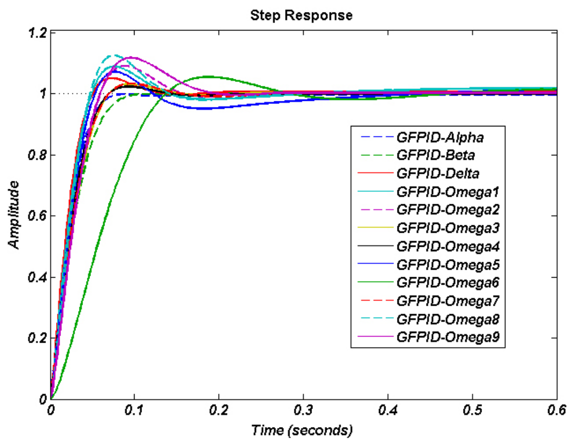


Figure 5: Step response all the twelve wolves

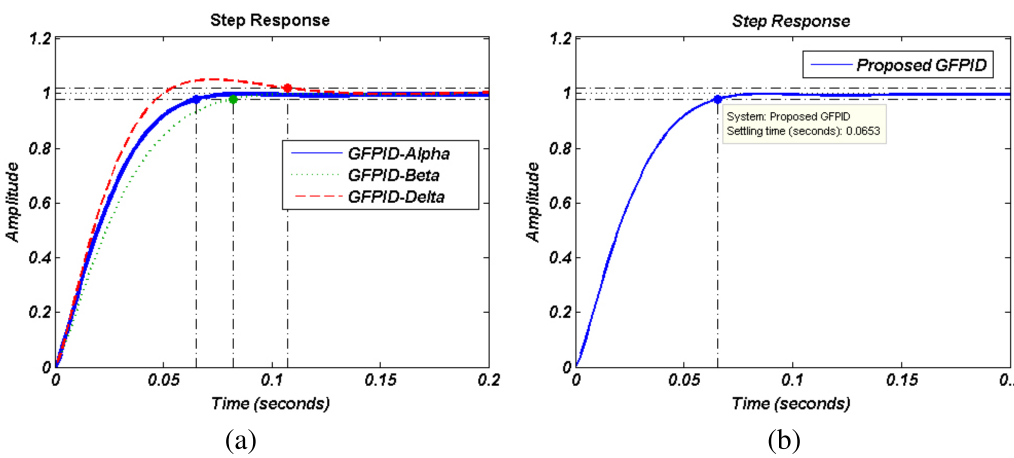


Figure 6: (a) Step response three best wolves of the hierarchy (i.e. alpha, beta, and delta), (b) Step response AVR system with proposed FOPID controller (i.e. leader of the hierarchy)

Although, the FOPID controller given by other membersof the hierarchy like delta and w_1 are more master in terms of rise-time but the settling time of this controller are much greater than the leader of the hierarchy.

Table 1: All specifications of FOPID controller using MGWO

Grey wolf	K_p	K_i	K_d	λ	μ	Rise-time	Settling time	Peak overshoot	G.M.	P.M.
α	11.2902	1.1035	0.4155	0.8714	1.7281	0.0427	0.0653	0	29.8	71.0
γ	8.1816	1.3680	0.4084	1.0105	1.6914	0.0528	0.0821	0	30.2	71.7
δ	11.4158	1.7858	0.5779	0.9706	1.6790	0.0341	0.1073	5.1110	27.2	64.9
ω_1	9.6182	1.4376	0.6895	1.0005	1.6208	0.0345	0.1245	8.9367	25.9	60.4
ω_2	9.5074	1.6425	0.5848	0.8582	1.5980	0.0417	0.1401	9.1540	26.4	59.3
ω_3	8.3947	1.1609	0.4944	1.0426	1.6442	0.0459	0.1204	3.2380	28.6	66.5
ω_4	8.9042	0.8009	0.4851	0.9811	1.6747	0.0442	0.1060	2.3623	4.62	16.1
ω_5	6.9650	1.4124	0.6547	0.9376	1.6140	0.0371	0.3083	7.1859	26.1	60.2
ω_6	3.3762	0.7789	0.3910	1.1637	1.4305	0.0937	0.2538	5.5107	29.0	62.2
ω_7	8.2377	1.6531	0.5014	0.9943	1.6551	0.0456	0.1238	3.3296	28.5	66.6
ω_8	9.6430	2.4695	0.7658	1.0468	1.5776	0.0342	0.1284	12.5826	25.0	56.7
ω_9	9.5857	1.2275	0.6429	0.9720	1.5759	0.0424	0.1693	11.7936	26.6	57.9

4.2.2. Comparison of proposed MGWO-FOPID Controller with other Controllers:

So far as many researchers had worked for designing the optimum controller for the same AVR system. The performance of proposed controller is broadly compared with other PID and FOPID controllers proposed in the literature. For comparison with PID controller, results of GA based PID developed in [52], PSO algorithm based PID developed in [37] and chaotic ant swarm (CAS) based PID developed in [51] is taken. Similarly, for comparing with other FOPID controllers results of GA based FOPID developed in [52], PSO algorithm based FOPID developed in [55] and chaotic ant swarm (CAS) based FOPID developed in [56] is taken.

