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Sum-α Stopping Criterion for Turbo Decoding

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***Abstract*—In this article, we propose a new stopping criterion for turbo codes. This criterion is based on the behaviour of the probabilistic values alpha '*α*' calculated in the forward recursion during turbo decoding. We called this criterion Sum-α. The simulation results show that the Bit Error Rates BER are very close to those of the Cross-Entropy CE criterion with the same average number of iterations.**

***Keywords*—stopping criterion, correct frames, sum-α criterion , cross-entropy, turbo decoding**

I. Introduction

TURBO codes [1] are attractive parallel concatenated codes, approaching the Shannon limit. Since their introduction, they are used in different digital communication standards because of their correction capacity. BER are decreased after each iteration. However, it is useless to treat a frame when it is decoded correctly. So, stopping the turbo decoding process is necessary to reduce computational complexity. Several methods based on Log-Likelihood Ratios LLRs have been proposed [2-11]. This requires the end of the processing of a frame to manipulate the LLRs and a large memory storage which depends on the length of the frame N (length before coding).

In this article, we present a simple method, which uses practically no memory storage (except an accumulated sum) and which provides an early decision to stop the processing of an iteration. It means, after the end of the alpha calculation $\alpha_m(t)$ of the forward recursion. We compare its efficiency with the Cross-Entropy CE criterion [2], which constitutes a good rule. We verify the application of the Sum-α criterion to the turbo decoder for several sizes of the interleaver. We also study the behaviour of this new criterion with several thresholds and we deduce the optimal value.

The document is structured as follows: In section 2, we present some stopping criteria proposed in the literature. In Section 3, we show the behaviour of alpha probabilities and in Section 4 we explain our idea of reducing the number of iterations. We call this stopping criterion Sum-α. In section 5, we present the results of our simulations.

II. Existing Stopping Techniques

We consider two Recursive Systematic Convolutional 'RSC' codes concatenated in parallel. The frame of information bits $\{u(k)\}$, $k = 1, \dots \dots, N$, is coded by this turbo code. Each information bit, after coding gives a systematic bit $u(k)$, and two redundant bits $x_1(k)$ and $x_2(k)$. After transmission over a Gaussian channel using BPSK modulation, the received samples are $\{y_u(k), y_1(k), y_2(k)\}$. The received frame is passed to the turbo decoder of the Fig. 1. INT and DEI represent respectively interleaver and the deinterleaver.

The famous criterion Cross-entropy CE uses the LLRs at the outputs of the two decoders [2,3]. Let $L_m^{(i)}(\hat{u}(k))$ be the LLR of bit $u(k)$ at the output of the decoder 'm' ($m=1, 2$) of the 'i^{th}' iteration at the moment 'k', and $Le_m^{(i)}(\hat{u}(k))$ his extrinsic information. The cross-entropy of the iteration 'i' is approximated by [2]

$$CE(i) \approx \sum_k \frac{\left|\Delta Le_2^{(i)}\left(\hat{u}(k)\right)\right|^2}{e^{\left|L_1^{(i)}(\hat{u}(k))\right|}} \tag{1}$$

where

$$\Delta Le_2^{(i)}\left(\hat{u}(k)\right) = Le_2^{(i)}\left(\hat{u}(k)\right) - Le_2^{(i-1)}\left(\hat{u}(k)\right) \tag{2}$$

Decoding is considered converged and stopped when the cross-Entropy $CE(i)$ is below a threshold 'ε' :

$$CE(i) < \varepsilon \tag{3}$$

Hagenauer [2] claims that a threshold 'ε' between (10^{-2} $CE(1) \le \varepsilon \le 10^{-4}$ $CE(1)$) is appropriate for stopping iterations. This criterion permits the stopping of the Turbo process after decoding of the frames with a very weak degradation of the performances.

Two other methods derived from the 'CE' criterion have been proposed. The first is the Sign Change Ratio criterion SCR [4]. It consists of counting the number of sign changes $C(i)$ of extrinsic information produced by the second decoder between the iteration 'i' and 'i-1'. Simulations show that we can stop the iterations when [4]:

$$C(i) \le (0.005 \sim 0.03)N \tag{4}$$

where N is the length of information frames before coding.

This criterion makes it possible to stop the turbo process with about the same average number of iterations and the same performances of the CE rule.

The second rule HDA (Hard-Decision-Aided) [4], compares the hard decisions at the output of the second decoder with those of the previous iteration. Decoding is stopped if all hard decisions remain the same. The overall performances of the simplified variants are close to that obtained by the original CE rule.

Another stopping criterion called Sign Difference Ratio (SDR) [5] is a variant of the SCR. In this case, we count the number of times D_{ji} where the signs of the a priori information and the extrinsic information of the same decoder differ at the iteration 'i'. The turbo process is stopped if:

$$D_{ji} \le p\,N \tag{5}$$

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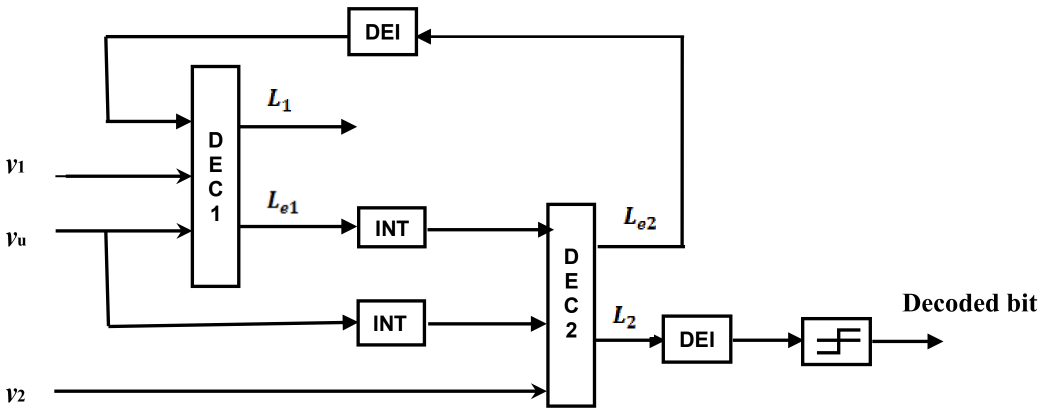

Fig. 1. Turbo decoder

where 'p' is a threshold that represents the sign difference ratio SDR , and:

$$10^{-3} \le p \le 10^{-2} \tag{6}$$

D_{ji} is also the number of sign differences between the extrinsic informations of the two decoders.

SDR achieves similar performance of SCR in terms of BER, FER, and the average number of iterations, while requiring lower complexity. The CE, SCR and SDR methods require an additional iteration on average than the ideal GENIE criterion [5]. For the GENIE criterion, the information bits are known and the iterations are stopped immediately after the frame is correctly decoded. GENIE is only a theoretical criterion.

The improved hard-decision-aided rule (IHDA) [6] modifies HDA to compare the hard decisions of the two decoders. All of these methods are based on the manipulation of LLRs calculated after each iteration. In addition, they require the end of frame processing and data storage.

Another contribution [7] uses a CRC code to check if the frame has been corrected, but it penalizes the information rate by adding redundancy. Other method use the identification of undecodable blocks for stopping the turbo process [8]. We cite other criteria that also use the LLR (or decisions) [9-14], but in this article, we consider for comparison the Cross-Entropy CE criterion [2-4], which constitutes a good criterion and offers good performances.

In this article, we propose a new method based on the behaviour of alpha $\alpha_m(t)$ probabilities. This criterion works after the end of the forward recursion of the Maximum A Posteriori MAP algorithm. It allows to know if the frame was decoded before the calculation of the probabilities Beta $\beta_m(t)$ of the backward recursion, and before the computation of the LLR. It is therefore considered as an ***early*** criterion. In addition, it requires practically no memory storage.

III. Behaviour of the Alpha Probabilities

During a MAP decoding, before calculating the LLR, the alpha probability of the state 'm' of the trellis at the moment 't' is computed by

$$\alpha_m(t) = \sum_{m'=1}^{M_{tr}} \alpha_{m'}(t-1)\, \gamma_{(m',m)}(t) \tag{7}$$

where M_{tr} is the number of states. $\gamma_{(m',m)}(t)$ are the probabilities of transition between the states (m',m) of the trellis. Note that the alpha value $\alpha_m(t)$ of the state 'm' is only the probability

$$\alpha_m(t) = \Pr ob\left(S_t = m, Y_1^t\right) \tag{8}$$

where Y_1^t is the sequence of samples received between the moment '1' and 't'.

[15] shows that the alphas $\alpha_m(t)$ of the corrected instants are characterized by a strong impulse of the most probable state. The other alphas are very weak. In addition, an erroneous moment is characterized by some concurrent alphas that have significant probabilities. This idea is used in [15] to improve a turbo decoding based on the M-BCJR algorithm [16]. The proposed algorithm in [15] is called the ZMAP algorithm.