All the controller's parameters (i.e. K_P, K_I, K_D, λ and μ) and performance characteristics (time-domain as well as frequency domain) are compared in Table 2 and Table 3 respectively. The proposed FOPID controller shows the better performance than the other PID and FOPID controller. The proposed FOPID controller can produce perfect gain margin and phase margin for the AVR system. Moreover, the time-domain characteristics of proposed FOPID controller are far better than the other controllers in the literature.

Table 2: Comparison of parameters of controllers designed using different techniques

Controller	K_P	K_I	K_D	λ	μ
		$\beta = 1$			
GA-PID	0.8861	0.7984	0.3158	–	–
PSO-PID	0.0657	0.5389	0.2458	–	–
CAS-PID	0.6746	0.6009	0.2618	–	–
CAS-FOPID	1.0537	0.4418	0.2510	1.1122	1.0624
GA-FOPID	1.3227	0.5398	0.2443	1.2790	1.1849
PSO-FOPID	1.2623	0.5531	0.2382	1.2555	1.1827
		$\beta = 1.5$			
GA-PID	0.7717	0.5930	0.3507	–	–
PSO-PID	06254	0.4577	0.2187	–	–
CAS-PID	0.6202	0.4531	0.2152	–	–
CAS-FOPID	0.9315	0.4776	0.2536	1.0838	1.0275
GA-FOPID	1.3715	0.5663	0.2395	1.2823	1.2892
PSO-FOPID	1.2623	0.5526	0.2381	1.2559	1.1832
MGWO-FOPID (Proposed)	**11.2902**	**1.1035**	**0.4155**	**0.8714**	**1.7281**

Table 3: Comparison of performance characteristics of controllers designed using different techniques

Controller	Rise time t_r (S)	Settling time t_s (S)	Maximum overshoot M_P (%)	Gain margin	Phase margin
No Controller	0.2607	6.9865	65.7226	4.62	16.1
		$\beta = 1$			
GA-PID	0.2019	0.5980	8.6644	23.5	59.2
PSO-PID	0.2767	0.4025	1.16	25.8	67.6
CAS-PID	0.2425	0.3550	1.7678	25.4	66.9
CAS-FOPID	0.2223	0.3037	0.1678	26.6	57.7
GA-FOPID	0.1402	0.561	0	29.8	62.8
PSO-FOPID	0.1604	0.2657	0.02	30.2	63.5
		$\beta = 1.5$			
GA-PID	0.2003	1.0517	3.6287	23.2	61.8
PSO-PID	0.2997	0.4156	0.4400	26.7	68.5
CAS-PID	0.3156	0.4212	0.4000	26.8	68.6
CAS-FOPID	0.2305	0.3187	0.0642	25.5	59
GA-FOPID	0.1444	0.4894	0.36	30.7	73.3
PSO-FOPID	0.1603	0.2655	0.01	30.2	63.6
MGWO-FOPID (Proposed)	**0.0427**	**0.0653**	**0**	**29.8**	**71**

4.3. Robustness Analysis of the proposed Controller

For robustness analysis, it is considered that there may be three different types of uncertainties in the AVR system. Hense we have analyzed the performance of the proposed FOPID controller by considering all the three uncertainties in the system one by one. Which are presented as:

4.3.1. Uncertainty in amplifier

Here, it is considered that the parameters of the amplifier may alter from original value $K_A = 10$, $\tau_A = 0.1$ to $K_A = 14$, $\tau_A = 0.007$. The step response of the closed-loop system with the altered value of amplifier constants is compared with the original AVR system in Figure 7. It is very clear that the performance of the proposed FOPID controller very similar in both the cases. Moreover, the rise-time of AVR system with the altered value of amplifier constants is less than the original system, i.e.; the overall system performs faster than the previous one.

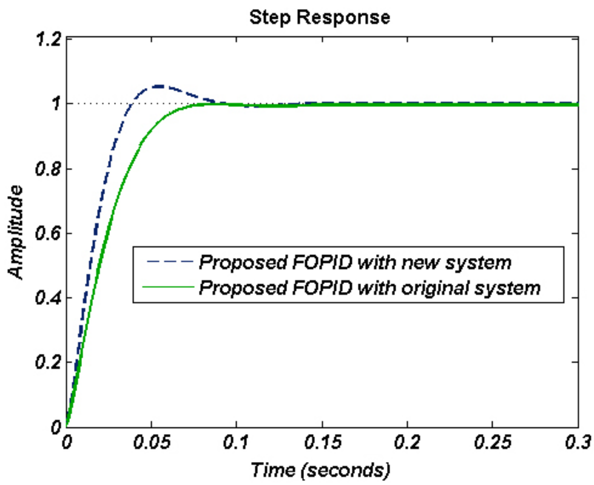


Figure 7: Comparison of step response with uncertainty in amplifier

4.3.2. Uncertainty in Exciter

It is assumed that the Exciter's settings will change from their original value of $K_E = 1$, $\tau_E = 0.4$ to $K_E = 1.2$, $\tau_E = 0.5$. Figure 8 compares the step response of the closed-loop system with exciter uncertainty to the original AVR system. The performance of the suggested FOPID controller is identical in both circumstances, as can be seen here.

4.3.3. Uncertainty in Generator:

Here, it is considered that the parameters of the Generator may change from original value $K_G = 1$, $\tau_G = 1$ to $K_G = 0.7$, $\tau_G = 1.6$. The step response of the

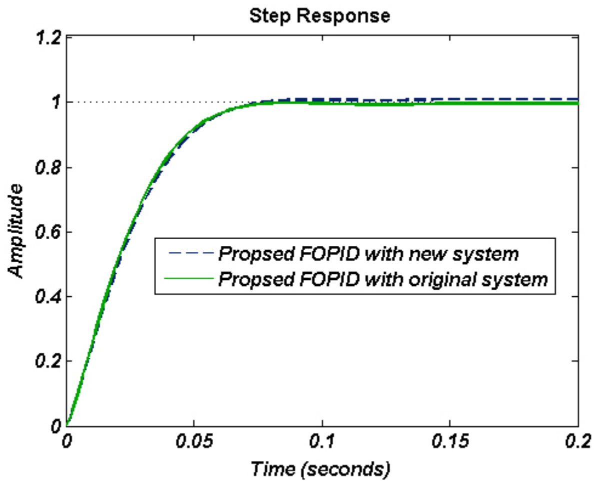


Figure 8: Comparison of step response with uncertainty in exciter

closed-loop system with the generator uncertainty is compared with the original AVR system in Figure 9. Here we can see that the performance of the proposed FOPID controller is almost similar in both the cases.

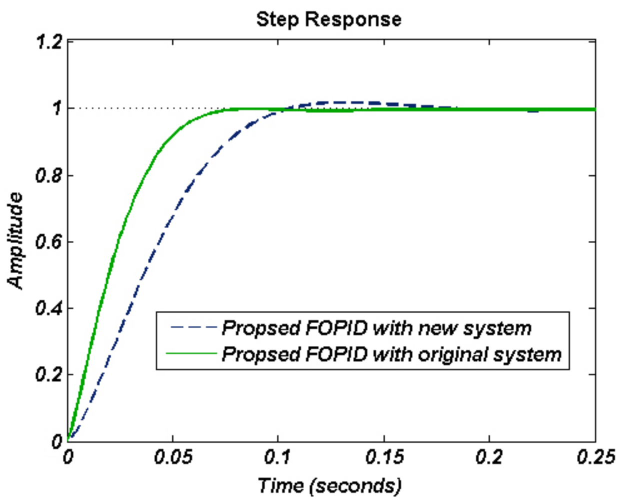


Figure 9: Comparison of step response with uncertainty in generator

By observing all three cases, we derive that the proposed FOPID controller performs well in all the three uncertainities in the system.

5. Conclusions

The paper has successfully designed and optimized the parameters of FOPID controller to control the terminal voltage of the AVR system. The FOPID controller parameters were optimized using a modified version of the grey wolf optimizer and a new fitness function. The algorithm is based on natural grey wolf meta-heuristics, which have the effective potential of avoiding local minima and suggesting the optimal global optimum parameter values in a pre-defined search space. The settings of the practical FOPID controller were optimized using a new fitness function that included the integral of time absolute error (ITAE), integral of time square error (ITSE), raise-time, settling-time, and maximum-overshoot. The step response of the closed-loop AVR system was used to determine the values of all these parameters.

The suggested FOPID controller's simulation results using a real-world AVR system verified its superior control performance. Furthermore, the suggested FOPID controller has greater control capability in the time-domain and frequency-domain than existing PID and FOPID controllers found in the literature. Furthermore, the suggested FOPID controller operates admirably in the face of all three externally induced uncertainties in the system. This demonstrates the controller's robustness. All of the simulation results were presented in the study and compared to various Figures and Tables.

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