Figure 2 shows the concurrent alphas of an erroneous moment of a trellis that has 16 states. This phenomenon can be used to stop the Turbo process after decoding a frame correctly. It makes it possible to formulate an early stopping criterion of the turbo decoding because this decision will be taken just after the alphas calculations, that is to say, just after the end of the forward recursion of the MAP algorithm. Moreover, we will show in the simulation results section that the proposed criterion ensures the same performances in terms of BER and average number of iterations of the CE criterion.

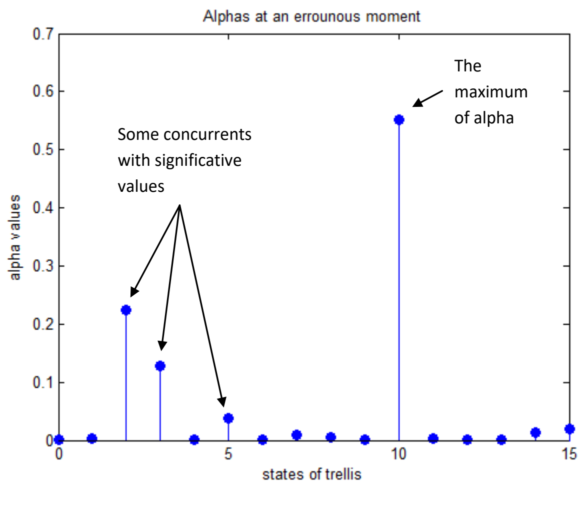

Fig. 2. Concurrent alphas of a trellis with 16 states.

IV. The Proposed Stopping Rule: The Sum-α criterion

We propose a very simple stopping criterion for the turbo decoding based on the behaviour of the probabilistic values $\alpha_m(t)$ calculated in the forward recursion. This criterion uses practically no memory storage.

For every moment '*t*', we call $\alpha_{max}(t)$ the probability of the state with the highest alpha value. It means that

$$\alpha_{\max}(t) = \max(\alpha_m(t)) \quad (9)$$

We call other alphas 'concurrents'.

The following remarks are observed during repeated simulations :

- The correct moments are characterized by a strong value of alpha $\alpha_{max}(t)$ of the most probable state. The alphas probabilities of other states are very weak.
- Erroneous instants are characterized by the concurrent alphas that have significant values. $\alpha_{max}(t)$ is therefore decreased.
- The correct frames are therefore characterized by $\alpha_{max}(t)$ who have great values and the concurrent alphas who have very low values. The sum of the alphas of the concurrents remains weak.
- Erroneous frames are characterized by concurrent alphas that have significant values (probabilities). The increase of the alpha values of the concurrent states will decrease the probability of the most probable states $\alpha_{max}(t)$,
- The decrease of the concurrent values can be used to signalize correct frames.

From these remarks, we can formulate a criterion for detection of the correct frames to stop the turbo process. We call this criterion '***Sum-α***'. it consists in :

1. At each moment '*t*', and after the calculation of the alphas in the forward direction of the MAP algorithm, calculate the sum of the concurrents, it means

$$Sum_c(t) = \left(\sum_{m=1}^{M_{tr}} \alpha_m(t)\right) - \alpha_{\max}(t) \quad (10)$$

Note that we can calculate $Sum_c(t)$ by :

$$Sum_c(t) = 1 - \alpha_{\max}(t) \quad (11)$$

because

$$\left(\sum_{m=1}^{M_{tr}} \alpha_m(t)\right) = 1 \quad (12)$$

2. Calculate the sum of '$Sum_c(t)$' of all moments normalized by the length of the frames '*N*',

$$Sum\alpha = \frac{\sum_{t=1}^{N} Sum_c(t)}{N} \quad (13)$$

3. If '$Sum\alpha$' is below a threshold '*T*', then, stop the Turbo process. It means that :

If $Sum\alpha < T$, then, stop the iterations.

For the 'T' threshold, a large value can considerably reduce the average number of iterations because it will quickly stop turbo decoding, but this will degrade the BER bit error rate. For an interleaver of size $N = 5120$, we will show that T=0.001 is a good value . This technique uses only one memory storage (great advantage) because the sum to the moment '*l*' is calculated by

$$\sum_{t=1}^{l} Sum_c(t) = \sum_{t=1}^{l-1} Sum_c(t) + Sum_c(l) \quad (14)$$

We called this reduction mechanism of iterations the Sum-α stopping criterion. Its implementation is simpler in terms of memory space required than those presented at the beginning of this article. In addition, we will show in the simulation part that it provides the same performance of the Cross-Entropy CE stopping criterion.

V. Simulation Results

We consider a parallel Turbo code with a nominal rate *R=1/3*, consisting of two RSC codes with the octal representation [1, 35/23]. We use a pseudo-random interleaver S-random with different sizes. The encoded bits are transmitted with BPSK modulation over a Gaussian channel. At the receiver, the turbo decoder uses a maximum of 10 iterations. The number of transmitted frames is 3000 (3000 × 5120 information bits, or 3000 × 5120 × 3 coded bits).

We compare the performance of the CE criterion with that of the Sum-α using a threshold T=0.001. Then, we consider another interleaver of small size (1280) with several thresholds. The thresholds of the Sum-α criterion chosen for the second part of the simulation are T=0.1, 0.01, 0.001 and 0.0001. For all cases, the threshold used for the CE criterion is same. We chose the threshold ε = 10^{-3} CE(1).

Figure 3 shows the performance in terms of BER of the turbo decoder controlled by the Sum-α criterion. The used interleaved has the size N=5120. For a threshold equals to T = 0.001, the performances of the turbo decoder MAP with the Sum-α criterion are almost the same as those of the Cross-Entropy CE criterion.

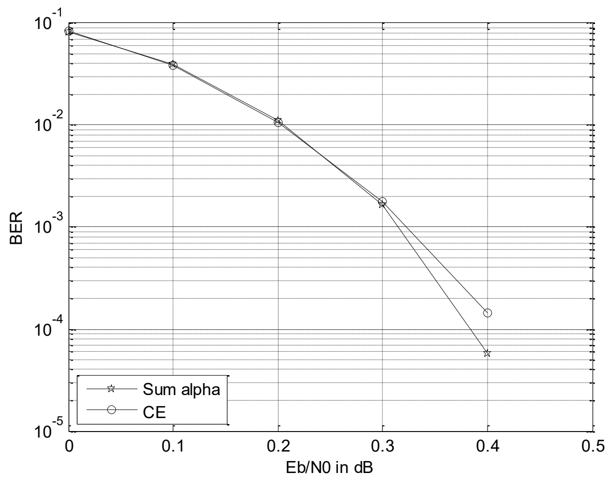

Fig. 3. BER of the MAP turbo decoder using the '*Sum-α'* and 'CE' stopping criteria (Interleaver 5120).

Similarly for Figure 4 which shows a comparison of the Frame Error Rate FER of the two criteria. The two criteria CE and Sum-α ensure the same FER. The results of the two figures prove that the Sum-α rule is equivalent in performance to the CE criterion. Moreover, it is considered as an ***early*** criterion (just after the forward recursion), without memory storage.

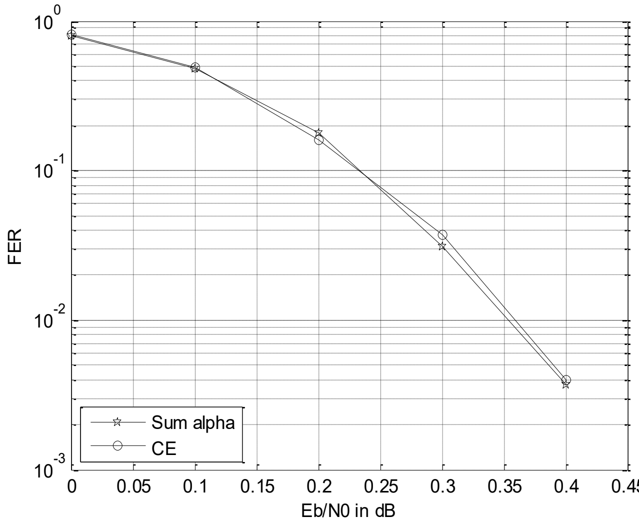

Fig. 4. FER of the turbo decoder MAP using the criteria 'Sum-α' and 'CE' (Interleaver 5120).

It remains for us to check if the Sum-α criterion allows to stop the Turbo process just after the correct decoding of the frames. To do this, Fig. 5 plots the average number of iterations of the turbo decoder using the Sum-α criterion. This criterion uses the same average number of iterations of the CE. Consequently, this result shows the importance of the Sum-α criterion. In addition, Sum-α is an early criterion because it allows us to make a decision before calculating the LLR. This makes the Sum-α criterion attractive and efficient.

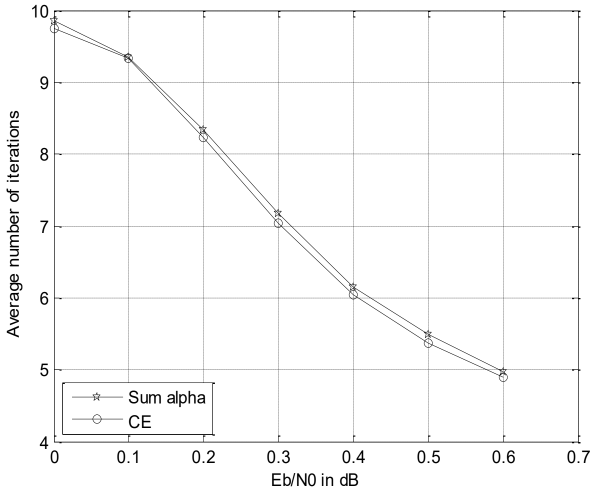

Fig. 5. Average number of iterations of the MAP turbo decoder using the 'Sum-α' criterion (Interleaver 5120).

When we use interleavers sizes smaller than 5120, the BER and FER of the Sum-α remain the same as those of the CE. However, the average number of iterations increases slightly. For example, consider a pseudo-random interleaver (S-random) of size 2560. Figure 6 shows that the average number of iteration of the turbo decoder using the Sum-α is increased by 0.5 iteration on the average over the CE at high Signal to Noise Ratio SNR. This represents the cost to be paid if we consider small interleavers.

In this second part, we compare the results obtained with two interleavers for different thresholds. Figure 7 plots the BER of the turbo decoders using two interleavers of sizes 5120 and 1280 for the thresholds of the Sum-α criterion 0.1, 0.01, 0.001 and 0.0001. The figure shows that the Sum-α criterion also works with small and large interleavers.

For the small size interleaver 1280, the Sum-α criterion with the 4 thresholds gives almost the same BER. When we increase the size of the interleaver (5120), we always get the good performances. For this, the BERs of the turbo decoder using the interleaver with the size 5120 are better than those of the turbo decoder using the interleaver of size 1280. On the other hand, the use of a low threshold (T = 0.001 and T = 0.0001) ensures low bit error rates BER.

Figure 7 also shows that the use of high threshold (0.1) quickly degrades performances (Interleaver 5120), as it leads to the turbo process stopping before decoding is complete.

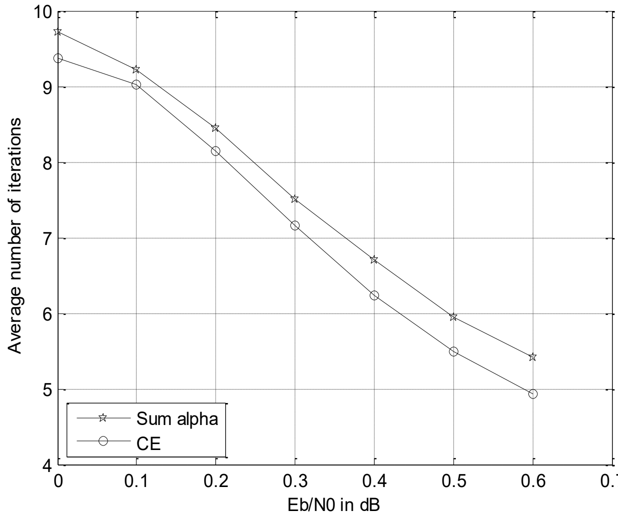

Fig. 6. Average number of iterations of the MAP turbo decoder using the 'Sum-α' criterion (Interleaver 2560)

Regarding the FER (Fig. 8), the high values of the threshold T (T = 0.1) degrades the Frame Error Rate FER for the two interleavers. The best FER are obtained for low thresholds (0.001 and 0.0001) using a large interleaver (5120).

These choices of thresholds have a great effect on the computational complexity. By analyzing Fig. 9, which plots the average number of iterations, we see that the large threshold 0.1 gives the lowest complexity but deteriorates the BER and FER (Fig. 7 and Fig. 8) because it stops the turbo decoding. We also see that the low threshold 0.0001 wasted a lot of iterations.

In conclusion, the analysis of these three figures (BER, FER and average number of iterations) shows that the best threshold of the new criterion Sum-α is $T = 0.001$.

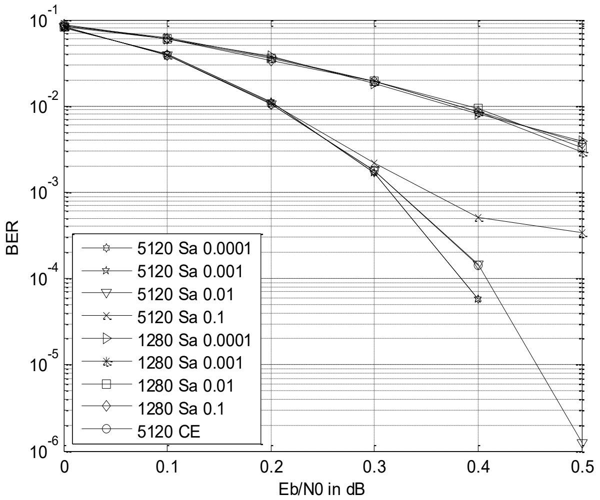

Fig. 7. BER of the MAP turbo decoder using the '*Sum-α*' and 'CE' stopping criteria for two interleavers and different thresholds.

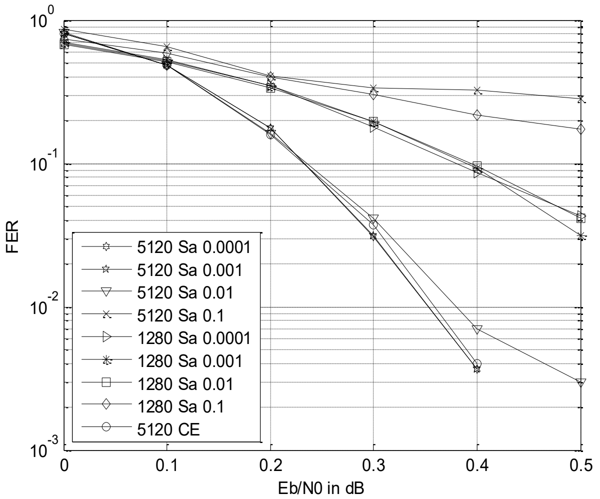

Fig. 8. FER of the MAP turbo decoder using the '*Sum-α*' and 'CE' stopping criteria for two interleavers and different thresholds.

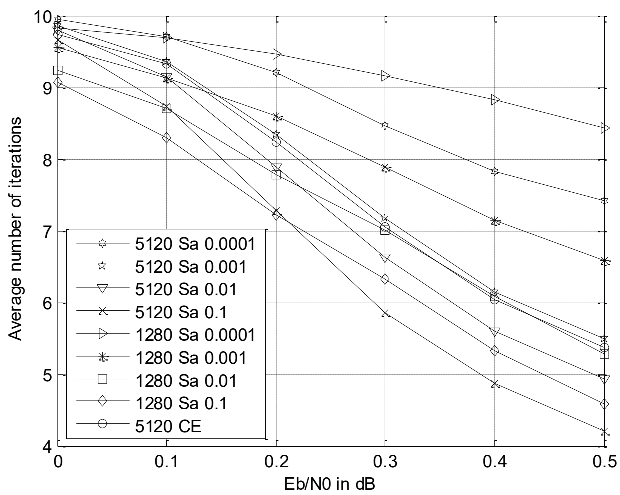

Fig. 9. Average number of iterations of the MAP turbo decoder using the 'Sum-α' criterion for two interleavers and different thresholds

VI. CONCLUSION

In this article, we presented a new stopping criterion of iterations for turbo codes. This criterion called Sum-α, allows to take an early decision that uses practically no memory storage. The simulations have shown that this criterion ensures the same performances in terms of BER, FER and average number of iterations of the Cross-Entropy CE criterion. For small interleavers, the average number of iterations is slightly increased. So, we can say that the Sum-α criterion is a good and early stopping technique for turbo decoding without any storage memory.

The use of a low threshold of the Sum-α criterion ensures the weakest BER and FER. Moreover, the use of high threshold degrades performances. We also notice that high thresholds gives the lowest complexity but degrade performances and low thresholds increase the number of iterations.

Finally, the optimal threshold which ensures a good compromise between decoding quality and complexity is the threshold $T_{opt} = 0.001$.

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